J. KSIAM Vol.19, No.3, 305–325, 2015 http://dx.doi.org/10.12941/jksiam.2015.19.305

POLYNOMIAL FUNCTION BASED GUIDANCE FOR IMPACT ANGLE AND TIME CONTROL

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ABSTRACT. In this paper, missile homing guidance laws to control the impact angle and time are proposed based on the polynomial function. To derive the guidance commands, we first assume that the acceleration command profile can be represented as a polynomial function with unknown coefficients. After that, the unknown coefficients are determined to achieve the given terminal constrains. Using the determined coefficients, we can finally obtain the state feedback guidance command. The suggested approach to design the guidance laws is simple and provides the more generalized optimal solutions of the impact angle and time control guidance.

1. INTRODUCTION

Advanced guidance laws with several terminal constraints as well as zero miss-distance have been proposed for the guidance performance improvement, high kill probability, and warhead effectiveness maximization, and survivability enhancement. To achieve these objectives, the impact angle and time have been considered as important terminal state constraints for homing missiles.

For anti-tank missiles or air-to-surface missiles, the impact angle constraint is needed for the purpose of maximizing the warhead effect and hitting weak spots on the target. For decades, many impact angle control guidance laws have been proposed based on the optimal control theory or nonlinear control design methods, etc. [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. The authors of [1] proposed a suboptimal guidance law with a terminal body attitude constraint for reentry vehicles. An impact angle control guidance law for varying velocity missiles and maneuvering targets was derived in [2], where the proposed law based on the optimal control theory is combined with a target tracking filter for real applications. In [3], a generalized form of energy optimal guidance for arbitrary order missile dynamics was proposed, which achieves both the desired impact angle and zero miss-distance. As an extension of this study, another type of the optimal guidance law was developed in [4], by solving a linear quadratic optimal control problem with a time-to-go weighted energy cost. Impact angle constrained guidance laws with additional constraint on terminal acceleration were suggested in [5] and [13], where the proposed laws can nullify the terminal maneuvering

Received by the editors August 6 2015; Revised August 17 2015; Accepted in revised form August 19 2015; Published online September 24 2015.

acceleration for command saturation avoidance and robustness against time-to-go estimation errors. Modified proportional navigation guidance (PNG) laws to control the impact angle were also devised in [6, 7, 8]. In [6], a time-varying biased PNG law with a bias term, which is a function of time-to-go, was suggested. In a surface-to-surface planar engagement, the authors of [7, 8] proposed composite guidance laws for intercepting a stationary or moving target with a desired impact angle, on the basis of the conventional PNG law. These proposed laws consist of an orientation guidance for covering various terminal angles and the classical PNG law with a specific navigation constant. They also proposed an guidance law by using the state-dependent Riccati equation (SDRE) technique [9]. In addition to the above guidance laws, other guidance laws to satisfy the terminal angle constraint have been developed: terminal body angle constrained guidance law based on the linear quadratic terminal control problem [10], impact angle controller for minimum flight time [11], impact angle control law derived by the backstepping control method [12], and optimal solution to a simple rendezvous problem which can be used to control the impact angle [15, 16].

Modern battleships are equipped with advanced defense systems such as anti-air defense missile systems, ECM systems, and CIWS (close-in weapon system). These defensive weapons dramatically reduce the survivability of anti-ship missiles, so that sometimes the missiles cannot accomplish their missions. In order to enhance the survivability against the defense systems, therefore, the guidance laws with terminal time constraint, which can be used for the salvo attack or cooperative attack missions, have been devised. Despite a number of studies on impact angle control guidance problems for decades, studies on impact time control guidance laws have attracted more attention in recent years. The proposal in [19, 27] for a suboptimal guidance law with the impact time constraint appears to be the first attempt to design an impact time control guidance law. The authors also proposed a new guidance law to control both of the impact time and angle, by using the jerk control term and optimal control theory [20, 28]. In [21], the PNG law with a time-varying navigation constant, called cooperative proportional navigation (CPN), was suggested to make the missiles performing the cooperative attack mission intercept the target at the same arrival time. Based on nonlinear control design methods, impact time and angle control laws were derived in [22, 23, 24]. In [22], the feedback linearization method is used to obtain the impact time control law, and the backstepping control method and modified PNG law are utilized to design the impact time and angle controller in [23]. The impact time and angle control law in [24] was derived from the proposed line-of-sight rate shaping technique and second-order sliding mode control, in which both the line-of-sight (LOS) angle and rate profiles are determined to satisfy given terminal constraints and then sliding mode control is designed to track the obtained LOS rate profile. A homing guidance law, consisting of the well-known optimal impact angle control law and an additional command to meet the impact time constraint, was presented in [25].

In this paper, the simple approach, which was first proposed by Tahk of [14], is applied to design homing guidance laws with impact angle and time constraints. This approach is that firstly the guidance command is assumed to be represented as a polynomial function with unknown coefficients and then the coefficients of the assumed function are determined to satisfy the given terminal constraints. From these procedures, we can finally derive a feedback form of the guidance command that can achieve the given constraints. As compared with the optimal control theory or nonlinear control design methods, Tahks design approach is very simple and easy for developing the terminal state constrained guidance laws, and it can also give a more generalized optimal solution of the guidance law.

This paper includes two major topics: polynomial function based impact-angle-control guidance law and impact-time-control guidance law. Therefore, this paper is organized as follows: Section 2 presents impact angle controller, which can hit a stationary target with a designated impact angle. In Section 3, the impact time constrained guidance is proposed using the linearized homing problem with constraint on the homing trajectory length. In Section 4, the performance and characteristics of the proposed guidance laws are demonstrated through nonlinear simulations with various engagement conditions. The final section discusses the conclusions of this work.

2. Polynomial Guidance with Impact Angle Constraint

The impact angle control guidance for a stationary target is proposed in this section. The proposed guidance is able to achieve both terminal zero acceleration and angle constraints. In order to derive the guidance law, we first assume that the guidance command can be represented as the time-to-go polynomial function with two unknown coefficients. And then we determine the unknown coefficients which can satisfy the given terminal constraints. Finally, the feedback acceleration command can be obtained from the determined coefficients and polynomial function. In this section, we also propose a time-to-go calculation method for the implementation of the proposed guidance law.

2.1. **Problem Statement.** Let us consider the two-dimensional engagement geometry against a stationary target, as depicted in Fig. 1. The inertial frame is denoted as (X_I, Y_I) , and the rotated frame by the desired impact angle, γ_f , is denoted as (X_f, Y_f) . The subscripts M and T represent the missile and target. V_M, a_M and γ_M represent the missile velocity, acceleration perpendicular to the velocity, and flight path angle in inertial frame, respectively. γ denotes the impact angle error, which is the flight path angle w.r.t X_f-axis. The other variables in Fig. 1 are self-explanatory.

Assuming the constant missile velocity and small impact angle error, the linearized equations of the motion for this engagement can be derived as

$$\begin{aligned} \dot{y} \approx V_M \gamma = \nu, \quad & y(t_0) = y_0 \\ \dot{\nu} = a_M, \quad & v(t_0) = V_M \gamma_0 = \nu_0 \end{aligned} \tag{2.1}$$

where y and v are the lateral position and velocity perpendicular to X_f-axis. The subscript 0 presents the initial time. In this engagement kinematics, we neglect the gravitational force and autopilot lag.

In the optimal guidance problems, the acceleration command solutions, including PNG, are represented as time-to-go t_{go} polynomial functions. In view of this observation, let the function of the acceleration command, which can control the terminal angle, be defined as the following

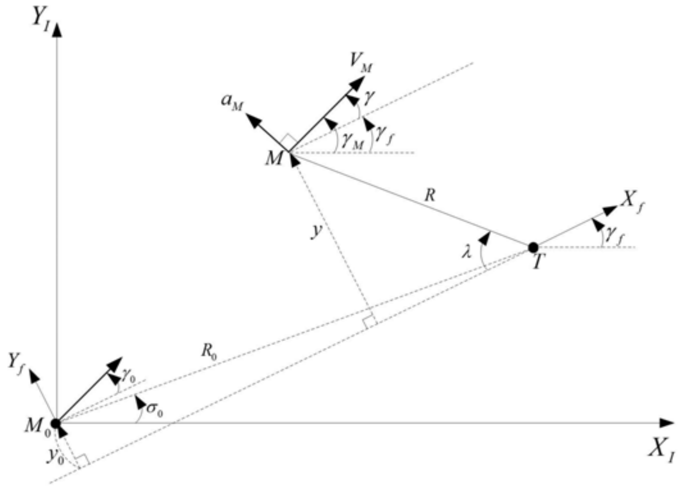


FIGURE 1. Two-dimensional Engagement Geometry

polynomial function

$$a_M(t) = c_m t_{go}^m + c_n t_{go}^n \tag{2.2}$$

where $t_{go} = t_f - t$ and guidance gains m, n are positive constants with $n > m \geq 0$. Using this assumed function, the states feedback command for the impact angle control can be derived.

2.2. **Guidance Command for Impact-Angle-Control.** In Eq.(2.2), only two coefficients c_m and c_n are used to determine a unique solution because the two terminal conditions, $y(t_f) = \nu(t_f) = 0$, are required for the impact angle control problem. Note that the acceleration at the final time, $a_M(t_f)$, can be zero when $m > 0$ in Eq. (2.2).

When substituting Eq. (2.2) into Eq. (2.1), and thereafter integrating the resulting equation using initial conditions, the lateral position and velocity at t_f are calculated as

$$\begin{aligned} y(t_f) &= y_0 + \nu_0 \hat{t}_{go} + c_m \left(\frac{\hat{t}_{go}^{m+2}}{m+2} \right) + c_n \left(\frac{\hat{t}_{go}^{n+2}}{n+2} \right) \\ \nu(t_f) &= \nu_0 + c_m \left(\frac{\hat{t}_{go}^{m+1}}{m+1} \right) + c_n \left(\frac{\hat{t}_{go}^{n+1}}{n+1} \right) \end{aligned} \tag{2.3}$$

where $\hat{t}_{go} = t_f - t_0$. The coefficients are then determined from the terminal constraints, $y(t_f) = \nu(t_f) = 0$, as follows:

$$\begin{aligned} c_m &= \frac{(m+1)(m+2)}{(n-m)\hat{t}_{go}^{m+2}} \left[(n+2)y_0 + \hat{t}_{go}\nu_0\right] \\ c_n &= \frac{(n+1)(n+2)}{(m-n)\hat{t}_{go}^{n+2}} \left[(m+2)y_0 + \hat{t}_{go}\nu_0\right] \end{aligned} \tag{2.4}$$

The guidance command at $t = t_0$ can be obtained by inserting Eq. (2.4) into Eq. (2.2). If the coefficients are recalculated at each time step, the state feedback command for the impact-angle-control can be finally expressed as

$$a_{IAC}(t) = -\frac{(m+2)(n+2)}{t_{go}^2} y(t) - \frac{(m+n+3)}{t_{go}} \nu(t) \tag{2.5}$$

where m and n are chosen to be any positive real value. If missiles are equipped with a passive seeker which provides LOS angle measurements, σ, the command of Eq. (2.5) can be rewritten by substituting $y \approx R(\gamma_f - \sigma)$ and $R \approx V_M t_{go}$ into Eq. (2.5) as

$$a_{IAC}(t) = -\frac{V_M}{t_{go}} \left[-(m+2)(n+2)\sigma(t) + (m+n+3)\gamma_M(t) + (m+1)(n+1)\gamma_f\right] \tag{2.6}$$

where σ and γ_M are measured from the built-in seeker and INS on board the missile, and γ_f is predetermined before the launch. However, t_{go} cannot be directly measured from any equipment, therefore we should estimate the time-to-go using an appropriate method with INS information and the estimated relative range. The time-to-go calculation method for the implementation is discussed in Sec. 2.3.

The closed-form trajectory solutions of the proposed law can be easily obtained by solving the second-order ordinary differential equation, which is calculated by substituting Eq. (2.5) into Eq. (2.1). The closed-form solutions of differential equation, the same form as Euler-Cauchy equation in [26], are expressed in two time-to-go terms.

$$\begin{aligned} y(t) &= C_1 t_{go}^{n+2} + C_2 t_{go}^{m+2} \\ \nu(t) &= -C_1(n+2)t_{go}^{n+1} - C_2(m+2)t_{go}^{m+1} \\ a_M(t) &= C_1(n+2)(n+1)t_{go}^{n} + C_2(m+2)(m+1)t_{go}^{m} \end{aligned} \tag{2.7}$$

where the constants of integration C_1 and C_2 are

$$C_1 = \frac{(m+2)y_0 + \hat{t}_{go}\nu_0}{(m-n)\hat{t}_{go}^{n+2}}, \quad C_2 = \frac{(n+2)y_0 + \hat{t}_{go}\nu_0}{(n-m)\hat{t}_{go}^{m+2}} \tag{2.8}$$

From the solutions given in Eq. (2.7), the proposed guidance can be regarded as a more general form of the impact angle controller because the choice of m and n are not restricted; that is, both the impact angle error and miss-distance converge to zero as $t \to t_f$ for the nonnegative real values of the gains. According to several combinations of m and n, the proposed acceleration command also involve the various optimal impact angle controllers previously studied in the literature [3, 4]. If $m = 0$ and $n = 1$, Eq. (2.5) is the same as the optimal control law in [3], and also identical to the optimal guidance law proposed in [4] when the guidance gains have integer values with $n = m + 1$ relation, as shown in Table. 1

TABLE 1. Examples of Proposed Impact Angle Control Law

Gains	Guidance Command	Note
$m = 0, n = 1$	$-\frac{6}{t_{go}^2}y(t) - \frac{4}{t_{go}}\nu(t)$	Same as [3]
$m = 0.5, n = 1$	$-\frac{7.5}{t_{go}^2}y(t) - \frac{4.5}{t_{go}}\nu(t)$	New form
$m = 1, n = 2$	$-\frac{12}{t_{go}^2}y(t) - \frac{6}{t_{go}}\nu(t)$	Same as [4]
$m = 1, n = 3$	$-\frac{15}{t_{go}^2}y(t) - \frac{7}{t_{go}}\nu(t)$	New form
$m = 2, n = 3$	$-\frac{20}{t_{go}^2}y(t) - \frac{8}{t_{go}}v(t)$	Same as [5]

Using the inverse problem approach of the optimal control theory [18, 29, 30], we can find the performance index of the proposed guidance inversely from the command in Eq. (2.5). As a result, the performance index associated with Eq. (2.5) is obtained as

$$J = \frac{1}{2}x^T(t_f)S_f x(t_f) + \frac{1}{2}\int_0^{t_f}(x^T Qx + ru^2)dt, \quad x = \begin{bmatrix} y, & \nu \end{bmatrix}^T, \quad u = a_M \tag{2.9}$$

where

$$\begin{aligned} S_f &= \lim_{t_{go}\to 0}\begin{bmatrix} \frac{(m+2)(n+2)(n+1)}{t_{go}^{m+3}} & \frac{(m+2)(n+2)}{t_{go}^{m+2}} \\ \frac{(m+2)(n+2)}{t_{go}^{m+2}} & \frac{(n+m+3)}{t_{go}^{m+1}} \end{bmatrix} = \infty \\ Q &= \begin{bmatrix} \frac{(m+2)(n+2)(n-m-1)}{t_{go}^{m+4}} & 0 \\ 0 & \frac{(n+2)(n-m-1)}{t_{go}^{m+2}} \end{bmatrix} \\ r &= \frac{1}{t_{go}^m} \end{aligned} \tag{2.10}$$

From the above performance index, it can be observed that, if $m = 0,\ n = 1$, the performance index $J = \frac{1}{2}\int_0^{t_f} u^2 dt$, and if $n = m + 1, m > 0$, then $J = \int_0^{t_f} \frac{u^2}{t_{go}^m} dt$. Therefore, the suggested guidance includes the time-to-go weighted optimal impact angle controller as well as the energy optimal angle controller, as expected; that is, we can regard the proposed guidance as a more generalized optimal impact angle control law even though the guidance was derived on the basis of the time-to-go polynomial function of the acceleration command.

2.3. **Time-to-go Calculation Method.** As mentioned in the previous section, we need to calculate the accuracy time-to-go using the relative range estimate for the implementation of the proposed guidance law. The most widely used calculation method is R/V_c which is defined by the relative range over the closing velocity. This method can give a good time-to-go estimate in case the homing trajectory is near to the collision course. However, this method may not be suitable for the impact angle control problem since the trajectory of impact angle controller is generally curved and far from the collision course. In [3], a time-to-go computation method was suggested exclusively for the energy optimal terminal angle constrained law. In the case of the proposed guidance with arbitrary gains, the method of [3] is inadequate because the homing trajectory generated by the proposed law is different according to the guidance gain selections. We therefore propose a more appropriate time-to-go calculation method based on the predicted trajectory generated by the proposed guidance with arbitrary m and n values.

Let us consider the closed-form solution of ν in Eq. (2.7). This can be approximated to a function of range η by substituting $t \approx \eta/V_M, t_f \approx R_0/V_M$ and $t_0 = 0$ into Eq. (2.7). It can then be transformed to the angle error using the definition of $\nu = V_M\gamma$.

$$\gamma(\eta) = -C_3(n+2)(R_0-\eta)^{n+1} - C_4\left(\frac{m+2}{n-m}\right)(R_0-\eta)^{m+1} \tag{2.11}$$

where $\eta \in [0, R_0]$ and

$$\begin{aligned} C_3 &= \frac{1}{(m-n)R_0^{n+1}}\left[(m+2)\lambda_0+\gamma_0\right] \\ C_4 &= \frac{1}{R_0^{m+1}}\left[(n+2)\lambda_0+\gamma_0\right] \end{aligned} \tag{2.12}$$

If the $\gamma+\lambda$ angle is small, the length of the predicted trajectory from t_0 to t_f can be defined in the initial LOS frame

$$S = \int_0^{R_0}\sqrt{1+[\gamma(\eta)+\lambda]^2}d\eta \approx R_0 + \frac{1}{2}\int_0^{R_0}\gamma^2(\eta)d\eta + \lambda_0\int_0^{R_0}\gamma(\eta)d\eta + \frac{1}{2}\lambda_0^2R_0 \tag{2.13}$$

By substituting Eq. (2.11) into the right-hand-side of Eq. (2.13) and manipulating the resulting equations, we obtain

$$\begin{aligned} \frac{1}{2}\int_0^{R_0}\gamma^2(\eta)d\eta =& \frac{R_0}{(2n+3)(2m+3)(m+n+3)}\left\{\left[(m+2)(n+2)\lambda_0+\frac{1}{2}\gamma_0\right]^2\right. \\ &\left.+\left(m+\frac{3}{2}\right)\left(n+\frac{3}{2}\right)\gamma_0^2\right\} \\ \int_0^{R_0}\gamma(\eta)d\eta =& -R_0\lambda_0 \end{aligned} \tag{2.14}$$

The total predicted trajectory length at t_0 is then approximated as

$$S = R_0\left\{1+p_1\left[\left(p_2\lambda_0+\frac{1}{2}\gamma_0\right)^2+p_3\gamma_0^2\right]-\frac{1}{2}\lambda_0^2\right\} \tag{2.15}$$

where

$$
\begin{aligned}
p_1 &= 1/\left[(2m+3)(2n+3)(m+n+3)\right] \\
p_2 &= (m+2)(n+2) \\
p_3 &= (m+3/2)(n+3/2)
\end{aligned} \tag{2.16}
$$

From Eq. (2.15), we know that S includes the length increment due to the trajectory curvature with guidance gains.

Because the constant missile velocity is assumed, the terminal time, t_f, can be calculated by the length of the predicted curve over the velocity as

$$
t_f = \frac{S}{V_M} = \frac{R_0}{V_M}\left\{1 + p_1\left[\left(p_2\lambda_0 + \frac{1}{2}\gamma_0\right)^2 + p_3\gamma_0^2\right] - \frac{1}{2}\lambda_0^2\right\} \tag{2.17}
$$

If t_f in Eq. (2.17) is initialized at the each step of time, the terminal time can be replaced by the time-to-go with $\lambda = \gamma_f - \sigma$ and $\gamma = \gamma_M - \gamma_f$. Thus,

$$
t_{go} = \frac{R}{V_M}\left\{1 + p_1\left[\left(p_2(\gamma_f - \sigma) + \frac{1}{2}(\gamma_M - \gamma_f)\right)^2 + p_3(\gamma_M - \gamma_f)^2\right] - \frac{1}{2}(\gamma_f - \sigma)^2\right\} \tag{2.18}
$$

Note that p_1 converges to zero as the guidance gains increase, so the time-to-go can be approximated to $t_{go} \approx R/V_M$ when the guidance gains are large and small angle assumption is valid. This tendency implies that the total homing trajectory of the proposed guidance law is near to the collision course as the gains grow.

Based on the time-to-go polynomial function, we proposed the new guidance law which satisfies the terminal angle constraint. In Section 4, the characteristic of the proposed law is discussed through various numerical simulations.

3. Polynomial Guidance for Impact-Time-Control

Using the similar approach proposed in Section 2, we derive a more generalized missile guidance law that can intercept a stationary target at a desired flight time, where the desired flight time is given before the missile is launched.

3.1. **Problem Statement.** Let us consider the two-dimensional engagement geometry as depicted in Fig. 2. As the previous engagement conditions in Section 2, we assume that V_M is constant and the autopilot lag is neglected. From Fig. 2, the nonlinear equations of motion w.r.t the flight time, t, for this impact-time-control problem are given as

$$
\begin{aligned}
\dot{x} &= V_M \cos\gamma_M, \quad x(t_0) = x_0, \quad x(t_d) = x_f \\
\dot{y} &= V_M \sin\gamma_M, \quad y(t_0) = y_0, \quad y(t_d) = y_f \\
\dot{\gamma_M} &= a_M/V_M, \quad \gamma_M(t_0) = \gamma_{M_0}
\end{aligned} \tag{3.1}
$$

where t_0 and t_d are the initial and desired impact time, respectively. In Eq. (3.1), the terminal constraints, $x(t_d) = x_f,\ y(t_d) = y_f$, are defined for the successful interception at the desired impact time; the missile positions at the desired impact time should be equal to the target positions.

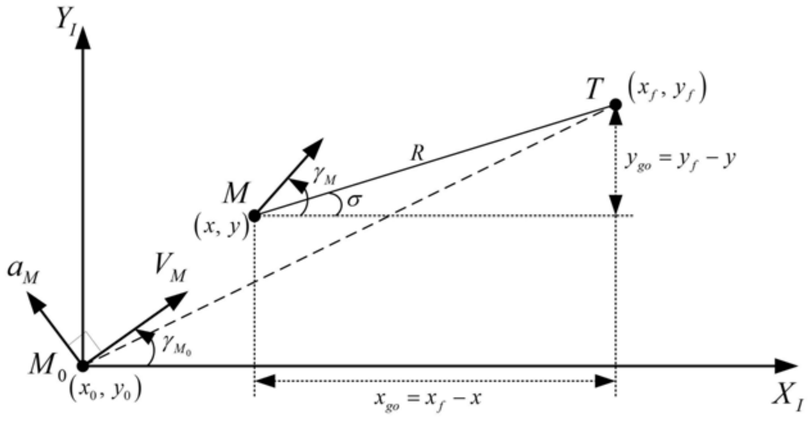


FIGURE 2. Homing Engagement for Impact-Time-Control

If the initial LOS angle, σ_0, and flight path angle, γ_M, are small during the flight, the linearized equations w.r.t the downrange, x, can be obtained as

$$\begin{bmatrix} y' \\ \gamma_M' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \gamma_M \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{3.2}$$

where the prime $'$ represents the derivative w.r.t x and the command is defined by $u = a_M/V_M^2$. The linearized equations can help to solve the impact-time-control problem easily. The initial and final boundary conditions for this equation are given as

$$y(x_0) = y_0, \quad \gamma_M(x_0) = \gamma_{M_0}, \quad y(x_f) = y_f \tag{3.3}$$

Note that the final boundary condition, $y(x_f) = y_f$, is required to hit the target at $x = x_f$. The terminal time constraints given in the nonlinear equations can be replaced by a path constraint that the total length of the homing trajectory should be the same as the desired distance-to-go, $\hat{R}^*_{go}$; where $\hat{R}^*_{go} = V_M(t_d - t_0)$, and this path constraint is valid because V_M is constant.

In addition to the final boundary condition of Eq. (3.3), therefore, we consider the following trajectory length constraint to obtain the impact-time-control guidance law.

$$S = \int_{x_0}^{x_f} \sqrt{1 + (y')^2} d\eta \approx \int_{x_0}^{x_f} 1 + \frac{1}{2}\gamma_M^2(\eta) d\eta = \hat{R}^*_{go} \tag{3.4}$$

where S is the total length of homing trajectory from x_0 to x_f, and this approximation is valid when the flight path angle is small.

In the impact time constrained problem, there are two terminal boundary conditions: zero miss-distance of Eq. (3.3) and the terminal time constraint (i.e., the path constraint given by Eq. (3.4). In order to derive a unique solution of the guidance law as in the previous chapter, we assume that the guidance command history for the impact-time-control is defined as the following polynomial function with two unknown coefficients

$$u(x) = c_n x_{go}^n + c_m, \quad n > 0 \tag{3.5}$$

where n is an arbitrary positive constant. Note that, as shown in Eq. (3.5), the guidance command profile is the function of downrange-to-go which has the similar property of the time-to-go, and it includes two unknown coefficients, $c_n,\ c_m$; the coefficient c_n in the first term of the assumed polynomial function is considered to satisfy zero miss-distance, and the second constant c_m is for achieving the desired impact time (i.e., the path constraint).

Using the assumed guidance command history defined in Eq. (3.5), we derive a state feedback guidance law, that can satisfy the terminal constraints on the miss-distance and impact time, in next sections.

3.2. **Guidance Command for Target Interception.**

3.2.1. *State Feedback Command.* To obtain the state feedback guidance law for successful interception of the target, we should first determine the coefficient c_n in the assumed guidance command history. Let us substitute Eq. (3.5) into Eq. (3.2) and then integrate the resulting equation with the initial conditions. After that, we can get

$$y(x) = \frac{1}{2}c_m x^2 + \frac{1}{(n+1)(n+2)}c_n x_{go}^{n+2} + c_\gamma x + c_y \tag{3.6}$$

where c_γ and c_y are the constants of integration, which are defined as

$$\begin{aligned} c_\gamma &= \gamma_{M_0} - c_m x_0 + \frac{1}{(n+1)}c_n (x_f - x_0)^{n+1} \\ c_y &= y_0 - \frac{1}{2}c_m x_0^2 - \frac{1}{(n+1)(n+2)}c_n (x_f - x_0)^{n+2} - c_\gamma x_0 \end{aligned} \tag{3.7}$$

From Eq. (3.6) and Eq. (3.7), the missile lateral position at $x = x_f$ is

$$\begin{aligned} y(x_f) &= \frac{1}{2}c_m x_f^2 + c_\gamma x_f + c_y \\ &= \frac{1}{2}c_m \hat{x}_{go}^2 + \gamma_{M0}\hat{x}_{go} + \frac{1}{(n+2)}c_n \hat{x}_{go}^{n+2} + y_0 \end{aligned} \tag{3.8}$$

where $\hat{x}_{go}^2 = x_f - x_0$. For the terminal zero miss-distance, the lateral position $y(x_f)$ should satisfy the final boundary condition, $y(x_f) = y_f$. Using Eq. (3.8) and this boundary condition, we can determine the coefficient c_n to intercept the target at $x = x_0$ as follows:

$$c_n = \frac{(n+2)}{\hat{x}_{go}^{n+2}} \left(\hat{y}_{go} - \gamma_{M_0}\hat{x}_{go} - \frac{1}{2}c_m \hat{x}_{go}^2 \right) \tag{3.9}$$

Hence, substituting Eq. (3.9) into Eq. (3.5) yields the guidance command for target interception at the initial x_0 as

$$u_{ITC}(x_0) = (n+2)\frac{(\hat{y}_{go} - \gamma_{M_0}\hat{x}_{go})}{\hat{x}_{go}^2} - \frac{n}{2}c_m \tag{3.10}$$

If the coefficient c_n is initialized and recalculated at each step of x, the guidance command given in Eq. (3.10) can be expressed in the form of the state feedback as

$$u_{ITC}(x) = u_p(x) - \frac{n}{2}c_m \tag{3.11}$$

where

$$u_p(x) = (n+2)\frac{(y_{go} - \gamma_M x_{go})}{x_{go}^2} \tag{3.12}$$

and $y_{go} = y_f - y$. Note that, the feedback command of Eq. (3.11) enables the missile to hit the target even though the coefficient c_m has any arbitrary constant. This result can be demonstrated by obtaining the closed-form trajectory solutions.

3.2.2. *Closed-form Solutions.* The closed-form solutions of u_{ITC} can be obtained by solving the following linear second-order ordinary differential equation (ODE), which is determined by substituting Eq. (3.11) into Eq. (3.2).

$$y'' + \frac{(n+2)}{(x_f - x)}y' - \frac{(n+2)}{(x_f - x)^2}(y_f - y) = -\frac{n}{2}c_m \tag{3.13}$$

For convenience, the above second-order ODE is rewritten by using $Y = y_f - y$ and $\tau = x_f - x$.

$$D^2Y - \frac{(n+2)}{\tau}DY + \frac{(n+2)}{\tau^2}Y = \frac{n}{2}c_m \tag{3.14}$$

where D is a differentiation operator w.r.t the independent variable τ. This equation is the same form as the Euler-Cauchy equation, so we can easily find the solutions of Eq. (3.14)

The homogeneous solution to the ODE is

$$Y_h(\tau) = c_1\tau^{n+2} - c_2\frac{1}{(n+2)}\tau \tag{3.15}$$

and the particular solution to the ODE is

$$Y_p(\tau) = -\frac{1}{2}c_m\tau^2 \tag{3.16}$$

Hence, from these equations, the general solutions of Eq. (3.13) can be obtained as

$$
\begin{aligned}
y(x) &= y_f - c_1 x_{go}^{n+2} + c_2 \frac{1}{(n+1)} x_{go} + \frac{1}{2} c_m x_{go}^2 \\
\gamma_M(x) &= y'(x) = c_1 (n+2) x_{go}^{n+1} - c_2 \frac{1}{(n+1)} - c_m x_{go} \\
u(x) &= \gamma_M'(x) = -c_1 (n+2)(n+1) x_{go}^n + c_m = c_n x_{go}^n + c_m
\end{aligned} \tag{3.17}
$$

where c_1 and c_2 are the constants of integration

$$
\begin{aligned}
c_1 &= \frac{-1}{(n+1)(n+2)\hat{x}_{go}^n} u_p(x_0) + \frac{1}{2(n+1)\hat{x}_{go}^n} c_m \\
c_2 &= -\hat{x}_{go} u_p(x_0) - (n+1)\gamma_{M_0} - \frac{n}{2} c_m \hat{x}_{go}
\end{aligned} \tag{3.18}
$$

From Eq. (3.17), it can be seen that the final boundary condition given by Eq. (3.3) can be always satisfied for an arbitrary c_m, therefore, the feedback command u_{ITC} can achieve the terminal zero miss-distance. It is also known that the missile flight path angle is dependent on the constant coefficient c_m; this result implies that the flight trajectory can be adjusted by choosing the coefficient c_m, therefore, we determine the appropriate coefficient c_m to satisfy the path constraint for the impact time control.

3.3. **Guidance Command for Impact-Time-Control.**

3.3.1. *c_m to Control Impact Time.* In order to determine the coefficient c_m that can satisfy the path constraint, we first derive the total intercept trajectory length of u_{ITC}. From Eq. (3.4), the total trajectory length estimated from the initial x_0 to x_f can be rewritten as

$$
S = \int_{x_0}^{x_f} \sqrt{1 + \gamma_M^2(\eta)} d\eta \approx \hat{x}_{go} + \frac{1}{2} \int_{x_0}^{x_f} \gamma_M^2(\eta) d\eta \tag{3.19}
$$

By substituting the solution γ_M of Eq. (3.17) into Eq. (3.19) and manipulating the resulting equations, the integral term of the right-hand-side of Eq. (3.19) can be obtained as

$$
\begin{aligned}
\frac{1}{2} \int_{x_0}^{x_f} \gamma_M^2(\eta) d\eta = {} & \frac{n^2}{12(2n+3)(n+3)} \hat{x}_{go}^3 c_m^2 + \frac{n}{2(n+2)(n+3)(2n+3)} \hat{x}_{go}^3 u_p(x_0) c_m \\
& + \frac{1}{(2n+3)(n+2)} \hat{x}_{go}^3 u_p^2(x_0) + \frac{1}{(n+2)} \hat{x}_{go}^2 \gamma_{M_0} u_p(x_0) + \frac{1}{2} \gamma_M^2 \hat{x}_{go}
\end{aligned} \tag{3.20}
$$

Hence, from Eqs. (3.19) and (3.20), we have

$$
S = p_1 \hat{x}_{go}^3 c_m^2 + p_2 \hat{x}_{go}^3 u_p(x_0) c_m + \hat{R}_{go}\big|_{c_m=0} \tag{3.21}
$$

where

$$p_1 = n^2/\left[12(2n+3)(n+3)\right]$$
$$p_2 = n/\left[2(n+2)(n+3)(2n+3)\right]$$
$$\hat{R}_{go}\big|_{c_m=0} = \frac{1}{(2n+3)(n+2)}\hat{x}_{go}^3 u_p^2(x_0) + \frac{1}{(n+2)}\hat{x}_{go}^2 \gamma_{M_0} u_p(x_0) + \left(1 + \frac{1}{2}\gamma_{M_0}^2\right)\hat{x}_{go} \tag{3.22}$$

In Eq. (3.21), $\hat{R}_{go}\big|_{c_m=0}$ is the length of the intercept trajectory guided by u_{ITC} with $c_m = 0$; that is, $\hat{R}_{go}\big|_{c_m=0}$ is the estimated distance-to-go using u_p at the initial x_0.

As mentioned in the previous section, S should be equal to the desired distance-to-go, $\hat{R}_{go}^*$, in order to make the missile hit the target at the designated impact time. Therefore, from Eqs. (3.21) and (3.22), we can obtain the following equation for determining the coefficient c_m

$$p_1\hat{x}_{go}^3 c_m^2 + p_2\hat{x}_{go}^3 u_p(x_0)c_m - \hat{\varepsilon} = 0 \tag{3.23}$$

where the distance-to-go error $\hat{\varepsilon} = \hat{R}_{go}^* - \hat{R}_{go}\big|_{c_m=0}$, and this equation is a quadratic equation in terms of c_m. Hence, the solutions are given by

$$\begin{aligned} c_m &= \frac{1}{2}\left[-\frac{p_2}{p_1}u_p(x_0) \pm \sqrt{\left(\frac{p_2}{p_1}\right)^2 u_p^2(x_0) + \frac{4}{p_1\hat{x}_{go}^3}\hat{\varepsilon}}\,\right] \\ &= \frac{1}{2}\left[-\frac{p_2}{p_1}u_p(x_0) \pm \left|u_p(x_0)\right|\sqrt{\left(\frac{p_2}{p_1}\right)^{+} \frac{4}{p_1 u_p^2(x_0)\hat{x}_{go}^3}\hat{\varepsilon}}\,\right] \end{aligned} \tag{3.24}$$

The coefficient c_m has two solutions, as shown in Eq. (3.24). If $\hat{R}_{go}\big|_{c_m=0} = \hat{R}_{go}^*$, then c_m is not required for the impact time control; that is, c_m when $\hat{\varepsilon} = 0$. From this condition, we can choose the unique c_m as follows

$$c_m = -\frac{p_2}{2p_1}u_p(x_0)\left[1 - \sqrt{1 + \frac{4p_1}{p_2^2 u_p^2(x_0)\hat{x}_{go}^3}\hat{\varepsilon}}\,\right] \tag{3.25}$$

The solution of c_m obtained in Eq. (3.25) can achieve the path constraint defined in Eq. (3.4), so it can be used for intercepting the target at the desired impact time by employing c_m solution in u_{ITC} given by Eq. (3.11). However, since the c_m solution contains the square root term, the c_m may have an imaginary solution when $\hat{\varepsilon} < 0$. To avoid this unacceptable result, the desired impact time t_d should be selected to be larger than $\hat{R}_{go}\big|_{c_m=0}/V_M$. This requirement implies that the desired distance-to-go, $\hat{R}_{go}^*$, should be larger than the total length of the intercept trajectory generated by $u_p(x)$.

3.3.2. *Guidance Command for Practical Implementation.* From Eq. (3.25), c_m is calculated using the initial engagement conditions, but this result contains an error due to the approximation of S. Therefore, in order to reduce the approximation error, the coefficient c_m should be applied in a feedback form as follows:

$$c_m = -\frac{3}{n(n+2)} u_p(x) \left[1 - \sqrt{1 + \frac{4(n+2)^2(n+3)(2n+3)}{3u_p^2(x)\hat{x}_{go}^3} \varepsilon} \right] \tag{3.26}$$

where

$$\begin{aligned} \varepsilon &= R_{go}^* - R_{go}\big|_{c_m=0} \\ R_{go}^* &= V_M(t_d - t) \\ R_{go}\big|_{c_m=0} &= \frac{1}{(2n+3)(n+2)} x_{go}^3 u_p^2(x) + \frac{1}{(n+2)} x_{go}^2 \gamma_M u_p(x) + \left(1 + \frac{1}{2}\gamma_M^2\right) x_{go} \end{aligned} \tag{3.27}$$

Note that c_m given by Eq. (3.26) is no longer constant and is updated at each step of x.

Using Eqs. (3.11) and (3.26), we can finally obtain the guidance command to achieve the terminal zero miss-distance as well as the impact time control as follows

$$\begin{aligned} u_{ITC}(x) &= u_p(x) - \frac{n}{2} c_m \\ &= u_p(x) \left[\frac{2n+7}{2(n+2)} - \frac{3}{2(n+2)} \sqrt{1 + \frac{4(n+2)^2(n+3)(2n+3)}{3u_p^2(x)x_{go}^3} \varepsilon} \right] \end{aligned} \tag{3.28}$$

From the proposed guidance law in Eq. (3.28), it is seen that the u_{ITC} command is gradually converted to the u_p command as the distance-to-go error, $\varepsilon \to 0$. As discussed in the previous subsection, the desired impact time should be larger than $\hat{R}_{go}\big|_{c_m=0}/V_M$ to avoid the imaginary solution, so the distance-to-go error at the beginning of the homing phase is always a positive value. Also, the distance-to-go error gradually decreases and finally reaches the zero as the missile approaches the target. For the practical implementation without numerical instability, therefore, we determine the distance-to-go error by $\varepsilon = \max(\varepsilon, \varepsilon_{min})$, where $\varepsilon_{min} = -3u_p^2(x)x_{go}^3/4(n+2)^2(n+3)(2n+3)$. Figure 3 illustrates the guidance loop of the proposed impact time controller which consists of two feedback loops: the inner u_p feedback loop is to reduce the miss-distance and the outer feedback loop is to reduce the distance-to-go error for achieving the impact time constraint.

To implement the proposed guidance law to a realistic missile system, the guidance command of Eq. (3.28) should be transformed to the missile acceleration with measurable units in the time domain. Under the assumption that γ_M is small and $x_{go} \gg y_{go}$, the line-of-sight angle and its derivative can be approximated as

$$\sigma \approx y_{go}/x_{go}, \quad d\sigma/dx \approx (y_{go} - \gamma_M x_{go})/x_{go}^2 \tag{3.29}$$

Using the definition of $a_M = V_M^2 u$ and the above equations, $u_p(x)$ can be expressed as

$$a_p(t) = V_M^2 u_p(x) = (n+2)V_M(dx/dt)(d\lambda/dx) = (n+2)V_M\dot{\sigma} \tag{3.30}$$

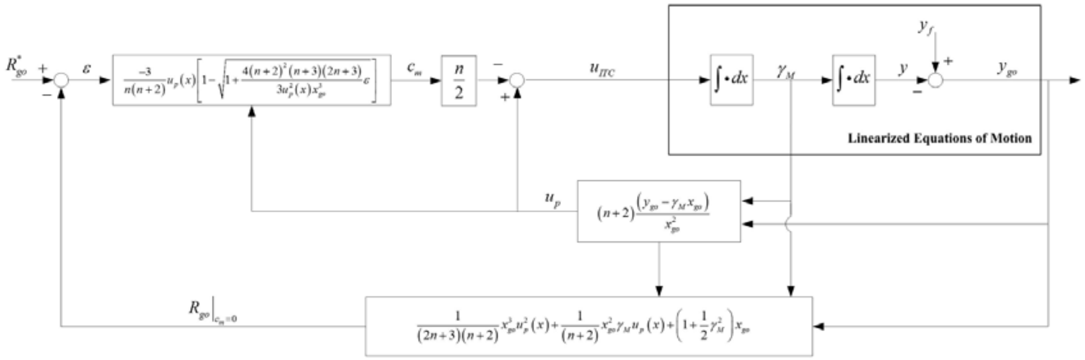


FIGURE 3. Guidance Loop of Impact Time Control Law

From Eq. (3.30), the missile acceleration command corresponding to Eq. (3.28) in the time domain is derived as

$$a_{ITC}(t) = a_p(t)\left[\frac{2n+7}{2(n+2)} - \frac{3}{2(n+2)}\sqrt{1+\frac{4(n+2)^2(n+3)(2n+3)}{3a_p^2(t)R^3}V_M^4\varepsilon}\right] \tag{3.31}$$

where R is the relative range between the missile and target. The estimated range-to-go can also be expressed in terms of the line-of-sight and the relative range as

$$R_{go}\big|_{c_m=0} \approx R\left[1+\frac{1}{2(2n+3)}(\gamma_M-\sigma)^2\right] \tag{3.32}$$

Note that, the a_p command corresponding to u_p is the PNG law with navigation constant of $n+2$, therefore the proposed law given by Eq. (3.31) is converged to the PNG as $\varepsilon \to 0$ and $R \to 0$. Examples of the proposed impact time control law for several values of n are shown in Table 2. It is noted that the command with $n = 1$ is identical to the suboptimal guidance law proposed in [19], and we can regard the proposed guidance as the more generalized impact-time-control guidance law.

The characteristic and performance of the derived guidance law are demonstrated by performing various numerical simulations, and the results are described in the next chapter.

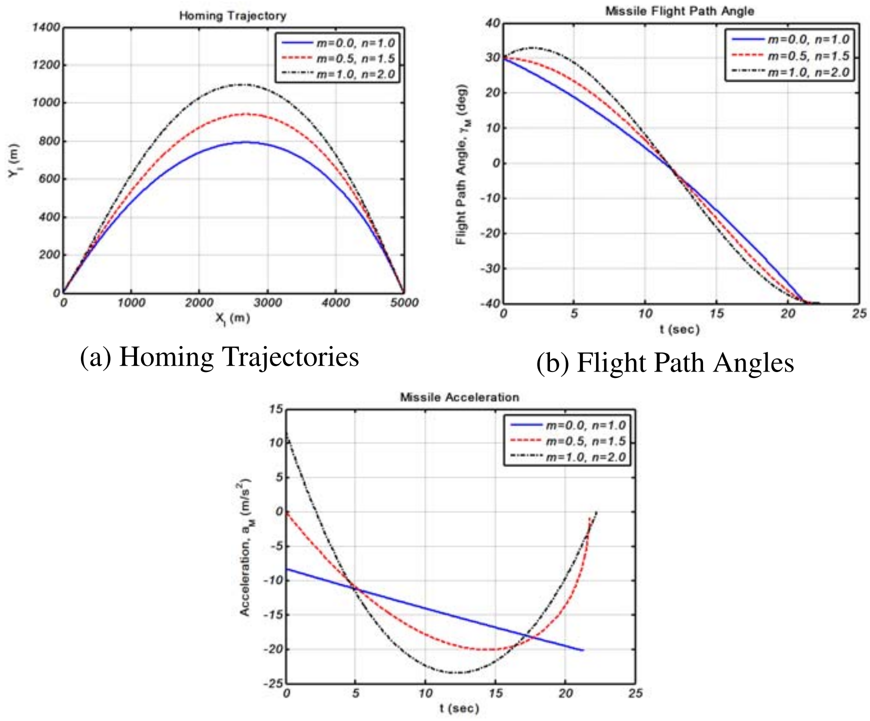
TABLE 2. Examples of Proposed Impact-Time-Control Law

n	Acceleration Command, $a_{ITC}(t)$
0.5	$2.5V\dot{\sigma}\left(\frac{8}{5}-\frac{3}{5}\sqrt{1+\frac{350}{3\cdot(2.5V_M\dot{\sigma})^2R^3}V_M^4\varepsilon}\right)$
1	$3V\dot{\sigma}\left(\frac{3}{2}-\frac{1}{2}\sqrt{1+\frac{240}{(3V_M\dot{\sigma})^2R^3}V_M^4\varepsilon}\right)$
2	$4V\dot{\sigma}\left(\frac{11}{8}-\frac{3}{8}\sqrt{1+\frac{2240}{3\cdot(4V_M\dot{\sigma})^2R^3}V_M^4\varepsilon}\right)$

4. Numerical Simulations

To investigate the characteristics and performance of the proposed guidance laws, we perform several nonlinear simulations for the various gains and terminal conditions. In these simulations, we assume that the missile has a constant speed of $250m/s$, and is located at the origin. The initial flight path angle is given by 30deg. The stationary target positions in the inertial frame are $(5000, 0)m$, and all simulations are terminated when the missile-target relative range is less than $0.5m$.

4.1. **Impact-Angle-Control Guidance.** For the purpose of demonstrating the basic properties for various guidance gains of the proposed impact-angle-control guidance law described in the chapter 2, the nonlinear guidance simulations with $m = 0,\ 0.5,\ 1.0$ and $n = m + 1$ are carried out, where the desired terminal angle, -40deg is imposed. The homing trajectories, flight path angles, and the required acceleration commands for the different gains are presented in Figs 4.(a)-(c).



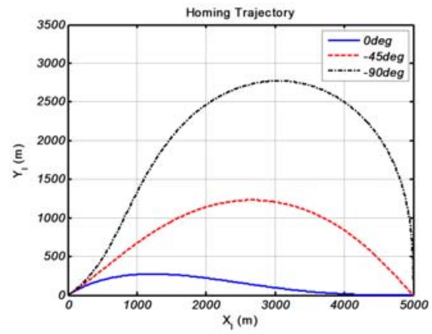
(a) Homing Trajectories (b) Flight Path Angles

(c) Missile Accelerations

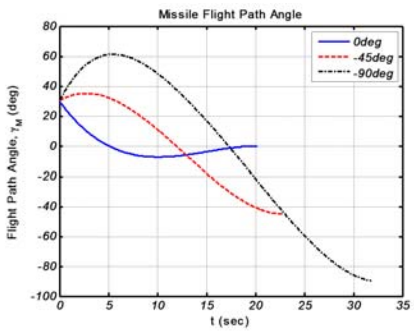
FIGURE 4. Simulation Results of Impact-Angle-Control Law with Various Gains

From the figure 4.(b), it can be seen that all guidance laws with different gains satisfy the given terminal impact angle, -40deg. From the figures, it can be also known that, as the guidance gains increase, the larger acceleration command in the beginning phase is required whereas the smaller acceleration is generated during the terminal phase of the engagement. Therefore, the proposed impact-angle-controller with large guidance gains produces the higher homing trajectory that converges rapidly to the collision course.

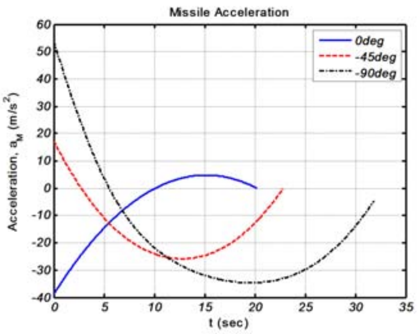
With $m = 1.0,\ n = 2.0$ gains, the nonlinear simulations for the desired impact angles, $\gamma_f = 0,\ -45,\ -90$deg, are performed and the results are presented in Fig. 5. As shown in the figure, the proposed guidance law can achieve the higher impact angle constraints, even though the proposed law is derived using the linearized equations and some assumptions.



(a) Homing Trajectories



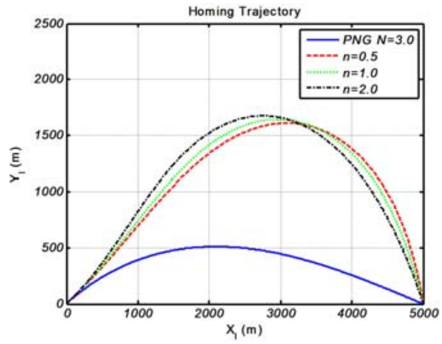
(b) Flight Path Angles



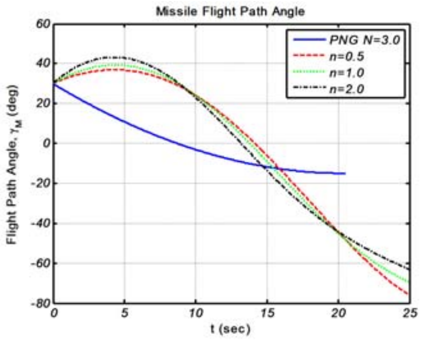
(c) Missile Accelerations

FIGURE 5. Simulation Results for Various Desired Impact Angles

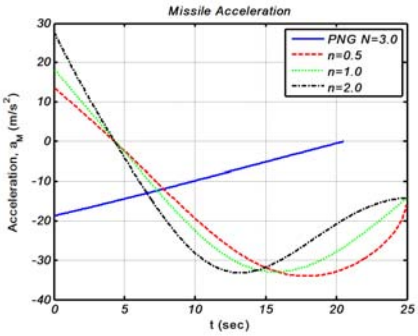
4.2. **Impact-Time-Control Guidance.** Using the same simulation conditions in the previous section, we investigate the characteristics and performance of the suggested impact-time-control guidance law. Firstly, we carry out the nonlinear simulations with difference guidance gains, $n = 0.5,\ 1.0,\ 2.0$, in which the desired impact time constraint $t_d = 25$sec is imposed. Figs. 6.(a) to (c) present the simulation results: the homing trajectories, flight path angles, and required acceleration commands resulting from the proposed impact time control law with various gains. As shown in figures, the missile guided by PNG can intercept the target but it cannot satisfy the given terminal time constraint. For all cases with various guidance gains, however, the proposed impact-time-control law enables the missile to hit the target at the designated impact time, 25sec, by increasing the homing trajectory length. From the figures, it can be seen that the guidance with larger guidance gain increases the initial missile acceleration command, whereas it decreases the required acceleration in the terminal homing phase. It can also be seen that increasing the guidance gain results in the larger maximum acceleration required and higher intercept trajectory. As compared with PNG law, the proposed guidance to control the impact time demands more acceleration capability.



(a) Homing Trajectories



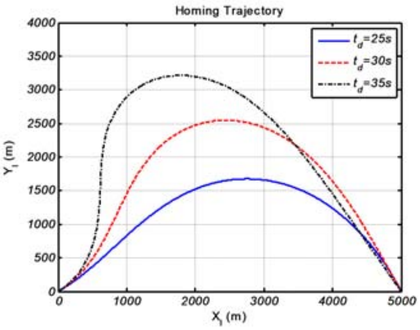
(b) Flight Path Angles



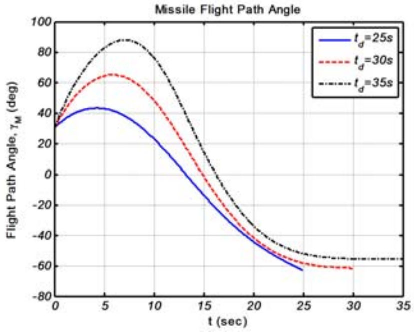
(c) Missile Accelerations

FIGURE 6. Simulation Results of Impact-Time-Control Law with Various Gains

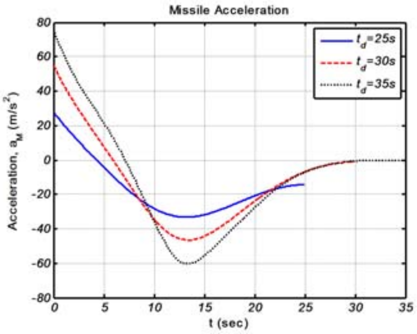
Next, we also perform the nonlinear simulations for the different impact times, $t_d = 25$, 30, 35sec, and $n = 2.0$, where the other parameters are the same as the previous one. The intercept trajectories, flight path angles, and acceleration commands for all the cases are presented in Figs. 7.(a) to (c). As shown in these figures, the results prove the capability of the proposed law for satisfying the various impact time constraints, even though the proposed law is obtained on the basis of the linearized equations and some assumptions.



(a) Homing Trajectories



(b) Flight Path Angles



(c) Missile Accelerations

FIGURE 7. Simulation Results for Various Desired Impact Times

5. CONCLUSION

In this paper, we proposed the polynomial function based guidance laws with terminal angle and time constraints. To derive the guidance laws, we suggested the simple approach, where the guidance command is assumed as a time-to-go and/or downrange-to-go polynomial function and then the polynomial function coefficients are determined to satisfy the terminal constraints given in homing problems. This approach is a very easy method to derive the homing guidance law with terminal state constraints, as compared with other approaches such as optimal control theory or nonlinear control design method. Unlike the conventional optimal control problem for missiles guidance law design, this approach can also provide a more generalized optimal solution of the guidance law with terminal constraints.

Using this approach, we proposed the two homing guidance laws; impact-angle-control law and impact-time-control law. The derived guidance laws have arbitrary guidance gains, and can be represented as the form of the optimal and/or suboptimal guidance law by choosing the proper guidance gains. Since the proposed guidance laws include arbitrary gains as design parameters, we can also select the appropriate guidance command form considering the homing engagement conditions and missiles capability. The performance and characteristics of the suggested guidance laws were investigated through the various nonlinear simulations with different guidance gains and terminal constraints. In future studies, several factors such as acceleration controller lag or aerodynamic model of missiles should be considered for the practical implementation.

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