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A prediction method of ice breaking resistance using a multiple regression analysis

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ABSTRACT: *The two most important tasks of icebreakers are first to secure a sailing route by breaking the thick sea ice and second to sail efficiently herself for purposes of exploration and transportation in the polar seas. The resistance of icebreakers is a priority factor at the preliminary design stage; not only must their sailing efficiency be satisfied, but the design of the propulsion system will be directly affected. Therefore, the performance of icebreakers must be accurately calculated and evaluated through the use of model tests in an ice tank before construction starts. In this paper, a new procedure is developed, based on model tests, to estimate a ship's ice breaking resistance during continuous ice-breaking in ice. Some of the factors associated with crushing failures are systematically considered in order to correctly estimate her ice-breaking resistance. This study is intended to contribute to the improvement of the techniques for ice resistance prediction with ice breaking ships.*

***KEY WORDS**:* Ice breaking ships; Multiple regression analysis; Ice model test; Ice resistance; Crushing failure.

NOMENCLATURES

α	Waterline angle (*Deg.*)	E	Elastic modulus (*Pa*)
μ	Frictional coefficient (-)	h	Ice thickness (*m*)
σ_f	Flexural strength (*Pa*)	L	Ship's length (*m*)
φ	Stem angle (*Deg.*)	R_{BR}	Ice breaking resistance (*N*)
B	Beam (*m*)	T	Draft (*m*)
C_{BR}	Coefficient of ice breaking resistance (-)	v	Ship's velocity (*m/s*)

INTRODUCTION

The hull and propulsion system of ice breaking ships used to be designed and constructed largely on the basis of sailing experience in the Baltic and Norwegian Seas, or else by making adaptations to existing vessels designed for the open-water. Then, in the early 1960s, a design for an ice-breaker was tested in model ice (Corlett and Snaith, 1964). This was the Polish ice-

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breaker, the Perkun, launched in 1964. To meet their particular requirements, the bow of ice breaking ships needs to be sharp, with a small stem angle, so as to break the ice sheets easily, while her propulsion system needs to be 2~3 times stronger than that of an open-water vessel.

By the late 1960's, many studies had been conducted with model tests in ice and new hull types for breaking sea ice efficiently (Kashteljan et al., 1968; Lewis and Edwards, 1970; Edwards et al., 1972; Enkvist, 1972; Vance, 1975). These model tests were followed by ice trial tests. By this time, the purpose of ice breaking ships was changing from pure research to exploration and transportation, due to the discovery of crude oil in Alaska (White, 1970). This discovery prompted an important leap forward in the study of polar engineering.

In the 1980's, it was reported that enormous oil and natural gas resources were lying under the area above the Arctic Circle. However, arctic platforms that could endure ice loads would be required for the oil drilling, and many strong icebreakers and expensive ice-strengthened vessels would be needed for the safe transportation of the oil. Furthermore, the price of oil was only gradually stabilizing across the globe. For these reasons, there was very little activity in arctic development projects at that time.

Since the 2000's the economic efficiency of the Northern Sea Route (NSR) has improved, due to a decrease in the extent of the sea ice in the Arctic; consequently research into the development of the Arctic is steadily moving forward, and the value of the area's natural resources becomes increasingly significant. Therefore, large ice-breaking merchant vessels that can make their own way through ice, and arctic platforms for the drilling and production of oil are now under construction (Niini et al., 2007; Clarke et al., 2005). In addition, more reliable testing techniques for using ships in ice have been developed through improvements in the measurement techniques used with ice model tests, and with new model test methods and the further development of sea ice trials (Cho et al., 2011; 2013; Kim et al., 2014).

In this paper, some major studies on ice resistance done in the 1970's are briefly reviewed and analyzed, and the merits and demerits of each study are identified. We have also developed a new procedure for estimating ice breaking resistance, based on model tests with three medium ice breaking ships, which were built recently and exhibit good ice breaking performances. Through dimensional and regression analysis, an equation that includes various influencing factors and a logical procedure for deciding on weights is proposed. The equation is compared with previous studies, and is evaluated against the results of the model test.

Previous studies

Much literature related to ice resistance has been published since 1960, however Kashteljan et al. (1968) proposed the first detailed equation to analyze the level of ice resistance by breaking it down into its components. They divided total ice resistance into the resistance due to breaking the ice, the resistance due to forces connected with weight, and the resistance due to passage through broken ice, but omitted the open-water resistance, as shown in Table 1. Their equation was developed from a model and then full-scale tests were done with the Ermak.

Table 1 Various equations used for the estimation of ice breaking resistance in previous studies.

Published by	Year	Ice-breaking resistance
Kashteljan et al.	1968	$C_{BR}\sigma_f Bh\mu_o$
Lewis et al.	1970	$C_{BR}\sigma_f h^2$
Edwards et al.	1972	$C_{BR}\sigma_f Bh$
Enkvist	1972	$C_{BR}\frac{\sigma_f}{E}Bh\sigma_f$
Vance	1975	$C_{BR}\sigma_f Bh$

In North America, many strong icebreakers and ice-strengthened vessels were needed for transportation once crude oil was discovered at North Slope in Alaska. Lewis and Edwards (1970) published a technique using regression methods whereby the icebreaking resistance was formulated and a final regression equation was created using data from several different model tests and from full-scale data (Table 1). In 1972, Edwards et al. proposed a prediction technique, once again based on regression analysis; however, some changes were made to their previous equation, as is shown in Table 1.

Enkvist (1972) also presented a full-scale resistance formulation from an analysis of model data. His equation (Table 1) is probably the most complete to date, however, he neglected to include the effect of the ship's length. He was not only able to isolate the velocity dependent term by using a model test at low speed, but also derived a velocity resistance, by using the law of conservation of energy.

Vance (1974) became interested in the procedure for estimating ice resistance when he conducted a review of earlier studies. He used a multiple regression technique (MRA) and statistical analysis to develop a new ice resistance equation, which can be applied to all kinds of ice breaking vessels. He proposed an optimum regression equation from five sets of model and full-scale data, as shown in Table 1.

The crushing phenomenon that happens when ice breaking ships strike against an ice sheet had been recognized by the earlier studies reviewed, however, the magnitude of this resistance was assumed to be small, and the resistance due to crushing failure was not included in their equations. In addition, they derived these equations from a model and full-scale data only, due to the lack of a technique for model testing in ice at that time. However, the full-scale data obtained contained huge errors because the measurement of sea ice properties is more difficult than the measurement of model ice properties. Besides, some equations did not consider the effect of ice resistance, since a dimensional analysis was not included in their procedure. Lastly, while the other equations were derived from a dimensional analysis, the total ice resistance was considered to be one factor, because the model tests and sea trials were conducted without classification of the ice resistance components. Consequently, the error of the MRA increased, and the accuracy of the ice resistance estimation became lower.

A PROCEDURE FOR ESTIMATING ICE BREAKING RESISTANCE

Overview of model tests

Data from three model tests, conducted at the National Research Council-Ocean Coastal and River Engineering (NRC-OCRE) ice model basin, are used in this study, and the principle specifications and photos are shown in Table 2. The three model tests (Derradji and Van, 2004; Jones, 2005; Lau and Akinturk, 2011) were conducted using the same procedures, which consisted of a level ice test, a pre-sawn ice test, creep test in pre-sawn ice, and an open-water test. The results of the various ice model tests enabled ice-breaking, buoyancy, ice-clearing and open-water resistance to be separated and calculated experimentally. In addition, the three model ships were coated with the same paint so that they had the same frictional coefficient (0.05) between the model ship and the model ice. Tables 3 and 4 show the measured model ice properties and the results of the model test with C.C.G.S. Terry Fox, respectively. The model ice properties with three different thicknesses are also shown in Table 5, and the U.S.C.G.C Healy's results with each thickness are presented in Tables 6~8. Lastly, Table 9 shows the measurements for the model ice properties, and Tables 10~12 show the results of the model test with the Araon and three different thicknesses of model ice.

Table 2 Summary of the three ships used in this study, their major specifications, with photos.

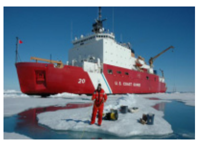
Ship (Name)	CCGS (Terry Fox)	USCGC (Healy)	Korean RV (Araon)
Country	Canada	U.S.	Korea
Year Built	1983	1999	2009
Length (*m*)	75	128	109
Beam (*m*)	17.2	24.8	19.0
Draft (*m*)	8.02	8.53	6.8
Tonnage (*ton*)	6,910	16,000	6,950
Stem angle (°)	23	20	34
L/B	4.286	5.120	5.642
Photo			

Table 3 Model ice properties for C.C.G.S. Terry Fox.

Model ice	
Thickness (*m*)	0.04
Strength (*kPa*)	35.0
Elastic modulus (*MPa*)	24.1
Density (kg/m^3)	940.0

Table 4 Model test results for C.C.G.S. Terry Fox.

Items		Unit	Value				
Ship's Velocity		*m/s*	0.020*	0.100	0.200	0.400	0.600
Test results	R_{BR}	N	1.794	19.962	27.871	30.703	29.450
	R_B	N	10.787	10.787	10.787	10.787	10.787
	R_C	N	0.309	3.472	6.326	11.515	16.330
	R_{OW}	N	2.050	0.319	1.016	3.485	7.393
	R_T	N	14.940	34.540	46.000	56.490	63.960

*Creep speed in pre-sawn ice channel

Table 5 Model ice properties for U.S.C.G.C. Healy.

Model ice	1st	2nd	3rd
Thickness (*m*)	0.0295	0.0401	0.0579
Strength (*kPa*)	31.5	34.1	36.0
Density (kg/m^3)	852.0	844.0	843.0

Table 6 Model test results for U.S.C.G.C. Healy with an ice sheet of 29.5 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.020*	0.099	0.397	0.595	0.795
Test results	R_{BR}	N	-	18.760	25.780	28.290	30.230
	R_B	N	19.340	19.340	19.340	19.340	19.340
	R_C	N	-	1.010	6.630	11.510	17.040
	R_{OW}	N	-	0.303	3.851	8.423	14.795
	R_T	N	-	39.413	55.601	67.563	81.405

Table 7 Model test results for U.S.C.G.C. Healy with an ice sheet of 40.1 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.020*	0.096	0.389	0.587	0.788
Test results	R_{BR}	N	-	35.960	49.080	53.880	57.660
	R_B	N	27.690	27.690	27.690	27.690	27.690
	R_C	N	-	1.450	9.610	16.710	24.980
	R_{OW}	N	-	0.293	3.723	8.151	14.506
	R_T	N	-	65.393	90.103	106.431	124.836

Table 8 Model test results for U.S.C.G.C. Healy with an ice sheet of 57.9 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.020*	0.091	0.180	0.375	0.578
Test Results	R_{BR}	N	-	73.760	86.230	102.010	112.630
	R_B	N	40.230	40.230	40.230	40.230	40.230
	R_C	N	-	2.160	5.460	14.790	26.610
	R_{OW}	N	-	0.263	0.876	3.454	7.937
	R_T	N	-	116.410	132.796	160.484	187.407

Table 9 Model ice properties for RV Araon.

Model ice	1st	2nd	3rd
thickness (*m*)	0.02	0.04	0.06
strength (*kPa*)	37.0	25.0	34.0
density (kg/m^3)	890.0	879.0	864.0

Table 10 Model test results for Korean RV Araon with an ice sheet of 20 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.050*	0.100	0.200	0.400	0.600
Test results	R_{BR}	N	1.004	12.844	16.857	22.859	38.255
	R_B	N	10.787	10.787	10.787	10.787	10.787
	R_C	N	1.099	1.979	3.606	6.564	9.308
	R_{OW}	N	2.050	1.790	1.950	4.890	11.350
	R_T	N	14.940	27.400	33.200	45.100	69.700

*Creep speed in pre-sawn ice channel

Table 11 Model test results for Korean RV Araon with an ice sheet of 40 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.050*	0.100	0.200	0.400	0.600
Test results	R_{BR}	N	8.504	19.251	37.616	42.121	56.340
	R_B	N	23.684	23.684	23.684	23.684	23.684
	R_C	N	3.222	5.805	10.570	19.245	27.296
	R_{OW}	N	2.050	1.790	1.950	4.890	11.350
	R_T	N	37.460	50.530	73.820	89.940	118.670

Table 12 Model test results for Korean RV Araon with an ice sheet of 60 *mm*.

Items		Unit	Value				
Ship's velocity		*m/s*	0.050*	0.100	0.200	0.400	0.600
Test results	R_{BR}	N	10.893	102.02	126.164	146.858	160.590
	R_B	N	39.842	39.842	39.842	39.842	39.842
	R_C	N	5.985	10.795	19.634	35.790	50.688
	R_{OW}	N	2.050	1.950	1.950	4.880	11.350
	R_T	N	58.770	154.61	187.590	227.370	262.470

Influencing factors

The forces at work around an icebreaker when she sails in ice-covered waters are very complex and are made up of many parameters; therefore, it is not easy to calculate ice resistance accurately. To arrive at an accurate estimate of ice resistance, the factors affecting ice resistance need to be identified, and the weight of each of these influencing factors needs to be analyzed by the use of both model and full-scale tests.

We classified these parameters according to ice and ship information derived from previous studies, and from our own experience. Some environmental information, such as the viscosity of the water, currents and wind, as well as some hull information, like the waterline angle and the displacement, were not considered in this study.

In terms of ice information, ice thickness is the most important parameter. The flexural strength of ice also causes strong resistance in the breaking of ice sheets. The elastic modulus affects the bending of the ice sheet, as does the density of the ice. In terms of the ship information, the effect of the hull geometry is reflected through information concerning the length, breadth, and draft of the ships. Ice breaking resistance is calculated for each ship speed. In particular, the mass and velocity of a ship experiencing the crushing failure effect are considered in ice-breaking resistance, while relationships between conditions related to the weights of the influence factors are derived from dimensional analysis to increase the accuracy of the MRA.

Dimensional analysis

Lindqvist (1989) insisted that estimated breaking resistance, where an ice sheet is broken only by bending failure, was smaller than the real resistance. The other researchers discussed in chapter 2 had recognized that ice-breaking resistance was due to the crushing failure of the ice sheet, however they estimated that ice-breaking resistance is caused only by bending failure, because the resistance due to crushing failure is too small to matter. Ice-breaking resistance due to crushing failure occurs when an icebreaker moves into ice-covered water, and it could be huge if the ice sheet is thick and the stem angle of the ship is large.

This crushing failure brings about a change in the momentum of the icebreaker, which is moving with the mass of the ship, M, and with velocity, v. Therefore, when a collision between the icebreaker and an ice sheet results in a crushing failure, the momentum can be expressed as a function of the mass and the velocity of the ship. For example, icebreakers move quickly when in ramming mode, or when sailing with a fully loaded draught, where ballast tanks are filled with sea water, or when she is locked in thick ice or passes through a big ridge.

When an icebreaker meets a thick ice sheet that does not break, a crushing failure will result in her getting onto the ice sheet and pushing it down with her own weight. Consequently, the ice sheet eventually breaks, due to bending failure, and this bending procedure is explained by the theory of a beam on an elastic foundation (Hetenyi, 1946). The resistance due to bending failure can be calculated as the potential energy of the ice sheet. Therefore, ice-breaking resistance needs to be expressed by taking into account both crushing and bending failure.

$$R_{BR} = C_{BR}\sigma_f^P h^Q B^R v^S M^T \tag{1}$$

To equate the dimensions of both sides of the Eq. (1), the following indeterminate equations were made.

$$\begin{bmatrix} P+T=1 \\ -P+Q+R+S=1 \\ 2P+S=2 \end{bmatrix} \tag{2}$$

Multiple regression analysis

The ice-breaking resistance depends not only on the flexural strength of the ice sheet, the ice thickness, and the beam of the icebreaker, but also on the ship's velocity and the mass of the ship. Therefore, these independent variables are separated by using a natural logarithm for the MRA:

$$\ln R_{BR} = \ln C_{BR} + P\ln\sigma_f + Q\ln h + R\ln B + S\ln v + T\ln M \tag{3}$$

The Eq. (2) can be written again as follows:

$$\begin{bmatrix} T = 1-P \\ P = 1-\frac{1}{2}S \\ R = 2-\frac{3}{2}S-Q \end{bmatrix} \tag{4}$$

The weighting factors T, P, and R, are automatically fixed if the weighting factors Q and S are determined, therefore the MRA was carried out by using the results of model tests, as discussed in chapter 3.1, once the ice thickness and ship's velocity had been put in as independent variables. The coefficient of determination and the adjusted coefficient of determination are 0.768 and 0.750 respectively, so the multiple regression model shows an explanation of 76.8%, and it is very stable because the difference between them is so small. Table 13 shows each coefficient, the standard errors, t-statistics, p-values, the lower 95% values, and the upper 95% values. In particular, the explanation provided by the multiple regression model is supported because all the p-values are smaller than the significance level of 0.05.

Table 13 Results of regression analysis for ice breaking resistance in model-scale.

	Coefficients	Standard errors	t-statistics	P-value	Lower 95% value	Upper 95% value
Y intercept	9.52711	0.64047	14.87513	6.33E-14	8.20803	10.84618
ln(h)	1.65609	0.19069	8.68469	5.09E-09	1.26336	2.04883
ln(v)	0.29455	0.09082	3.24308	0.00334	0.10749	0.48160

The coefficients for each weighting factor can be calculated by rounding off the fractions to four decimal places in Table 13, so that Q=1.656 and S=0.295. Other weighting factors can also be determined by using the Eq. (5) as follows:

$$\begin{bmatrix} P = 0.853 \\ Q = 1.656 \\ R = -0.099 \\ S = 0.295 \\ T = 0.147 \end{bmatrix} \tag{5}$$

By substituting into Eq. (1), we have:

$$R_{BR} = C_{BR}\sigma_f^{0.853} h^{1.656} v^{0.295} M^{0.147} \tag{6}$$

In Eq. (6), the final equation for ice-breaking resistance is expressed, now including the mass component in the coefficient of ice-breaking resistance, since the mass information of ships is generally inaccurate. The final equation for the ice-breaking resistance is indicated as follows (Eq. (7)).

$$R_{BR} = C_{BR}\sigma_f^{0.853} h^{1.656} v^{0.295} \tag{7}$$

Coefficient by curve fitting method

To obtain a coefficient of ice breaking resistance (C_{BR}), a curve fitting method was carried out as shown in Table 14. If ice breaking, buoyancy, and ice clearing resistance is decided perfectly, each coefficient can be calculated through the second MRA. At this stage, however, the coefficient of ice breaking resistance from the curve fitting method was used, and the final ice breaking resistance equation was derived as follows:

$$R_{BR} = 1.8\sigma_f^{0.853} h^{1.656} v^{0.295} \tag{8}$$

Table 14 Calculated resistance coefficients in model-scale by curve fitting.

	Terry Fox	Healy	Araon	Average
Cbr	2.5	1.8	1.1	1.8

VERIFICATIONS

C.C.G.S. Terry Fox

Fig. 1 shows various estimated ice resistances plotted against the ship's velocity in model-scale for the C.C.G.S. Terry Fox. The present equation shows the good tendency when compared with the results from the model tests even though there is a small difference in this estimate. In addition, the present equation is under-estimated because the coefficient is changed into a small value for the Terry Fox as shown in Table 14. However, all others method did not change with the ship's velocity because they did not consider the crushing failure effect. Even though the Lewis and Vance methods make a close ice breaking resistance, it's a little hard to use them.

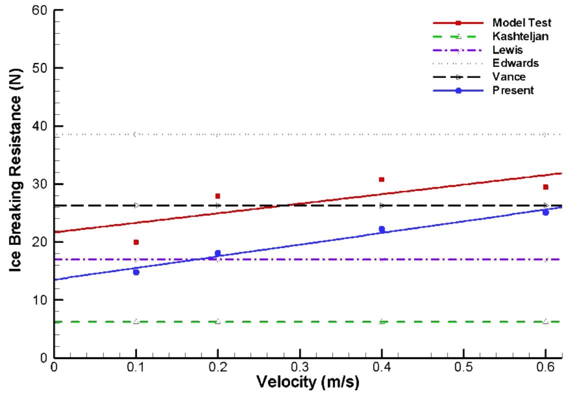
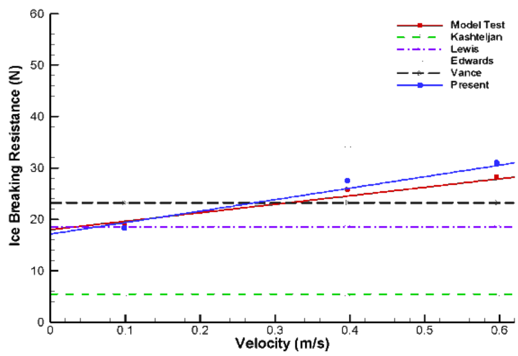


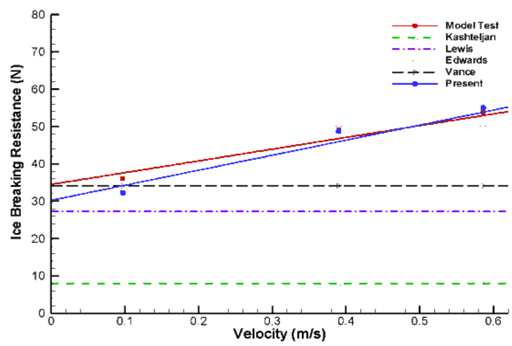
Fig. 1 Comparison of various equations for the Terry Fox in model-scale (40 *mm*).

The U.S.C.G.C. Healy

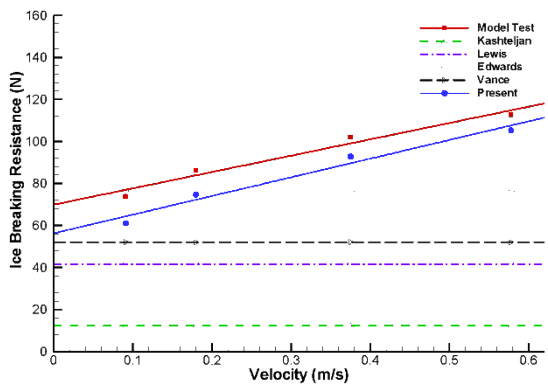
The estimated resistance in the model-scale for the U.S.C.G.C. Healy is calculated by using Eq. (8), as plotted in Fig. 2. The equation developed by this study not only gave the best result for all thicknesses and velocities, but was also the closest to the results of the model test. The reason is that the coefficient used this equation is exactly same with the coefficient by curve fitting for the Healy. All others method, on the other hand, did not change with the ship's velocity, too. The Lewis and Vance methods calculated slightly lower values than the value estimated from the model test at thickness of 29.5 *mm*, as well as showing a big difference when the ice sheet was thick.



(a) 29.5 *mm.*



(b) 40.1 *mm.*

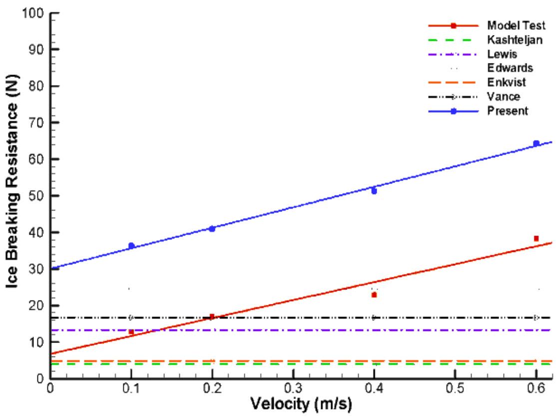


(c) 57.9 *mm.*

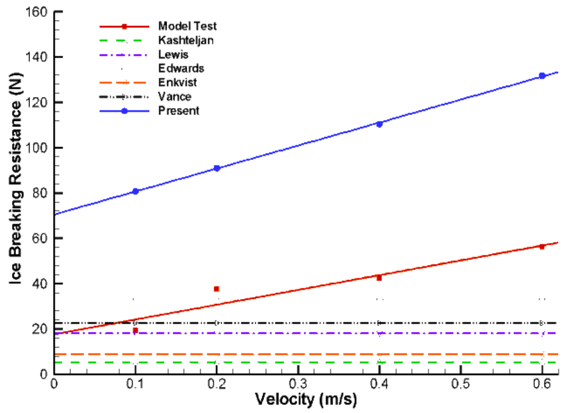
Fig. 2 Comparison of various equations for the Healy in model-scale.

Korean RV Araon

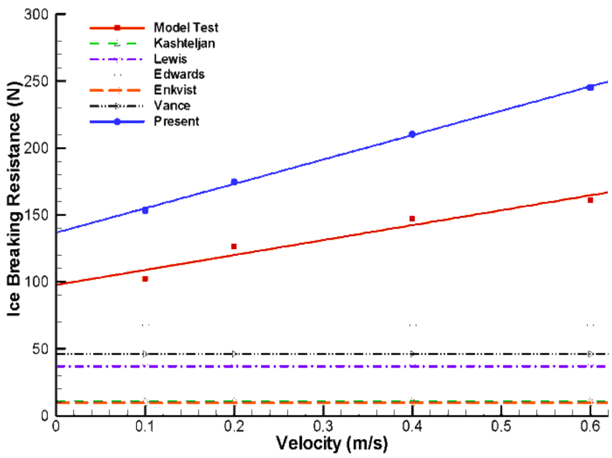
Fig. 3 shows various estimated ice resistances plotted against the ship's velocity in the model-scale for the Korean RV Araon. In general, the present equation shows a good tendency compared with the results of the model tests even though there is a big error because an average coefficient that is higher than the value from the result of curve fitting is used. All others method, however, calculated very lower values than the result of model test and the present equation. Because we already have detail hull parameters of Araon, Enkvist method can be applied only for the Araon.



(a) 20 *mm*.



(b) 40 *mm*.



(c) 60 *mm*.

Fig. 3 Comparison of various equations for the Araon in model-scale.

C.C.G.S. R-Class Vessel

The C.C.G.C. R-Class vessel is a medium Arctic and Gulf icebreaker of the Canadian Coast Guard (Timco et al., 2004) and her hull is similar with three ships we studied in this research. To evaluate the effectiveness of the present equation, the ice breaking resistance for the R-Class vessel is calculated.

Fig. 4 shows estimated resistances calculated by using Eq. (8) of this study in relation to the model test for the Tatinclaux (1984) in ice thickness of 23.8 *mm* and its flexural strength of 30 *kPa*. An error of 15% occurs with the result of model test at all velocities, and the present equation shows an accurate tendency, even though it slightly under-estimates. We therefore concluded that this equation can be applied to estimate ice resistances for medium sized icebreakers, even when a ship has not been analyzed by MRA.

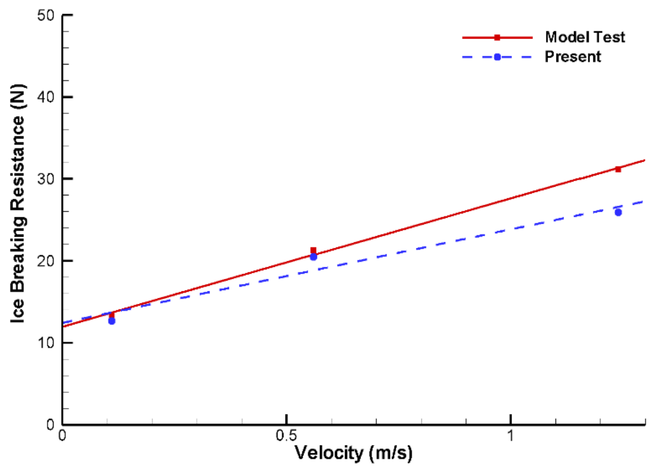


Fig. 4 Comparison of measured resistance from the model test (Tatinclaux, 1984) and computed resistance from this study in model-scale for an R-class vessel (h=23.8 *mm*, σ=30 *kPa*).

CONCLUSIONS

Ship ice resistance cannot be described in simple terms because the forces around an icebreaker when she sails into ice-covered water are very complex and are made up of many parameters. In previous studies, the ice-breaking resistance was estimated merely from the bending failure of the ice, however in this study we created an equation that also took into consideration the ice crushing effect. As a result, the new equation includes the ship's mass and velocity as related to the ship's momentum. This equation shows a better tendency and agreement than those of previous studies.

Previous studies have shown large differences between the estimated and the measured values for ice breaking resistance because they created their equations by many assumptions. In this paper, the ice breaking resistance was estimated with a good tendency since an MRA was conducted using a state-of-the-art hull and the results of a model test. The new equation, which takes all these factors into account, thus becomes a very efficient tool for estimating ice resistance.

The equation developed in this research can estimate the ice breaking resistance with errors of less than 10% in the design speed of a ship, and it is a very simple tool with which to calculate ice breaking resistance for the medium icebreaker. It can be used to evaluate various hulls and ice thicknesses at the primary design stage of ship-building, and to further develop the accuracy of the ice model test by prediction of its results in advance. In this way we expect to arrive at a still more accurate equation included a buoyancy and ice clearing resistances in future.

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