

## Trajectory tracking control of underactuated USV based on modified backstepping approach

Zaopeng Dong<sup>1,2</sup>, Lei Wan<sup>1,2</sup>, Yueming Li<sup>1,2</sup>, Tao Liu<sup>1,3</sup> and Guocheng Zhang<sup>1,2</sup>

<sup>1</sup>*National Key Laboratory of Science and Technology on Autonomous Underwater Vehicle, Harbin Engineering University, Harbin, China*

<sup>2</sup>*College of Shipbuilding Engineering, Harbin Engineering University, Harbin, China*

<sup>3</sup>*College of Automation, Harbin Engineering University, Harbin, China*

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**ABSTRACT:** *This paper presents a state feedback based backstepping control algorithm to address the trajectory tracking problem of an underactuated Unmanned Surface Vessel (USV) in the horizontal plane. A nonlinear three Degree of Freedom (DOF) underactuated dynamic model for USV is considered, and trajectory tracking controller that can track both curve trajectory and straight line trajectory with high accuracy is designed as the well known Persistent Exciting (PE) conditions of yaw velocity is completely relaxed in our study. The proposed controller has further been enriched by incorporating an integral action additionally for enhancing the steady state performance and control precision of the USV trajectory tracking control system. Global stability of the overall system is proved by Lyapunov theory and Barbalat's Lemma, and then simulation experiments are carried out to demonstrate the effectiveness of the controller designed.*

**KEY WORDS:** Unmanned surface vessel (USV); Trajectory tracking; Underactuated control; Backstepping approach; Lyapunov theory.

### INTRODUCTION

Unmanned Surface Vessel (USV) is attracting more and more attention from researchers all over the world because of its extensive applications in military reconnaissance, homeland security, shallow-water surveys, environmental monitoring and coordinating working with Autonomous Underwater Vehicle (AUV) (Campbell et al., 2012; Martin, 2013; Sharma et al., 2014). As USV is usually controlled remotely by humans, an effective and reliable motion controller for its autonomous sailing is very important. A typical motion control problem for USV is trajectory tracking, which is concerned with the design of control laws that force USV to reach and follow a time parameterized reference trajectory. Most of the deployed and developing USVs are underactuated as they are not actuated in the sway axis for economic and practical considerations. We can see in (Sharma et al., 2014; Do et al., 2004; Fredriksen and Pettersen, 2006) that trajectory tracking controller design for fully actuated vehicle is not so hard while it is especially a challenging for underactuated USV because of its nonholonomic constraints and can not be fully feedback linearized.

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Corresponding author: *Lei Wan*, e-mail: [wanlei103@163.com](mailto:wanglei103@163.com)

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In order to overcome the difficulties of trajectory tracking control of underactuated USV, different nonlinear control methods have been proposed in last few years, such as Sliding Mode Control (SMC) (Cheng et al., 2007; Ashrafiun et al., 2008; Soltan et al., 2009; Yu et al., 2012; Fahimi and Van Kleeck, 2013), backstepping technique (Do and Pan, 2005; Chen and Tan, 2013; Sonnenburg and Woolsey, 2013; Liao et al., 2014), Lyapunov's direct method (Ma and Xie, 2013), Dynamic Surface Control (DSC) (Chwa, 2011), robust control method (Gierusz et al., 2007; Yang et al., 2014), intelligent control technology (Gierusz et al., 2007; Zhang et al., 2011) and Hybrid control technology (Liu et al., 2014), etc (Harmouche et al., 2014; Katayama and Aoki, 2014; Serrano et al., 2014; Wu et al., 2014).

SMC is one of the most widely used trajectory tracking control methods for USV. Considering the limitation of full state feedback linearization for trajectory tracking problem of USV, a multivariable SMC controller is designed in (Cheng et al., 2007), and stability of the control law is proved by Lyapunov theory. A first-order sliding surface in terms of surge tracking errors and a second-order surface in terms of lateral motion tracking errors are introduced into the SMC law in (Ashrafiun et al., 2008), where it guarantees the position tracking errors of USV converge to zero and meanwhile the rotational motion remains bounded. Furthermore in paper (Yu et al., 2012), uncertainty associated with the hydrodynamic damping coefficients of the ship is discussed while controller design method is the same with that in (Ashrafiun et al., 2008). Moreover in paper (Soltan et al., 2009), a set of two Ordinary Differential Equations (ODEs) in terms of the position state feedback is used for transitional trajectory between the USV's initial condition and the desired trajectory such that the ODE solution converges to the desired trajectory path, which solve the limitations factor of SMC law designed that it can only guarantee position tracking as long as the USV's initial conditions are on the desired trajectory. A nonlinear robust model-based sliding mode controller is designed in (Fahimi and Van Kleeck, 2013), where the concept of shifting the control point is first tried for trajectory tracking of underactuated surface vessels.

Backstepping technique is another frequently used nonlinear control method for trajectory tracking of USV. A backstepping technique based controller that forces position and orientation of underactuated ship to globally track a reference trajectory is designed in (Do and Pan, 2005), which not required that the reference trajectory be generated by a ship model. A nonlinear backstepping controller which show excellent trajectory tracking performance even for aggressive and variable speed trajectories is proposed in (Sonnenburg and Woolsey, 2013), which is more reliable than the PD cascade approach. An adaptive backstepping controller is proposed in (Chen and Tan, 2013) for fully actuated surface vessels with the option of high-gain observer for output feedback control, where the stability of the closed-loop systems is explored through Lyapunov theory. Moreover in (Liao et al., 2014), under the transformation of tracking control problem into stabilization problem of trajectory tracking error equation, a nonlinear state feedback controller based on backstepping technique is developed and the stability of the system is proved by Lyapunov theory.

In addition, A trajectory tracking controller which achieve global  $k$ -exponential convergence of state to the desired reference trajectory is designed based on Lyapunov's direct method in (Ma and Xie, 2013), where Persistent Exciting (PE) conditions is needed. A global trajectory tracking controller based on DSC for underactuated ship is proposed in (Chwa, 2011), where the controller is designed using the linearization of kinematic and dynamic systems similarly as in the backstepping technique. A complex trajectory tracking control system based on two different controllers connected in parallel, one is robust controller and the other is fuzzy logic controller, is presented in (Gierusz et al., 2007) for autonomous model of the Very Large Crude Carrier (VLCC). A trajectory tracking robust control law has been designed for fully actuated surface vessels in the presence of uncertain time-variant disturbances in (Yang et al., 2014), where vectorial backstepping technique based disturbance observer is employed to compensate disturbance uncertainties. Considering the coupling interactions among forces from each Degree of Freedom (DOF) and nonlinear characteristics of the hydrodynamic damping, a Neural Network Feedback Feedforward Compensator (NNFFC) controller is designed in (Zhang et al., 2011) for trajectory tracking control of a surface ship.

A hybrid controller based on adaptive technique and hierarchical SMC is presented in (Liu et al., 2014), where adaptive technique is employed to deal with the uncertainties of the mathematical model while hierarchical SMC is used to deal with the underactuation of surface vessels. In paper (Wu et al., 2014), a novel finite-time switching controller based on the inherent cascaded interconnected structure of the ship dynamics is developed for the ship tracking a reference trajectory generated by a virtual ship. A global tracking controller based on saturated-state feedback control method for underactuated ship is designed in (Harmouche et al., 2014), where the controller still works and remains stable with observers in the absence of velocity measurements. The problem of straight line trajectory tracking control of underactuated ships with both state and output feedback controllers is addressed and analyzed by introducing a nonlinear sampled-data control theory in (Katayama and Aoki,

2014), where state feedback controllers and reduced-order observers based on Euler approximate models are combined to obtain output feedback controllers. A trajectory tracking controller for underactuated ships is designed based on searching for conditions under which the system of linear equations had exact solution in (Serrano et al., 2014), where the method proposed does not need a coordinate transformation and the algorithm is implemented directly on the ship’s microcontroller.

Though researchers have made a lot of contributions and proposed many pioneering methods for trajectory tracking control of USV in the literature mentioned above, we can still see some limitations in them. In paper (Cheng et al., 2007; Chen and Tan, 2013; Yang et al., 2014), only fully actuated controller is designed for trajectory tracking, it is not suitable for most USV as they are underactuated. In addition in paper (Soltan et al., 2009; Ma et al., 2013; Harmouche et al., 2014; Liao et al., 2014), the well know assumption that PE conditions of yaw velocity is needed, so a straight line reference cannot be tracked, while in paper (Katayama and Aoki, 2014), only straight line trajectory tracking can be achieved. Moreover in paper (Fahimi and Van Kleeck, 2013; Sonnenburg and Woolsey, 2013), trajectory tracking problem is decomposed into several sub-problems, such as separately considering of course control and position control, thus it would lead to the loss of global stability of the overall system and then the system would only be stable in some certain conditions. Though Wu Y. Q. and Zhang Z. C. try to relax the PE conditions by designing finite-time switching controller in paper (Wu et al., 2014), the control performance is not good enough as steady position tracking errors appears.

Motivated by the above considerations, this paper aims to provide and prove a nonlinear backstepping trajectory tracking method which can track an arbitrary reference trajectory for underactuated USV. For most USV in the horizontal plane, only the yaw and surge are directly actuated while the sway axis is not actuated, so a challenging underactuated problem has been studied here. The well know assumption that PE conditions of yaw velocity is completely relaxed for trajectory tracking control of USV in this paper, thus a controller that can track both curve trajectory and straight line trajectory with high accuracy is designed. Controller for trajectory tracking of USV is designed based on the overall system and the global stability is proved by Lyapunov theory and Barbalat’s Lemma, which is improved and derived from separate controller design in (Chen and Tan, 2013; Fahimi and Van Kleeck, 2013). In order to enhance the steady state performance and precision of the trajectory tracking controller for USV, an integral action is added into the backstepping control law. Simulation results are presented to demonstrate the effectiveness of the proposed control schemes.

## PROBLEM FORMULATION

### USV modeling

Establish inertial reference coordinate system  $\{n\}$  with origin defined on Earth and body-fixed reference  $\{b\}$  with origin chosen to coincide with USV’s center of mass as show in Fig. 1, the mathematical model of an underactuated USV moving in the horizontal plane can be described as follows (Fossen, 2011):

$$\begin{cases} \dot{\boldsymbol{\eta}} = \mathbf{R}(\varphi)\mathbf{v} \\ \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}\mathbf{v} = \boldsymbol{\tau} \end{cases} \quad (1)$$

with  $\boldsymbol{\eta} = [x \ y \ \varphi]^T$ ,  $\mathbf{v} = [u \ v \ r]^T$ ,  $\boldsymbol{\tau} = [\tau_u \ 0 \ \tau_r]^T$ ,  $\mathbf{M} = \text{diag}\{m_{11} \ m_{22} \ m_{33}\}$ ,  $\mathbf{D} = \text{diag}\{d_{11} \ d_{22} \ d_{33}\}$ ,

$$\mathbf{R}(\varphi) = \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{C}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & -m_{22}v \\ 0 & 0 & m_{11}u \\ m_{22}v & -m_{11}u & 0 \end{bmatrix} \quad (2)$$

where  $x$  and  $y$  are the Cartesian coordinated of USV’s center of mass in  $\{n\}$ ,  $\varphi$  denotes yaw angle in  $\{n\}$ ,  $u$ ,  $v$ ,  $r$  denote surge, sway and yaw velocities expressed in  $\{b\}$ , the controls  $\tau_u$  and  $\tau_r$  are the surge force and yaw moment.  $m_{11}$ ,  $m_{22}$ ,  $m_{33}$  are USV’s inertia coefficients including added mass effects,  $d_{11}$ ,  $d_{22}$ ,  $d_{33}$  are hydrodynamic damping coefficients in surge, sway, and yaw.

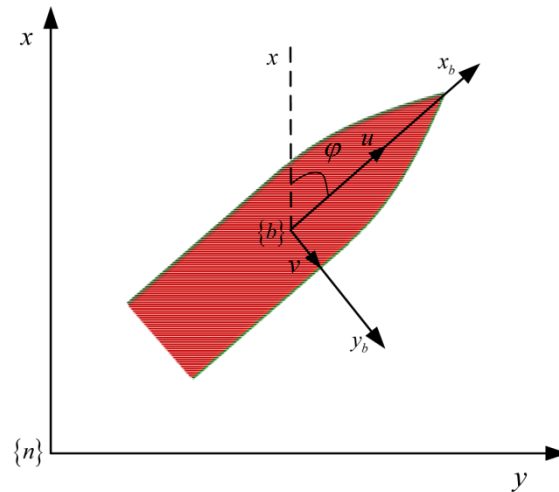


Fig. 1 Inertial reference frame  $\{n\}$  and body-fixed reference  $\{b\}$ .

**Problem formulation**

The general control problem of trajectory tracking for USV that we consider in this paper can be formulated as follows: Considering an arbitrary trajectory expressed in  $\{n\}$  with desired state  $\eta_d = [x_d(t) \ y_d(t) \ \varphi_d(t)]^T$  and directly given surge and yaw reference velocities  $u_d(t)$  and  $r_d(t)$ , while the reference sway velocity  $v_d(t)$  is generated by a virtual USV as described in (Do et al., 2002):

$$\begin{cases} \dot{x}_d(t) = u_d(t) \cos(\varphi_d(t)) - v_d(t) \sin(\varphi_d(t)) \\ \dot{y}_d(t) = u_d(t) \sin(\varphi_d(t)) + v_d(t) \cos(\varphi_d(t)) \\ \dot{\varphi}_d(t) = r_d(t) \\ \dot{v}_d(t) = -\frac{m_{11}}{m_{22}} u_d(t) r_d(t) - \frac{d_{22}}{m_{22}} v_d(t) \end{cases} \tag{3}$$

Define the trajectory tracking errors:

$$\begin{cases} \eta_e = [x_e(t) \ y_e(t) \ \varphi_e(t)]^T = [x(t) - x_d(t) \ y(t) - y_d(t) \ \varphi(t) - \varphi_d(t)]^T \\ \mathbf{v}_e = [u_e(t) \ v_e(t) \ r_e(t)]^T = [u(t) - u_d(t) \ v(t) - v_d(t) \ r(t) - r_d(t)]^T \end{cases} \tag{4}$$

Thus the control objective for trajectory tracking of USV is to design control law  $\tau_u$  and  $\tau_r$  to ensure the tracking errors  $\eta_e$  and  $\mathbf{v}_e$  converge to an arbitrarily small neighborhood of zero as  $t \rightarrow \infty$ .

**CONTROLLER DESIGN**

**Coordinate transformation**

Coordinate transformation that need be carried out in order to facilitate the controller design is made in this section. Differentiating both sides of the first equation of Eq. (1) results in:

$$\ddot{\eta} = \dot{\mathbf{R}}(\varphi)\mathbf{v} + \mathbf{R}(\varphi)\dot{\mathbf{v}} \tag{5}$$

The second equation of Eq. (1) can be rewritten as:

$$\dot{\mathbf{v}} = -\mathbf{M}^{-1} [\mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}\mathbf{v}] + \mathbf{M}^{-1}\boldsymbol{\tau} \tag{6}$$

Substituting Eq. (6) into Eq. (5) yields:

$$\ddot{\boldsymbol{\eta}} = \dot{\mathbf{R}}(\varphi)\mathbf{v} + \mathbf{R}(\varphi) \{ -\mathbf{M}^{-1} [\mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}\mathbf{v}] + \mathbf{M}^{-1}\boldsymbol{\tau} \} \tag{7}$$

and Eq. (7) can be rewritten as:

$$\mathbf{MR}^{-1}(\varphi)\ddot{\boldsymbol{\eta}} - [\mathbf{MR}^{-1}(\varphi)\dot{\mathbf{R}}(\varphi) - \mathbf{C}(\mathbf{v}) - \mathbf{D}]\mathbf{v} = \boldsymbol{\tau} \tag{8}$$

Expanding Eq. (8) leads to:

$$\begin{cases} m_{11} \cos(\varphi)\ddot{x} + m_{11} \sin(\varphi)\ddot{y} + d_{11}u + (m_{11} - m_{22})vr = \tau_u \\ -m_{22} \sin(\varphi)\ddot{x} + m_{22} \cos(\varphi)\ddot{y} + d_{22}v + (m_{11} - m_{22})ur = 0 \\ m_{33}\ddot{\varphi} + d_{33}r - (m_{11} - m_{22})uv = \tau_r \end{cases} \tag{9}$$

Motivated by (Gierusz et al., 2007; Harmouche et al., 2014), define the following variables  $\boldsymbol{\eta}^*$  and  $\mathbf{v}^*$  :

$$\begin{cases} \boldsymbol{\eta}^* = [x_1 \ x_2 \ x_3]^T = [x \ y \ \varphi]^T \\ \mathbf{v}^* = [x_4 \ x_5 \ x_6]^T = [\dot{x} \ \dot{y} \ \dot{\varphi}]^T \end{cases} \tag{10}$$

Design the following input transformation:

$$\begin{cases} u_1 = \frac{1}{m_{11}} \cos(x_3) \tau_u - \frac{1}{m_{11}} \cos(x_3) \{ [d_{11}x_4 + (m_{11} - m_{22})x_5x_6] \cos(x_3) + [d_{11}x_5 - (m_{11} - m_{22})x_4x_6] \sin(x_3) \} \\ \quad + \frac{\sin(x_3)}{m_{22}} \{ [d_{22}x_5 + (m_{11} - m_{22})x_4x_6] \cos(x_3) + [(m_{11} - m_{22})x_5x_6 - d_{22}x_4] \sin(x_3) \} \\ u_2 = \frac{1}{m_{33}} \tau_r - \frac{1}{m_{33}} \{ d_{33}x_6 - (m_{11} - m_{22})x_4x_5 \cos(2x_3) - \frac{(m_{11} - m_{22})(x_5^2 - x_4^2) \sin(2x_3)}{2} \} \end{cases} \tag{11}$$

Then system (9) can be transformed into:

$$\begin{cases} \dot{x}_1 = x_4 \\ \dot{x}_2 = x_5 \\ \dot{x}_3 = x_6 \\ \dot{x}_4 = u_1 \\ \dot{x}_5 = -\frac{1}{m_{22}} \sec(x_3) \chi + u_1 \tan(x_3) \\ \dot{x}_6 = u_2 \end{cases} \tag{12}$$

and

$$\chi = [d_{22}x_5 + (m_{11} - m_{22})x_4x_6] \cos(x_3) + [(m_{11} - m_{22})x_5x_6 - d_{22}x_4] \sin(x_3) \quad (13)$$

Then the desired trajectory expressed in Eq. (3) can be transformed into the following form in the same way:

$$\begin{cases} \dot{x}_{1d} = x_{4d} \\ \dot{x}_{2d} = x_{5d} \\ \dot{x}_{3d} = x_{6d} \\ \dot{x}_{4d} = u_{1d} \\ \dot{x}_{5d} = -\frac{1}{m_{22}} \sec(x_{3d}) \chi_d + u_{1d} \tan(x_{3d}) \\ \dot{x}_{6d} = u_{2d} \end{cases} \quad (14)$$

Motivated by (Do et al., 2005), define new tracking errors  $e_i, i = 1, \dots, 6$  as follows:

$$e_i = x_i - x_{id}, i = 1, \dots, 6 \quad (15)$$

Differentiating both sides of Eq. (15) results in:

$$\begin{cases} \dot{e}_1 = e_4 \\ \dot{e}_2 = e_5 \\ \dot{e}_3 = e_6 \\ \dot{e}_4 = \tau_1 \\ \dot{e}_5 = \delta + \tau_1 \tan(e_3 + x_{3d}) \\ \dot{e}_6 = \tau_2 \end{cases} \quad (16)$$

with

$$\begin{cases} \tau_1 = u_1 - u_{1d} \\ \tau_2 = u_2 - u_{2d} \end{cases} \quad (17)$$

and

$$\begin{aligned} \delta &= \left\{ -\frac{1}{m_{22}} \sec(e_3 + x_{3d}) \chi + u_{1d} \tan(e_3 + x_{3d}) - \left[ -\frac{1}{m_{22}} \sec(x_{3d}) \chi_d + u_{1d} \tan(x_{3d}) \right] \right\} \\ &= -\frac{1}{m_{22}} [d_{22}(e_5 + x_{5d}) + (m_{11} - m_{22})(e_4 + x_{4d})(e_6 + x_{6d})] - \frac{1}{m_{22}} [(m_{11} - m_{22})(e_5 + x_{5d})(e_6 + x_{6d}) - d_{22}(e_4 + x_{4d})] \tan(e_3 + x_{3d}) \\ &\quad + u_{1d} \tan(e_3 + x_{3d}) - \left[ -\frac{1}{m_{22}} \sec(x_{3d}) \chi_d + u_{1d} \tan(x_{3d}) \right] \end{aligned} \quad (18)$$

and

$$\chi_d = [d_{22}x_{5d} + (m_{11} - m_{22})x_{4d}x_{6d}] \cos(x_{3d}) + [(m_{11} - m_{22})x_{5d}x_{6d} - d_{22}x_{4d}] \sin(x_{3d}) \quad (19)$$

**Controller design**

Motivated by (Do et al., 2004; Rudra et al., 2013) and considering the expression (16), define the following error:

$$\varepsilon_1 = e_1 - k_1[e_2 + e_5 - e_4 \tan(e_3 + x_{3d})] \tag{20}$$

where  $k_1$  is a positive constant. Differentiating both sides of Eq. (20) leads to:

$$\begin{aligned} \dot{\varepsilon}_1 &= \dot{e}_1 - k_1[\dot{e}_2 + \dot{e}_5 - \dot{e}_4 \tan(e_3 + x_{3d}) - e_4(e_6 + x_{6d})\sec^2(e_3 + x_{3d})] \\ &= e_4 - k_1[e_5 + \delta + \tau_1 \tan(e_3 + x_{3d}) - \tau_1 \tan(e_3 + x_{3d}) - e_4(e_6 + x_{6d})\sec^2(e_3 + x_{3d})] \\ &= e_4 - k_1[e_5 + \delta - e_4(e_6 + x_{6d})\sec^2(e_3 + x_{3d})] \\ &= e_4 - k_1\phi \end{aligned} \tag{21}$$

and

$$\phi = e_5 + \delta - e_4(e_6 + x_{6d})\sec^2(e_3 + x_{3d}) \tag{22}$$

Define the following new variables:

$$\gamma_1 = \int_0^t \varepsilon_1 dt \tag{23}$$

$$o_1 = -\rho_1 \varepsilon_1 - \lambda_1 \gamma_1 + k_1 \phi \tag{24}$$

$$\varepsilon_2 = e_4 - o_1 \tag{25}$$

where  $\rho_1, \lambda_1$  are positive constant. Substituting Eq. (24) and Eq. (25) into Eq. (21) gives:

$$\dot{\varepsilon}_1 = \varepsilon_2 + o_1 - k_1 \phi = \varepsilon_2 - \rho_1 \varepsilon_1 - \lambda_1 \gamma_1 + k_1 \phi - k_1 \phi = \varepsilon_2 - \rho_1 \varepsilon_1 - \lambda_1 \gamma_1 \tag{26}$$

Differentiating both sides of Eq. (25) leads to:

$$\begin{aligned} \dot{\varepsilon}_2 &= \dot{e}_4 - \dot{o}_1 = \tau_1 + \rho_1 \dot{\varepsilon}_1 + \lambda_1 \dot{\gamma}_1 - k_1 \dot{\phi} = \tau_1 + \rho_1 \dot{\varepsilon}_1 + \lambda_1 \dot{\gamma}_1 - k_1 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_4} \dot{e}_4 + \frac{\partial \phi}{\partial e_5} \dot{e}_5 + \frac{\partial \phi}{\partial e_6} \dot{e}_6 \right. \\ &\quad \left. + \frac{\partial \phi}{\partial x_{1d}} \dot{x}_{1d} + \frac{\partial \phi}{\partial x_{2d}} \dot{x}_{2d} + \frac{\partial \phi}{\partial x_{3d}} \dot{x}_{3d} + \frac{\partial \phi}{\partial x_{4d}} \dot{x}_{4d} + \frac{\partial \phi}{\partial x_{5d}} \dot{x}_{5d} + \frac{\partial \phi}{\partial x_{6d}} \dot{x}_{6d} \right) \\ &= [1 - k_1 \frac{\partial \phi}{\partial e_4} - k_1 \frac{\partial \phi}{\partial e_5} \tan(e_3 + x_{3d})] \tau_1 + k_1 \left( -\frac{\partial \phi}{\partial e_6} \tau_2 + \rho_1 \dot{\varepsilon}_1 + \lambda_1 \dot{\gamma}_1 - k_1 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_5} \delta + \varpi_d \right) \right) \\ &= \omega_1 \tau_1 + \omega_2 \tau_2 + \rho_1 \dot{\varepsilon}_1 + \lambda_1 \dot{\gamma}_1 + \Omega_1 \end{aligned} \tag{27}$$

and

$$\begin{cases} \omega_1 = 1 - k_1 \frac{\partial \phi}{\partial e_4} - k_1 \frac{\partial \phi}{\partial e_5} \tan(e_3 + x_{3d}) \\ \varpi_2 = -k_1 \frac{\partial \phi}{\partial e_6} \\ \Omega_1 = -k_1 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_5} \delta + \varpi_d \right) \end{cases} \quad (28)$$

and

$$\varpi_d = \frac{\partial \phi}{\partial x_{1d}} \dot{x}_{1d} + \frac{\partial \phi}{\partial x_{2d}} \dot{x}_{2d} + \frac{\partial \phi}{\partial x_{3d}} \dot{x}_{3d} + \frac{\partial \phi}{\partial x_{4d}} \dot{x}_{4d} + \frac{\partial \phi}{\partial x_{5d}} \dot{x}_{5d} + \frac{\partial \phi}{\partial x_{6d}} \dot{x}_{6d} \quad (29)$$

and

$$\begin{cases} \frac{\partial \phi}{\partial e_1} = \frac{\partial \phi}{\partial e_2} = 0 \\ \frac{\partial \phi}{\partial e_3} = \frac{d_{22}}{m_{22}} (e_4 + x_{4d}) \sec^2(e_3 + x_{3d}) + u_{1d} \sec^2(e_6 + x_{6d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_5 + x_{5d}) (e_6 + x_{6d}) \sec^2(e_3 + x_{3d}) \\ \quad - 2e_4 (e_6 + x_{6d}) \sec^2(e_3 + x_{3d}) \tan(e_3 + x_{3d}) \\ \frac{\partial \phi}{\partial e_4} = \frac{d_{22}}{m_{22}} \tan(e_3 + x_{3d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_6 + x_{6d}) - (e_6 + x_{6d}) \sec^2(e_3 + x_{3d}) \\ \frac{\partial \phi}{\partial e_5} = 1 - \frac{d_{22}}{m_{22}} - \frac{m_{11} - m_{22}}{m_{22}} (e_6 + x_{6d}) \tan(e_3 + x_{3d}) \\ \frac{\partial \phi}{\partial e_6} = -\frac{m_{11} - m_{22}}{m_{22}} (e_4 + x_{4d}) - e_4 \sec^2(e_3 + x_{3d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_5 + x_{5d}) \tan(e_3 + x_{3d}) \end{cases} \quad (30)$$

and

$$\begin{cases} \frac{\partial \phi}{\partial x_{1d}} = \frac{\partial \phi}{\partial x_{2d}} = 0 \\ \frac{\partial \phi}{\partial x_{3d}} = \frac{d_{22}}{m_{22}} (e_4 + x_{4d}) \sec^2(e_3 + x_{3d}) + u_{1d} \sec^2(e_6 + x_{6d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_5 + x_{5d}) (e_6 + x_{6d}) \sec^2(e_3 + x_{3d}) \\ \quad + \frac{1}{m_{22}} \sec(x_{3d}) \tan(x_{3d}) \chi_d + \frac{1}{m_{22}} \sec(x_{3d}) \frac{\partial \chi_d}{\partial x_{3d}} - 2e_4 (e_6 + x_{6d}) \sec^2(e_3 + x_{3d}) \tan(e_3 + x_{3d}) - u_{1d} \sec^2(x_{3d}) \\ \frac{\partial \phi}{\partial x_{4d}} = \frac{d_{22}}{m_{22}} \tan(e_3 + x_{3d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_6 + x_{6d}) + \frac{1}{m_{22}} \sec(x_{3d}) \frac{\partial \chi_d}{\partial x_{4d}} \\ \frac{\partial \phi}{\partial x_{5d}} = -\frac{d_{22}}{m_{22}} - \frac{m_{11} - m_{22}}{m_{22}} (e_6 + x_{6d}) \tan(e_3 + x_{3d}) + \frac{1}{m_{22}} \sec(x_{3d}) \frac{\partial \chi_d}{\partial x_{5d}} \\ \frac{\partial \phi}{\partial x_{6d}} = -\frac{m_{11} - m_{22}}{m_{22}} (e_4 + x_{4d}) - e_4 \sec^2(e_3 + x_{3d}) - \frac{m_{11} - m_{22}}{m_{22}} (e_5 + x_{5d}) \tan(e_3 + x_{3d}) + \frac{1}{m_{22}} \sec(x_{3d}) \frac{\partial \chi_d}{\partial x_{6d}} \end{cases} \quad (31)$$

and



$$\begin{cases} \frac{\partial \mathcal{X}_d}{\partial x_{3d}} = -[d_{22}x_{5d} + (m_{11} - m_{22})x_{4d}x_{6d}] \sin(x_{3d}) + [(m_{11} - m_{22})x_{5d}x_{6d} - d_{22}x_{4d}] \cos(x_{3d}) \\ \frac{\partial \mathcal{X}_d}{\partial x_{4d}} = (m_{11} - m_{22})x_{6d} \cos(x_{3d}) - d_{22} \sin(x_{3d}) \\ \frac{\partial \mathcal{X}_d}{\partial x_{5d}} = d_{22} \cos(x_{3d}) + (m_{11} - m_{22})x_{6d} \sin(x_{3d}) \\ \frac{\partial \mathcal{X}_d}{\partial x_{6d}} = (m_{11} - m_{22})[x_{4d} \cos(x_{3d}) + x_{5d} \sin(x_{3d})] \end{cases} \quad (32)$$

Motivated by (Do et al., 2004; Rudra et al., 2013) and considering the expression (16), define the following error:

$$\varepsilon_3 = e_3 - k_2[e_2 + e_5 - e_4 \tan(e_3 + x_{3d})] \quad (33)$$

where  $k_2$  is a positive constant. Differentiating both sides of Eq. (33) leads to:

$$\begin{aligned} \dot{\varepsilon}_3 &= \dot{e}_3 - k_2[\dot{e}_2 + \dot{e}_5 - \dot{e}_4 \tan(e_3 + x_{3d}) - x_6 e_4 \sec^2(e_3 + x_{3d})] \\ &= e_6 - k_2[e_5 + \delta + \tau_1 \tan(e_3 + x_{3d}) - \tau_1 \tan(e_3 + x_{3d}) - e_4 (e_6 + x_{6d}) \sec^2(e_3 + x_{3d})] \\ &= e_6 - k_2[e_5 + \delta - e_4 (e_6 + x_{6d}) \sec^2(e_3 + x_{3d})] \\ &= e_6 - k_2 \phi \end{aligned} \quad (34)$$

Define:

$$\gamma_2 = \int_0^t \varepsilon_3 dt \quad (35)$$

$$o_2 = -\rho_2 \varepsilon_3 - \lambda_2 \gamma_2 + k_2 \phi \quad (36)$$

$$\varepsilon_4 = e_6 - o_2 \quad (37)$$

where  $\rho_2, \lambda_2$  are positive constants. Substituting Eq. (36) and Eq. (37) into Eq. (34) gives:

$$\dot{\varepsilon}_3 = \varepsilon_4 + o_2 - k_2 \phi = \varepsilon_4 - \rho_2 \varepsilon_3 - \lambda_2 \gamma_2 + k_2 \phi - k_2 \phi = \varepsilon_4 - \rho_2 \varepsilon_3 - \lambda_2 \gamma_2 \quad (38)$$

Differentiating both sides of Eq. (37) leads to:

$$\begin{aligned} \dot{\varepsilon}_4 &= \dot{e}_6 - \dot{o}_2 = \tau_2 + \rho_2 \dot{\varepsilon}_3 + \lambda_2 \dot{\gamma}_2 - k_2 \dot{\phi} = \tau_2 + \rho_2 \dot{\varepsilon}_3 + \lambda_2 \dot{\gamma}_2 - k_2 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_4} \dot{e}_4 + \frac{\partial \phi}{\partial e_5} \dot{e}_5 + \frac{\partial \phi}{\partial e_6} \dot{e}_6 \right. \\ &\quad \left. + \frac{\partial \phi}{\partial x_{1d}} \dot{x}_{1d} + \frac{\partial \phi}{\partial x_{2d}} \dot{x}_{2d} + \frac{\partial \phi}{\partial x_{3d}} \dot{x}_{3d} + \frac{\partial \phi}{\partial x_{4d}} \dot{x}_{4d} + \frac{\partial \phi}{\partial x_{5d}} \dot{x}_{5d} + \frac{\partial \phi}{\partial x_{6d}} \dot{x}_{6d} \right) \\ &= [-k_2 \frac{\partial \phi}{\partial e_4} - k_2 \frac{\partial \phi}{\partial e_5} \tan(e_3 + x_{3d})] \tau_1 + (1 - k_2 \frac{\partial \phi}{\partial e_6}) \tau_2 + \rho_2 \dot{\varepsilon}_3 + \lambda_2 \dot{\gamma}_2 - k_2 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_5} \delta + \varpi_d \right) \\ &= \omega_3 \tau_1 + \omega_4 \tau_2 + \rho_2 \dot{\varepsilon}_3 + \lambda_2 \dot{\gamma}_2 + \Omega_2 \end{aligned} \quad (39)$$

and

$$\begin{cases} \omega_3 = -k_2 \frac{\partial \phi}{\partial e_4} - k_2 \frac{\partial \phi}{\partial e_5} \tan(e_3 + x_{3d}) \\ \omega_4 = 1 - k_2 \frac{\partial \phi}{\partial e_6} \\ \Omega_2 = -k_2 \left( \frac{\partial \phi}{\partial e_1} \dot{e}_1 + \frac{\partial \phi}{\partial e_2} \dot{e}_2 + \frac{\partial \phi}{\partial e_3} \dot{e}_3 + \frac{\partial \phi}{\partial e_5} \delta + \varpi_d \right) \end{cases} \quad (40)$$

From Eq. (27) and Eq. (39), we choose the controls  $\tau_1$  and  $\tau_2$  as:

$$\begin{cases} \tau_1 = \frac{1}{\omega_1 \omega_4 - \omega_2 \omega_3} [\omega_2 (\rho_2 \dot{e}_3 + \lambda_2 \dot{\gamma}_2 + \Omega_2 + \varepsilon_3 + \rho_4 \varepsilon_4) - \omega_4 (\rho_1 \dot{e}_1 + \lambda_1 \dot{\gamma}_1 + \Omega_1 + \varepsilon_1 + \rho_3 \varepsilon_2)] \\ \tau_2 = \frac{1}{\omega_1 \omega_4 - \omega_2 \omega_3} [\omega_3 (\rho_1 \dot{e}_1 + \lambda_1 \dot{\gamma}_1 + \Omega_1 + \varepsilon_1 + \rho_3 \varepsilon_2) - \omega_1 (\rho_2 \dot{e}_3 + \lambda_2 \dot{\gamma}_2 + \Omega_2 + \varepsilon_3 + \rho_4 \varepsilon_4)] \end{cases} \quad (41)$$

where  $\rho_3, \rho_4$  are positive constants. Combining expression (11) and (17) we can obtain the control input  $\tau_u$  and  $\tau_r$  for underactuated USV as follows:

$$\begin{cases} \tau_u = m_{11} \sec(x_3) (\tau_1 + u_{1d}) + (m_{11} - m_{22}) x_5 x_6 \cos(x_3) + d_{11} x_4 \cos(x_3) + [d_{11} x_5 - (m_{11} - m_{22}) x_4 x_6] \sin(x_3) \\ \quad - \frac{m_{11}}{m_{22}} \tan(x_3) \{ [d_{22} x_5 + (m_{11} - m_{22}) x_4 x_6] \cos(x_3) + [(m_{11} - m_{22}) x_5 x_6 - d_{22} x_4] \sin(x_3) \} \\ \tau_r = m_{33} (\tau_2 + u_{2d}) + d_{33} x_6 - (m_{11} - m_{22}) x_4 x_5 \cos(2x_3) - \frac{(m_{11} - m_{22}) (x_5^2 - x_4^2) \sin(2x_3)}{2} \end{cases} \quad (42)$$

**Stability analysis**

**Theorem 3.1** The controls  $\tau_u$  and  $\tau_r$  given in (42) would achieve the trajectory tracking of arbitrary reference trajectory for USV with the dynamics given in (1). In particular, for any initial conditions  $\boldsymbol{\eta}(0) = [x(0) \ y(0) \ \varphi(0)]^T$  and  $\boldsymbol{\nu}(0) = [u(0) \ v(0) \ r(0)]^T$ , the trajectory tracking errors  $\boldsymbol{\eta}_e = [x_e(t) \ y_e(t) \ \varphi_e(t)]^T$  and  $\boldsymbol{\nu}_e = [u_e(t) \ v_e(t) \ r_e(t)]^T$  would globally asymptotic converge to zero as  $t \rightarrow \infty$  under the operation of the control law given in Eq. (42).

Proof:

We prove **Theorem 3.1** in three steps. The first step is to prove that the closed loop system consisting of Eqs. (26), (27), (38), (39) is asymptotic stabilization under the control input  $\tau_1$  and  $\tau_2$  as expressed in (41). In the second step, we prove convergence of the tracking errors  $e_i, i = 1, \dots, 6$  described in Eq. (15) to zero as  $t \rightarrow \infty$ . Finally, we prove that the trajectory tracking errors  $\boldsymbol{\eta}_e = [x_e(t) \ y_e(t) \ \varphi_e(t)]^T$  and  $\boldsymbol{\nu}_e = [u_e(t) \ v_e(t) \ r_e(t)]^T$  globally asymptotic converge to zero in the third step.

Step 1:

Substituting Eq. (41) into Eq. (27) and Eq. (39), combining Eqs. (23), (26), (35) and (38) gives:

$$\begin{cases} \dot{\gamma}_1 = \varepsilon_1 \\ \dot{\gamma}_2 = \varepsilon_2 \\ \dot{\varepsilon}_1 = \varepsilon_2 - \rho_1 \varepsilon_1 - \lambda_1 \gamma_1 \\ \dot{\varepsilon}_2 = -\varepsilon_1 - \rho_3 \varepsilon_2 \\ \dot{\varepsilon}_3 = \varepsilon_4 - \rho_2 \varepsilon_3 - \lambda_2 \gamma_2 \\ \dot{\varepsilon}_4 = -\varepsilon_3 - \rho_4 \varepsilon_4 \end{cases} \quad (43)$$

We consider the following Lyapunov function candidate:

$$V = \frac{1}{2} \lambda_1 \gamma_1^2 + \frac{1}{2} \lambda_2 \gamma_2^2 + \frac{1}{2} \varepsilon_1^2 + \frac{1}{2} \varepsilon_2^2 + \frac{1}{2} \varepsilon_3^2 + \frac{1}{2} \varepsilon_4^2 \quad (44)$$

Differentiating both sides of Eq. (44) along the solutions of system Eq. (43) results in:

$$\dot{V} = -\rho_1 \varepsilon_1^2 - \rho_3 \varepsilon_2^2 - \rho_2 \varepsilon_3^2 - \rho_4 \varepsilon_4^2 \leq 0 \quad (45)$$

Expression of Eq. (45) implies that  $V(t) < V(0)$ , therefore,  $\gamma_1, \gamma_2, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$  are bounded, and then  $\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3, \dot{\varepsilon}_4$  are bounded as they satisfy the last four equations of Eq. (43). The derivative of  $\dot{V}$  can be expressed as:

$$\ddot{V} = -2\rho_1 \varepsilon_1 \dot{\varepsilon}_1 - 2\rho_3 \varepsilon_2 \dot{\varepsilon}_2 - 2\rho_2 \varepsilon_3 \dot{\varepsilon}_3 - 2\rho_4 \varepsilon_4 \dot{\varepsilon}_4 \quad (46)$$

Since  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$  and  $\dot{\varepsilon}_1, \dot{\varepsilon}_2, \dot{\varepsilon}_3, \dot{\varepsilon}_4$  are all bounded, expression (46) implies the fact that  $\ddot{V}$  is bounded. Hence, with the application of Barbalat's Lemma (Isidori, 1995) it can be prove that  $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$  converge to zero as  $t \rightarrow \infty$ .

Step 2:

This step aims to prove that the tracking errors  $e_i, i = 1, \dots, 6$  described in Eq. (15) converge to zero. As  $\varepsilon_1$  converge to zero and  $k_1$  is just an arbitrarily positive parameter, Eq. (20) implies that:

$$\lim_{t \rightarrow \infty} e_1 = \lim_{t \rightarrow \infty} [e_2 + e_5 - e_4 \tan(e_3 + x_{3d})] = 0 \quad (47)$$

Similarly, Eq. (33) implies that:

$$\lim_{t \rightarrow \infty} e_3 = \lim_{t \rightarrow \infty} [e_2 + e_5 - e_4 \tan(e_3 + x_{3d})] = 0 \quad (48)$$

Eqs. (24), (25), (36) and (37) imply that:

$$\lim_{t \rightarrow \infty} e_4 = \lim_{t \rightarrow \infty} e_6 = \lim_{t \rightarrow \infty} \phi = 0 \quad (49)$$

Eqs. (18), (19), (22), (47), (48) and (49) imply that:

$$\lim_{t \rightarrow \infty} e_2 = \lim_{t \rightarrow \infty} e_5 = 0 \quad (50)$$

So that the tracking errors  $e_i, i = 1, \dots, 6$  described in (15) converge to zero is proved.

Step 3:

In this step, we aim to prove that the control law for surge force  $\tau_u$  and torque  $\tau_r$  expressed in Eq. (42) can have the trajectory tracking errors  $\boldsymbol{\eta}_e = [x_e(t) \ y_e(t) \ \varphi_e(t)]^T$  and  $\boldsymbol{v}_e = [u_e(t) \ v_e(t) \ r_e(t)]^T$  converge to zero as  $t \rightarrow \infty$ .

As  $e_i, i = 1, 2, 3$  converge to zero is proved in step 2, so the tracking errors  $\boldsymbol{\eta}_e = [x_e(t) \ y_e(t) \ \varphi_e(t)]^T$  converge to zero as  $t \rightarrow \infty$ . The first equation of (1) can be rewritten as:

$$\begin{cases} \dot{x} = u \cos(\varphi) - v \sin(\varphi) \\ \dot{y} = u \sin(\varphi) + v \cos(\varphi) \\ \dot{\varphi} = r \end{cases} \quad (51)$$

and then

$$\begin{cases} u = \dot{x} \cos(\varphi) + \dot{y} \sin(\varphi) \\ v = \dot{y} \cos(\varphi) - \dot{x} \sin(\varphi) \\ r = \dot{\varphi} \end{cases} \quad (52)$$

So  $e_i, i = 3, 4, 5, 6$  converge to zero imply  $\varphi, \dot{x}, \dot{y}, \dot{\varphi}$  converge to  $\varphi_d, \dot{x}_d, \dot{y}_d, \dot{\varphi}_d$ , and then  $u, v, r$  converge to  $u_d, v_d, r_d$ , so  $\boldsymbol{v}_e = [u_e(t) \ v_e(t) \ r_e(t)]^T$  converge to zero as  $t \rightarrow \infty$  is proved.

The proof of **Theorem 3.1** is complete.

### SIMULATION EXPERIMENT

In order to verify and illustrate the performance of the control schemes proposed for trajectory tracking control of under-actuated USV, computer simulations were carried out on a model USV with hydrodynamic parameters show in Table 1, which are derived from the hydrodynamic calculation based on FLUENT. The USV model used is a 1:5 model of XX USV developed by National Key Laboratory of Science and Technology on Autonomous Underwater Vehicle affiliated to Harbin Engineering University, China.

Table 1 Parameters of USV model.

Parameters of USV model							
$m$ (mass)	45 kg	$L$ (length)	1.2 m	$X'_u$	$-2.89 \times 10^{-3}$	$Y'_v$	$-5.68 \times 10^{-2}$
$N'_r$	$-4.42 \times 10^{-3}$	$X'_u$	$-1.87 \times 10^{-2}$	$Y'_v$	$-6.97 \times 10^{-2}$	$N'_r$	$-2.62 \times 10^{-2}$
$I_z$	$8.1 \text{ kg} \times \text{m}^2$	$m_{11} = m - X'_u, \ m_{22} = m - Y'_v, \ m_{22} = I_z - N'_r, \ d_{11} = -X_u, \ d_{22} = -Y_v, \ d_{33} = -N_r$					

Three cases of experiments are carried out in this section as below case1 to case 3. The initial position and heading angle of USV is  $x(0) = 10 \text{ m}, y(0) = 10 \text{ m}, \varphi(0) = 0.5 \text{ rad}$  with initial velocities  $u(0) = v(0) = r(0) = 0$ . Parameters of the controller designed above are chosen as  $k_1 = 10, k_2 = 10, \lambda_1 = 2, \lambda_2 = 2, \rho_1 = 1, \rho_2 = 1, \rho_3 = 1, \rho_4 = 1$ .

Case 1: In this case, the surge and yaw reference velocities  $u_d(t)$  and  $r_d(t)$  are chosen as  $u_d(t) = 5 \text{ m/s}$  and  $r_d(t) = 0.1 \text{ rad/s}$ , the initial desired position and heading angle is chosen as  $x_d(0) = y_d(0) = \varphi_d(0) = 0$  with initial desired velocity  $v_d(0) = 0$ , and then the reference trajectory is a circle. Simulation results are shown below in Fig. 2.

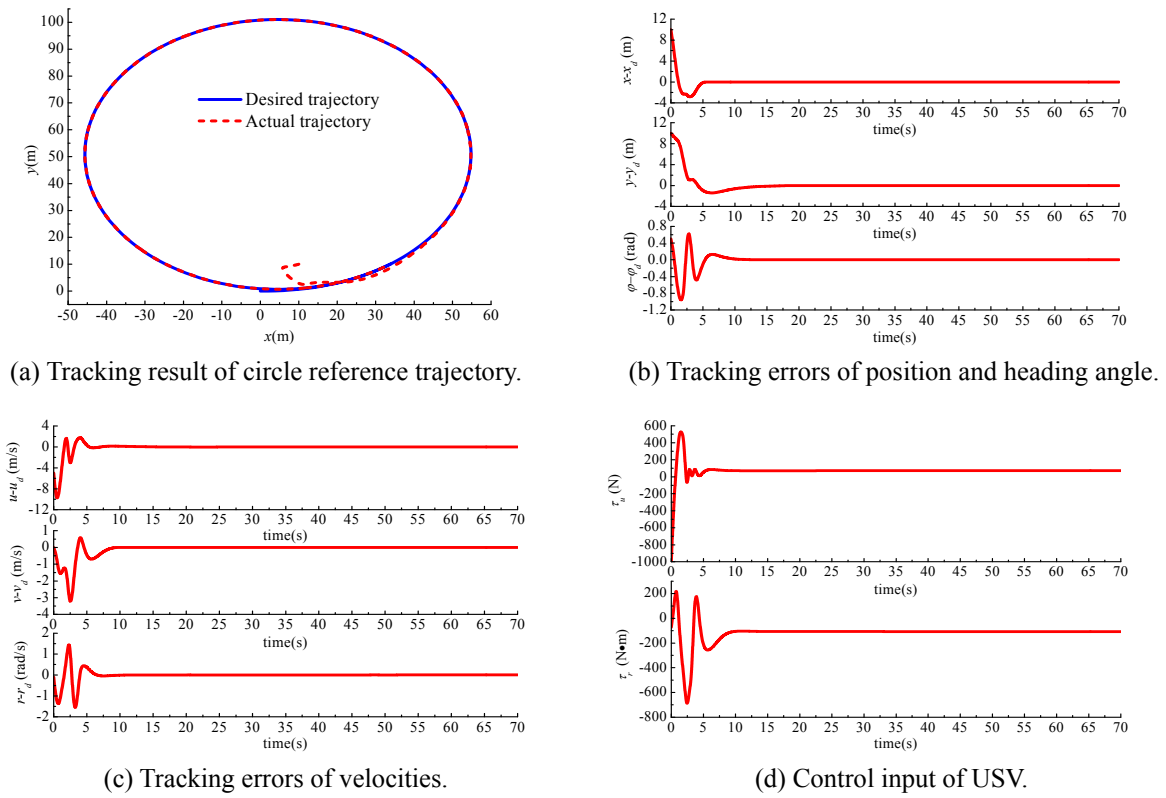


Fig. 2 Trajectory tracking results of circle reference trajectory.

Case 2: In this case, the surge and yaw reference velocities  $u_d(t)$  and  $r_d(t)$  are chosen as  $u_d(t)=5\text{ m/s}$  and  $r_d(t)=0$ , the initial desired position and heading angle is chosen as  $x_d(0)=y_d(0)=\varphi_d(0)=0$  with initial desired velocity  $v_d(0)=0$ , and then the reference trajectory is a straight line. Simulation results are shown below in Fig. 3.

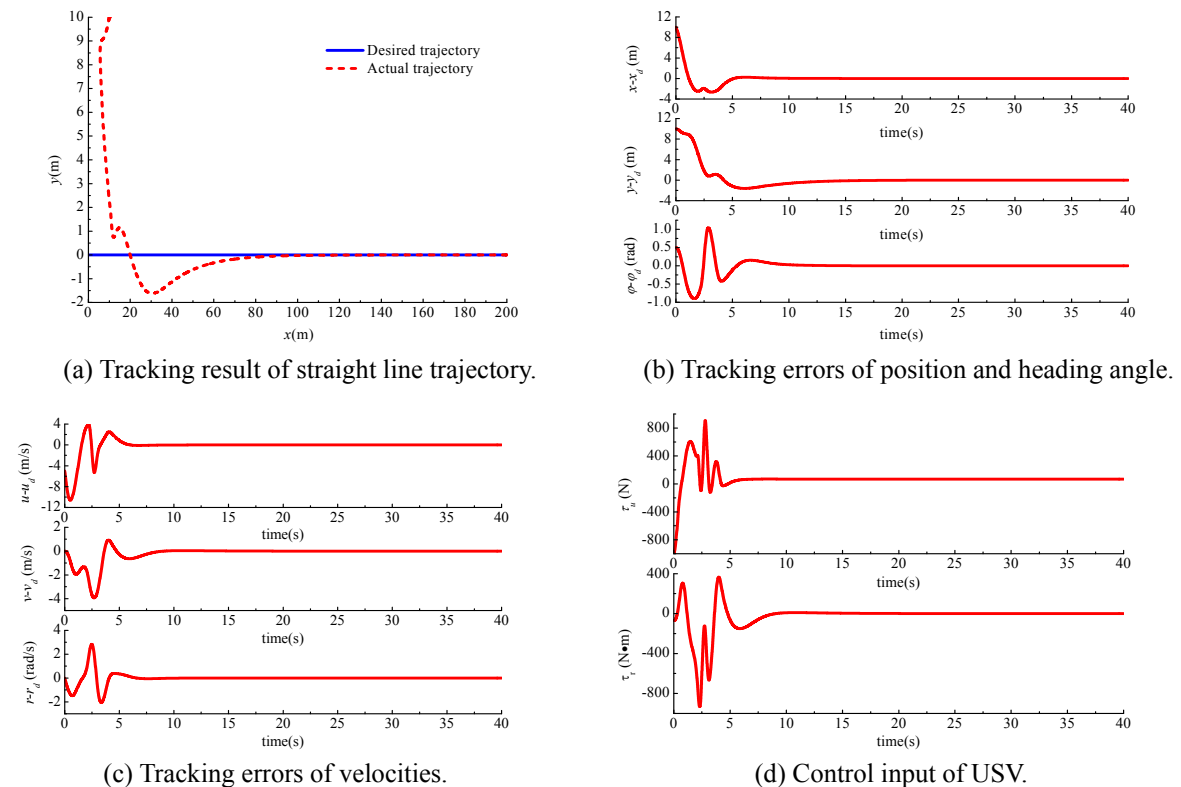
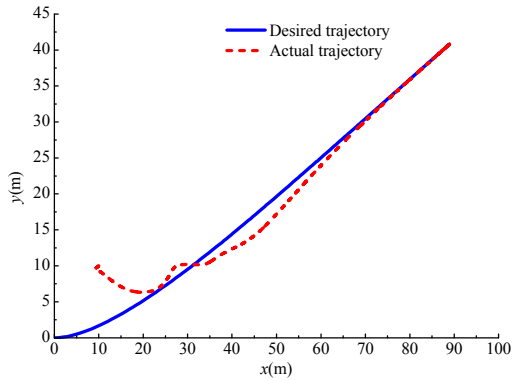
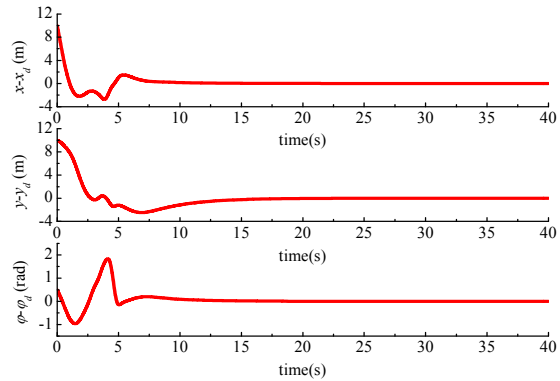


Fig. 3 Trajectory tracking results of straight line trajectory.

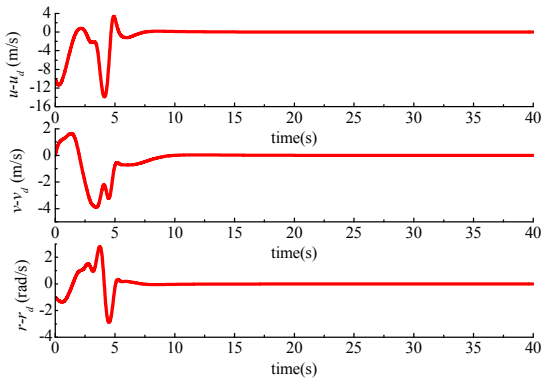
Case 3: In this case, the surge and yaw reference velocities  $u_d(t)$  and  $r_d(t)$  are chosen as  $u_d(t) = 10e^{-0.1t}$  m/s and  $r_d(t) = e^{-2t}$  rad/s, the initial desired position and heading angle is chosen as  $x_d(0) = y_d(0) = \varphi_d(0) = 0$  with initial desired velocity  $v_d(0) = 0$ , and then the reference trajectory is a general curved path. Simulation results are shown below in Fig. 4.



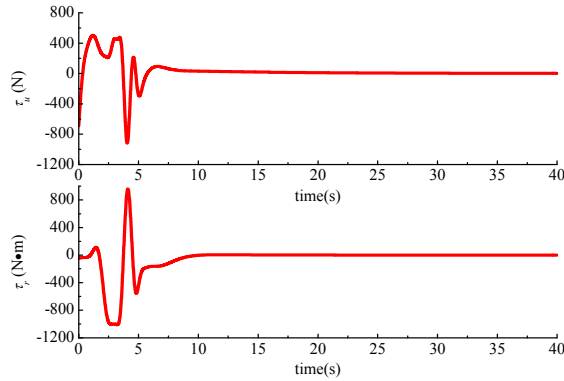
(a) Tracking result of a general curved trajectory.



(b) Tracking errors of position and heading angle.



(c) Tracking errors of velocities.



(d) Control input of USV.

Fig. 4 Trajectory tracking results of a general curved trajectory.

Three cases of experiments are carried out to verify the schemes proposed and demonstrate the effectiveness of the control algorithm designed in this paper. Simulation results in Figs. 2 to 4 show that:

- 1) The control algorithm proposed could achieve trajectory tracking control of USV without the role of side thrusters, therefore underactuated control of USV is obtained.
- 2) PE condition is completely relaxed in controller designed in this paper, and the controller guaranteeing that the USV can track an arbitrary reference trajectory, including a circle trajectory, a straight line trajectory and a general curved trajectory.
- 3) Controller designed could have the actual trajectory rapidly convergence to desired trajectory in all three simulation experiments above.
- 4) Global stability and precise control performance of the controller designed is shown in the simulation experiment results.

CONCLUSION

A novel model based backstepping controller is designed in this paper for trajectory tracking control of underactuated USV in the horizontal plane. The well known PE conditions of yaw velocity is completely relaxed in our study, and the controller designed could drive USV tracking an arbitrary trajectory, including a circle trajectory, a straight line trajectory and a general curved trajectory. An integral action is added into the backstepping controller in order to improve the tracking control performance. Global stability is proved in theory by Lyapunov theory and Barbalat's Lemma, and the control algorithm is verified and illustrated in three cases of simulation experiments.

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