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Optimal Operation for Green Supply Chain with Quality of Recyclable Parts and Contract for Recycling Activity

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ABSTRACT

This study discusses a contract to promote collection and recycling of used products in a green supply chain (GSC). A collection incentive contract is combined with a reward-penalty contract. The collection incentive contract for used products is made between a retailer and a manufacturer. The reward-penalty contract for recycling used products is made between a manufacturer and an external institution. A retailer pays an incentive for collecting used products from customers and delivers them to a manufacturer with a product order quantity under uncertainty in product demand. A manufacturer remanufactures products using recyclable parts with acceptable quality levels and covers a part of the retailer's incentive from the recycled parts by sharing the reward from an external institution. Product demand information is assumed as (i) the distribution is known (ii) mean and variance are known. Besides, the optimal decisions for product quantity, collection incentive of used products and lower limit of quality level for recyclable parts under decentralized integrated GSCs. The analysis numerically investigates how (1) contract for recycling activity, (ii) product demand information and (iii) quality of recyclable parts affect the optimal operation for each GSC. Supply chain coordination to shift IGSC is discussed by adopting Nash Bargaining solution.

Keywords: Green Supply Chain, Quality, Collection Incentive, Reward-Penalty Contract, Demand Information, Game Theory

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1. INTRODUCTION

In recent years, for the purpose of solving problems about environment protection and resource saving, several evaluation measures and policies have been promoted in order to establish a new supply chain management which incorporates reverse chains/logistics into traditional forward chains/logistics. The traditional forward chains/logistics are composed of the flows from procurement of new materials through production of new products to selling them. The reverse chains/logistics consists of the flows from collection of used products through recycling parts from the used products to reuse the recycled parts. Theoretical analyses and the marginal insights obtained from numerical examples on the reverse chains/logistics are discussed in the following previous papers: Aras *et al*., 2004; Behret and Korugan, 2009; Ferguson *et al*., 2009; Fleischman *et al*., 1997; Guide and Wassenhove, 2001; Inderfurth, 2005; Konstantaras *et al*., 2010; Mukhopadhyay and Ma, 2009; Nenes *et al*., 2010;

Pokharel and Liang, 2012; Teunter and Flapper, 2011; Wei *et al*., 2011; Wu, 2012.

Also, a supply chain which organizes the forward chains and the reverse chains has been called a closed-supply chain, reverse supply chain or a green supply chain (GSC), which is defined in the following previous papers: Bakal and Akcali, 2006; Fleischman *et al*., 1997; Guide *et al*., 2003; Inderfurth, 2005; Kaya, 2010; Lee *et al*., 2011; Shi *et al.*, 2010, 2011; Tagaras and Zikopoulos, 2008; Thierry *et al*., 1995; Van Wassenhove and Zikopoulos, 2010; Wei *et al.*, 2012; Yan and Sun, 2012; Zikopoulos and Tagaras, 2007, 2008. In this study, the supply chain that has the forward chains and the reverse chains is called a GSC. The manufacturing to reuse recycled parts is called the remanufacturing. It is necessary to take some measures and policies in order to promote 3R activities (Reduce-Reuse-Recycle) in the GSC.

Several previous papers have dealt with the optimal operations for GSC, and the uncertainty in remanufacturing has been attracting more attention in recent papers.

The incorporation of the uncertainty in demands of products/parts and collection quantity of used products into GSC have been discussed by Inderfurth (2005), Lee *et al*. (2011), Mukhopadhyay and Ma (2009), Shi *et al.* (2010, 2011), and Wei *et al.* (2011).

The incorporation of the price-sensitivity in collection quantity of used products and demands of products/parts into the optimal tactical production planning for a GSC have been discussed in the following previous papers: Bakal and Akcali (2006), Pokharel and Liang (2012), Shi *et al*. (2010), Teunter and Flapper (2011), Wei *et al.* (2012), and Yan and Sun (2012). They proposed theoretical analyses to determine the optimal operation in a GSC on above topic and provided the marginal insights obtained from numerical examples.

Also, the effects of inspection and sorting of used products on the optimal tactical production planning in GSC have been discussed in some previous papers. Theoretical analyses and the marginal insights obtained from numerical examples on above topic are discussed in the following previous papers: Aras *et al.* (2004), Behret and Korugan (2009), Ferguson *et al*. (2009), Guide *et al.* (2003), Konstantaras *et al*. (2010), Nenes *et al.* (2010), Tagaras and Zikopoulos (2008), Van Wassen-hove and Zikopoulos (2010), and Zikopoulos and Tagaras (2007, 2008).

In dealing with the GSC, it is necessary to consider a variety of qualities of used products collected from the market. Some authors have discussed the optimal tactical production planning by incorporating uncertainty in the quality of used products into the GSC. Aras *et al.* (2004) investigated the issue of the stochastic nature of product returns and found conditions under which quality-based categorization was most cost effective. Zikopoulos and Tagaras (2007) investigated how the profitability of reuse activities was affected by uncertainty regarding the quality of returned products in two collection sites and determined the unique optimal solution (procurement and production quantities). In Guide *et al*. (2003) and Ferguson *et al*. (2009), returned products were assumed to have N quality categories, and the procurement prices and the remanufacturing costs were different based on the corresponding quality level. Behret and Korugan (2009) discussed a remanufacturing stage with uncertainties in the quality of remanufacturing products, return rates, and return times of returned products. After returned products were classified by considering quality uncertainties, remanufacturing processing times, material recovery rates, the remanufacturing costs, and disposal costs were determined by using the ARENA simulation program. Mukhopadhyay and Ma (2009) discussed a GSC consisting of a retailer who sold a single product and a manufacturer who collected used products from the market, remanufactured parts from the used products and then produced products. They assumed two situations for the remanufacturing ratio between reuse parts and used products: a constant situation and an uncertain situation. Under each situation, they proposed the optimal production strategy for the procurement quantity of used products, the remanufacturing quantity of parts from used products and the production quantity of new parts from new materials. Nenes *et al*. (2010) observed that both quality and quantity of returns (used products) were unfortunately highly stochastic, and investigated the optimal policies for ordering of new products and remanufacturing of products so as to maximize the companies' performance, such as minimizing their expected cost or maximizing their expected profit. Teunter and Flapper (2011) discussed how quality of cores (i.e., products supplied for remanufacturing) could vary significantly, affecting the cost of remanufacturing, and derived the optimal policies regarding acquisition and remanufacturing for both deterministic and uncertain demand.

The effect of incentive for collection of end-of-used products which is paid from either a retailer or a manufacturer to customers on the optimal tactical strategy in a GSC has been discussed in Kaya (2010), Aras and Aksen (2008), Aras *et al.* (2008) and Asghari *et al.* (2014). Kaya (2010) discussed a manufacturer producing original products using virgin materials and remanufactured products using returns from the market. The amount of returns depended on the incentive offered by the manufacturer. The optimal value of this incentive and the optimal production quantities of both remanufacturing parts and new parts in a stochastic demand were determined. Aras and Aksen (2008) discussed the problem of locating collection centers of a company that aimed to collect used products from consumers. Returned items were categorized with respect to their quality level into classes called return types, and a different incentive was offered for each return type from the company to customers. Both the optimal locations of the collection centers and the optimal incentive values for each return type so as to maximize the company's profit from the returns. Aras *et al.* (2008) extended Aras and

Aksen (2008) by considering a pick-up policy with capacitated vehicles. Asghari *et al.* (2014) discussed the incentive effect on return quantity of used products between the collection centers and recovery facilities. A dynamic nonlinear programming model of reverse logistics network design was proposed to manage the used products allocation by coordinating the collection centers and recovery facilities to warrant economic efficiency.

The effect of incentive for quality of collected end-of-used products on the optimal tactical production planning in a GSC has been discussed in Veldman and Gaalman (2014) and Lee *et al*. (2013). Veldman and Gaalman (2014) discussed investigated the effects of strategic incentives for product quality and process improvement using a game theoretic model that considers two owner-manager pairs in competition. Lee *et al*. (2013) discussed the quality-compensation contract, in which the manufacturer compensates the retailer for defective products that are inadvertently sold to consumers.

The effect of incentive for collection of end-of-used products under Extended Producer Responsibility (EPR) on the optimal tactical production planning in a GSC has been discussed in Li *et al*. (2014). Li *et al.* (2014) discussed the collection outsourcing phenomena under EPR. They studied a contract design problem for a manufacturer who consigned the used product collection to a collector, while the manufacturer only had incomplete information on the collector's cost. On the basis of the incentive theory, optimal contracts were developed to minimize the cost and satisfy the collection constraints prescribed by EPR.

The effect of incentive for the collection of end-of-used products which is paid from a manufacturer to a retailer on the optimal tactical production planning in a GSC has been discussed in Lee *et al*. (2011). Lee *et al.* (2011) discussed a model that integrated pricing, production, and inventory decisions in a reverse production system (RPS) with retailer collection. The returned products were assumed valuable to the manufacturer for creating as-new products, but the retailer had to divide effort between selling the new product and collecting the returns. The manufacturer offered incentives to the retailer to participate in the overall system.

The effects of two types of incentives has been discussed in Watanabe *et al*. (2013) and Watanabe and Kusukawa (2014). One is the incentive which is paid from a retailer to customers, the other is the incentive which is paid a manufacturer from a retailer. Two types of incentive enabled to promote the collection and the recycling of used products. The optimal decisions were made for the product quantity, the collection incentive of used products and the lower limit of quality level for recycling of used products under both the decentralized GSC (DGSC) and the integrated GSC (IGSC).

It is necessary to determine the optimal operations to establish a GSC to obtain its profitability. In a DGSC, all members in the GSC determine the optimal operations so as to maximize their profits. As one of the optimal decision-making approaches under a DGSC, the Stackelberg game has been adopted in several previous papers. In the Stackelberg game, there is a single leader of the decision-making and a single (multiple) follower(s) of the decision-making of the leader. The leader of the decision-making determines the optimal strategy so as to maximize the leader's (expected) profit. The follower(s) of the decision-making determine(s) the optimal strategy so as to maximize the follower(s)'s (expected) profit under the optimal strategy determined by the leader of the decision-making. Theoretical analyses and the marginal insights obtained from numerical examples on above topics are discussed in the following previous papers: Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Lee *et al*., 2011; Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Sun, 2012.

In a supply chain management, the optimal decisions under an integrated supply chain maximizing the whole supply chain's expected profit can bring the more expected profit to the whole supply chain than those under a decentralized supply chain maximizing the expected profit of each member in a supply chain. So, from the aspect of the total optimization in supply chain management, it is preferable for all members in supply chain to shift the optimal decisions under the integrated supply chain. In this case, it is the absolute requirement for all members under the integrated supply chain to obtain the more expected profits than those under the decentralized supply chain. In order to achieve the increases in profits of all members under the integrated supply chain, a variety of supply chain coordination approaches between all members have been discussed in the following papers: Cachon and Netessine (2004), Du *et al*. (2011), Kaya (2010), Tsay *et al*. (1999), Wei *et al*. (2012), Wu (2012), Yan and Sun (2012), and Yano and Gilbert (2004). The incorporation of the game theory into not only the optimal pricing strategies, but also the supply chain coordination in a GSC have been discussed by Kaya (2010), Wei *et al*. (2012), Wu (2012), Yan and Sun (2012), and Du *et al*. (2011). They proposed theoretical analyses on supply chain coordination on above topic and provide the marginal insights obtained from numerical examples.

In above previous studies mentioned above, it was assumed that it was possible for a retailer who placed an order to know the full information of the product demand regarding the probability density function with two parameters: mean and variance of the product demand. In a real situation for the GSC, it may be impossible for a retailer to know the full information of the product demand, but possible to know the limited demand information of product regarding only both mean and variance of the product demand. Under the limited demand information, the distribution-free approach (DFA) to optimize a product order quantity in a newsboy problem handling with a single product in a single period is discussed in the following previous papers: Callego and Moon (1993); Moon and Gallego (1994), Moon and Choi (1995), Alfares and Elmorra (2005). Gallego and Moon (1993) and Moon and Gallego (1994) proposed DFA to determine the optimal product order quantity in a newsboy problem handling a single product in a single period, but this paper did not consider the shortage penalty cost between the product demand and the product order quantity. Alfares and Elmorra (2005) extended Gallego and Moon (1993) to determine the optimal product order quantity in a newsboy problem handling with a single product in a single period, considering both the inventory holding cost and the shortage penalty cost. The above previous papers regarding DFA provided the following the marginal insights: DFA can provide the optimal product order quantity which maximizes the expected profit against the worst possible distribution of the product demand with mean and variance. Concretely, DFA can derive a simple upper bound on the expected inventory holding cost and the shortage penalty cost between the product demand and the product order quantity in terms of all possible distributions of the product demand. Therefore, it can derive a simple lower bound on the expected profit in terms of all possible distributions of the product demand. Thus, DFA is effective when the optimal product order quantity is made under the limited demand information.

Watanabe and Kusukawa (2014) incorporated DFA into an optimal operational policy for a GSC. Concretely, this previous study proposed an optimal operational policy for GSC where a retailer paid an incentive for collection of used products from customers and determined the optimal order quantity of a single product under uncertainty in product demand. A manufacturer produced the optimal order quantity of product using recyclable parts with acceptable quality levels and covered a part of the retailer's incentive from the recycled parts. Here, as demand information, two scenarios for the product demand were assumed as: the distribution of product demand was known, and only both mean and variance were known. According to demand information, this previous study developed mathematical models to find how the product order quantity, the collection incentive of used products and the lower limit of quality level for recycling affect the expected profits of each member and the whole system chain under both DGSC and IGSC.

However, only the collection incentive contract between a retailer and a manufacturer in a GSC was negative to collect and recycle the used products with low quality.

The effect of penalty contract on the optimal tactical production planning in a GSC has been discussed in Yoo (2014). Yoo (2014) identified the relationship between return policy and product quality decisions in a decentralized system. It considered the penalty contract based on external failure to control the supplier's action in a decentralized system.

The effect of a target rebate-punish contract for the collection of end-of-used products on the optimal tactical production planning in a GSC has been discussed in Yan and Sun (2012). They discussed a closed-loop supply chain (CLSC) with a manufacturer and a third-party reverse logistics provider (3PRLP). They considered the impacts of environmental legislation on scrap recycling in a GSC. In order to encourage a 3PRLP to exert him to the col-lection activities of end-of-used products, a target rebate- punish contract was designed between a manufacturer and a 3PRLP under both stochastic price-dependent demands and stochastic effort-dependent returns.

The effect of compensation from the government contract on the optimal tactical production planning in a GSC has been discussed in Hong and Ke (2011). Hong and Ke (2011) discussed that advanced recycling fees (ARFs) and government subsidies played important roles in encouraging or curtailing the flows of recycled items. A decentralized reverse supply chain consisted of the government, a group of manufacturers, importers, and sellers (MISs) and a group of recyclers. ARFs and socially optimal subsidy fees were optimized. To maximize social welfare, the government determined the ARFs paid by MIS and the subsidy fees for recyclers when MIS sold new products and recyclers process EOL products.

However, previous paper has not been discussed how the reward-penalty (punish) contract between a manufacturer and an external institution such as governments impact the optimal tactical production planning in a GSC. It is expected that the reward-penalty (punish) contract between them enables to promote the collection activity and the recycling activity under the environmental legislation.

The motivation of this paper to verify how the optimal operations under a DGSC and IGSC is affected by the contract to promote collection and recycling of used products and by the demand information of product with uncertain demand. Besides, this paper verifies how two contract combining the collection incentive contract and the reward-penalty contract and supply chain coordination affect the promotion of the collection and the recycling of used products between a retailer and a manufacturer in a GSC.

For practitioners, academic researchers and real-world policymakers regarding operations in a GSC, this paper deals with the following questions to discuss the operation of a GSC: 1) how much the collection incentive should be paid to encourage to collect used products from customers in a GSC, 2) how the quality of recyclable parts after disassembly of used products affect the recycling activity in a GSC and the profits of GSC members and the whole system, 3) how two contracts combining the collection incentive contract and the reward-penalty contract can promote the recycling activity between a retailer and a manufacturer, 4) how the optimal product order quantity can be determined as to the either the full demand information of the product or the limited demand information. This study tries to answer the above questions by theoretical analysis, the numerical calculation and the numerical analysis.

Concretely, this study discusses a GSC with material flows from collection of used products to sales of a single product reusing a single recycled parts in a market. A GSC consisting of a retailer, a manufacturer and an external institution is dealt with and the optimal operation in the GSC is presented in a single period. The collection incentive contract is combined with the reward-penalty contract in a GSC in order to promote the collection and the recycling of used products with low quality level as well as those with high quality level. The collection incentive contract for used products is made between a retailer and a manufacturer. The reward-penalty contract for recycling used products is made between a manufacturer and an external institution. The reward-penalty contract proposed in this paper is shown as follows: an external institution provides a target of quality level for recycling of used products. According to the magnitude relation between the target for recycling of recyclable parts and the lower level of recyclable parts in used products, the following three events are occurred in a GSC: (I) the manufacturer can receive the reward from the external institution, (II) the manufacturer pays the penalty to the external institution and (III) the manufacturer incurs neither the reward nor the penalty.

According to the combined contract in a GSC, a retailer pays an incentive for collecting used products from customers and delivers them to a manufacturer with a product order quantity under uncertainty in product demand. A manufacturer remanufactures products using recyclable parts with acceptable quality levels and covers a part of the retailer's incentive from the recycled parts by sharing the reward from the external institution.

Also, two scenarios for the demand information of a single product are assumed as (i) the demand distribution of a single product is known and (ii) the demand distribution is unknown and only both mean and variance of the demand are known. This paper applies DFA into optimization in a GSC with a single product and a single period.

As the optimal operations in a GSC, this paper focuses on a product order quantity, collection incentive of used products and lower limit of quality level for recyclable parts extracted from used products. This paper proposes two types of optimal operations in a GSC. One is the optimal operations under DGSC to maximize both the each member's expected profit and the lower limit of the each member's expected profit in a GSC. The other is that under IGSC to maximize both the whole system's expected profit and the lower limit of the whole system's expected profit in a GSC.

The analysis numerically investigates how three factors: (i) the contract combined a collection incentive contract with a reward-penalty contract to promote the recycling activity, (ii) the demand information of a single product and (iii) quality of the recyclable parts extracted from used products-affect not only the optimal operations for a product order quantity, the collection incentive, a lower limit of quality level, but also the expected profits under DGSC and IGSC. The optimal operations and the expected profits under DGSC are compared with those under IGSC. As supply chain coordination, the effect of profit sharing on the expected profits of members for the optimal operations under IGSC is investigated by adopting Nash Bargaining solution.

The contribution of this paper is to provide the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to academic researchers and real-world policymakers regarding operations in a GSC:

- Combination of the collection contract of used products between a retailer and a manufacturer and the reward-penalty contract between a manufacturer and an external institution enables to not only promote both activities of the collection and the recycling of used products, but also guarantee both the expected profits and the lower limit of the expected profits of a retailer, a manufacturer and the whole system in a GSC.
- The reward-penalty contract can promote the collection and the recycling of used products when it is incorporated into IGSC.
- When the demand distribution of a single products is unknown, but the mean and the variance are known, the optimal product order quantity in a GSC is derived by the theoretical analysis adopting DFA.
- Incorporating supply chain coordination into the optimal operation under IGSC with the contract combining the collection contract and the reward-penalty contract enables to encourage to guarantee to improve both the expected profits and the lower limit of the expected profits of all members and the whole system in a GSC as well as to promote the aggressive eco-activity among all members in a GSC.

The rest of this paper is organized as follows: in Section 2, notation used in our model is defined. In Section 3, model descriptions including the operational flows of a GSC and the model assumptions are provided. Section 4 formulates both the expected profits in GSC and the lower limit of the expected profits as to the demand information of a single products. Section 5 proposes the decision procedures for optimal operations under DGSC. Section 6 presents those for the optimal operations under DGSC. Section 7 discusses incorporation of profit sharing approach into IGSC as supply chain coordination. In Section 8, numerical examples are provided, and the numerical analysis conducts to illustrate the results of the optimal operation under DGSC and IGSC. In Section 9, conclusions, managerial insights obtained from this paper and future researches for this paper are summarized.

2. NOTATION

• **General Notation**

Q : product order quantity
t : collection incentive (purchasing cost) per used product, referred to collection incentive
u : lower limit of quality level to remanufacture recyclable parts after disassembly of used products, referred to lower limit of quality level ($0 \le u \le 1$)
$A(t)$: collection quantity of product for collection incentive t
$R(t)$: compensation per used product paid to a retailer from a manufacturer for the amount of used products which are remanufactured
c_t : delivery cost per unit of used products collected from a market, to a manufacturer
ℓ : quality level of recyclable parts $(0 \le \ell \le 1)$
$g(\ell)$: probability density function of quality level ℓ
$c_r(\ell)$: remanufacturing cost per unit of recyclable parts with quality level ℓ
c_d : disposal cost per un-reused part
c_n : procurement cost per new part
c_m : production cost per product
m_a : margin obtained from wholesale per product
w : wholesale price of product, referred to unit wholesale price
p : sales price per product, referred to unit sales price
t_U : upper limit of collection incentive t
s : shortage penalty cost per product of which demand is unsatisfied
h_r : inventory holding cost per unsold products
s_a : salvage cost per unsold products
T : the target of quality level for recycling of used products provided by an external institution $(0 < T < 1)$
τ_r : the reward of the recycling effort which a manufacturer receives from an external institution when $u < T$
τ_p : the penalty of lack of recycling effort which a manufacturer pays to an external institution when $u > T$
x : demand of product in a market
$f(x)$: probability density function of demand x
μ : mean of demand x
σ^2 : variance of demand x
i : Index of scenario for distribution information of demand
$i = 1$: the situation where demand x follows a probabilistic distributions and the probability density function with mean μ and variance σ^2 of x are known
$i = 2$: the situation where a probabilistic distribution of demand x is unknown, but mean and variance of x are known
$E^i[\cdot]$: the expected value in scenario $i(=1, 2)$
$E_L^2[\cdot]$: the lower limit of the expected value in scenario 2

• **Notation regarding DGSC**

$Q_D^i (i = 1, 2)$: the optimal product order quantity under DGSC in scenario $i(=1, 2)$
t_D : the optimal collection incentive under DGSC
$u_D(t)$: the provisional lower limit of quality level under DGSC determined for collection incentive t
u_D : the optimal lower limit of quality level under DGSC

• **Notation regarding IGSC**

$Q_C^i (i = 1, 2)$: the optimal product order quantity under IGSC in scenario $i(=1, 2)$
t_C : the optimal collection incentive under IGSC
u_C : the optimal lower limit of quality level under IGSC

3. MODEL DESCRIPTIONS

3.1 Operational Flows of a GSC

This paper addresses a GSC with a retailer, a manufacturer and an external institution. Figure 1 depicts the following operational flows of the GSC. Figure 1 consists of the flows from collection of used products through remanufacturing of parts from the used products to reuse in production of a single product and sales of a single products by reusing the remanufactured parts.

(1) A retailer collects used products from customers by paying the collection incentive t and delivers the collection quantity $A(t)$ of the used products at the unit cost c_t to the manufacturer. Also, the retailer places an order for the quantity Q of a single product for a single period with the manufacturer.
(2) A manufacturer disassembles and inspects the used products at the unit cost c_a, and then all the recyclable parts into quality level ℓ $(0 \le \ell \le 1)$ are classified. The manufacturer remanufactures all the recyclable parts with higher quality levels than the lower limit of quality level u. All the un-recyclable parts with lower quality levels than u are disposed at the unit cost c_d.

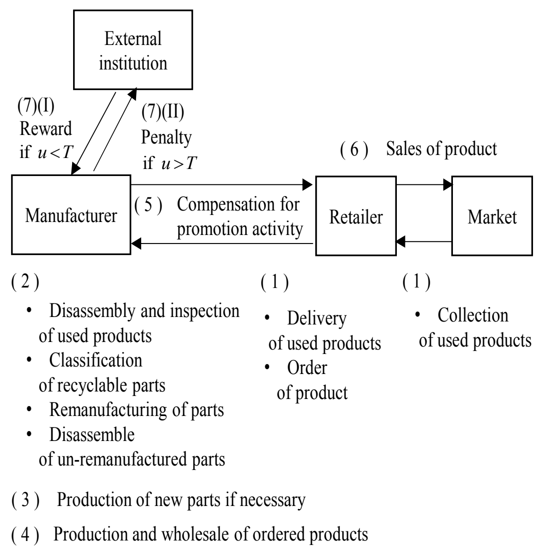

Figure 1. Operational flows of a GSC.

(3) The manufacturer produces the required quantity of new parts at the unit cost c_n if the quantity of the recycled parts is unsatisfied with the required quantity of parts to produce the product order quantity Q from the retailer.
(4) The manufacturer produces order quantity Q of a single products at the unit cost c_m and sells them to retailer at the unit wholesale price w.
(5) The manufacturer gives a compensation to the retailer to covers a part of the retailer's collection incentive to collect the used products cooperatively and aggressively according to the quantity of the recycled parts.
(6) The retailer sells the product in a market at the unit sales price p during a single period. The retailer incurs the unit shortage penalty cost s of the unsatisfied product demand, while the retailer incurs the unit inventory holding cost h_r and the unit salvage cost s_a of the unsold products.
(7) Define T as the target of quality level for recycling of used products $(0<T<1)$ provided by an external institution. As the criterion of the target T for recycling of recyclable parts, (I) when $u<T$, a manufacturer can receive the reward τ_r from an external institution. (II) when $u>T$, a manufacturer pays the penalty τ_p to an external institution. (III) when $u=T$, a manufacturer incurs neither τ_r nor τ_p.

3.2 Model Assumptions

(1) In scenario 1 (i = 1) of the demand information for a single product, the demand x follows a probabilistic distribution and the probability density function (PDF) of x, $f(x)$, is known. In scenario 2 (i = 2), PDF of x is unknown and only both mean μ and variance σ^2 of x are known. Here, $\mu>0$, $\sigma>0$ and $\sigma^2>0$.
(2) It is assumed that a single product such as consumer electronics (mobile phone, personal computer) is produced and is sold in a market. Some of the used products are collected from customers by the retailer's cooperation. A single recyclable part is extracted from the unit of used products. A manufacturer remanufactures products using a single type of recyclable parts with acceptable quality levels.
(3) Regarding collecting the used products, a retailer pays the collection incentive t to collect the used products from customers/a market. Here, the collection quantity of the used products $A(t)$ varies according to the collection incentive t. In general, the higher the collection incentive t is, the more a retailer can collect the used products from customers, where the collection incentive t has the upper limit t_U ($0\le t\le t_U<p$). The manufacturer pays a compensation to the retailer to covers a part of the retailer's collection incentive to collect the used products cooperatively and aggressively according to the quantity of the recycled parts. Concretely, the manufacturer pays the compensation $R(t)$ to the retailer who paid the collection incentive t according to the quantity of the recycled parts from the used products.
Here, the collection quantity of used products $A(t)$ is not enough to satisfy the quantity of parts required to produce the order quantity of a single products even if retailer pays the upper limit t_U of t.
(4) The unit wholesale price w is calculated from the unit procurement cost c_n of new parts, the unit production cost c_m of product and the unit margin m_a obtained from the manufacturer's wholesale per product.
(5) The variability of quality level ℓ of the recyclable parts is modeled as a probabilistic distribution with the probability density function (PDF) $g(\ell)$.
(6) The unit remanufacturing cost $c_r(\ell)$ to a recycled part from a recyclable part with quality level ℓ varies as to quality level ℓ $(0\le\ell\le1)$. The lower quality level ℓ is, the higher the unit remanufactured cost $c_r(\ell)$ is. Here, $\ell=0$ indicates the worst quality level of the recyclable parts, meanwhile $\ell=1$ indicates the best quality level of the recyclable products. Thus, it is assumed that $c_r(\ell)$ is a monotone decreasing function in terms of quality level ℓ. Note that each quality of the recycled parts produced from recyclable parts is as good as that of new parts produced from new materials.

4. EXPECTED PROFITS IN GSC

The manufacturer's profit consists of the product wholesale, the disassembly and the inspection costs of used products, the remanufacturing cost of recyclable parts, the compensation cost to a retailer, the disposal cost of un-recycled parts, the procurement cost of new parts, the production cost of product and either the reward from an external institution $(u<T)$ or the penalty to an external institution $(u>T)$.

The manufacturer's expected profit for Q, t and u is formulated as

$$
\begin{aligned}
E\left[\pi_M(Q,t,u)\right] &= wQ - c_a A(t) - A(t)\int_u^1 c_r(\ell)g(\ell)d\ell \\
&-R(t)A(t)\int_u^1 g(\ell)d\ell \\
&-c_d A(t)\int_0^u g(\ell)d\ell - c_n\left\{Q - A(t)\int_u^1 g(\ell)d\ell\right\} - c_m Q \\
&+\tau_r A(t)\left[\int_u^1 g(\ell)d\ell - \int_T^1 g(\ell)d\ell\right]^+ \quad (u<T) \\
&-\tau_p A(t)\left[\int_T^1 g(\ell)d\ell - \int_u^1 g(\ell)d\ell\right]^+ \quad (u>T). \qquad (1)
\end{aligned}
$$

In Eq. (1), the first term is the product wholesales, the second term is the disassembly and the inspection cost of the used products, the third term is the remanufacturing cost of recyclable parts after disassembly of the used products, the fourth term is the compensation cost to a retailer, the fifth term is the disposal cost of the un-recycled parts, the sixth term is the procurement cost of new parts, the seventh term is the reward of the recycling effort which a manufacturer receives from an external institution when $u < T$, the eighth term is the penalty of lack of recycling effort which a manufacturer pays to an external institution when $u < T$ and the final term is the production cost of products.

Here, Eq. (1) shows that the manufacturer's expected profit is unaffected by the demand information of a single product. Also, from Eq. (1), notice that the magnitude relationship between the lower limit of quality level u and the target T of quality level for recycling of used products indicates the magnitude relation between the remanufacturing quantity of parts products and the target quantity of the remanufacturing.

4.1 Case of Scenario 1 of Demand Information of a Single Product

The retailer's expected profit in scenario 1 (i = 1) of the demand information of a single product is discussed.

From the operational flows of a GSC in 2, the retailer's profit consists of the collection cost and the delivery cost of used products, the procurement cost of product, the compensation revenue from a manufacturer, the product sales, the inventory holding cost and the salvage cost of the unsold products and the shortage penalty cost of the unsatisfied demand in a market.

The retailer's expected profit in scenario 1 (i = 1) for the product order quantity Q, the collection incentive t and the lower limit of quality level u is formulated as

$$\begin{aligned}E^1\left[\pi_R(Q,t,u)\right] &= -tA(t) - c_t A(t) - wQ + R(t)A(t)\int_u^1 g(\ell)d\ell \\ &+p\left\{\int_0^Q x f(x)dx + Q\int_Q^\infty f(x)dx\right\} \\ &-\left(h_r + s_a\right)\int_0^Q (Q-x)f(x)dx - s\int_Q^\infty (x-Q)f(x)dx. \qquad (2)\end{aligned}$$

In Eq. (2), the first term is the collection cost of the used products from customers, the second term is the delivery cost of the used products to a manufacturer, the third term is the procurement cost of products, the fourth term is the compensation revenue from a manufacturer to a retailer, the fifth term is the expected product sales, the sixth term is the expectation of the sum of the inventory holding cost and the salvage cost of the unsold products, and the final term is the expected shortage penalty cost for unsatisfied product demand in a market.

The whole system's expected profit in scenario 1 (i = 1) of the demand information of a single product for Q, t and u is obtained as the following sum of the both members' expected profits in Eq. (1) and Eq. (5).

$$\begin{aligned}E^1\left[\pi_S(Q,t,u)\right] &= E^1\left[\pi_R(Q,t,u)\right] + E\left[\pi_M(Q,t,u)\right] \\ &= -tA(t) - c_t A(t) \\ &-c_a A(t) - A(t)\int_u^1 c_r(\ell)g(\ell)d\ell \\ &-c_d A(t)\int_0^u g(\ell)d\ell \\ &-c_n\left\{Q - \int_u^1 A(t)g(\ell)d\ell\right\} - c_m Q \\ &+\tau_r A(t)\left[\int_u^1 g(l)dl - \int_T^1 g(l)dl\right]^+ \quad (u<T) \\ &-\tau_p A(t)\left[\int_T^1 g(l)dl - \int_u^1 g(l)dl\right]^+ \quad (u>T) \\ &+p\left\{\int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx\right\} \\ &-\left(h_r + s_a\right)\int_0^Q (Q-x)f(x)dx \\ &-s\int_Q^\infty (x-Q)f(x)dx. \qquad (3)\end{aligned}$$

In Eq. (3), the first term is the collection cost of the used products, the second term is the delivery cost of the used products, the third term is the disassembly and the inspection cost of the used products, the fourth term is the remanufacturing cost of the recyclable parts after disassembly of the used products, the fifth term is the disposal cost of un-recycled parts, the sixth term is the procurement cost of new parts, the seventh term is the production cost of products, eighth term is the reward of the recycling effort which a manufacturer receives from an external institution when $u < T$, the ninth term is the penalty of lack of recycling effort which a manufacturer pays to an external institution when $u < T$, the tenth term is the expected product sales, the eleventh term is the expectation of the sum of the inventory holding cost and the salvage cost of the unsold products, the final term is the expected shortage penalty cost for unsatisfied demand in a market.

4.2 Case of Scenario 2 of Demand Information of a Single Product

The retailer's expected profit in scenario 2 (i = 2) of the demand information of a single product is discussed.

Next, the retailer's expected profit in scenario 2 (i = 2) of the product demand is discussed. Here, the retailer's expected profit $E^1\left[\pi_R(Q,t,u)\right]$ in scenario 1 (i = 1) of the product demand for Q, t and u in Eq. (3) can be rewritten as follows:

$$\begin{aligned}E^1\left[\pi_R(Q,t,u)\right] &= -tA(t) - c_t A(t) \\ &+R(t)\int_u^1 g(\ell)A(t)d\ell + \left(p-w\right)Q \\ &-\left\{p + \left(h_r + s_a\right)\right\}\int_0^Q (Q-x)f(x)dx \\ &-s\int_Q^\infty (x-Q)f(x)dx. \qquad (4)\end{aligned}$$

The elicitation process of Eq. (4) is shown in Appendix A.

In scenario 2, mean μ and variance σ^2 of the demand x are known. Using the distribution-free approach (DFA) (Gallego and Moon, 1993; Moon and Gallego, 1994; Moon and Choi, 1995; Alfares and Elmorra, 2005), the upper limits of both the expected excessive inventory quantity and the expected shortage quantity between the demand x and the product order quantity Q are derived as

$$E[x-Q]^+ \leq \left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2 \tag{5}$$

$$E[Q-x]^+ \leq \left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2. \tag{6}$$

The elicitation processes of Eq. (4) and Eq. (5) are shown in Gallego and Moon (1993) and Alfares and Elmorra (2005).

The lower limit of the retailer's expected profit in $i = 2$ can be obtained by applying DFA into the retailer's expected profit in $i = 1$. Concretely, the lower limit of the retailer's expected profit in $i = 2$ for Q, t and u can be derived by substituting the upper limits of both the expected excessive inventory quantity and the expected shortage quantity between the product order quantity Q ($i = 1, 2$) and demand x derived in Eq. (5) and Eq. (6) into the expected excessive inventory quantity and the expected shortage quantity in Eq. (1). In this case, the upper limits of both the expectation of the sum of the inventory holding cost and the salvage cost of unsold products and the expected shortage penalty cost are derived respectively as follows:

- The upper limit of the expectation of the sum of the inventory holding cost and the salvage cost of unsold products:

$$-\{p+(h_r+s_a)\}\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2 .$$

- The upper limit of expected shortage penalty cost

$$-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2 .$$

Therefore, the lower limit of the retailer's expected profit in $i = 2$ for Q, t and u can be obtained as

$$\begin{aligned}E^2\left[\pi_R(Q,t,u)\right] &= -tA(t)-c_tA(t)\\&+R(t)\int_u^1 g(\ell)A(t)d\ell+(p-w)Q\\&-\{p+(h_r+s_a)\}\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2\\&-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2 .\end{aligned} \tag{7}$$

The lower limit of the whole system's expected profit in scenario 2 ($i = 2$) for Q, t and u is obtained as the following sum of the manufacturer's expected profit in Eq. (1) and the lower limit of the retailer's expected profit in scenario 2 ($i = 2$) in Eq. (7):

$$\begin{aligned}E_L^2\left[\pi_S(Q,t,u)\right] &= E_L^2\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]\\&= -tA(t)-c_tA(t)-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell\\&-c_dA(t)\int_0^u g(\ell)d\ell-c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ\\&+\tau_rA(t)\left[\int_u^1 g(l)dl-\int_T^1 g(l)dl\right]^+ \quad (u<T)\\&-\tau_pA(t)\left[\int_T^1 g(l)dl-\int_u^1 g(l)dl\right]^+ \quad (u>T)\\&-p\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2\\&-(h_r+s_a)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2\\&-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2.\end{aligned} \tag{8}$$

5. OPTIMAL OPERATIONS FOR DECENTRALIZED GREEN SUPPLY CHAIN

For the optimal operations under a decentralized GSC (DGSC), the optimal decision approach of the Stackelberg game is adopted in some previous papers: Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Sun, 2012; Watanabe *et al*., 2013. The reason why the Stackelberg game is adopted under DGSC of this paper is shown as follows: the optimal decision in the Stackelberg game is made under a situation consisting of one leader of the decision-making and one (multiple) follower(s). First, a leader of the decision-making makes the optimal decision so as to the leader's profit. Next, one (multiple) follower(s) make(s) the optimal decision(s) so as to maximize the follower(s)' profit(s) under the optimal decision made by the leader of the decision-making. Suppose that decision variable(s) of supply chain members affect(s) not only the optimal decision so as to maximize the profit of a supply chain member, but also that (those) of the other supply chain member(s), interacting between supply chain members' profit. Under the situation, the optimal decision approach in the Stackelberg game is adopted effectively among supply chain members.

In a GSC addressed in this paper, the retailer's optimal decision for collection incentive affect not only the maximization of the retailer's expected profit, but also the manufacturer's optimal decision for the lower limit of quality level so as to maximize the manufacturer's expected profit, and vice versa. Accordingly, this paper determines the optimal operation under DGSC by adopting the Stackelberg game. In this paper, a retailer is the leader of the decision-making under DGSC and a manufacturer is the follower of the decision-making. This is because the following situations is considered in this paper: a retailer not only pays the collection incentive t to collect used products from a market so as to cooperate the encouragement of the manufacturer's recycling activity of used products, but also faces stochastic demand of products in a market, sells the product in the market and earns the most profit of the whole system in a GSC.

The retailer determines the optimal product order quantity Q_D^i (i = 1, 2) as to scenario i(= 1, 2) of the demand information of a single products and the optimal collection incentive t_D so as to maximize the own expected profit. The manufacturer determines the optimal lower limit of quality level u_D so as to maximize the own expected profit under the optimal product order quantity Q_D^i and the optimal collection incentive t_D determined by the retailer who is the leader of the decision-making under DGSC. Next, the manufacturer produces the same quality Q_D^i of a single product and sells it to the retailer at the unit wholesale price w.

The procedures to determine the optimal operations (Q_D^i, t_D, u_D) under DGSC is shown as following subsections.

5.1 Optimal Decision for Product Order Quantity

5.1.1 Case of Scenario 1 of Demand Information

The optimal product order quantity Q_D^1 under DGSC in scenario 1 is discussed. The optimal product order quantity under DGSC Q_D^1 in scenario 1 is determined so as to maximize the retailer's expected profit in Eq. (2) under the collection incentive t and the lower limit of quality level u. A manufacturer follows the optimal product order quantity under DGSC Q_D^1 in scenario 1 determined by the retailer who is the leader of the decision-making under DGSC.

Proposition 1. The retailer's expected profit in scenario 1 in Eq. (2) is the concave function in terms of the product order quantity Q under the collection incentive t and the lower limit of quality level u.

Proof. The first- and second-order differential equations between the product order quantity Q and the expected profit $E^1\left[\pi_R\left(Q|t,u\right)\right]$ of the retailer in scenario 1 in Eq. (2) under the collection incentive t and the lower limit of quality level u are derived as follows:

$$dE^1\left[\pi_R\left(Q|t,u\right)\right]\Big/dQ = -w+p+s-(p+\left(h_r+s_a\right)+s)\int_0^Q f(x)dx, \tag{9}$$

$$d^2E^1\left[\pi_R\left(Q|t,u\right)\right]\Big/dQ^2 = -\left\{p+\left(h_r+s_a\right)+s\right\}f(Q). \tag{10}$$

The elicitation process of Eq. (9) is shown in Appendix B. It is derived that Eq. (10) is negative since it is natural to satisfy the condition $p>0$, $h_r>0$, $s>0$, either $s_a \ge 0$ or $\left(h_r+s_a\right)>0$ when $s_a<0$. Therefore, the theoretical analysis results in Proposition 1.

Proposition 2. The optimal product order quantity Q_D^1 in scenario 1 can be obtained as the following unique solution to maximize Eq. (2):

$$Q_D^1 = F^{-1}\left(\frac{-w+p+s}{p+\left(h_r+s_a\right)+s}\right). \tag{11}$$

From Eq. (11), the optimal decision for the product order quantity Q under DGSC in scenario 1 is unaffected by t and u.

Proof. Substituting 0 into Eq. (9), the solution of $dE^1\left[\pi_R\left(Q|t,u\right)\right]\Big/dQ=0$ results in Proposition 2.

5.1.2 Case of Scenario 2 of Demand Information

The optimal product order quantity Q_D^2 under DGSC in scenario 2 is discussed. The optimal product order quantity under DGSC Q_D^2 in scenario 2 is determined so as to maximize the lower limit of the retailer's expected profit in Eq. (7) under the collection incentive t and the lower limit of quality level u. A manufacturer follows the optimal product order quantity under DGSC Q_D^2 in scenario 2 determined by the retailer who is the leader of the decision-making under DGSC.

Proposition 3. The lower limit of the retailer's expected profit in scenario 2 in Eq. (7) is the concave function in terms of the product order quantity Q under the collection incentive t and the lower limit od quality level u. Also, the optimal decision for Q under DGSC in scenario 2 is unaffected by t and u.

Proof. The first- and second-order differential equations between the product order quantity Q and the lower limit of the retailer's expected profit $E_L^2\left[\pi_R\left(Q|t,u\right)\right]$ in scenario 2 in Eq. (7) under t and u are derived as follows:

$$\begin{aligned} &dE_L^2\left[\pi_R\left(Q|t,u\right)\right]\Big/dQ \\ &=\frac{1}{2}(p+s-\left(h_r+s_a\right)-2w) \\ &-\frac{1}{2}\left(p+s+\left(h_r+s_a\right)\right)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}, \end{aligned} \tag{12}$$

$$\frac{d^2E_L^2\left[\pi_R(Q,t,u)\right]}{dQ^2} = -\frac{\sigma^2\left(p+s+\left(h_r+s_a\right)\right)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}}. \tag{13}$$

The elicitation processes of Eq. (12) and Eq. (13) are shown in Appendix C. It is derived that Eq. (13) is negative since it is natural to satisfy the condition $p>0$, h_r >0, $s>0$, either $s_a \ge 0$ or $\left(h_r+s_a\right)>0$ when $s_a<0$. Therefore, the theoretical analysis results in Proposition 3.

Proposition 4. The optimal product order quantity Q_D^2 in scenario 2 can be obtained as the following unique solution to maximize Eq. (14) and Eq. (15):

$$Q_D^2 = \frac{\mu + \sigma y_D}{\sqrt{1 - y_D^2}}, \tag{14}$$

$$y_D = \frac{p + s - h_r - 2w}{p + (h_r + s_a) + s}. \tag{15}$$

From Eq. (14) and Eq. (15), the optimal decision for the product order quantity Q under DGSC in scenario 2 is unaffected by t and u.

Proof. The solution of $dE_L^2\left[\pi_R(Q|t, u)\right]/dQ = 0$ substituting 0 into Eq. (12) results in Proposition 4.

5.2 Optimal Lower Limit of Quality Level and Optimal Collection Incentive

In a GSC addressed in this paper, the retailer's optimal decision for collection incentive affects not only the maximization of the retailer's expected profit, but also the manufacturer's optimal decision for the lower limit of quality level so as to maximize the manufacturer's expected profit, and vice versa. Under the optimal product order quantity in each scenario i (= 1, 2) of the demand information, Q_D^i (i =1, 2), in Eq. (8) and Eq. (9), the optimal collection incentive t_D and the optimal lower limit of quality level u_D under DGSC are determined independently from standpoints where the retailer is the leader of the decision-making under DGSC and the manufacturer is the follower of the decision-making.

The decision procedures for the optimal collection incentive t_D and the optimal lower limit of quality level u_D under DGSC are shown as follows:

According to the magnitude relationship between u and T, the following first-order differential equation between the lower limit of quality level u and the expected profit of the manufacturer $E\left[\pi_M(u)\middle| Q_D^i, t\right] (i = 1, 2)$ in Eq. (1) under Q_D^i and t are obtained as

$$\frac{dE\left[\pi_M(u)\middle| Q_D^i, t\right]}{du}(i = 1, 2)$$
$$= \begin{cases} A(t)g(u)\{c_r(u) + R(t) - c_d - c_n - \tau_r\} & (u < T) \quad (16) \\ A(t)g(u)\{c_r(u) + R(t) - c_d - c_n - \tau_p\} & (u > T) \quad (17) \\ A(t)g(u)\{c_r(u) + R(t) - c_d - c_n\}. & (u = T) \quad (18) \end{cases}$$

The elicitation processes of Eq. (16)-Eq. (18) is shown in Appendix D.

Here, Eq. (16) when $u < T$ is zero if and only if to satisfy the following condition:

$$c_r(u) + R(t) - c_d - c_n - \tau_r = 0. \tag{19}$$

Eq. (17) when $u < T$ is zero if and only if to satisfy the following condition:

$$c_r(u) + R(t) - c_d - c_n - \tau_p = 0. \tag{20}$$

Eq. (18) when $u = T$ is zero if and only if to satisfy the following condition:

$$c_r(u) + R(t) - c_d - c_n = 0. \tag{21}$$

Also, from Eq. (16)-Eq. (18), the optimal decision for the lower level of quality level is impacted by the collection incentive t, but is unaffected by the product order quantity.

According to the magnitude relation between u and T, the lower limit of quality level u to satisfy either Eq. (19)-Eq. (21) under t is defined as the provisional lower limit of quality level $u_D(t)$ determined under t.

The provisional lower limit of quality level for recyclable parts $u_D(t)$ under t can be determined so as to satisfy either Eq. (19)-Eq. (21) under t according to the magnitude relation between u and T.

Here, from (6) in Section 3.2 model assumptions, it can be seen that there is the unique lower limit of quality level u to satisfy either Eq. (19)-Eq. (21) under t as to the magnitude relation between u and T.

From Eq. (1), notice that the magnitude relationship between the lower limit of quality level u and the target T of quality level for recycling of used products indicates the magnitude relation between the remanufacturing quantity of parts and the target quantity of the remanufacturing. Therefore, $u_D(t)$, which is determined from either Eq. (19)-Eq. (21) as to the magnitude relation between the remanufacturing quantity of parts products and the target quantity of the remanufacturing, maximizes the manufacturer's expected profit $E\left[\pi_M(u)\middle| Q_D^i, t\right] (i = 1, 2)$ under Q_D^i and t.

The decision procedures of the optimal collection incentive and the optimal lower limit of quality level under DGSC are shown as follows:

[Step 1] Substitute the optimal product order quantity Q_D^1 in Eq. (11) in scenario 1 into the retailer's expected profit in Eq. (2). Also, substitute the optimal pro-duct order quantity Q_D^2 in Eq. (14) and Eq. (15) into the lower limit of the retailer's expected profit in Eq. (7).

[Step 2] Substitute t and $u_D(t)$ in either Eq. (19)-Eq. (21) according to the magnitude relation between the remanufacturing quantity of parts and the target quantity of the remanufacturing into Eq. (2) and Eq. (7) under $Q_D^i (i = 1, 2)$ as to scenario i of the demand information in [Step 1].

[Step 3] Vary t at step size 0.01 within the range where $0 \le t \le t_U$.

[Step 4] Find the optimal combination (t_D, u_D) as the combination $(t, u_D(t))$ so as to maximize the retailer's expected profit $E^1\left[\pi_R(t, u_D(t))\middle| Q_D^1\right]$ in Eq. (2) under Q_D^1 in scenario 1, and the lower limit of the retailer's expected profit $E_L^2\left[\pi_R(t, u_D(t))\middle| Q_D^2\right]$ in Eq. (7) under Q_D^2 in scenario 2.

[Step 5] Substitute the optimal collection inventive t_D into the manufacturer's expected profit in Eq. (1).

[Step 6] Determine finally the optimal lower limit of quality level u_D under DGSC as $u_D(t_D)$ which maximizes the manufacturer's expected profit in Eq. (1) when $u_D(t_D)$ determined in either Eq. (19)-Eq. (21).

Thus, the optimal collection incentive t_D and the optimal lower limit of quality level u_D are determined mutually between a retailer who is the leader of the decision-making and a manufacturer who is the follower of the decision-making of the retailer under DGSC, maximizing the individual expected profit.

5.3 The Expected Profits under DGSC

The expected profits of a retailer and a manufacturer and the whole system under DGSC can be obtained by using the optimal decisions (Q_D^i, t_D, u_D) under DGSC as to scenario i (= 1, 2) of the demand and the magnitude relationship between u and T.

6. OPTIMAL OPERATIONS FOR INTEGRATED GREEN SUPPLY CHAIN

Under an integrated GSC (IGSC), the optimal operations for product order quantity $Q_C^i\,(i$ = 1, 2) as to scenario i(= 1, 2) of the demand information of a single products, the collection incentive t_C and the lower limit of quality level u_C are made so as to maximize the whole system's expected profit in scenario 1 and maximize the lower limit of the whole system's expected profit in scenario 2. A retailer and a manufacturer follows the optimal operations under IGSC.

The procedures for the optimal decision-making $(Q_C^i, t_C, u_C)(i=1,2)$ under IGSC as to scenario i(= 1, 2) are shown as follows.

6.1 Optimal Decision for Product Order Quantity

6.1.1 Case of Scenario 1 of Demand Information

The optimal product order quantity Q_C^1 under IGSC in scenario 1 is discussed. The optimal product order quantity under IGSC Q_C^1 in scenario 1 is determined so as to maximize the expected profit of the whole system in Eq. (3) under the collection incentive t and the lower limit of quality level u.

Proposition 5. The whole system's expected profit in scenario 1 in Eq. (3) is the concave function in terms of product order quantity Q under the collection incentive t and the lower limit of quality level u.

Proof. The first- and second-order differential equations between the product order quantity Q and the expected profit $E^1\left[\pi_S\left(Q\middle|t, u\right)\right]$ of the whole system in scenario 1 in Eq. (3) under the collection incentive t and the lower limit of quality level u are derived as follows:

$$dE^1\left[\pi_S\left(Q\middle|\, t, u\right)\right]\Big/dQ$$
$$= -c_n - c_m + p + s - (p + (h_r + s_a) + s)\int_0^Q f(x)dx, \qquad (22)$$

$$d^2E^1\left[\pi_R\left(Q\middle|t,u\right)\right]\Big/dQ^2 = -\left\{p + (h_r + s_a) + s\right\}f(Q). \qquad (23)$$

The elicitation process of Eq. (22) is shown in Appendix E. It is derived that Eq. (23) is negative since it is natural to satisfy the condition $p>0, h_r>0$ either $s_a \ge 0$ or $(h_r + s_a) > 0$ when $s_a < 0$. Therefore, the theoretical analysis results in Proposition 5.

Proposition 6: The optimal product order quantity Q_C^1 in scenario 1 can be obtained as the following unique solution to maximize Eq. (3):

$$Q_C^1 = F^{-1}\left(\frac{-c_n - c_m + p + s}{p + (h_r + s_a) + s}\right). \qquad (24)$$

From Eq. (24), the optimal decision for the product order quantity Q under IGSC in scenario 1 is unaffected by t and u.

Proof. Substituting 0 into Eq. (23), the solution of $dE^1\left[\pi_S\left(Q\middle|t, u\right)\right]\Big/dQ = 0$ results in Proposition 6.

6.1.2 Case of Scenario 2 of Demand Information

The optimal product order quantity Q_C^2 under IGSC in scenario 2 is discussed. The optimal product order quantity under IGSC Q_C^2 in scenario 2 is determined so as to maximize the lower limit of the whole system's expected profit in Eq. (8) under the collection incentive t and the lower limit of quality level u.

Proposition 7. The lower limit of the whole system's expected profit in scenario 2 in Eq. (8) is the concave function in terms of the product order quantity Q under the collection incentive t and the lower limit of quality level u. Also, the optimal decision for Q under IGSC in scenario 2 is unaffected by t and u.

Proof. The first- and second-order differential equations between the product order quantity Q and the lower limit of the whole system's expected profit $E_L^2\left[\pi_S\left(Q\middle| t, u\right)\right]$ in scenario 2 in Eq. (8) under t and u are derived as follows:

$$dE_L^2\left[\pi_S\left(Q\middle|t,u\right)\right]\Big/dQ$$
$$=\frac{1}{2}\left(p+s-\left(h_r+s_a\right)-2\left(c_m+c_n\right)\right)$$
$$-\frac{1}{2}\left(p+s+\left(h_r+s_a\right)\right)\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}, \tag{25}$$

$$\frac{d^2E_L^2\left[\pi_S(Q,t,u)\right]}{dQ^2}=-\frac{\sigma^2\left(p+s+\left(h_r+s_a\right)\right)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}}. \tag{26}$$

Thus, it is verified that the theoretical results of Eq. (26) is same as that in Eq. (13). The elicitation processes of Eq. (25) and Eq. (26) are shown in Appendix F.

It is derived that Eq. (26) is negative since it is natural to satisfy the condition $p>0, h_r>0, s>0,$ either $s_a \geq 0$ or $\left(h_r+s_a\right)>0$ when $s_a<0$. Therefore, the theoretical analysis results in Proposition 7.

Proposition 8. The optimal product order quantity Q_D^2 in scenario 2 can be obtained as the following unique solution to maximize Eq. (14) and Eq. (15):

$$Q_C^2=\frac{\mu+\sigma y_C}{\sqrt{1-y_C^2}}, \tag{27}$$

$$y_C=\frac{p+s-\left(h_r+s_a\right)-2(c_n+c_m)}{p+\left(h_r+s_a\right)+s}. \tag{28}$$

From Eq. (27) and Eq. (28), the optimal decision for the product order quantity Q under IGSC in scenario 2 is unaffected by t and u.

Proof. The solution of $dE_L^2\left[\pi_S\left(Q\middle|t,u\right)\right]\Big/dQ=0$ substituting 0 into Eq. (25) results in Proposition 4.

6.2 Optimal Lower Limit of Quality Level

Under the optimal product order quantity in each scenario i (= 1, 2) of the demand information, Q_C^i (i = 1, 2), in Eq. (8) and Eq. (9), the optimal lower limit of quality level u_C under IGSC is determined which maximizes the whole system's expected profit in scenario 1 and maximizes the lower limit of the whole system's expected profit in scenario 2.

The decision procedure of the optimal lower limit of quality level u_C under IGSC is shown as follows:

As to the magnitude relationship between u and T, the following first-order differential equation between u and the expected profit of the whole system $E^1\left[\pi_S(u)\middle|Q_C^1,t\right]$ in Eq. (3) and the lower limit of it $E_L^2\left[\pi_S(u)\middle|Q_C^2,t\right]$ in Eq. (8) under $Q_C^i\ (i=1,2)$ and t are obtained as

$$\frac{dE\left[\pi_S(u)\middle|Q_C^1,t\right]}{du}=\frac{dE_L^2\left[\pi_S(u)\middle|Q_C^2,t\right]}{du}$$

$$=\begin{cases}A(t)g(u)\left\{c_r(u)-c_d-c_n-\tau_r\right\} & \left(u<T\right) \quad (29)\\ A(t)g(u)\left\{c_r(u)-c_d-c_n-\tau_p\right\} & \left(u>T\right) \quad (30)\\ A(t)g(u)\left\{c_r(u)-c_d-c_n\right\} & \left(u=T\right) \quad (31)\end{cases}$$

The elicitation processes of Eq. (29)-Eq. (31) is shown in Appendix G.

Here, Eq. (29) when $u<T$ is zero if and only if to satisfy the following condition:

$$c_r(u)-c_d-c_n-\tau_r=0. \tag{32}$$

Eq. (30) when $u>T$ is zero if and only if to satisfy the following condition:

$$c_r(u)-c_d-c_n-\tau_p=0. \tag{33}$$

Eq. (31) when $u=T$ is zero if and only if to satisfy the following condition:

$$c_r(u)-c_d-c_n=0. \tag{34}$$

Here, from (6) in Section 3.2 model assumptions, it can be seen that there is the unique lower limit of quality level u to satisfy either Eq. (32)-Eq. (34) under t according to the magnitude relation between the remanufacturing quantity of parts and the target quantity of the remanufacturing. Also, from Eq. (32)-Eq. (34), the optimal lower limit of quality level under IGSC is unaffected by the product order quantity Q as to the demand information of the product and the collection incentive t.

The optimal lower limit of quality level u_C under IGSC so as to maximize the whole system's expected profit in Eq. (3) and the lower limit of the whole system's expected profit in Eq. (8) is determined as the lower limit of quality level u which satisfies either Eq. (32)-Eq. (34) according to the magnitude relation between the remanufacturing quantity of parts and the target quantity of the remanufacturing.

6.3 Optimal Collection Incentive

The decision procedure of the optimal collection incentive t_C under IGSC is shown as follows:

[Step 1] Substitute the optimal product order quantity Q_C^1 in Eq. (24) in scenario 1 into the whole system's expected profit in Eq. (3). Also, substitute the optimal product order quantity Q_C^2 in Eq. (27) and Eq. (28) into the lower limit of the whole system's expected profit in Eq. (8).

[Step 2] Substitute the optimal lower limit of quality level u_C in either Eq. (29)-Eq. (31) according to the magnitude relation between the remanufacturing quantity of parts and the target quantity of the remanufacturing into Eq. (3) and Eq. (8) under $Q_C^i(i=1,2)$ as to scenario i of the demand information in [Step 1].

[Step 3] Vary t at step size 0.01 within the range where $0 \le t \le t_U$.

[Step 4] Find the optimal unit collection inventive t_C which maximizes the expected profit of the whole system $E^1\left[\pi_s\left(t \middle| Q_C^1, u_C\right)\right]$ in Eq. (3) under Q_C^1 and u_C and the lower limit of the whole system's expected profit $E_L^2\left[\pi_s\left(t \middle| Q_C^2, u_C\right)\right]$ in Eq. (8) under Q_C^2 and u_C.

6.4 The Expected Profits under IGSC

The expected profits of a retailer and a manufacturer and the whole system under IGSC can be obtained by using the optimal decisions (Q_C^i, t_C, u_C) under IGSC as to scenario i (= 1, 2) of the demand information and the magnitude relationship between u and T.

7. INCORPORATION OF PROFIT SHARING APPROACH INTO IGSC AS SUPPLY CHAIN COORDINATION

When the optimal operation in a GSC shifts from DGSC to IGSC, the whole system's expected profit can be improved under IGSC. However, it is not always guaranteed that the expected profits of a retailer and a manufacturer under IGSC can be improved even if the whole system's expected profit increases under IGSC.

Nevertheless, it is desirable to shift the optimal operation under IGSC from that under DGSC from the aspect of the total optimization maximizing the expected profit of the whole system in a GSC. In order to enable that, it is necessary to guarantee that all members in a GSC under IGSC can obtain the more expected profits than under DGSC.

This section discusses supply chain coordination to guarantee the profit improvements for a retailer and a manufacturer under IGSC. Here, the effect of profit sharing approach as supply chain coordination on the expected profit of each member under IGSC in scenario i(= 1, 2) of the demand information are discussed by adopting the Nash bargaining solutions (Nagarajan and Sosic, 2008; Du *et al*., 2011).

In this paper, the unit wholesale price w^i and compensation per used product $R^i(t)$ in i(= 1, 2) are coordinated between both members under IGSC. w and $R(t)$ are set as $R(t) = (1+\alpha)\, t$ and $w = w(m_a) = c_n + c_m + m_a$.

In this paper, the degree α of compensation for the retailer's collection incentive t and the unit margin m_a for wholesale per product are coordinated as α^{iN} and m_a^{iN} by adopting the Nash bargaining solutions. The unit wholesale price w^i and the compensation $R^i(t)$ are calculated by substituting α^{iN} and m_a^{iN} into w^i and $R^i(t)$. α^{iN} and m_a^{iN} are determined by satisfying the following equations:

$$\begin{aligned} &Max\ T\left(\alpha^{iN}, m_a^{iN}\right)(i=1,2) \\ &= \left\{E^i\left[\pi_R\left(\alpha^{iN}, m_a^{iN} \middle| Q_C^i, t_C, u_C\right)\right] - E^i\left[\pi_R\left(\alpha, m_a \middle| Q_D^i, t_D, u_D\right)\right]\right\} \\ &\times \left\{E^i\left[\pi_M\left(\alpha^{iN}, m_a^{iN} \middle| Q_C^i, t_C, u_C\right)\right] - E^i\left[\pi_M\left(\alpha, m_a \middle| Q_D^i, t_D, u_D\right)\right]\right\} \end{aligned} \quad (35)$$

subject to

$$E^i\left[\pi_R\left(\alpha^{iN}, m_a^{iN} \middle| Q_C^i, t_C, u_C\right)\right] - E^i\left[\pi_R\left(\alpha, m_a \middle| Q_D^i, t_D, u_D\right)\right] > 0 \quad (36)$$

$$E^i\left[\pi_M\left(\alpha^{iN}, m_a^{iN} \middle| Q_C^i, t_C, u_C\right)\right] - E^i\left[\pi_M\left(\alpha, m_a \middle| Q_D^i, t_D, u_D\right)\right] > 0\ . \quad (37)$$

Eq. (36) and Eq. (37) are the constraint conditions to guarantee that the expected profit of each member under IGSC with supply chain coordination as to in scenario i(= 1, 2) of the demand information is always higher than that under DGSC.

The details of Eq. (35)-Eq. (37) are provided as follows: Eq. (35) coordinates the unit wholesale price w and the unit compensation $R(t)$ of the used products as α^{iN} and m_a^{iN} under the optimal ordering policy of IGSC as to scenario i (= 1, 2) of the demand information. α^{iN} and m_a^{iN} are determined so as to maximize the multiplication of (the difference of the expected profit of a retailer for α^{iN} and m_a^{iN} coordinated under the optimal decision of IGSC as to scenario i(= 1, 2) of the demand information and that for $R(t)$ and w not coordinated under the optimal decision of DGSC as to scenario i(= 1, 2) of the demand information) and (the difference of the expected profit of a manufacturer for α^{iN} and m_a^{iN} coordinated under the optimal decision of IGSC as to scenario i(= 1, 2) of the demand information and that for $R(t)$ and w not coordinated under the optimal decision of DGSC as to scenario i(= 1, 2) of the demand information.

Eq. (36) is the constrained condition to guarantee the situation where the expected profit of a retailer for α^{iN} and m_a^{iN} coordinated under the optimal decision of IGSC as to scenario i(= 1, 2) of the demand information is always higher than that for $R(t)$ and w not coordinated under the optimal decision of DGSC as to scenario i(= 1, 2) of the demand information.

As with Eq. (36), Eq. (37) is the constrained condition to guarantee the situation where the expected profit of a manufacturer for α^{iN} and m_a^{iN} coordinated under the optimal decision of IGSC as to scenario i(= 1, 2) of the demand information is always higher than that for $R(t)$ and w not coordinated under the optimal decision of DGSC as to scenario i(= 1, 2) of the demand information.

8. NUMERICAL ANALYSIS

The analysis numerically verifies how three factors: (i) the contract combined a collection incentive contract with a reward-penalty contract to promote the recycling activity, (ii) the demand information of a single product and (iii) quality of the recyclable parts extracted from used products-affect not only the optimal operations for a product order quantity, the collection incentive, a lower limit of quality level, but also the expected profits under DGSC and IGSC. Two scenarios of the demand information of the product are taken as scenario $1(i = 1)$: the demand distribution of a single product is known and scenario $2(i = 2)$: both mean and variance of the demand are known. The optimal operations and the expected profits under DGSC are compared with those under IGSC. Moreover, as supply chain coordination, the effect of the profit sharing on the expected profits of members for the optimal operations under IGSC is investigated by adopting Nash Bargaining solution.

The following data sources are used as the numerical examples of system parameters: p = 150, s = 175, $(h_r + s_a)$ = 15, c_a = 1, c_d = 1, c_t = 1, c_n = 35, c_m = 2, m_a = 15.

In scenario $1(i = 1)$ of the demand information, demand x follows the normal distribution with mean μ = 1,000 and variance $\sigma^2 = 300^2$. In scenario 2 (i = 2), mean and variance of x are known as μ= 1,000 and $\sigma^2 = 300^2$.

$A(t)$, w and $c_r(\ell)$ are set as $A(t) = 100 + 50t$ $(0 \le t \le t_U,\ t_U = 0.1p)$, $w = c_n + c_m + m_a$, $c_r(\ell) = 40(1 - 0.9\ell)$, satisfying the conditions of (3), (4) and (6) of model assumptions in 3.2.

$R(t)$ is set as $R(t) = (1+\alpha)t$, where α denotes the degree of compensation for the collection incentive from a manufacturer to a retailer without any supply chain coordination. Here, α = 0.7 is set in aspect of the manufacturer's profit.

The target of quality level for recycling of used products T is set at 0.0, 0.1, 0.3, 0.5, 0.7 and 0.9. The reward τ_r and the penalty τ_p are equally set at 0, 1, 2 and 3. This is because the events of the reward and the penalty, which occurs between a manufacturer and an external institution, does not occur simultaneously, and to illustrate fairly and clearly the effect of τ_r and τ_p on the optimal operation of a GSC.

As shown in Figure 2, we assume some shapes of the distribution of quality level ℓ $(0 \le \ell \le 1)$ of recyclable parts in used products. We model each shape of the distribution of quality level ℓ $(0 \le \ell \le 1)$ of recyclable parts by using the beta distribution. This is the reason why we use the beta distribution is not only because it's possible to express various shapes, but more important, it's widely used to measure relative parameters like level ℓ $(0 \le \ell \le 1)$, or anything that is between 0-1.

Concretely, the beta distribution can express various shapes of distribution of recyclable parts in used products such as the uniform distribution-type shape, the normal distribution-type shape, the exponential distribution-type shape, the left-biased distribution shape, the right-biased distribution shape, by using the following probability density function with parameters (a, b) :

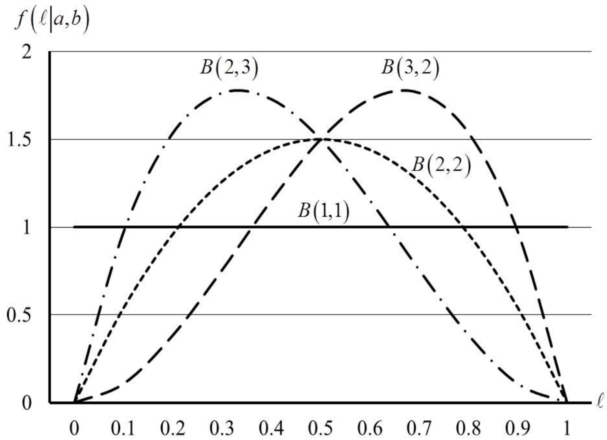

Figure 2. Four cases of distribution of quality level $\ell(0 \le \ell \le 1)$ of recyclable parts in used products modeled as the beta distribution $B(\ell|a, b)$.

$$f(\ell|a, b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \ell^{a-1}(1-\ell)^{b-1}, \tag{38}$$

where $\Gamma(\cdot)$ denotes the gamma function. As shown in Figure 1, we provide four cases of the beta distribution:

Case 1 $B(\ell|1, 1)$: the situation where each quality of recyclable parts are uniformly distributed, corresponding to the uniform distribution-type shape for quality level $\ell(0 \le \ell \le 1)$,

Case 2 $B(\ell|2, 2)$: the situation where there are the more recyclable parts with the middle quality and each quality of recyclable parts are symmetrically distributed, corresponding to the normal distribution-type shape for quality level $\ell(0 \le \ell \le 1)$,

Case 3 $B(\ell|3, 2)$: the situation where there are the more recyclable parts with the relatively high quality, corresponding to the right-biased distribution shape for quality level $\ell(0 \le \ell \le 1)$,

Case 4 $B(\ell|2, 3)$: the situation where there are the more recyclable parts with the relatively low quality, corresponding to the left-biased distribution shape for quality level $\ell(0 \le \ell \le 1)$.

By changing parameters (a, b) of the probability density function of the beta distribution in Eq. (38), we can see how the results of the optimal operations in the GSC change.

A computer programming was developed by using Visual Studio C# in Visual Studio Express 2013 for Windows Desktop in order to conduct numerical experiments and obtain the results for the optimal operations under DGSC and IGSC by the numerical calculation and the numerical search. In the development of the computer programming and implementation of the numerical experiment, the following computer: the Dell computer, Vostro 260s model, CPU: Intel(R) Core(TR) i5-2400, 3.10 GHz: Memory: 4 GB, OS: Windows 7 Professional 32 bit was used in this paper.

• Effect of the demand information on the optimal product order quantity

First, as to each scenario of the demand information, the optimal product order quantity Q_D^i under DGSC are compared with the optimal product order quantity Q_C^i under IGSC. $(Q_D^2, Q_C^2) = (1228, 1289)$ in scenario 2 (i = 2) are smaller than $(Q_D^1, Q_C^1) = (1256, 1307)$ in scenario 1(i = 1). This is because the available demand information is limited in scenario 2(i = 2) and (Q_D^2, Q_C^2) are determined so as to maximize the lowest expected profits applying DFA in scenario 2(i = 2). Moreover, despite scenario i(= 1, 2), Q_C^i under IGSC are larger than Q_D^i under DGSC. This reason is clear from the comparison of the analysis results in Eq. (8) and Eq. (13) in scenario 1(i = 1) with those in Eq. (9) and Eq. (14) in scenario 2(i = 2). Concretely, from the aspect of the manufacturer's profit, the condition $w > c_n + c_m$ is generally satisfied. Under the condition, Q_D^i is affected by the unit wholesale price of product w, meanwhile Q_C^i is affected by the sum of the unit procurement cost of new parts and the production cost of product $c_n + c_m$.

Next, it is investigated how the following three factors: (i) the demand information, (ii) the distribution of quality level of recyclable parts and (iii) the contract combined the collection incentive contract with the reward-penalty contract-affect the optimal lower limit of quality level under DGSC and IGSC. This investigation uses the analytical results in Eq. (19)-Eq. (21) under DGSC and Eq. (32)-Eq. (34) under IGSC.

Table 1 shows the results of (u_D, u_C) under DGSC and IGSC without and with the reward-penalty contract. From Table 1, the following results can be seen: For the effect of the demand information, (u_D, u_C) is unaffected by $(Q_D^i, Q_C^i)(i = 1, 2)$ as to scenario of the demand information.

For the effect of the collection incentive contract, in DGSC, the optimal lower limit of quality level u_D is affected by the optimal collection incentive t_D. t_D is impacted by the quality distribution of recyclable parts. Therefore, u_D is affected by the quality distribution of it. In IGSC, the optimal lower limit of quality level u_C is unaffected by the optimal collection incentive t_C. Thus, u_C can be determined as lower values than u_D, despite the quality distribution of recyclable parts.

For the effect of the reward-penalty contract, (u_D, u_C) with RPC ($T > 0,\ \tau_r > 0,\ \tau_p > 0$) is lower than ($u_D$, u_C) without RPC ($T = 0,\ \tau_r = 0,\ \tau_p = 0$). Also, RPC can enhance (u_D, u_C) as lower values even if there are the more recyclable parts with low quality shown as Case 4 of the quality distribution of recyclable parts. As the reward τ_r and the penalty τ_p become higher, u_C with RPC has a higher degree of improvement than u_D. It is because u_D is affected by t_D determined by the retailer.

Furthermore, it is investigated how the following three factors: (i) the demand information, (ii) the distribution of quality level of recyclable parts and (iii) the contract combined the collection incentive contract with the reward-penalty contract-affect the optimal collection incentive under DGSC and IGSC.

Table 2 shows (t_D, t_C) under DGSC and IGSC with-

Table 1. Optimal lower limits of quality level under DGSC and IGSC without and with the reward-penalty contract

Reward-penalty contract with external institution		Quality distribution of quality level ℓ $(0 \le \ell \le 1)$ of recyclable parts							
		Case 1: $B(1, 1)$		Case 2: $B(2, 2)$		Case 3: $B(3, 2)$		Case 4: $B(2, 3)$	
		DGSC	IGSC	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC
T	τ_r τ_p	u_D	u_C	u_D	u_C	u_D	u_C	u_D	u_C
0	0	0.25	0.11	0.3	0.11	0.4	0.11	0.22	0.11
0.1	1	0.25	0.08	0.29	0.08	0.39	0.08	0.21	0.08
0.3		0.25	0.08	0.29	0.08	0.39	0.08	0.21	0.08
0.5		0.25	0.08	0.29	0.08	0.39	0.08	0.21	0.08
0.7		0.25	0.08	0.29	0.08	0.39	0.08	0.21	0.08
0.9		0.25	0.08	0.29	0.08	0.39	0.08	0.21	0.08
0.1	2	0.24	0.06	0.29	0.06	0.39	0.06	0.21	0.06
0.3		0.24	0.06	0.29	0.06	0.39	0.06	0.21	0.06
0.5		0.24	0.06	0.29	0.06	0.39	0.06	0.21	0.06
0.7		0.24	0.06	0.29	0.06	0.39	0.06	0.21	0.06
0.9		0.24	0.06	0.29	0.06	0.39	0.06	0.21	0.06
0.1	3	0.23	0.03	0.28	0.03	0.38	0.03	0.2	0.03
0.3		0.23	0.03	0.28	0.03	0.38	0.03	0.2	0.03
0.5		0.23	0.03	0.28	0.03	0.38	0.03	0.2	0.03
0.7		0.23	0.03	0.28	0.03	0.38	0.03	0.2	0.03
0.9		0.23	0.03	0.28	0.03	0.38	0.03	0.2	0.03

out and with RPC.

From Table 2, the following results can be seen:

For the effect of the demand information, it is confirmed that (t_D, t_C) are unaffected by each scenario of the demand information. t_C is higher than t_D. It is because only t_D is affected by the compensation from a manufacturer. Therefore, t_D is affected more highly by the quality distribution of recyclable parts than t_C.

For the effects of the collection incentive contract and the reward-penalty contract, as the reward τ_r and the penalty τ_p become higher, t_D with the reward-penalty contract ($T>0,\ \tau_r>0,\ \tau_p>0$) has a higher degree of improvement than t_C. It is because t_D is affected by the compensation from a manufacturer. As the target T become higher, t_C is higher. t_D is unaffected by the target T. Thus, even if there are the more recyclable parts with low quality shown as Case 4 of the quality distribution of recyclable parts, RPC can enhance t_D as a higher value, meanwhile the contract combined the collection incentive contract and the reward-penalty contract can enhance t_C.

Thus, the above results verifies that the contract combined the collection incentive contract with the reward-penalty contract can encourage both the collection and the recycling of the used products under DGSC and IGSC.

Moreover, the expected profits of a retailer, a manufacturer and the whole system under DGSC are compared with those under IGSC for scenario 1 of the demand information and Cases 3 and 4 of the quality distribution of recyclable parts.

Table 3 shows Effects of profit sharing as SCC in each quality distribution of recyclable parts in scenario 1 of the demand information under Cases 3 and 4 of the quality distribution, $T = 0.3$ and $\tau_r = \tau_p = 2$. Table 4 shows the results in scenario 2 of the demand information.

From Table 3 and Table 4, regardless of the demand information, the following results can be seen:

In Case 3 of the quality distribution, there are the more recyclable parts with the relatively high quality, meanwhile in Case 4 of the quality distribution, there are the more recyclable parts with the relatively low quality.

In Case 3 of the quality distribution, all the expected profits of a retailer, a manufacturer and the whole system under IGSC are higher than those under DGSC. However, the manufacturer has the more increment of the profit obtained under IGSC than the retailer has under IGSC. This implies that the increment of the expected profit for each member under IGSC does not reflect the size of the expected profit of each member. These results are same in Case 1 and Case 2 of the quality distribution.

In Case 4 of the quality distribution, only the retailer's expected profit is lower than that under IGSC.

From Table 3 and Table 4, it is verified that it is difficult for a retailer to shift the optimal decisions under IGSC to enhance the expected profit of the whole system.

As a supply chain coordination under IGSC, any

Table 2. Optimal collection incentive under DGSC and IGSC without and with the reward-penalty contract

Reward-penalty contract with external institution		Quality distribution of quality level $\ell\ (0 \le \ell \le 1)$ of recyclable parts							
		Case 1: $B(1, 1)$		Case 2: $B(2, 2)$		Case 3: $B(3, 2)$		Case 4: $B(2, 3)$	
		DGSC	IGSC	DGSC	IGSC	DGSC	IGSC	DGSC	IGSC
T	τ_r τ_p	t_D	t_C	t_D	t_C	t_D	t_C	t_D	t_C
0	0	2.94	4.61	3.94	4.52	6.13	6.3	2.27	2.74
0.1	1	3.44	4.61	4.4	4.52	6.6	6.3	2.77	2.75
0.3		3.44	4.71	4.4	4.62	6.6	6.34	2.77	2.89
0.5		3.44	4.81	4.4	4.76	6.6	6.46	2.77	3.06
0.7		3.44	4.91	4.4	4.9	6.6	6.63	2.77	3.18
0.9		3.44	5.01	4.4	5	6.6	6.77	2.77	3.22
0.1	2	3.92	4.63	4.86	4.53	7.07	6.3	3.24	2.76
0.3		3.92	4.83	4.86	4.72	7.07	6.38	3.24	3.05
0.5		3.92	5.03	4.86	5	7.07	6.61	3.24	3.39
0.7		3.92	5.23	4.86	5.29	7.07	6.95	3.24	3.62
0.9		3.92	5.43	4.86	5.48	7.07	7.25	3.24	3.7
0.1	3	4.38	4.66	5.32	4.54	7.54	6.31	3.71	2.78
0.3		4.38	4.96	5.32	4.82	7.54	6.43	3.71	3.22
0.5		4.38	5.26	5.32	5.25	7.54	6.77	3.71	3.73
0.7		4.38	5.56	5.32	5.68	7.54	7.28	3.71	4.08
0.9		4.38	5.86	5.32	5.96	7.54	7.72	3.71	4.2

Table 3. Effects of profit sharing as supply chain coordination (SCC) in each quality distribution of recyclable parts in scenario 1 of the demand information under Cases 3 and 4 of the quality distribution, $T = 0.3$ and $\tau_r = \tau_p = 2$

Case 3: B(3, 2)	Expected profits	DGSC	No SCC	SCC
			IGSC	IGSC
Retailer		70509	70660(+151)	71010(+501)
Manufacturer		20822	21673(+851)	21322(+500)
Whole system		91331	92333(+1002)	92333(1001)
Coordinated degree of compensation α^{1N}		0.7	0.7	0.88
Coordinated margin m_a^{1N}		15	15	15.1

Case 4: B(2, 3)	Expected profits	DGSC	No SCC	SCC
			IGSC	IGSC
Retailer		69682	69475(-207)	69933(+251)
Manufacturer		19912	20621(+709)	20162(+250)
Whole system		89594	90096(+502)	90095(+501)
Coordinated degree of compensation α^{1N}		0.7	0.7	0.96
Coordinated margin m_a^{1N}		15	15	14.8

Table 4. Effects of profit sharing as supply chain coordination in each quality distribution of recyclable parts in scenario 2 of the demand information under Cases 3 and 4 of the quality distribution, $T = 0.3$ and $\tau_r = \tau_p = 2$

Case 3: B(3, 2)	Expected profits	DGSC	No profit sharing	Supply chain coordination
			No SCC	SCC
Retailer		58306	58399(+93)	58846(+540)
Manufacturer		20416	21402(+986)	20956(+540)
Whole system		78722	79802(+1079)	79802(+1080)
Coordinated degree of compensation α^{2N}		0.7	0.7	2.94
Coordinated margin m_a^{2N}		15	15	19.3

Case 4: B(2, 3)	Expected profits	DGSC	No SCC	SCC
			IGSC	IGSC
Retailer		57479	57213(-266)	57774(+295)
Manufacturer		19505	20351(846)	19790(+285)
Whole system		76984	77564(846)	77564(+580)
Coordinated degree of compensation α^{2N}		0.7	0.7	0.08
Coordinated margin m_a^{2N}		15	15	14.2

reasonable profit sharing is necessary for members under IGSC to shift the optimal operations under IGSC, guaranteeing more profits to members under IGSC than those under DGSC. The effects of profit sharing under IGSC on the expected profits of the retailer and the manufacturer are investigated. Table 4 shows the effects of profit sharing as a supply chain coordination in Case 4 of quality distribution of recyclable parts in scenario 1. For Case 4 of the quality distribution of recyclable parts in scenario 1 (i = 1) where the expected profit of the retailer under IGSC is lower than that under DGSC.

From Tables 3 and 4, regardless of the demand information, the following results can be seen:

It can be seen that the expected profits of all members under IGSC adopting profit sharing are higher than those under DGSC in Case 3 and Case 4 of the quality distribution of recyclable parts. Adopting a supply chain coordination under the optimal operations under IGSC can guarantee the more profit to both members under IGSC, and it can encourage the shift the optimal operations under IGSC from that under DGSC.

It is confirmed that the increment of the expected profit obtained under IGSC is shared almost equally between a retailer and a manufacturer when the unit whole sales price and the compensation of collection cost for the recycled parts are adjusted by Nash bargaining solutions. This is the property of the Nash bargaining solution.

However, it is more desirable for members under IGSC to incorporate the alternative supply chain coordination to reflect the size of either the expected profit of each member under IGSC in scenario 1 or the lower level of that in scenario 2 on the amount of profit sharing for each member under IGSC.

9. CONCLUSIONS

This study discussed a green supply chain (GSC) consisting of a retailer, a manufacturer and an external institution, and presented the optimal operation in the GSC for a single product in a single period. Two contracts were combined to promote the collection and the recycling of used products in GSC. The collection incentive contract for used products was made between a retailer and a manufacturer. The reward-penalty contract for recycling used products was made between a manufacturer and an external institution. The demand information were assumed as (i) the demand distribution was known and (ii) only both mean and variance of the demand were known. The distribution-free approach (DFA) was adopted to analyze. (iii) A product order quantity, the collection incentive of used products and lower limit of quality level for recyclable parts were optimized under a decentralized GSC (DGSC) and an integrated GSC (IGSC). The analysis numerically investigated how (i) the combined contract, (ii) the demand information and (iii) quality of the recyclable parts-affected the optimal operations and the expected profits under DGSC and IGSC.

Numerical results illustrated the following marginal insights regarding the optimal operations in a GSC for academic researchers and real-world policymakers regarding operations in a GSC:

- It is possible to promote both activities of the collection and the recycling of used products, guaranteeing the expected profits of a retailer, a manufacturer and the whole system in the GSC regardless of the demand information when the contract combining the collection contract and the reward-penalty contract is incorporated into a GSC.
- When the reward-penalty contract is incorporated into IGSC, the optimal collection incentive and the lower limit of quality level are improved more than those under DGSC.
- The optimal product order quantity in scenario 2, where the demand distribution of a single product is unknown, but the mean and the variance are known, is determined as a lower value than that in scenario 1 where the demand distribution is known. This is because the optimal product order quantity in scenario 2 is determined by adopting DFA which maximizes the expected profit against the worst possible distribution of the product demand with mean and variance.
- Incorporation of supply chain coordination into the optimal operation under the integrated green supply chain with the contract combining the collection contract and the reward-penalty contract enables to encourage to guarantee to improve both the expected profits and the lower limits of the expected profits of all members and the whole system in a GSC as well as to promote the aggressive eco-activity among all members in a GSC.

Therefore, it is highly expected that research outcomes in this paper would provide not only the optimal solution and its practices to construct a GSC to encourage both aggressive eco-activities of the collection and the remanufacturing of used products to firms, but also informative motivations for researchers and policymakers regarding operations in a GSC.

This paper investigated how the reward-penalty contract between a manufacturer and an external institution affected the optimal operation for a GSC. However, this paper did not discuss the limitation or the optimal decision for the reward which an external institution assigned to a manufacturer. The unrestraint reward from an external institution to a manufacturer brings an inadequate operational flows in a GSC. In this case, a manufacturer tends to recycle the used products even if the manufacturer's recycling activity has a deficit, leaning the reward from an external institution excessively.

As the extendable consideration including the above issue, it will be necessary to discuss the following issues to analyze the optimal operation for a GSC:

- Setting of limitation of the reward to a manufacturer from an external institution
- Effect of recycled quantity of used products on the unit reward and the unit penalty for an external institution.
- A situation of uncertainty in the collection quantity of the used products
- Limitation of information regarding quality distribution of used products/ recyclable parts
- Alternative supply chain coordination between a retailer and a manufacturer to evaluate the profit balance and cost effectiveness of members in GSC.

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APPENDIX A

- The elicitation process of Eq. (4)

$$
\begin{aligned}
&E^1\left[\pi_R(Q,t,u)\right]=-tA(t)-c_tA(t)\\
&\quad -wQ+R(t)\int_u^1 g(\ell)A(t)d\ell\\
&\quad +p\int_0^Q\left\{Q-\left(Q-x\right)\right\}f(x)dx+pQ\int_Q^\infty f(x)dx\\
&\quad -\left(h_r+s_a\right)\int_0^Q(Q-x)f(x)dx-s\int_Q^\infty(x-Q)f(x)dx\\
&\quad =-t\,A(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)d\ell\\
&\quad -wQ+pQ\int_0^Q f(x)dx+pQ\int_Q^\infty f(x)dx\\
&\quad -p\int_0^Q\left(Q-x\right)f(x)dx\\
&\quad -\left(h_r+s_a\right)\int_0^Q(Q-x)f(x)dx\ -s\int_Q^\infty(x-Q)f(x)dx\\
&\quad =-tA(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)d\ell-wQ+pQ\\
&\quad -\left\{p+\left(h_r+s_a\right)\right\}\int_0^Q\left(Q-x\right)f(x)dx\\
&\quad -s\int_Q^\infty(x-Q)f(x)dx. \qquad (4)
\end{aligned}
$$

Therefore, the elicitation process of Eq. (4) can be shown.

APPENDIX B

- The elicitation process of Eq. (9)

The first-order differential equation between the product order quantity Q and the expected profit $E^1\left[\pi_R\left(Q|t,u\right)\right]$ of the retailer in scenario 1 in Eq. (2) under the collection incentive t and the lower limit of quality level u is derived as follows:

$$
\begin{aligned}
&\frac{dE^1\left[\pi_R\left(Q,t,u\right)\right]}{dQ}=-w+p\frac{d}{dQ}Q\int_Q^\infty f(x)dx\\
&+p\frac{d}{dQ}\int_0^Q xf(x)dx-\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q(Q-x)f(x)dx\\
&-s\frac{d}{dQ}\int_Q^\infty(x-Q)f(x)dx\\
&=-w\\
&+p\frac{d}{dQ}Q\int_Q^\infty f(x)dx+p\frac{d}{dQ}\int_0^Q\{Q-(Q-x)\}f(x)dx\\
&-\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q Qf(x)dx+\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q xf(x)dx\\
&-s\frac{d}{dQ}\int_Q^\infty xf(x)dx+s\frac{d}{dQ}\int_Q^\infty Qf(x)dx\\
&=-w\\
&+p\frac{d}{dQ}\int_Q^\infty Qf(x)dx+p\frac{d}{dQ}\int_0^Q Qf(x)dx\\
&-p\frac{d}{dQ}\int_0^Q(Q-x)f(x)dx\\
&-\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q Qf(x)dx+\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q xf(x)dx\\
&-s\frac{d}{dQ}\int_Q^\infty xf(x)dx+s\frac{d}{dQ}\int_Q^\infty Qf(x)dx\\
&=-w+p\frac{d}{dQ}\int_0^\infty Qf(x)dx\\
&-p\frac{d}{dQ}\int_0^Q Qf(x)dx+p\frac{d}{dQ}\int_0^Q xf(x)dx\\
&-\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q Qf(x)dx+\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q xf(x)dx\\
&-s\frac{d}{dQ}\int_Q^\infty xf(x)dx+s\frac{d}{dQ}\int_Q^\infty Qf(x)dx\\
&=-w+p\\
&-p\int_0^Q f(x)dx-pQf(Q)+pQf(Q)\\
&-\left(h_r+s_a\right)\int_0^Q f(x)dx-\left(h_r+s_a\right)Qf(Q)+\left(h_r+s_a\right)Qf(Q)\\
&+sQf(Q)+s\int_Q^\infty f(x)dx-sQf(Q)\\
&=-w+p\\
&-p\int_0^Q f(x)dx-\left(h_r+s_a\right)\int_0^Q f(x)dx+s\left\{1-\int_0^Q f(x)dx\right\}\\
&=-w+p+s\\
&-p\int_0^Q f(x)dx-\left(h_r+s_a\right)\int_0^Q f(x)dx-s\int_0^Q f(x)dx\\
&=-w+p+s-\left\{p+\left(h_r+s_a\right)+s\right\}\int_0^Q f(x)dx. \qquad (9)
\end{aligned}
$$

Therefore, the elicitation process of Eq. (9) can be shown.

APPENDIX C

- The elicitation process of Eq. (12)
  Eq. (7) is shown as

$$E_L^2\left[\pi_R(Q,t,u)\right]=-tA(t)-c_tA(t)$$
$$+R(t)\int_u^1 g(\ell)A(t)d\ell+(p-w)Q$$
$$-\{p+(h_r+s_a)\}\frac{\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}-(\mu-Q)}{2}$$
$$-s\frac{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}-(Q-\mu)}{2}. \tag{7}$$

Here, Eq. (7) can be rewritten as

$$E_L^2\left[\pi_R(Q,t,u)\right]=-tA(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)d\ell$$
$$+\frac{1}{2}\left\{2(p-w)Q-\{p+(h_r+s_a)\}\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right.$$
$$\left.+(p+h_r)(\mu-Q)-s\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}+s(Q-\mu)\right\}$$
$$=-tA(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)d\ell$$
$$+\frac{1}{2}\left\{(2p-2w)Q+\{p+(h_r+s_a)-s\}(\mu-Q)\right.$$
$$\left.-\{p+(h_r+s_a)+s\}\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right\}$$
$$=-tA(t)-c_tA(t)+R(t)\int_u^1 g(\ell)A(t)d\ell$$
$$+\frac{1}{2}\left\{(p+s-(h_r+s_a)-2w)Q+\{p+(h_r+s_a)-s\}\mu\right.$$
$$\left.-(p+s+(h_r+s_a))\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}. \tag{C-1}$$

The first-order differential equation between the product order quantity Q and the lower limit of the retailer's expected profit $E_L^2\left[\pi_R(Q|t,u)\right]$ in scenario 2 in Eq. (7) under t and u is derived as

$$\frac{dE_L^2\left[\pi_R(Q|t,u)\right]}{dQ}$$
$$=\frac{1}{2}\{p+s-(h_r+s_a)-2w\}$$
$$-\frac{1}{2}\frac{d}{dQ}\{p+s+(h_r+s_a)\}\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}$$
$$=\frac{1}{2}\{p+s-(h_r+s_a)-2w\}$$
$$-\frac{1}{2}\{p+s+(h_r+s_a)\}\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}. \tag{12}$$

Therefore, the elicitation process of Eq. (12) can be shown.

- The elicitation process of Eq. (13)

The second-order differential equation between the product order quantity Q and the lower limit of the retailer's expected profit $E_L^2\left[\pi_R(Q|t,u)\right]$ in scenario 2 in Eq. (7) under t and u is derived as

$$\frac{d^2E_L^2\left[\pi_R(Q|t,u)\right]}{dQ^2}$$
$$=\frac{1}{2}\cdot\frac{d}{dQ}\left\{(p+s-(h_r+s_a)-2w)\right.$$
$$\left.-(p+s+(h_r+s_a))\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\}$$
$$=-\frac{d}{dQ}\frac{\{p+s+(h_r+s_a)\}(Q-\mu)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}$$
$$=-\frac{\sigma^2\{p+s+(h_r+s_a)\}}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}}. \tag{13}$$

Therefore, the elicitation process of Eq. (13) can be shown.

APPENDIX D

- The elicitation process of Eq. (16)

When the magnitude relation between the lower limit of quality level u and the target of quality level for recycling of used products T satisfies $u<T$, a manufacturer receives the reward of the recycling effort from an external institution. Therefore, when $u<T$, the manufacturer's expected profit for Q, t and u is obtained as

$$E\left[\pi_M(Q,t,u)\right]=wQ-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell$$
$$-R(t)\int_u^1 g(\ell)A(t)d\ell$$
$$-c_dA(t)\int_0^u g(\ell)d\ell-c_n\left\{Q-A(t)\int_u^1 g(\ell)d\ell\right\}-c_mQ$$
$$+\tau_r A(t)\left[\int_u^1 g(l)dl-\int_T^1 g(l)dl\right]^+ \quad (u<T). \tag{D-1}$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the manufacturer $E\left[\pi_M(u)\middle|Q_D^i, t\right](i=1,2)$ in Eq. (D-1) under the optimal product order quantity Q_D^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$\begin{aligned}&\frac{dE\left[\pi_M\left(u|Q_i,t\right)\right]}{du}(i=1,2)\\&=R(t)g(u)A(t)+A(t)c_r(u)g(u)\\&-c_dA(t)g(u)-c_nA(t)g(u)-\tau_rA(t)g(u)\\&=A(t)g(u)\left\{R(t)+c_r(u)-c_d-c_n-\tau_r\right\}.\end{aligned}\tag{16}$$

Therefore, the elicitation process of Eq. (16) can be shown.

- The elicitation process of Eq. (17)

When $u>T$, a manufacturer needs to pay the penalty of lack of recycling effort to an external institution.

Therefore, when $u>T$, the manufacturer's expected profit for Q, t and u is obtained as

$$\begin{aligned}&E\left[\pi_M(Q,t,u)\right]=wQ-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell\\&-R(t)\int_u^1 g(\ell)A(t)d\ell\\&-c_dA(t)\int_0^u g(\ell)d\ell-c_n\left\{Q-A(t)\int_u^1 g(\ell)d\ell\right\}-c_mQ\\&-\tau_pA(t)\left[\int_T^1 g(l)dl-\int_u^1 g(l)dl\right]^+\quad(u>T).\end{aligned}\tag{D-2}$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the manufacturer $E\left[\pi_M(u)\middle|Q_D^i, t\right](i=1,2)$ in Eq. (D-2) under the optimal product order quantity Q_D^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$\begin{aligned}&\frac{dE\left[\pi_M\left(u|Q_i,t\right)\right]}{du}(i=1,2)\\&=R(t)g(u)A(t)+A(t)c_r(u)g(u)\\&-c_dA(t)g(u)-c_nA(t)g(u)-\tau_pA(t)g(u)\\&=A(t)g(u)\left\{R(t)+c_r(u)-c_d-c_n-\tau_p\right\}.\end{aligned}\tag{17}$$

Therefore, the elicitation process of Eq. (17) can be shown.

- The elicitation process of Eq. (18)

When $u=T$, a manufacturer neither receives the reward of the recycling effort from an external institution nor needs to pay the penalty of lack of recycling effort to an external institution.

Therefore, when $u=T$, the manufacturer's expected profit for Q, t and u is obtained as

$$\begin{aligned}&E\left[\pi_M(Q,t,u)\right]=wQ-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell\\&-R(t)\int_u^1 g(\ell)A(t)d\ell\\&-c_dA(t)\int_0^u g(\ell)d\ell-c_n\left\{Q-A(t)\int_u^1 g(\ell)d\ell\right\}-c_mQ.\end{aligned}\tag{D-3}$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the manufacturer $E\left[\pi_M(u)\middle|Q_D^i, t\right](i=1,2)$ in Eq. (D-3) under the optimal product order quantity Q_D^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$\begin{aligned}&\frac{dE\left[\pi_M\left(u|Q_i,t\right)\right]}{du}(i=1,2)\\&=R(t)g(u)A(t)+A(t)c_r(u)g(u)\\&-c_dA(t)g(u)-c_nA(t)g(u)\\&=A(t)g(u)\left\{R(t)+c_r(u)-c_d-c_n\right\}.\end{aligned}\tag{18}$$

Therefore, the elicitation process of Eq. (18) can be shown.

APPENDIX E

- The elicitation process of Eq. (22)

The first-order differential equation between the product order quantity Q and the expected profit $E^1\left[\pi_S\left(Q|t,u\right)\right]$ of the whole system in scenario 1 in Eq. (3) under the collection incentive t and the lower limit of quality level u is derived as follows:

$$\begin{aligned}&\frac{dE^1\left[\pi_S\left(Q,t,u\right)\right]}{dQ}\\&=-c_n-c_m+p\frac{d}{dQ}Q\int_Q^\infty f(x)dx+p\frac{d}{dQ}\int_0^Q xf(x)dx\\&-\left(h_r+s_a\right)\frac{d}{dQ}\int_0^Q(Q-x)f(x)dx-s\frac{d}{dQ}\int_Q^\infty(x-Q)f(x)dx\\&=-c_n-c_m+p+s-\left\{p+\left(h_r+s_a\right)+s\right\}\int_0^Q f(x)dx.\end{aligned}\tag{22}$$

Therefore, the elicitation process of Eq. (22) can be shown.

APPENDIX F

- The elicitation process of Eq. (25)

By replacing w in Eq. (7) with (c_m+c_n), Eq. (8) can be rewritten as

$$E^2\left[\pi_S(Q,t,u)\right]=-tA(t)-c_tA(t)$$
$$+\{p-(c_m+c_n)\}Q$$
$$-\{p+(h_r+s_a)\}\frac{\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}-(\mu-Q)}{2}$$
$$-s\frac{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}-(Q-\mu)}{2}$$
$$=-tA(t)-c_tA(t)$$
$$+\frac{1}{2}\left\{2\{p-(c_m+c_n)\}Q-\{p+(h_r+s_a)\}\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right.$$
$$\left.+\{p+(h_r+s_a)\}(\mu-Q)+s(Q-\mu)-s\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}$$
$$=-tA(t)-c_tA(t)$$
$$+\frac{1}{2}\left\{2\{p-(c_m+c_n)\}Q+\{p+(h_r+s_a)-s\}(\mu-Q)\right.$$
$$\left.-\{p+(h_r+s_a)+s\}\left[\sigma^2+(\mu-Q)^2\right]^{\frac{1}{2}}\right\}$$
$$=-t\,A(t)-c_tA(t)$$
$$+\frac{1}{2}\left\{\{p+s-(h_r+s_a)-2(c_m+c_n)\}Q+\{p+(h_r+s_a)-s\}\mu\right.$$
$$\left.-\{p+s+(h_r+s_a)\}\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}. \tag{F-1}$$

Using the relation between (C-1) and (F-1), Eq. (25) is obtained easily as follows: by replacing w in Eq. (12) with (c_m+c_n), the first-order differential equation between the product order quantity Q and the lower limit of the expected profit $E_L^2\left[\pi_S(Q|t,u)\right]$ of the whole system in scenario 2 in Eq. (8) under the collection incentive t and the lower limit of quality level u is derived as

$$\frac{dE_L^2\left[\pi_S(Q|t,u)\right]}{dQ}$$
$$=\frac{1}{2}\left\{\{p+s-(h_r+s_a)-2(c_m+c_n)\}\right.$$
$$\left.-\frac{d}{dQ}\{p+s+(h_r+s_a)\}\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}\right\}$$
$$=\frac{1}{2}\{p+s-(h_r+s_a)-2(c_m+c_n)\}$$
$$-\frac{1}{2}\{p+s+(h_r+s_a)\}\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}. \tag{25}$$

Therefore, the elicitation process of Eq. (25) can be shown.

- The elicitation process of Eq. (26)

The second-order differential equation between the product order quantity Q and the lower limit of the expected profit $E_L^2\left[\pi_S(Q|t,u)\right]$ of the whole system in scenario 2 in Eq. (8) under the collection incentive t and the lower limit of quality level u is derived as

$$\frac{d^2E_L^2\left[\pi_S(Q|t,u)\right]}{dQ^2}$$
$$=\frac{1}{2}\cdot\frac{d}{dQ}\left\{\{p+s-(h_r+s_a)-2(c_m+c_n)\}\right.$$
$$\left.-\{p+s+(h_r+s_a)\}\frac{(Q-\mu)}{\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}\right\}$$
$$=-\frac{d}{dQ}\frac{\{p+s+(h_r+s_a)\}(Q-\mu)}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{1}{2}}}$$
$$=-\frac{\sigma^2\{p+s+(h_r+s_a)\}}{2\left[\sigma^2+(Q-\mu)^2\right]^{\frac{3}{2}}}. \tag{26}$$

Thus, it is verified that the elicitation process of Eq. (26) is same as that in Eq. (13). Therefore, the elicitation process of Eq. (26) can be shown.

APPENDIX G

- The elicitation process of Eq. (29)

When the magnitude relation between the lower limit of quality level u and the target of quality level for recycling of used products T satisfies $u<T$, a manufacturer receives the reward of the recycling effort from an external institution.

Therefore, when $u<T$, the whole system's expected profit for Q, t and u is obtained as

$$E^1\left[\pi_S(Q,t,u)\right]=E^1\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]$$
$$=-tA(t)-c_tA(t)$$
$$-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell$$
$$-c_dA(t)\int_0^u g(\ell)d\ell$$
$$-c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ$$
$$+\tau_rA(t)\left[\int_u^1 g(l)dl-\int_T^1 g(l)dl\right]^+ \quad (u<T)$$
$$+p\left\{\int_0^Q xf(x)dx+\int_Q^\infty Qf(x)dx\right\}$$

$$-\left(h_r+s_a\right)\int_0^Q (Q-x) f(x) dx$$
$$-s\int_Q^\infty (x-Q) f(x) dx. \tag{G-1}$$

Also, when $u<T$, the lower limit of the whole system's expected profit for Q, t and u is obtained as

$$E_L^2\left[\pi_S(Q,t,u)\right]=E_L^2\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]$$
$$=-tA(t)-c_tA(t)$$
$$-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell$$
$$-c_dA(t)\int_0^u g(\ell)d\ell$$
$$-c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ$$
$$-p\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2$$
$$-\left(h_r+s_a\right)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2$$
$$-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2$$
$$+\tau_rA(t)\left[\int_u^1 g(l)dl-\int_T^1 g(l)dl\right]^+ (u<T). \tag{G-2}$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the whole system $E\left[\pi_S(u)\middle|Q_D^i,t\right](i=1,2)$ in Eq. (G-1) under the optimal product order quantity Q_C^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$\frac{dE\left[\pi_S(u)\middle|Q_C^1,t\right]}{du}=\frac{dE_L^2\left[\pi_S(u)\middle|Q_C^2,t\right]}{du}$$
$$=A(t)c_r(u)g(u)-c_dA(t)g(u)-c_nA(t)g(u)$$
$$-\tau_rA(t)g(u)$$
$$=A(t)g(u)\left\{c_r(u)-c_d-c_n-\tau_r\right\}. \tag{G-3}$$

Therefore, the elicitation process of Eq. (29) can be shown.

- The elicitation process of Eq. (30)

When $u>T$, a manufacturer needs to pay the penalty of lack of recycling effort to an external institution.

Therefore, when $u>T$, the whole system's expected profit for Q, t and u is obtained as

$$E^1\left[\pi_S(Q,t,u)\right]=E^1\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]$$
$$=-tA(t)-c_tA(t)$$
$$-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell$$
$$-c_dA(t)\int_0^u g(\ell)d\ell$$
$$-c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ$$
$$-\tau_pA(t)\left[\int_T^1 g(l)dl-\int_u^1 g(l)dl\right]^+ \quad (u>T)$$
$$+p\left\{\int_0^Q xf(x)dx+\int_Q^\infty Qf(x)dx\right\}$$
$$-\left(h_r+s_a\right)\int_0^Q (Q-x)f(x)dx$$
$$-s\int_Q^\infty (x-Q)f(x)dx. \tag{G-4}$$

Also, when $u>T$, the lower limit of the whole system's expected profit for Q, t and u is obtained as

$$E_L^2\left[\pi_S(Q,t,u)\right]=E_L^2\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]$$
$$=-tA(t)-c_tA(t)$$
$$-c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell$$
$$-c_dA(t)\int_0^u g(\ell)d\ell$$
$$-c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ$$
$$-p\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2$$
$$-\left(h_r+s_a\right)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2$$
$$-s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2$$
$$-\tau_pA(t)\left[\int_T^1 g(l)dl-\int_u^1 g(l)dl\right]^+ \quad (u>T). \tag{G-5}$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the whole system $E\left[\pi_S(u)\middle|Q_D^i,t\right](i=1,2)$ in Eq. (G-2) under the optimal product order quantity Q_C^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$\frac{dE\left[\pi_S(u)\middle|Q_C^1,t\right]}{du}=\frac{dE_L^2\left[\pi_S(u)\middle|Q_C^2,t\right]}{du}$$
$$=A(t)c_r(u)g(u)-c_dA(t)g(u)-c_nA(t)g(u)$$
$$-\tau_pA(t)g(u)$$
$$=A(t)g(u)\left\{c_r(u)-c_d-c_n-\tau_p\right\}. \tag{G-6}$$

Therefore, the elicitation process of Eq. (30) can be shown.

- The elicitation process of Eq. (31)

When $u=T$, a manufacturer neither receives the reward of the recycling effort from an external institution nor needs to pay the penalty of lack of recycling effort to an external institution.

Therefore, when $u=T$, the whole system's expected profit for Q, t and u is obtained as

$$
\begin{aligned}
E^{1}\left[\pi_S(Q,t,u)\right] &= E^{1}\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]\\
&= -tA(t)-c_tA(t)\\
&\quad -c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell\\
&\quad -c_dA(t)\int_0^u g(\ell)d\ell\\
&\quad -c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ\\
&\quad +p\left\{\int_0^Q xf(x)dx+\int_Q^\infty Qf(x)dx\right\}\\
&\quad -(h_r+s_a)\int_0^Q (Q-x)f(x)dx\\
&\quad -s\int_Q^\infty (x-Q)f(x)dx. \qquad \text{(G-7)}
\end{aligned}
$$

Also, when $u=T$, the lower limit of the whole system's expected profit for Q, t and u is obtained as

$$
\begin{aligned}
E_L^{2}\left[\pi_S(Q,t,u)\right] &= E_L^{2}\left[\pi_R(Q,t,u)\right]+E\left[\pi_M(Q,t,u)\right]\\
&= -tA(t)-c_tA(t)\\
&\quad -c_aA(t)-A(t)\int_u^1 c_r(\ell)g(\ell)d\ell\\
&\quad -c_dA(t)\int_0^u g(\ell)d\ell\\
&\quad -c_n\left\{Q-\int_u^1 A(t)g(\ell)d\ell\right\}-c_mQ\\
&\quad -p\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2\\
&\quad -(h_r+s_a)\left\{\sqrt{\sigma^2+(\mu-Q)^2}-(\mu-Q)\right\}\Big/2\\
&\quad -s\left\{\sqrt{\sigma^2+(Q-\mu)^2}-(Q-\mu)\right\}\Big/2. \qquad \text{(G-8)}
\end{aligned}
$$

The first-order differential equation between the lower limit of quality level u and the expected profit of the whole system $E\left[\pi_S(u)\middle|Q_D^i,t\right](i=1,2)$ in Eq. (G-3) under the optimal product order quantity Q_D^i $(i=1,2)$ in scenario $i(=1,2)$ and the collection incentive t are obtained as

$$
\begin{aligned}
\frac{dE\left[\pi_S(u)\middle|Q_C^1,t\right]}{du} &= \frac{dE_L^2\left[\pi_S(u)\middle|Q_C^2,t\right]}{du}\\
&= A(t)c_r(u)g(u)-c_dA(t)g(u)-c_nA(t)g(u)\\
&= A(t)g(u)\left\{c_r(u)-c_d-c_n\right\}. \qquad \text{(G-9)}
\end{aligned}
$$

Therefore, the elicitation process of Eq. (31) can be shown.