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LINEAR QUADRATIC OPTIMAL GUIDANCE WITH ARBITRARY WEIGHTING FUNCTIONS

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ABSTRACT. In this article, the linear quadratic (LQ) optimal guidance laws with arbitrary weighting functions are introduced. The optimal guidance problems in conjunction with the control effort weighed by arbitrary functions are formulated, and then the general solutions of these problems are determined. Based on these investigations, we can know a lot of previous optimal guidance laws belong to the proposed results. Additionally, the proposed results are compared with other results from the generalization standpoint. The potential importance on the proposed results is that a lot of useful new guidance laws providing their outstanding performance compared with existing works can be designed by choosing weighting functions properly. Accordingly, a new optimal guidance law is derived based on the proposed results as an illustrative example.

1. INTRODUCTION

Over the past couple of decades, the linear quadratic (LQ) optimization methodology [1, 2] has been developed. Since LQ optimization methodology was first introduced to public, this method has received a great attention from many researchers because it has some advantages what other methods do not have, as follows. First, LQ optimization methodology is systematically well-posed and well-formulated. In this methodology, the optimal result can be simply obtained by solving the predetermined LQ optimal problem. Second, a state feedback form of solution can be easily determined. For these reasons, LQ optimal methodology has been applied to various control and filtering problems recently. In the control field, it can be called as the linear quadratic optimal control or the linear quadratic regulator (LQR). LQ optimization method has successfully been used to automobile, airplane, and missile systems as studied in [3, 4, 5]. In the estimation field, this method is called as the linear quadratic estimator. The well-known Kalman filter [6] is also a one of the linear quadratic estimators. These successful applications of LQ optimal methodology to various problems have brought new potential applications of this method to the guidance problems.

During the past decade, it has been reported that LQ optimal methodology was successfully applied to various problems related with the guidance applications such mid-course [7], terminal homing [8, 9], impact angle control [10, 11, 12], and impact time control [13, 14]

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and so on. In recent years, it has become one of most commonly used methods to devise new guidance laws. In LQ optimal guidance problems, since the selection of the performance index (i.e., cost function) can decide overall performance and characteristic of an obtained guidance law, the form of the performance index is important. The performance index intended to minimize overall control effort has been widely accepted from many researchers. Until now, it seems that kind of performance index is reasonable choice for the general missile systems because the handling of control effort during the entire flight is an important issue in the missile systems to avoid a severe miss distance at the final time.

As the battlefield situation has been diversified and the maneuverability of the missile systems has been improved, the demand for the control effort minimization has been decreased recently in the guidance operation. In the other words, some different guidance factors are more important rather than the control effort minimization according to considering guidance objectives. For examples, in the anti-ballistic missile systems, the minimization of miss distance is more important than the usage of control effort [7]. The survivability is most important factor in the application of the anti-ship missile systems [15].

Therefore, a different perspective of performance index should be considered in the design of guidance law for such missile systems. One of such tries, the control effort weighted by an exponential function has been proposed in [16]. In that article, the proposed guidance law was supposed to shape the guidance command in order to cope with the decrease of the maneuverability as the operating altitude increases in the ground-to-air missile systems. In [9], the minimization of the control effort weighted by a polynomial function was studied to alleviate the guidance command transition during the handover from the mid-course to the terminal homing guidance. On the purpose of distributing the control effort during the entire flight evenly, the performance index based on Gaussian weighting function has also been reported in [17]. These approaches as mentioned above are called the weighted optimal guidance. Although only specified and restricted weighting functions were just handled in the previous works, several new guidance laws that can improve some guidance objectives have been successfully developed using the concept of the weighted optimal guidance.

Now, such successes can give us a hint that some different functions may also be possible for the weighting function in order to devise a new guidance law in the weighted optimal guidance framework. Accordingly, in this paper, we try to find new possibilities in the weighted optimal guidance. In order for that, we consider LQ optimal guidance problems in conjunction with the control effort weighted by arbitrary functions. Then, we determine the general solutions of the weighted optimal guidance problems with respect to arbitrary weighting functions. For the impact angle control problem, a similar work already has performed by these authors in [18]. Therefore, we mainly focus on the homing problem in this paper.

Based on our investigations, we can show that previous optimal guidance laws as discussed before belong to the proposed results. Hence, the proposed results can be regarded as more general forms of the optimal guidance laws compared with previous optimal guidance laws. Additionally, the proposed results are also compared with the results in [24] (which is one of general solutions of optimal impact angle control) from the generalization standpoint. In conclusion, our findings say that any arbitrary functions can be used for the weighting function in the weighted optimal guidance framework. The potential importance of our findings is that the proposed results open a new way to design new guidance laws and provide a good start point to derive a lot of useful guidance laws according to selections of weighting functions. As an illustrative example, a new guidance law providing the insensitivity against the initial heading error is also designed using the proposed results.

This paper is structured in five sections. In section 2, the considering problem is discussed. The main results of this paper are given in Section 3. The simulation results are provided in order to show the usefulness of our findings in section 4. Finally, we conclude our study in section 5.

2. Problem Formulation

In this section, the homing engagement geometry and the engagement kinematics equation which are used for the derivation of the proposed results are discussed. And then, LQ optimal guidance problems to be considered in this paper are presented.

2.1. **Engagement Kinematics.** Figure 1 represents the planar homing engagement geometry for a fixed target or slowly moving target.

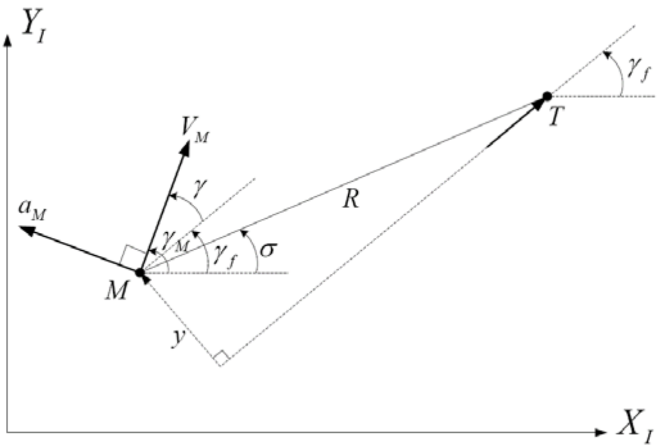


FIGURE 1. The planar homing engagement geometry

In this figure, the inertia reference frame is denoted by (X_I, Y_I). The notations of M and T represent the missile and the target, respectively, which can be regarded as the point mass models. The variables R and σ represent the relative distance and the line-of-sight (LOS) angle between the missile and the target. The flight path angle (i.e., the direction of velocity vector) with respect to the inertia reference frame is denoted by γ_M. The desired flight direction at the final time called the impact angle is defined by γ_f, which is usually a constant value. The angle difference between γ_M and γ_f is denoted by γ, which can be regarded as the flight path angle with respect to the impact course. The notation of represents the lateral distance in perpendicular to the impact course. Additionally, the variables V_M and a_M are the missile velocity and the acceleration, respectively. In the missile systems, the acceleration is widely chosen as the control input of those systems in order to achieve a fast response and the direction of this acceleration is always normal to the missile velocity as shown in Fig. 1. Under the assumption that the missile velocity is constant, the change of the missile velocity vector due to the missile acceleration can be expressed as:

$$\dot{\gamma_M} = \dot{\gamma} = \frac{a_M}{V_M} \tag{2.1}$$

From Fig. 1, the rate of lateral distance with respect to the impact course can be easily obtained as

$$\dot{y} = V_M \sin(\gamma_M - \gamma_f) = V_M \sin\gamma \tag{2.2}$$

Under the assumption that a proper midcourse guidance can lead to a small value of γ during the guidance handover, we can linearize the above equation as follows:

$$\dot{y} \approx V_M \gamma \triangleq \nu \tag{2.3}$$

Hereafter, let us introduce a new variable ν for convenience. The physical meaning of this variable is the lateral velocity with respect to the impact course. Using the expression as shown in Eq. (2.3) introduces:

$$\dot{\nu} = a_M \tag{2.4}$$

Let the state variables and the control input be defined as follows

$$x \triangleq \begin{bmatrix} x_1, & x_2 \end{bmatrix}^T = \begin{bmatrix} y, & \nu \end{bmatrix}^T, \quad u = a_M \tag{2.5}$$

Then, the linearized engagement kinematics in the matrix form is obtained by combining Eqs. (2.3) and (2.4).

$$\dot{x} = Fx + Gu \tag{2.6}$$

where

$$F \triangleq \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad G \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2.7}$$

In this linearized engagement kinematics, the terminal constraint for the interception is given by

$$x_1(t_f) = 0 \tag{2.8}$$

In order to provide both zero miss distance and the desired impact angle, the following conditions should be accomplished at the final time.

$$x_1(t_f) = 0, \quad x_2(t_f) = 0 \tag{2.9}$$

Note that the linearized engagement kinematics presented herein is most commonly used equation for derivation of new guidance laws in the optimal control approach [16, 17, 18, 19, 20].

2.2. **LQ Optimal Homing Problem with Arbitrary Weighting Functions.** The general form of LQ optimal guidance problem can be expressed as follows:

$$\min_u J = \frac{1}{2}\left[x(t_f) - x_f\right]^T S_f \left[x(t_f) - x_f\right] + \frac{1}{2}\int_{t_0}^{t_f} x^T Q x + R u^2(\tau) d\tau,\ S_f \geq 0,\ Q,\ R > 0 \tag{2.10}$$

where S_f, Q, and R represent the weighting parameters for the terminal conditions, the state variables, and the control input, respectively. The notation of x_f denotes the terminal constraints of the state variables. In our consideration, these values are given by Eq. (2.8) for the interception or Eq. (2.9) for the impact angle control. In addition, the terminal conditions are considered as the soft constraints in this formulation. If $S_f \to \infty$ in order to consider the terminal conditions as the hard constraints, then the above problem can be converted as follows:

$$\min_u J = \frac{1}{2}\int_{t_0}^{t_f} x^T Q x + R u^2(\tau) d\tau, \quad Q,\ R > 0 \tag{2.11}$$

subject to $x(t_f) = x_f$. Letting $Q = 0$ in order to further remove the weighting parameter related with the state variables leads to:

$$\min_u J = \frac{1}{2}\int_{t_0}^{t_f} R u^2(\tau) d\tau, \quad R > 0 \tag{2.12}$$

In this equation, if we choose $R = 1$, then it is well-known the control effort minimization problem.

$$\min_u J = \frac{1}{2}\int_{t_0}^{t_f} u^2(\tau) d\tau \tag{2.13}$$

As stated in the introduction, this performance index has been widely applied for the derivations of the optimal guidance laws [8, 10, 11]. However, in the derivations of optimal guidance laws using this performance index as shown in Eq. (2.13), it cannot provide an additional degree of freedom in shaping the guidance command or the flight trajectory to accomplish some specific guidance goals. Hence, in this paper, we derive the guidance laws based on LQ optimal problem with arbitrary weighting functions R given by Eq. (2.12).

3. Guidance Law Design

In this section, the solutions of LQ optimal problem as mentioned above are provided. We first solve the homing guidance problems using the weighted performance index and suggest the solutions of the impact angle control guidance problems in a similar way. Additionally, an illustrative example of the proposed guidance law is provided in order to show the usefulness of the proposed results. Finally, the solutions of the proposed approach are compared with other approach [24] from the generalization standpoint.

3.1. **Reviews of Schwarz Inequality.** There are some approaches to solve the optimal guidance problems such as the variation of calculus [1], Riccati approach [2], the Schwarz's inequality [17, 18, 19, 20, 21], and so on. Among these ways, Schwarz's inequality approach is taken in this paper because it can easily give the solution of such class of problem. In this section, we simply review Schwarzs inequality for the readers who are not familiar with this concept.

Let $\phi_1(\tau)$ and $\phi_2(\tau)$ be any two real integrable functions in range $[t_1, t_2]$, then Schwarz's inequality is given by

$$\left[\int_{t_1}^{t_2} \phi_1(\tau)\phi_2(\tau)d\tau\right]^2 \leq \int_{t_1}^{t_2} \phi_1(\tau)d\tau \int_{t_1}^{t_2} \phi_2(\tau)d\tau \tag{3.1}$$

When $\phi_1(\tau) = \Gamma\phi_2(\tau)$ with Γ a constant from Eq. (3.1), the equality condition holds. This idea will be mainly used to solve the optimal guidance problems in the next section.

3.2. **Solutions of Homing Problems.** As shown in Eqs. (2.6) and (2.7), the linearized engagement kinematics can be regarded as a time-invariant system. Therefore, we can easily determine the general solution of this system using the well-known linear control theory as follows.

$$x(t_f) = \Phi(t_f - t_0)x(t_0) + \int_{t_0}^{t_f} \Phi(t_f - \tau)G(\tau)u(\tau)d\tau \tag{3.2}$$

where $\Phi(t)$ is the state transition matrix. If the system matrix F is a time-invariant, it can generally be given as follows:

$$\Phi(t) = L^{-1}\left[(sI - F)^{-1}\right] \tag{3.3}$$

In this equation, the notation of L^{-1} represents the inverse Laplace transform operator. Then, substituting Eq. (2.7) into Eq. (3.3) provides the following results.

$$\Phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \tag{3.4}$$

Changing the argument of $\Phi(t)$ simply yields:

$$\Phi(t_f - t_0) = \begin{bmatrix} 1 & t_f - t_0 \\ 0 & 1 \end{bmatrix} \tag{3.5}$$

By substituting Eqs. (2.7) and (3.5) into Eq. (3.2), we can further expand Eq. (3.2) as follows:

$$x_1(t_f) = x_1(t_0) + (t_f - t_0)x_2(t_0) + \int_{t_0}^{t_f} (t_f - \tau)u(\tau)d\tau \tag{3.6}$$

$$x_2(t_f) = x_2(t_0) + \int_{t_0}^{t_f} u(\tau)\tau \tag{3.7}$$

where $x_1(t_f)$ and $x_2(t_f)$ denote the lateral position and the lateral velocity at the final time, respectively. $x_1(t_0)$ and $x_2(t_0)$ are the lateral position and the lateral velocity in the beginning of the homing phase and the magnitudes of these values are decided according to the performance of a proper mid-course guidance. As shown in Eq. (2.8), the lateral position at the final time should be zero in order to achieve the perfect interception. Therefore, imposing this condition to Eq. (3.6) gives:

$$x_1(t_0) + (t_f - t_0)x_2(t_0) + \int_{t_0}^{t_f} -(t_f - \tau)u(\tau)d\tau \tag{3.8}$$

From Eq. (3.8), we introduce a slack variable $R(\tau)$ without change of formula as follows:

$$x_1(t_0) + (t_f - t_0)x_2(t_0) + \int_{t_0}^{t_f} -(t_f - \tau)R^{-1/2}(\tau)u(\tau)d\tau \tag{3.9}$$

In Eq. (3.9), let us set the following variables in order to apply Schwarz's inequality given by Eq. (3.1).

$$\phi_1(\tau) = R^{-1/2}(\tau)u(\tau) \tag{3.10}$$

$$\phi_2(\tau) = -(t_f - \tau)R^{-1/2}(\tau) \tag{3.11}$$

$$t_1 = t_0, \quad t_2 = t_f \tag{3.12}$$

Next, applying Schwarz's inequality leads to:

$$[x_1(t_0) + (t_f - t_0)x_2(t_0)]^2 \leq \int_{t_0}^{t_f} (t_f - \tau)^2 R^{-1}(\tau)d\tau \int_{t_0}^{t_f} R(\tau)u^2(\tau)d\tau \tag{3.13}$$

The equality condition holds under the following condition.

$$R^{1/2}u(\tau) = -\Gamma(t_f - \tau)R^{-1/2}(\tau) \tag{3.14}$$

The condition of Schwarzs inequality as given in Eq. (3.13) can be rewritten as follows after rearranging it.

$$\frac{[x_1(t_0) + (t_f - t_0)x_2(t_0)]^2}{2\int_t^{t_f}(t_f - \tau)^2 R^{-1}(\tau)d\tau} \leq \frac{1}{2}\int_t^{t_f} R(\tau)u^2(\tau)d\tau \tag{3.15}$$

From Eq. (3.14), the equality condition can also be rewritten as follows:

$$u(\tau) = -\Gamma(t_f - (\tau))R^{-1}(\tau) \tag{3.16}$$

Note that the right-hand side of Eq. (3.15) is identical to the performance index to be minimized in our problem as given in Eq. (2.12). And when the equality condition holds, the left-hand side of Eq. (3.15) can represent the minimum value of the performance index as follows:

$$J = \frac{[x_1(t_0) + (t_f - t_0)x_2(t_0)]^2}{2\int_t^{t_f}(t_f - \tau)^2 R^{-1}(\tau)d\tau} \tag{3.17}$$

Additionally, the control input as shown in Eq. (3.16) can be regarded as the optimal control input that minimizes the performance index given by Eq. (2.12). It is notated that a constant

Γ in Eq. (3.16) is not determined yet. Therefore, in order to determine Γ, we substitute Eq. (3.16) into Eq. (3.8).

$$x_1(t_0) + (t_f - t_0)x_2(t_0) = \Gamma \int_{t_0}^{t_f} (t_f - \tau)^2 R^{-1}(\tau) d\tau \tag{3.18}$$

From Eq. (3.18), we can obtain Γ as follows:

$$\Gamma = \frac{x_1(t_0) + (t_f - t_0)x_2(t_0)}{\int_{t_0}^{t_f} (t_f - tau)^2 R^{-1}(\tau) d\tau} \tag{3.19}$$

Then, substituting Γ into Eq. (3.16) provides

$$u(\tau) = \frac{-\left[x_1(t_0) + (t_f - t_0)x_2(t_0)\right](t_f - \tau)R^{-1}(\tau)}{\int_{t_0}^{t_f} (t_f - \tau)^2 R^{-1}(\tau) d\tau} \tag{3.20}$$

By initializing and recalculating the above command in the time domain, the optimal guidance command can be rewritten as

$$u(t) = \frac{-\left[x_1(t) + t_{go}x_2(t)\right] t_{go} R^{-1}(t)}{\int_{t}^{t_f} (t_f - \tau)^2 R^{-1}(\tau) d\tau} \tag{3.21}$$

where $t_{go} = t_f - t$ is called time-to-go, which means the remaining time of interception. By using the original expressions of state variables and control input, Eq. (3.21) can be expressed as follows:

$$a_M = \frac{-\left[y + \nu t_{go}\right] t_{go} R^{-1}(t)}{\int_{t}^{t_f} (t_f - \tau)^2 R^{-1}(\tau) d\tau} \tag{3.22}$$

For convenient, let us define a new variable as similar the navigation constant in the proportional navigation guidance (PNG), as follows:

$$N' = \frac{t_{go}^3 R^{-1}(t)}{\int_{t}^{t_f} (t_f - \tau)^2 R^{-1}(\tau) d\tau} \tag{3.23}$$

Hereafter, we call N' as the equivalent navigation constant in this paper for convenient. By using this expression, the above guidance command can be rewritten in a similar form of PNG in the linear formulation, as follows.

$$a_M = -N' \frac{(y + \nu t_{go})}{t_{go}^2} \tag{3.24}$$

For implementing, an alternative form of the proposed guidance law will be more useful because the guidance command presented in Eq. (3.24) still contain some variables (such as y and ν) that cannot be directly measureable. By using the geometry relation, we can replace those variables with measureable variables. From Fig. 1, the LOS angle can be approximate as follows:

$$\sigma \approx -\frac{y}{R} + \gamma_f \tag{3.25}$$

For a stationary target or a slowly moving target, the relative range between the missile and the target is also approximated as $R \approx V_M t_{go}$. Then, Eq. (3.25) can be rewritten as follows:

$$\sigma \approx -\frac{y}{V_M t_{go}} + \gamma_f \tag{3.26}$$

Taking time-derivation of Eq. (3.26) yields

$$\dot{\sigma} \approx -\frac{1}{V_M}\frac{(y + \nu t_{go})}{t_{go}^2} \tag{3.27}$$

Using this expression gives the alternative form of Eq. (3.24) as follows:

$$a_M = N' V_M \dot{\sigma} \tag{3.28}$$

Finally, we obtain the nonlinear form of PNG-like law with a time-varying navigation constant N'. In Eq. (3.28), the missile velocity and the LOS rate can be measured from an on-board inertial navigation system (INS) and a built-in seeker, respectively.

3.3. **Solutions of Impact Angle Control Problems.** The solutions of the impact angle control problems can also be determined in a similar way of the homing problems. In reference [18], we already determined that kind of problems. Accordingly, in this paper, we just provide the results of these problems. More detail explanations can be founded in reference [18]. The optimal impact angle guidance laws with arbitrary weighting functions in the linear formulation are expressed as follow.

$$a_M = -k_1 \frac{y}{t_{go}^2} - k_2 \frac{\nu}{t_{go}} \tag{3.29}$$

where k_1 and k_2 represent the equivalent guidance gains for the impact angle control, and these values are given by:

$$k_1 = \left(\frac{g_0 t_{go}^3 - g_1 t_{go}^2}{g_2 g_0 - g_1^2} \right) R^{-1}(t) \tag{3.30}$$

$$k_2 = \left(\frac{g_2 t_{go} + g_0 t_{go}^3 - 2 g_1 t_{go}^2}{g_2 g_0 - g_1^2} \right) R^{-1}(t) \tag{3.31}$$

where

$$g_j = \int_t^{t_f} (t_f - \tau)^j R^{-1}(\tau) d\tau \tag{3.32}$$

The guidance command form as shown in Eq. (3.29) can be rewritten in some implementing forms by replacing y and ν with the different parameters related to the geometry. From Fig. 1, the lateral velocity can be expressed as follows:

$$\gamma_M - \gamma_f = \frac{\nu}{V_M} \tag{3.33}$$

Using the relations given by Eq. (3.26) and (3.33) introduce an alternative form of the guidance law as follows:

$$a_M = -\frac{V_M}{t_{go}}\left[-k_1\sigma + k_2\gamma_M + (k_1 - k_2)\gamma_f\right] \tag{3.34}$$

This guidance command can also be rewritten in the term of LOS rate by using the expressions of Eq. (3.27) and (3.33).

$$a_M = k_2 V_M \dot{\sigma} + \frac{(k_1 - k_2)V_M}{t_{go}}(\sigma - \gamma_f) \tag{3.35}$$

In these guidance commands as shown in Eqs. (3.34) and (3.35), the presented parameters can be measured from INS and seeker except for the time-to-go. The time-to-go can also be estimated using the conventional method (i.e.,$t_{go} = R/V_M$) or the advanced method as presented in [11, 12, 22, 23].

3.4. **Discussion of Proposed Guidance Law.** It is noted that the equivalent navigation constant or the equivalent guidance gains can directly affect the guidance command as shown in Eq. (3.28) and (3.35). In the previous guidance laws, those values are fixed during the flight, therefore the guidance command shape and the flight trajectory are also fixed. Namely, the scalability or the potential of those guidance laws can be limited to achieve various guidance goals. However, as shown in Eqs. (3.23), (3.30), and (3.31), the equivalent navigation constant or the equivalent guidance gains in our findings are given by the function of time-to-go and arbitrary weighting functions. Hence, according to the selections of weighting functions, those values can be constants or time-varying values. There exist a lot of cases of guidance gains for various weighting functions. Therefore, the proposed results can provide an additional degree of freedom in the derivation of new guidance laws that can achieve the specific guidance objectives in a way of shaping the guidance command or the flight trajectory through appropriate choices in weighting functions. Accordingly, the proposed results can be regarded as the generalized solutions of homing and impact angle control guidance laws.

For example, if we choose $R(\tau) = 1$, we can simply obtain the energy optimal guidance laws for homing and impact angle control [11] as follows:

$$N' = 3, \quad k_1 = 6, \quad k_2 = 4 \tag{3.36}$$

Under the selection of a time-to-go weighted function as $R(\tau) = (t_f - (\tau))^{-n}$, the following results are directly obtained by substituting this weighting function to Eqs. (3.23), (3.30), and (3.31).

$$N' = N + 3, \quad k_1 = (N+3)(N+2), \quad k_2 = 2(N+2) \tag{3.37}$$

These results are identical to PNG with arbitrary navigation constants and the weighted impact angle control guidance laws as studied in [12], respectively. In a similar way, when we chose the following weighting function

$$R(\tau) = e^{\left[(t_f - (\tau)) - a\right]^2/(2b^2)} \tag{3.38}$$

where a and b are some constants, then the solution of homing guidance problem is the same as the result in [16] as

$$N' = \frac{t_{go}^3 e^{-\omega_1^2}}{b^2 \left[a e^{-\omega_2^2} - (t_{go} + a) e^{-\omega_1^2} + \sqrt{\frac{\pi}{2}} \frac{(a^2+b^2)}{b} (\text{erf}(\omega_1) + \text{erf}(\omega_2)) \right]} \tag{3.39}$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \tag{3.40}$$

$$\omega_1 = \frac{t_{go} - a}{\sqrt{2} b}, \quad \omega_2 = \frac{a}{\sqrt{2} b} \tag{3.41}$$

Next, the sinusoidal weighting function is chosen as follows

$$R(\tau) = \frac{1}{\sin(a(t_f - \tau) + c) + b} \tag{3.42}$$

where a, b and c are some constants. Then, the obtained guidance law is identical to the result in [20]. Lastly, under the following weighting function, we can obtain the optimal guidance law for anti-tank missile systems [19].

$$R^{-1}(\tau) \triangleq \begin{cases} \frac{b-a}{t_1} + a & : \ t_0 \leq \tau < t_1 \\ b & : \ t_1 \leq \tau \leq t_2 \\ \frac{c-b}{t_f - t_2}(\tau - t_2) + b & : \ t_2 < \tau \leq t_f \end{cases} \tag{3.43}$$

where a, b, c, t_1, and t_2 are some constants. As shown in above, the proposed results can contain previous optimal guidance laws according to the selections of weighting functions. Therefore, the proposed results can be regarded as more general solutions of the homing problems and the impact angle control problems compared with other guidance laws.

Hereafter, the proposed results are compared with other optimal guidance law with impact angle constraint as studied in [24] from the generalization standpoint. As shown in Eqs. (3.30) and (3.31), the sets of possible guidance gains according to the design parameter R in the proposed results can be expressed as follows:

$$U_1 = \left\{ k_1 = \left(\frac{g_0 t_{go}^3 - g_1 t_{go}^2}{g_2 g_0 - g_1^2} \right) R^{-1}(t), \ k_2 = \left(\frac{g_2 t_{go} + g_0 t_{go}^3 - 2 g_1 t_{go}^2}{g_2 g_0 - g_1^2} \right) R^{-1}(t), \right\} \tag{3.44}$$

where $R(\tau) > 0$.

In [24], the authors have suggested a kind of generalized optimal impact angle guidance law with arbitrary constant gains. The constant guidance gains for impact angle control as founded in [24] are given by three sets as follows:

$$U_2 = U_2^1 \cup U_2^2 \cup U_2^3 \tag{3.45}$$

where

$$U_2^1 = \left\{ k_1, k_2 \in \mathbb{R}^+ \middle| 2(k_2 - 1) \le k_1 \le \left(\frac{k_2+1}{2}\right)^2 \text{ and } k_2 > 3 \right\} \tag{3.46}$$

$$U_2^2 = \left\{ k_1, k_2 \in \mathbb{R}^+ \middle| k_1 = \left(\frac{k_2+1}{2}\right)^2 \text{ and } k_2 > 3 \right\} \tag{3.47}$$

$$U_2^3 = \left\{ k_1, k_2 \in \mathbb{R}^+ \middle| k_1 = \left(\frac{k_2+1}{2}\right)^2 \text{ and } k_2 \ge 3 \right\} \tag{3.48}$$

The authors have also determined the performance index that can introduce the above guidance gains using the inverse problem of linear optimal control methodology [25, 26]. Pursuant to [24], the performance index for the above solutions is expressed as

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left[x^T Q(\tau) x + R(\tau) u^2(\tau)\right] d\tau \tag{3.49}$$

In this equation, the weighting parameter of state variables is given by

$$Q(\tau) = \begin{bmatrix} \frac{k_1^2 - 3k_1^2/k_2}{(t_f-\tau)^4} R(\tau) - \frac{k_1^2/k_2}{(t_f-\tau)^3}\dot{R}(\tau) & \frac{k_1k_2 - k_1^2/k_2 - 2k_1}{(t_f-\tau)^3} R(\tau) - \frac{k_1}{(t_f-\tau)^2}\dot{R}(\tau) \\ \frac{k_1k_2 - k_1^2/k_2 - 2k_1}{(t_f-\tau)^3} R(\tau) - \frac{k_1}{(t_f-\tau)^2}\dot{R}(\tau) - q & \frac{k_2^2 - k_2 - 2k_1}{(t_f-\tau)^2} R(\tau) - \frac{k_2}{(t_f-\tau)}\dot{R}(\tau) \end{bmatrix} \tag{3.50}$$

where q is an arbitrary value. From Eq. (3.50), we can easily observe that $Q(\tau)$ is given by the function of the guidance gains (k_1 and k_2), the weighting function of the control input $R(\tau)$, and the slack variable q, respectively.

As shown in Eq. (3.49), the generalized impact angle guidance laws in [24] have been determined based on the different performance index compared with our choice in the performance index as shown in Eq. (2.12). In the above equation, if there exists $R(\tau)$ that introduces $Q(\tau) = 0_{2\times2}$ with arbitrary selections of guidance gains in the set of U_2, then we can show that the proposed results can contain the results of [24]. If not, there are some intersections of the guidance gains between the results of [24] and our findings. Therefore, in the remaining of this section, we try to find the existence of the weighting function $R^*(\tau)$ that makes $Q(\tau) = 0_{2\times2}$. Since $Q(\tau) = Q(\tau)^T$, imposing the condition of $Q(\tau) = 0_{2\times2}$ introduces the following equations.

$$\frac{k_1^2 - 3k_1^2/k_2}{(t_f-\tau)^4} R^*(\tau) - \frac{k_1^2/k_2}{(t_f-\tau)^3}\dot{R}^*(\tau) - \dot{q}^* = 0 \tag{3.51}$$

$$\frac{k_1k_2 - k_1^2/k_2 - 2k_1}{(t_f-\tau)^3} R^*(\tau) - \frac{k_1}{(t_f-\tau)^2}\dot{R}^*(\tau) - q^* = 0 \tag{3.52}$$

$$\frac{k_2^2 - k_2 - 2k_1}{(t_f-\tau)^2} R^*(\tau) - \frac{k_2}{(t_f-\tau)}\dot{R}^*(\tau) = 0 \tag{3.53}$$

For $R^*(\tau)$, all equations as shown in Eqs. (3.51), (3.52), and (3.53) should be satisfied simultaneously. Note that Eq. (3.53) is given by Cauchy–Euler equation with respect to $\dot{R}(\tau)$.

Therefore, we can guess the solution of this equation as follows:

$$R^*(\tau) = (t_f - \tau)^p \tag{3.54}$$

where p is some constant value. Substituting Eq. (3.54) into Eq. (3.53) gives

$$p = -\frac{k_2^2 - k_2 - 2k_1}{k_2} \tag{3.55}$$

Substituting Eq. (3.55) into Eq. (3.54) yields the solution of Eq. (3.53) as

$$R^*(\tau) = (t_f - \tau)^{-\frac{k_2^2 - k_2 - 2k_1}{k_2}} \tag{3.56}$$

Since this $R^*(\tau)$ should satisfy the other two equations, $R^*(\tau)$ is substituted into Eq. (3.52). Then, the slack variable is determined as:

$$q^* = \frac{k_1(k_1 - k_2)}{k_2}(t_f - \tau)^{-\frac{k_2^2 - k_2 - 2k_1}{k_2} - 3} \tag{3.57}$$

Finally, to achieve the result of $Q(\tau) = 0_{2\times 2}$, the remaining equation as given in Eq. (3.51) is also satisfied under the selections of $R^*(\tau)$ and $q^*(\tau)$. Therefore, substituting Eq. (3.56) and (3.57) into Eq. (3.51) leads to:

$$\frac{4k_1(k_1 - k_2)\left[k_1 - \left(\frac{k_2+1}{2}\right)^2 - \frac{1}{4}\right]}{k_2^2}(t_f - \tau)^{-\frac{k_2^2 - k_2 - 2k_1}{k_2} - 4} = 0 \tag{3.58}$$

Note that this condition should be satisfied with arbitrary selections of guidance gains in the set of U_2 to guarantee the existence of $R^*(\tau)$ as we want. However, this condition is only satisfied with the specified guidance gains as follows:

$$k_1 = 0, \quad k_1 = k_2, \quad \text{or} \quad k_1 = \left(\frac{k_2 + 1}{2}\right)^2 - \frac{1}{4} \tag{3.59}$$

It means we cannot find $R^*(\tau)$ that makes $Q(\tau) = 0_{2\times 2}$ with arbitrary selections of guidance gains in the set of U_2. Only confined set of guidance gains as given in Eq. (3.59) can introduce the result of $Q(\tau) = 0_{2\times 2}$. Since first two conditions (i.e., $k_1 = 0,\ k_1 = k_2$) are infeasible solutions for impact angle control from above results, we can obtain the set of guidance gains that introduces $Q(\tau) = 0_{2\times 2}$ as follows:

$$U_3 = \left\{k_1, k_2 \in |k_1 = \left(\frac{k_2 + 1}{2}\right)^2 - \frac{1}{4}\right\} \tag{3.60}$$

Therefore, the set of Eq. (3.60) can be regarded as the intersections of guidance gains between the proposed results and the results of [24] as

$$U_3 = U_1 \cap U_2 \tag{3.61}$$

It is also noted that the guidance gains in U_3 are identical to the findings in [12].

3.5. **Illustrative Examples of Proposed Guidance Law.** In this section, an illustrative example of the proposed results is provided. We first discuss the considering guidance objective, and then a new guidance law that accomplishes that objective is derived through an appropriate selection of weighting functions in the proposed framework.

During the guidance handover from the midcourse guidance to the terminal homing guidance, the presence of the initial heading error is unavoidable. And, the magnitude of the guidance command in the beginning of the terminal homing is generally proportional to this initial heading error. If the initial guidance command caused by the initial heading error is too huge, the sudden change of the guidance command during the guidance handover is expected. Accordingly, such an abrupt guidance command causes an unwanted missile motion, and an onboard seeker feels that a target is maneuvering. Namely, the seeker may provide incorrect measurements of target. Such incorrect measurements can generate incorrect guidance commands, and these mismatched commands can lead to unwanted missile motions and incorrect measurements of seeker in return. As a result, this parasitic loop effect caused by the missile motion coupling may lead to the instability of the homing loop. Therefore, a guidance law providing the insensitive against the initial heading error is needed to avoid the problem as mentioned above. In the case of well-known the terminal homing guidance law such as PNG, the initial guidance command due to the initial heading error is given by

$$a_M(t_0) \approx -N\frac{V_M\varepsilon_\theta}{t_f} \tag{3.62}$$

where ε_θ represents the initial heading error. As shown in Eq. (3.62), the initial guidance command is proportional to the navigation constant N and the missile velocity V_M . For a fixed V_M , we can predict that the sensitivity with respect to the initial heading error is mainly affected by the selection of the navigation constant N . To reduce the sensitivity, the small value of the navigation constant should be used. However, according to reference [27], the navigation constant ranging from 3 to 5 is suitable value in practice to guarantee the acceptable homing performance. Therefore, since the guidance properties of PNG, a severe sensitivity against the initial heading error is unavoidable in the implementation of PNG with $N \geq 3$

Hereafter, as a remedy, we try to derive a new guidance law which can overcome this problem by using the proposed results. In order to shape the guidance command providing a small value at the initial phase, we suggest a rational weighting function as follows:

$$R(\tau) = \frac{1}{t_f - \tau} + a^2(t_f - \tau), \quad \text{for} \quad a > 0 \tag{3.63}$$

where the variable a represents the design parameter of this weighting function. In the beginning of the homing phase (i.e. $t_f \gg \tau$), the second term in Eq. (3.63) is dominant. Therefore, the cost value around the initial time is expensive. Therefore, we can expect that the obtained guidance law will provide a small magnitude of guidance command. If $\tau \to \tau_f$, then the first term of Eq. (76) is more dominant than the second term. It also introduces a large value of $R(\tau)$. Hence the cost value is also expensive at the final time. From Eq. (3.63), we can expect that the proposed guidance law can generate a small guidance command at the initial and the final time. These properties are desirable for alleviating the sensitivity against the initial heading error and improving the terminal guidance performance. From Eq. (3.63), it is also noted that the minimum value is achieved in the proposed weighting function at $\tau_{min} = t_f - 1/a$ as

$$R(\tau_{\min}) = 2a \tag{3.64}$$

Next, the equivalent navigation constant and the equivalent guidance gains are determined using this weighting function. First, substituting Eq. (3.63) into Eq. (3.23) gives

$$N' = \frac{2\mu^4}{(\mu^2+1)(\mu^2 - \ln(\mu^2+1))} \tag{3.65}$$

where $\mu = at_{go}$. In a similar way, the equivalent guidance gains can be obtained by substituting Eq. (3.63) into Eq. (3.30) and (3.31) as follows:

$$k_1 = \frac{2\mu^3[\mu\ln(\mu^2+1) - 2(\mu - \tan^{-1}(\mu))]}{(\mu^2+1)[\mu^2\ln(\mu^2+1) - \ln(\mu^2+1)^2 - 4(\mu - \tan^{-1}(\mu))^2]} \tag{3.66}$$

$$k_2 = \frac{2\mu^2[\mu^2\ln(\mu^2+1) - \ln(\mu^2+1) - 3\mu^2 + 4\mu\tan^{-1}(\mu)]}{(\mu^2+1)[\mu^2\ln(\mu^2+1) - \ln(\mu^2+1)^2 - 4(\mu - \tan^{-1}(\mu))^2]} \tag{3.67}$$

These values are given by the combinations of the polynomial function, the logarithm function, and the tangent function of time-to-go. From these results, we can observe some interesting features. If the design parameter a approaches zero, then these guidance gains converge to following values.

$$\lim_{a\to 0} N' = 4, \quad \lim_{a\to 0} k_1 = 12, \quad \lim_{a\to 0} k_2 = 6 \tag{3.68}$$

Conversely, if $a \to \infty$, the following values are obtained as

$$\lim_{a\to \infty} N' = 2, \quad \lim_{a\to \infty} k_1 = 2, \quad \lim_{a\to \infty} k_2 = 2 \tag{3.69}$$

According to the selections of the design parameter a, the value of the equivalent navigation constant is changed from 2 to 4. In the case of the impact angle control, the equivalent guidance gains are laid on the values ranging from 2 to 12 for k_1 and 2 to 6 for k_2. As the design parameter increases, the values of guidance gains decrease. Therefore, small values of guidance gains can be achieved through an appropriate choice in design parameters, and these values can be helpful to reduce the sensitivity to the initial heading error. In addition, as the missile approaches a target (i.e., $t_{go} \to 0$), the following results are obtained under the proposed guidance law.

$$\lim_{t_{go}\to 0} N' = 4, \quad \lim_{t_{go}\to 0} k_1 = 12, \quad \lim_{t_{go}\to 0} k_2 = 6 \tag{3.70}$$

It is noted that these values are always achieved under the proposed guidance law, regardless of the design parameter a. These large values of guidance gains in the terminal homing phase are desirable for ensuring the terminal homing performance. Therefore, through the guidance command shaping, the proposed guidance law can simultaneously achieve two guidance objectives: alleviating the sensitivity against the initial heading error and improving the terminal guidance performance. In the next section, the performance of the proposed guidance law will be determined.

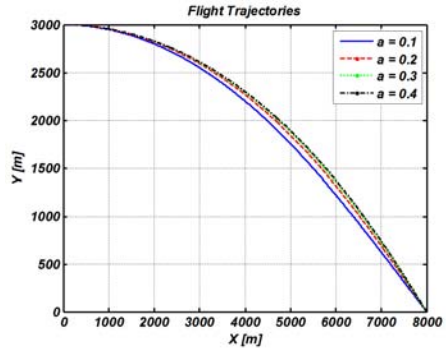
4. Simulation Results

In this section, we investigate the basic characteristics and the performance of the devised guidance law in Section 3.5. First, the basic properties of the proposed guidance law for various design parameters are determined. In the second simulation, the proposed guidance law is compared with the other guidance laws as studied in [21] and [11] to show the superiority of the proposed guidance law. The considering engagement conditions are given by Table 1.

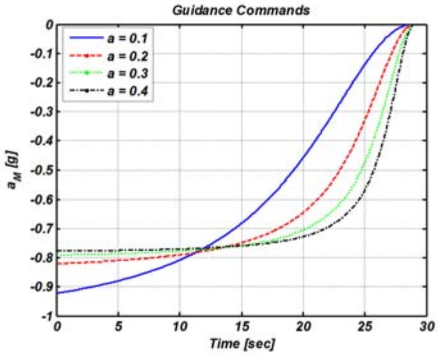
TABLE 1. Initial conditions for nonlinear simulations

Parameters	Values
Vehicle position, $X_M(0),\ Y_M(0)$	$(0, 2000)$m
Target position, $X_T(0),\ Y_T(0)$	$(8000, 0)$m
Vehicle velocity, V_M	300m/s
Initial flight path angle, $\gamma_M(0)$	$0°$
Desired impact angle, γ_f	$-70°$
Design Parameters, a	$0.1, 0.2, 0.3, 0.4$

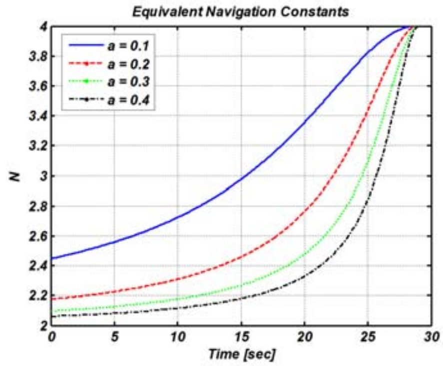
4.1. **Basic Properties of Proposed Guidance Law.** Figures 2 (a), (b), and (c) provide the flight trajectory, the guidance command, and the behavior of equivalent navigation constant for various design parameters in the interception case. As shown in Fig. 2 (a), in all cases, the proposed guidance law can successfully satisfy the terminal interception condition as well. From Fig. 2 (b), we can observe that the guidance commands at the initial phase are reduced as the design parameters increase. As mentioned before, this characteristic can be useful for reducing the sensitivity with respect to the initial heading error. In addition, we can observe an interesting result that the guidance commands under the proposed guidance law keep near constant values over the engagement. Namely, the proposed guidance law can evenly distribute the guidance effort during the entire flight. This property is also important to prevent the command saturation. As shown in Fig. 2 (c), the equivalent navigation constants approach the specific value as 4 regardless of selections in the design parameter. This result is well matched with the analytic result as studied in previous section. And, this result implies that the proposed guidance law can provide similar performance of PNG with $N = 4$ in the vicinity of a target. Namely, the proposed guidance law can ensure the terminal performance.



(a) Flight Trajectories



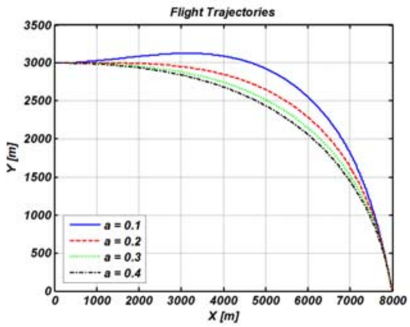
(b) Guidance Commands



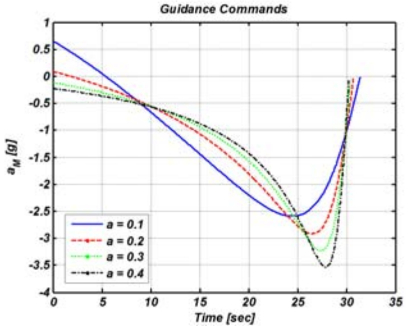
(c) Equivalent Navigation Constants

FIGURE 2. Simulation results of proposed homing guidance laws for various design parameters

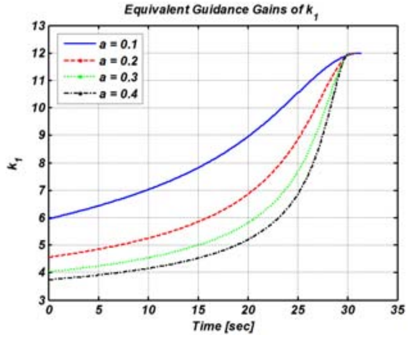
Figures 3 (a), (b), (c), (d), and (e) describe the flight trajectory, the guidance command, the guidance gain k_1, the guidance gain k_2, and the flight path angle, respectively. As shown in Fig. 3 (a) and (e), the proposed guidance law can successfully achieve the perfect interception and the satisfaction of the desired terminal impact angle. Additionally, from Fig. 3 (b), we can observe that the level of the initial guidance command can be handled by choosing an appropriate value of design parameter under the proposed guidance law. As shown in Fig. 3 (c) and (d), the guidance gains approach 12 for k_1 and 6 for k_2 , respectively. These results agree with the investigation in Section 3.5.



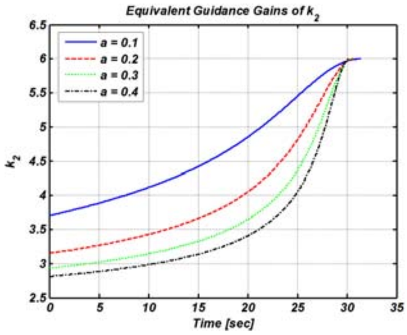
(a) Flight Trajectories



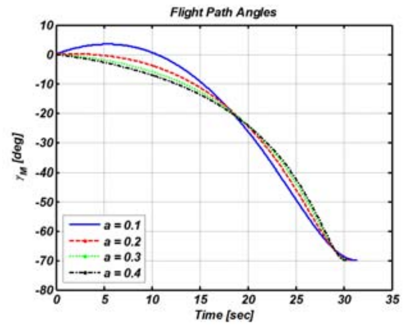
(b) Guidance Commands



(c) Equivalent Guidance Gain of k_1



(d) Equivalent Guidance Gain of k_2

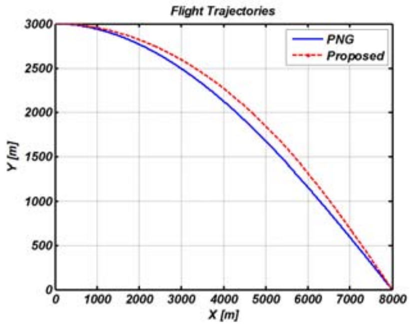


(e) Flight Path Angles

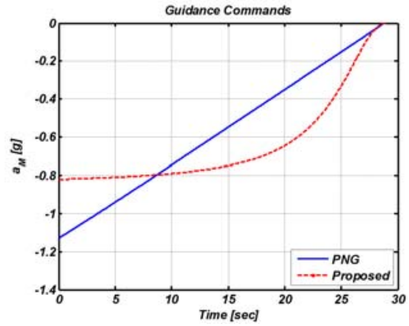
FIGURE 3. Simulation results of proposed impact angle control laws for various design parameters

4.2. **Comparisons of Other Guidance Laws.** In this section, we perform comparative studies between the proposed guidance laws and the existing guidance laws. The proposed guidance law in consideration the homing is compared with well-known PNG [21] with $N = 3$. The proposed guidance law for impact angle control is compared with optimal guidance law (OGL) [11]. In all simulations in this section, the design parameter of the proposed guidance law is chosen as $a = 0.2$.

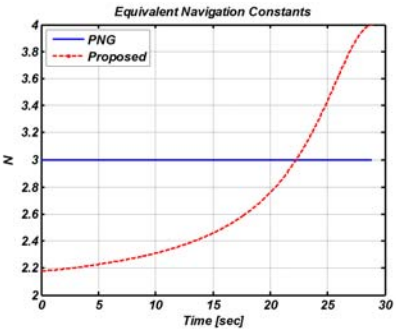
Figures 4 (a), (b), and (c) provide the flight trajectory, the guidance command, and the time history of the navigation constant under PNG and the proposed guidance law for the interception case. As shown in Fig. (c), the navigation constant at the initial phase under the proposed guidance law is smaller than the value of PNG. Therefore, we can predict that the proposed guidance law can reduce the magnitude of initial guidance command. This feature is desirable for reducing the command sensitivity against the initial heading error. In addition, the navigation constant of the proposed guidance law approaches $N = 4$ as $t_{go} \to 0$, therefore it implies that the proposed guidance law can provide a similar guidance performance of PNG in the terminal homing phase. Accordingly, the proposed guidance law can simultaneously accomplish the alleviation of the command sensitivity in the initial time and the interception of target with acceptable accuracy.



(a) Flight Trajectories



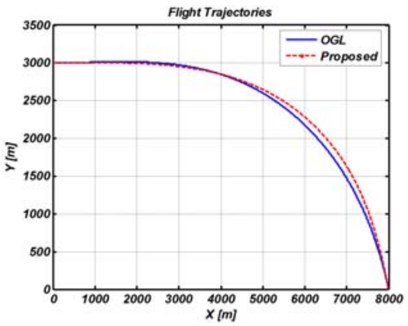
(b) Guidance Commands



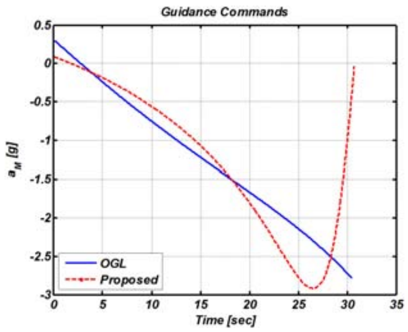
(c) Equivalent Navigation Constants

FIGURE 4. Simulation results under PNG and the proposed guidance law in the case of interception

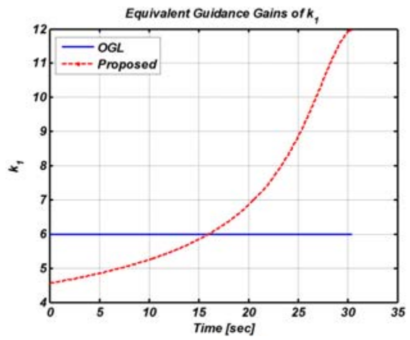
Figures 5 (a), (b), (c), (d), and (e) represent the simulation results under the circumstance of the impact angle control. From these results, we can see that the interesting features of the proposed guidance law as obtained in the interception case are also observed in the case of the impact angle control, compared with the previous guidance law as OGL. Additionally, the proposed guidance law can introduce a small guidance command in the vicinity of a target, unlike OGL. In practice, such characteristic of the proposed guidance law is desirable for saving the control effort to cope with the uncertainties or changes of engagement situations in the terminal phase.



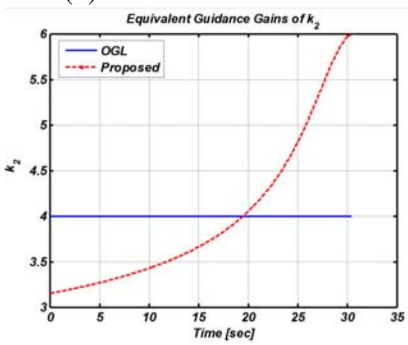
(a) Flight Trajectories



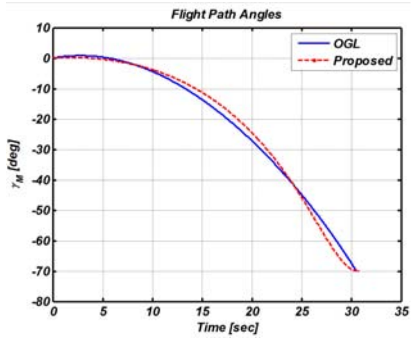
(b) Guidance Commands



(c) Equivalent Guidance Gain of k_1



(d) Equivalent Guidance Gain of k_2



(e) Equivalent Guidance Gain of

FIGURE 5. Simulation results under PNG and the proposed guidance law in the case of impact angle control

5. Conclusion

This paper dedicates the development of a new approach to guidance law design using the linear quadratic optimal control with arbitrary weighting functions. In order to provide an additional degree of freedom in designing a new guidance law, we consider the performance index which is given by the control effort weighted by arbitrary functions and determine the general solutions of optimal guidance problems in conjunction with that kind of performance index. Therefore, according to selections of arbitrary weighting functions, a lot of useful guidance laws can newly be derived based on the proposed results. In this paper, the proposed results are compared with other results from a view of generalization.

Additionally, a new guidance law providing the specific command shape is suggested as an illustrative example of our findings. Under the new guidance law, a small magnitude of the guidance command is generated in the beginning of the homing phase. Hence the sensitivity problem with respect to the initial heading error is alleviated under the new guidance law. The nonlinear simulations are also taken to investigate the characteristic of the new guidance law.

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