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Research on theoretical optimization and experimental verification of minimum resistance hull form based on Rankine source method

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ABSTRACT: *To obtain low resistance and high efficiency energy-saving ship, minimum total resistance hull form design method is studied based on potential flow theory of wave-making resistance and considering the effects of tail viscous separation. With the sum of wave resistance and viscous resistance as objective functions and the parameters of B-Spline function as design variables, mathematical models are built using Nonlinear Programming Method (NLP) ensuring the basic limit of displacement and considering rear viscous separation. We develop ship lines optimization procedures with intellectual property rights. Series60 is used as parent ship in optimization design to obtain improved ship (Series60-1) theoretically. Then drag tests for the improved ship (Series60-1) is made to get the actual minimum total resistance hull form.*

KEY WORDS: Rankine source method; Minimum resistance hull form; Theoretical optimization; Drag test.

INTRODUCTIONS

Minimum resistance hull form has been pursued by ship designers. Its design capabilities have a significant impact on technical and economic performance of ships. Optimization design of hull mainly depends on ship model towing test or experience of designers. But the best result still can't be found after a series of attempts, as shown in Fig. 1. This method is seriously contrary to the trend of modern ship design. Thus, there is an urgent need for a new approach to optimize hull form (Feng et al., 2008). With the fleeting progress of computer technology and numerical simulation technology, it becomes possible to apply hydrodynamic theory into optimization of hull form. The method has already been used for actual ship design in shipbuilding powers such as Europe, America, Japan, Korea and others and has achieved better economic efficiency (Zakerdoost et al., 2013; Peri and Campana, 2003). Rankine source method is a fast, efficient and high-precision numerical method of wave-making resistance in potential flow theory. It is widely used in design phase of medium-speed and high-speed ships for reducing wave-making resistance. In abroad, Suzuki K. (Suzuki and Iokamori, 1999) combined optimization algorithms with Rankine source method to design minimum wave resistance ship form and compared the results with experimental results. Hironori Yasukawawa (Hironori, 2000) got a new type of ship hull with better resistance performance by using total resistance as objection functions. He adopted the hull form modified functions provided by Suzuki K to

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achieve ship hull geometry reconstruction and used traditional genetic algorithm for optimization calculation. Kim (1995) and Park and Choi (2013) used the sum of flat friction resistance and wave-making resistance calculated by Dawson method as the objective function. They adopted Method of Moving Asymptote (MMA) method [8] to discuss minimum resistance hull form design. In China, Dr. Gao (2005) of Shanghai Jiao Tong University made forebody optimization with genetic algorithm based on Dawson method. Lan (2012) in Harbin Engineering University studied ship design with minimum wave resistance based on genetic algorithm in his master's thesis. He used wave-making resistance calculated by Dawson method as objective functions and the parameters of hull form modified functions provided by Suzuki K as design variables. The first author of this paper has studied the minimum resistance hull form based on Rankine source method since 2009 and has made optimization calculation with Nonlinear Programming Method (NLP), traditional genetic algorithms (SGA) and Niche Genetic Algorithm (NGA). In recent years, it has become possible to use Computational Fluid Dynamics (CFD) to evaluate or optimize ship hull. However, this method takes several hours. It will become even longer if it is combined with optimization method. It does not meet the urgent need of new ship hull in ship market. Therefore, we need to find an effective and quickly new ship hull design method. With B-Spline function parameters as design variables and the displacement as basic constraints, optimization design models with nonlinear programming method are established considering the effects of tail viscous separation. It is based on Rankine source method of potential flow theory of wave-making resistance. We developed whole ship lines optimization design procedures with intellectual property rights and verified the effectiveness of it by tests. It is of great significance for "digital shipbuilding", "green shipbuilding" and hull form design trending to knowledge.

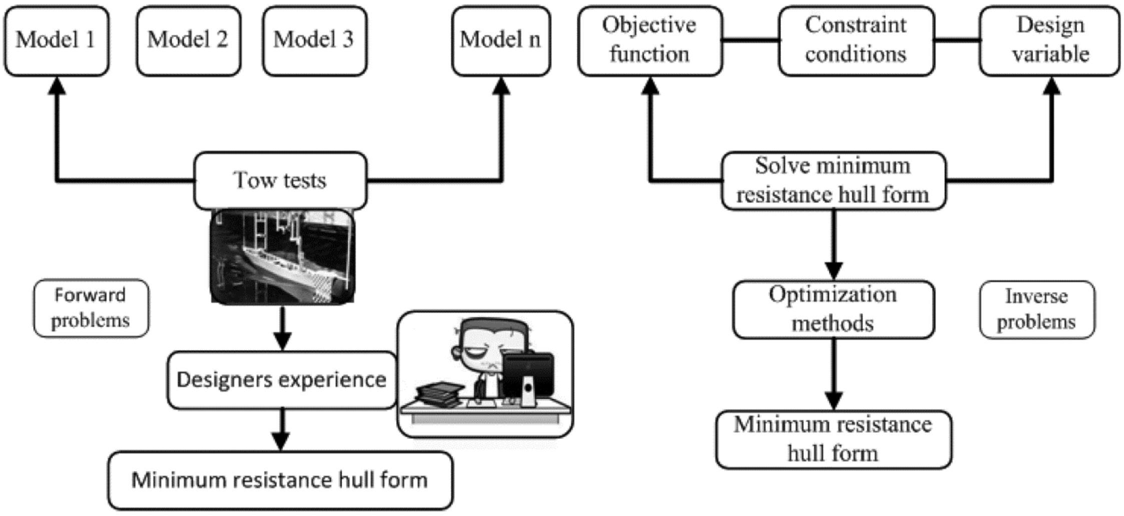


Fig. 1 Comparison of the ship design modes.

DETERMINATION OF MINIMUM RESISTANCE HULL FORM

With the sum of wave resistance and viscous resistance as objective function and parameters of B-Spline function as design variable, optimization calculation adopts Sequential Unconstrained Minimization Technique (SMUT) (Ma and Ichiro, 1997) of nonlinear planning method under the limit of displacement. Then, according to the adjustment of viscous separation, add annex constraints and calculate again to obtain minimum resistance hull form in theory. Finally, towing test is made to obtain actual minimum total resistance hull form.

Hull geometry reconstruction and parametric expression

Hull form functions of the improved ships can be expressed as the sum of functions of parent ship's hull form and variables relative to it.

$$y(x,z) = y_0(x,z) + \Delta y(x,z) \tag{1}$$

where

$$\Delta y(x,z) = \Delta y(x)_{z=WL1} + \Delta y(x)_{z=WL2} + \Delta y(x)_{z=WL3} + \Delta y(x)_{z=WL4} + ... + \Delta y(x)_{z=WLN} \tag{2}$$

Fix z along the direction of depth so that all functions of variables only related to x and expand the polynomial of them in x direction.

$$\begin{aligned} & a_{01} + a_{11}x + a_{21}x^2 + a_{31}x^3 + ... + a_{k1}x^k \Delta y(x)_{z=WL2} = a_{02} + a_{12}x + \\ & a_{22}x^2 + a_{32}x^3 + ... + a_{k2}x^k \Delta y(x)_{z=WL3} = a_{03} + a_{13}x + a_{23}x^2 + a_{33}x^3 + ... + a_{k3}x^k \\ & ....\Delta y(x)_{z=WLN} = a_{0N} + a_{1N}x + a_{2N}x^2 + a_{3N}x^3 + ... + a_{kN}x^k \end{aligned} \tag{3}$$

where $[A] = a_{01},\ a_{11}, \dots, a_{kN}$ is the parameters of each expanded variables.

If the parameter $[A]$ is given, each value of the functions of variables will be determined. Thus, the change of breadth along the x direction in water lines (WL1, WL2, WL3 ...... WLN) can be calculated. And the change of functions of variables at any waterline location can be obtained by cubic spline interpolation function. The interpolation function is based on the $N_{mi}(\zeta)$, $N_{mj}(\xi)$ and interpolated along the direction of depth.

$$\Delta y = \sum_{i=1}^{n+m} \sum_{j=1}^{k+m} c_{ij} N_{mi}(\varsigma) N_{mj}(\xi) \tag{4}$$

where N_{mi} and N_{mj} is the standard B-Spline function. n and k are the number of internal nodes among the modification range in the ξ and ζ direction (except the endpoint). m is the order of B-Spline function.
where m=4, n=3, k=2.

$$\xi_{-3} = \xi_{-2} = \xi_{-1} = \xi_0 < \xi_1 < \xi_2 < .... < \xi_{n+1} = \xi_{n+2} = \xi_{n+3} = \xi_{n+4}$$

$$\zeta_{-3} = \zeta_{-2} = \zeta_{-1} = \zeta_0 < \zeta_1 < \zeta_2 < .... < \zeta_{n+1} = \zeta_{n+2} = \zeta_{n+3} = \zeta_{n+4}$$

The number of design variables is 12 with B-spline function parameters as design variables, as shown in Table 1.

Table 1 B-Spline function parameters as design variables.

Cij	i=1	2	3	4	5	6	7
j=6	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	1.0	1.0	1.0	0.0	0.0
3	0.0	0.0	1.0	1.0	1.0	0.0	0.0
2	0.0	0.0	1.0	1.0	1.0	0.0	0.0
1	0.0	0.0	1.0	1.0	1.0	0.0	0.0

Objective function

In present study, total resistance R_T is selected as objective function in the optimization design process. R_T is the sum of wave making resistance R_W and viscous resistance $(1+k)R_F$.

$$R_T = R_{\mathrm{w}} + (k+1)\cdot R_F \rightarrow \min \tag{5}$$

The form factor k is calculated by the following formula

$$k = (\frac{V^{1/3}}{L})\cdot(0.5C_B + \frac{2\gamma^{1.3}}{C_B}) \tag{6}$$

where $\gamma = (b/L)/\{1.3(1-C_B)-0.031\cdot l_{cb}\}$. V is the displacement volume. b is the breath molded. C_B is the block coefficient. γ is the full degree of stern. l_{cb} is the longitudinal position of the center of buoyancy. V and l_{cb} are calculated by numerical integration, such as trapezoidal method , Simpson method and so on.

R_W is calculated by the Rankine source method (Lan, 2012).

$$R_W = \frac{1}{2}\rho\cdot U^2\cdot L^2\cdot C_W \tag{7}$$

where C_W is the wave resistance coefficient. U is the design speed. L is the ship length between perpendiculars and ρ is the density of the fluid.

The frictional resistance is calculated by the following flat plate resistance formula.

$$R_F = \frac{1}{2}\cdot\rho\cdot U^2\cdot S^2\cdot C_{f0} \tag{8}$$

Accurate computation of friction resistance requires an enormous computational effort in the process of optimization. For this reason, friction resistance is usually approximated by the drag generated by a turbulent layer flow over a flat plate with the same wetted area and length as ships. The formulation used in this work is the one accepted as a standard by the 1957 ITTC formula.

$$C_{f0} = \frac{0.075}{(\log \mathrm{Re}-2)^2} \tag{9}$$

where C_{f0} is the non-dimensional friction drag coefficient and Re is the Reynolds number based on the length of the ship.

Optimization

Numerical optimization can be carried out by systematic iteration of the objective function. In general, the objective functions are nonlinear with respect to the design variables and complex design constraints are needed. To overcome these problems, optimization of constraints is transformed into un-constrained optimization with SUMT interior-point method of NLP. Then gradient method may be used to minimize the wave making resistance.

In present study, SUMT interior-point method is selected as one of the NLP techniques to minimize the objective function under design constraints. Optimization is carried out so as to obtain a new hull form with lower resistance. The objective function is the total resistance, which may be calculated using Eq. (5).

The ship's hull was subjected to the following geometric and hydrodynamic constraints.

1) All offsets are nonnegative, namely $y(i, j) \geq 0$.
where $y(i, j)$ denotes the coordinate of the hull form.

2) The displacement should meet the demand of $\nabla \geq \nabla_0$.
where ∇, ∇_0 respectively denote the displacement of the improved hull form and the original hull.

In optimization procedure, variables to be altered are y coordinates of the ship's hull. The body shape is modified by Eqs. (1) - (4).

Viscous separation judgment

Minimum resistance hull form design based on potential flow theory often gets larger bulbous or ball-tailed. However, the change of tail shape may cause the changes of viscous resistance. Therefore, two-dimensional simple separation formula provided by Ma and Ichiro (1994) is used to judge the size of separated domain of the parent ship and the improved ship. Then optimize full ship lines with the control of separate domains as attached constraint conditions. Thus, viscous resistance is not increased obviously with wave-making resistance as main objective function in hull form optimization. The equation of separation judgment is shown as function (10) below.

$$C(s) \equiv \frac{1}{U^5}\frac{dU}{dS}\int_0^s U^4 ds \approx -2 \tag{10}$$

Separation judgment of each flow lines of hull surface is made by Eq. (10) to find out separation points. Connect these separation points of each flow lines to get separation domains.

$$\left| y(i,j) - y_0(i,j) \right| \leq h \tag{11}$$

where U is the flow velocity of outer edge of the boundary layer, which can be replaced by potential flow velocity calculate by Hess-Smith method. $y_0(i, j)$, $y(i, j)$ are ship hull surface coordinate values of the parent ship and the improved ship. Hull form optimization is carried out under the basic constraints. Then consider added constraints and optimize hull form again according to whether the improved ship-shape meets the design requirements. The value of b is based on the results of the stern separation judgment function or the author's experience.

Optimization calculation

Firstly, input files including parent ship's type and offsets, design range, number of design variables, design speed and initial parameter of optimization calculation. Then, express the parent ship with B-Spline function. Divide and arrange network grid for hull and free surface and make appropriate encryption for ship bow and stern. With the sum of wave resistance and viscous resistance as objective function and the parameters of B-Spline function as design variable, optimization calculation is made using Nonlinear Programming Method (NLP). The improved ship's displacement is not less than mother ship's and offsets are non-negative. Then, judge whether the total resistance is the smallest and whether clay separation occurs or not. If separation occurs, additional constraints will be added and re- optimization calculation will be made to obtain final theoretical improved hull form. Finally, make towing tests for the improved ship to determine actual minimum ship form. The frame of the design of hull form optimization is shown in Fig. 2.

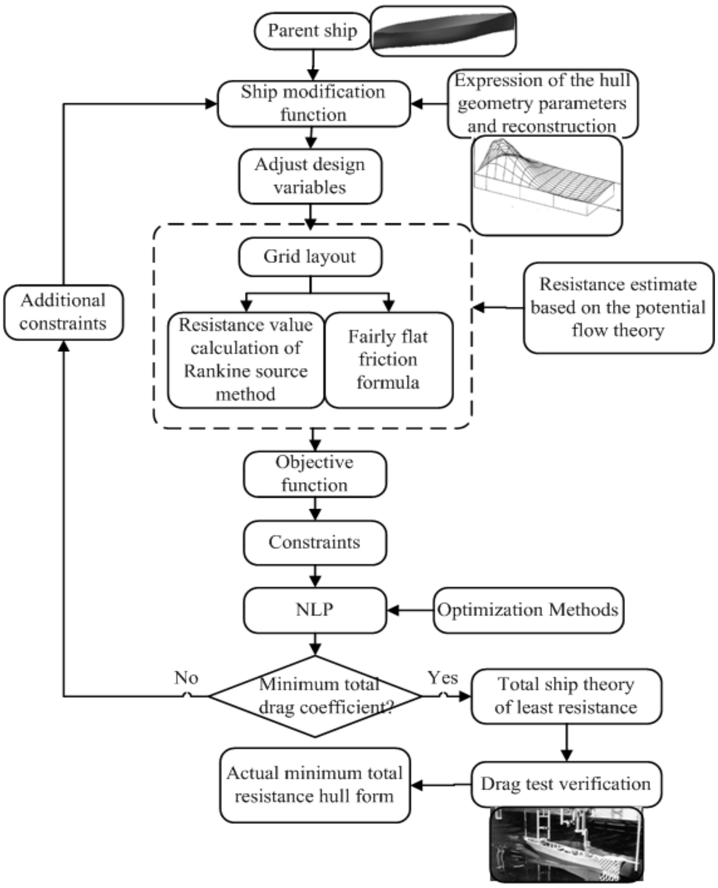
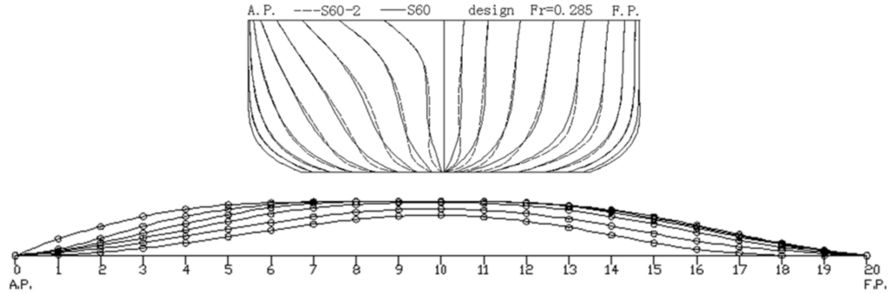


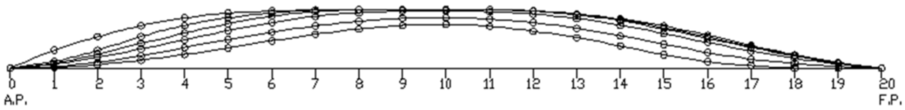
Fig. 2 Flow chart of the optimization calculation.

EXAMPLES

The parent ship of optimization calculation is series60 ship. Its main dimensions, hull grid and free surface meshing can be found in Zhang et al. (2009). Fig. 3 shows cross-sectional lines and waterlines of parent ship (Series60) and improved ship (Series60-1). Compared with the parent ship, the outward tendency of the improved ship's cross-sectional line and waterline is slightly reduced.



(a) The original hull form.



(b) The improved hull form.

Fig. 3 The comparison of the body lines and waterlines between the improved hull form (Series60-1) and the original hull form (Series60).

The results of optimization calculation are summarized in Table 2. R_w is the wave resistance of the improved ship. R_{wo} is the wave resistance of the parent ship. R_T is the total resistance of the improved ships. R_{T0} is the total resistance of the parent ship. V is the improved ship's displacement. V_0 is the parent ship’s displacement. S is wet surface area of the improved ship. S_0 is wet surface area of the parent ship. Wave resistance of the improved ship (Series 60-1) decreases by 13.2% and total resistance reduces by 4.5%.

Fig. 4 shows the comparison of wave resistance coefficient between the parent ship (Series60) and the improved ship (Series 60-1). The trend of the wave resistance coefficient between them is basically consistent. Wave- making resistance of the improved ship form gets some degree of reduction within a certain range of designed Froude number. Fig. 5 is the comparison of ship side's waveform. Wave height of ship side in bow and tail both reduces to some degree. Fig. 6 is the comparison of free surface's waveform. The waveform shows Kelvin wave obviously, which is consistent with test result. Fig. 7 is the history record of reduction of total resistance in the process of optimization. Fig. 8 is the comparison of cross-sectional area curves between them.

Table 2 The result of the optimal calculation based on Rankine source method.

Hull form	Constraints	Design Fr	R_W/R_{W0}	R_T/R_{T0}	V/V_0	S/S_0
Series60-1	(1), (2), (11)	0.285	86.8%	95.5%	1.003	1.004

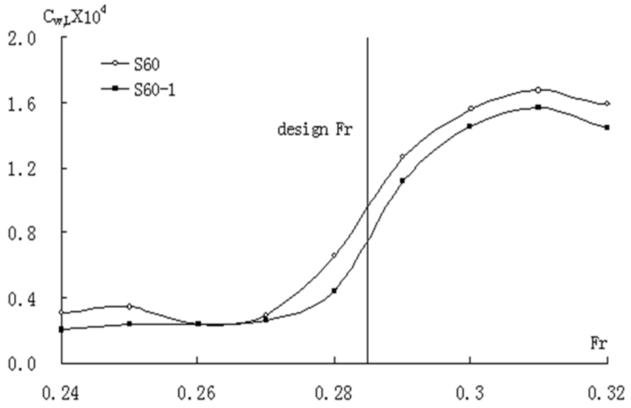


Fig. 4 Comparison of wave making resistance coefficient between the two improved hulls.

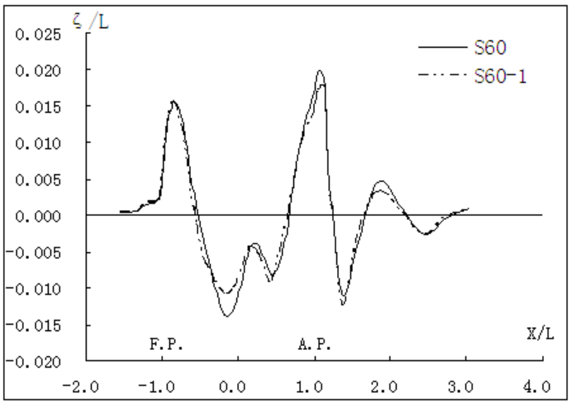


Fig. 5 Comparison of the wave profiles of the Series60 hull.

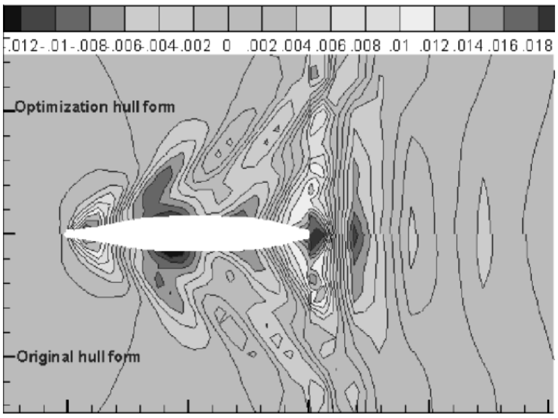


Fig. 6 The waveform contours of the original hull form and the improved hull form.

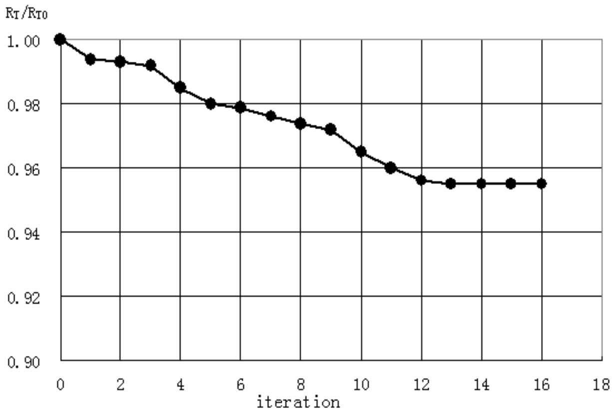


Fig. 7 History of optimization convergence.

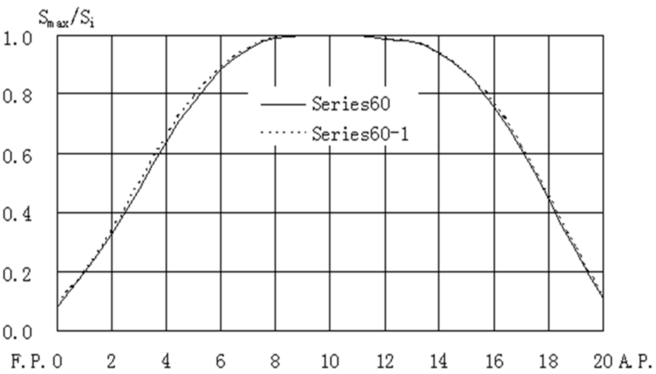


Fig. 8 Comparison of section area curve.

DRAG SHIP MODEL TEST RESULTS

To verify the reliability of the theory optimization theory, ship model towing tests were made in Shanghai Ship and Shipping Research Institute's towing tank. The objects of them are the parent ship (Series60) and the obtained minimum total resistance hull (Series60-1). Bounded ship model was used and the effect of heave and pitch was ignored. Main dimensions of the ship models are 3.0 *m* in length, 0.4005 *m* in width and 0.1605 *m* in draft. The transition of laminar flow to turbulent flow was simulated by setting 3 *mm* in height torrent nail at interval of 10 *mm* at 9.5 station of ship model.

The resistance value of ship model (R_{tm}) was measured by an electricity resistograph (NS-30) imported from Japan, which maximum range is 10 *kg* (100 *N*) and the accuracy is 0.1%. The recording system was composed by high-speed data acquisition card, amplifiers and IPC. Then, data collected was input into computers for calculation. Total resistance in test results was calculated by three-dimensional conversion method and the flat friction coefficient was calculated by the 1957 ITTC formula.

The total drag coefficient of ship model

$$C_{tm} = C_{fm}(1+k) + C_{wm} \tag{12}$$

The total drag coefficient of the ship

$$C_{ts} = C_{fs}(1+k) + C_{wm} \tag{13}$$

where the subscripts, m, and s, respect ship model and ship.

$$C_{wm} = C_{ws} \tag{14}$$

Therefore, we can get the total drag coefficient of the real ship. The total resistance of ship

$$R_{ts} = \frac{1}{2}\rho_s S_s v_s^2 C_{ts} \tag{15}$$

The wave resistance of ship

$$R_{ws} = R_{ts} - R_{fs} \tag{16}$$

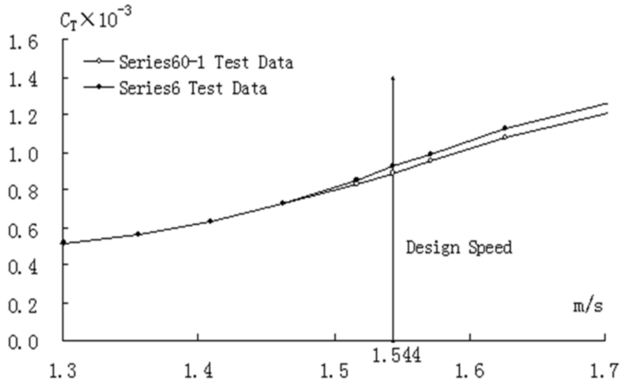


Fig. 9 Comparison of total resistance coefficient test results.

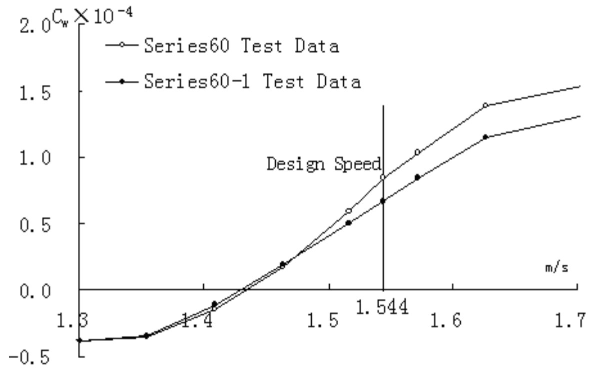


Fig. 10 Comparison of wave resistance coefficient test results.

The results of total resistance and wave resistance of the improved ship (Series 60-1) and the parent ship (Series60) are shown in Figs. 9 and 10. The total resistance and wave resistance of the improved ship reduced significantly by 3.5% and 21% around the design speed, which is consistent with theoretical calculation results and confirms the applicability of the method.

CONCLUSION

With the sum of wave resistance and viscous resistance calculated by Rankine source method as objective function and the parameters of B-Spline function as design variable, mathematical models are built using Nonlinear Programming Method (NLP) ensuring the basic limit of displacement and considering rear viscous separation. Drag tests of the parent ship and the improved ship confirm that reduction of the resistance of the improved ship is consistent with the result of optimization in theory. We will further make more research on the optimization of other ship types (such as container ships, luxury cruise ships, etc.).

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