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CLIQUE-TRANSVERSAL SETS IN LINE GRAPHS OF CUBIC GRAPHS AND TRIANGLE-FREE GRAPHS

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Abstract. A *clique-transversal set* D of a graph G is a set of vertices of G such that D meets all cliques of G. The *clique-transversal number* is the minimum cardinality of a clique-transversal set in G. For every cubic graph with at most two bridges, we first show that it has a perfect matching which contains exactly one edge of each triangle of it; by the result, we determine the exact value of the clique-transversal number of line graph of it. Also, we present a sharp upper bound on the clique-transversal number of line graph of a cubic graph. Furthermore, we prove that the clique-transversal number of line graph of a triangle-free graph is at most the chromatic number of complement of the triangle-free graph.

1. Introduction

All graphs considered here are finite, simple and nonempty. For standard terminology not given here we refer the reader to [7].

Let $G = (V, E)$ be a graph with *vertex set* V and *edge set* E. For a vertex $v \in V$, the *open neighborhood* $N_G(v)$ of v is defined as the set of vertices adjacent to v, i.e., $N_G(v) = \{u \mid uv \in E\}$. The *closed neighborhood* of v is $N_G[v] = N_G(v) \cup \{v\}$. For a subset $S \subseteq V$, the open neighborhood of S is $N_G(S) = \bigcup_{v\in S} N(v)$ and the closed neighborhood of S is $N_G[S] = \bigcup_{v\in S} N[v]$. The *degree* of v is equal to $|N_G(v)|$, denoted by $d_G(v)$ or simply $d(v)$. A graph is a *cubic* graph if every vertex has degree 3. For a subset $S \subseteq V$, the subgraph induced by S is denoted by $G[S]$. As usual, the complete bipartite graph $K_{1,3}$ is called a *claw* and the complete graph K_3 a *triangle*. For a given graph F, we say that a graph G is F-free if it does not contain F as an induced subgraph. In particular, $K_{1,3}$-free is also called *claw-free*. K_3-free is also called *Triangle-free*. For a family of graphs $\{F_1, \dots, F_k\}$ we say that G is $\{F_1, \dots, F_k\}$-free if it is F_i-free for all $i = 1, \dots, k$. A *bridge* is an edge of G whose removal increases

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the number of components of G. The *line graph* $L(G)$ of G is the graph whose vertices are the edges of G, and two vertices of $L(G)$ are connected if and only if the edges corresponding to them share a common vertex in G. As is well known, line graphs are a subclass of claw-free graphs.

A *matching* M in G is a set of pairwise non-adjacent edges; that is, no two edges share a common vertex. If M is a matching, the two ends of each edge of M are said to be *matched* under M, and each vertex incident with an edge of M is said to be *covered* by M. A *perfect matching* is one which covers every vertex of the graph. A *maximum matching* is a matching of G that contains the largest possible number of edges. The number of edges in a maximum matching of G is called the *matching number* of G and denoted $\alpha'(G)$. An *edge cover* of a graph G is defined as a set S of edges of G such that every vertex of G is incident with at least one of members of S. The *edge cover number*, denoted by $\beta'(G)$, is the number of edges in a minimum edge cover of G.

The clique-transversal set in graphs can be regarded as a special case of the transversal set in hypergraph theory [5]. A *clique* C of a graph G is a complete subgraph maximal under inclusion and having at least two vertices. According to this definition, isolated vertices are not considered to be cliques here. A clique of order m of G is called an *m-clique* of G. A set $D \subseteq V$ is called a *clique-transversal set* of G if D meets all cliques of G, i.e., $D \cap V(C) \neq \emptyset$ for any clique C of G. The *clique-transversal number*, denoted by $\tau_C(G)$, is the minimum cardinality of a clique-transversal set of G.

Erdős et al. [8] proved that the problem of finding a minimum clique-transversal set for a graph is NP-hard. It is therefore of interest to determine bounds on the clique-transversal number of a graph. In [8] the authors showed that any graph G of order n has $1 \leq \tau_C(G) \leq n - \sqrt{2n} + 3/2$ and they observed that $\tau_C(G)$ can be very close to $n = |V(G)|$, namely $\tau_C = n - o(n)$ can hold. On the basis of the fact above, it is reasonable to ask how drastically τ_C decreases or increases when some assumptions are imposed on the graph G. From this point of view, Tuza [14] and Andreae [1] established upper bounds on $\tau_C(G)$ for chordal graphs G. In [11] we established an upper bound and a sharp lower bound on $\tau_C(G)$ for connected k-regular graphs G. Bacsó and Tuza [4] found a tight upper bound on $\tau_C(G)$ for cubic graphs G. In [3] authors studied the upper bounds on τ_C for the classes of cographs and clique perfect graphs.

In 1991, Andreae, Schughart and Tuza [2] investigated the clique-transversal numbers for line graphs and complements of line graphs. They obtained the following result.

Theorem 1 (Andreae, Schughart and Tuza [2]). *Let G be a connected graph with at least two edges and assume that G is not an odd cycle. Then $\tau_C(L(G)) \leq |E(G)|/2$.*

This paper was motivated by the above result, we investigate the extremal behavior of $\tau_C(L(G))$ on the line graphs of cubic graphs and triangle-free graphs G. For every cubic graph with at most two bridges, we first show that it has a perfect matching which contains exactly one edge of each triangle of it; by the result, we can determine the exact value of the clique-transversal number of line graph of it. Furthermore, we present a tight upper bound on $\tau_C(G)$ for line graphs of cubic graphs. Finally, we prove that $\tau_C(L(G)) \leq \chi(\overline{G})$ for a triangle-free graph G without isolated vertices, and the equality holds if G has minimum degree at least two.

2. Line graphs of cubic graphs

For a cubic graph G, let T be a triangle of G and $|N_G(V(T)) \cap (V(G) - V(T))| = 3$. If $N_G(V(T)) \cap (V(G) - V(T))$ is an independent set of vertices in G, we call T an *isolated triangle* of G.

Lemma 2. *For every cubic graph G with no isolated triangles, there exists a maximum matching that contains exactly one edge of each triangle in G.*

Proof. Without loss of generality, we may assume that G is connected. Suppose that G contains no triangles, the assertion is trivial. Otherwise, G contains at least a triangle. Let M be a maximum matching of G that covers triangles of G as many as possible. Note that M contains at most one edge of each triangle in G. If M contains exactly one edge of each triangle in G, then there is nothing to show. Otherwise, there exists at least a triangle $T = u_1u_2u_3$ whose edges are not in M. The following proof is by contradiction.

Let $S = N_G(V(T)) \cap (V(G) - V(T))$ and let $S = \{x_1, x_2, x_3\}$ (x_i not necessarily distinct), where $x_iu_i \in E(G)$ for $1 \leq i \leq 3$. If $|S| = 1$, i.e., $x_1 = x_2 = x_3$, then G is isomorphic to K_4, the assertion immediately follows. If $|S| = 2$, without loss of generality, let $x_1 = x_2$. In this case, the induced subgraph $G[\{x_1\} \cup V(T)]$ is a diamond (the graph $K_4 - e$ obtained from K_4 by removing its one edge). By the maximality of M and $E(T) \cap M = \emptyset$, it is easy to see that exactly one of u_1x_1, u_2x_1 is in M and u_3 is covered by M. Without loss of generality, assume that $x_1u_1 \in M$. Let $M' = (M \setminus \{u_1x_1\}) \cup \{u_1u_2\}$. Then M' is a maximum matching of G that covers more triangles of G than M, which contradicts our assumption to M. So we may assumes that $|S| = 3$, that is, x_1, x_2, x_3 are distinct. By the maximality of M, there are at least two vertices, say u_1, u_2, of $V(T)$ that are covered by M. Thus $u_1x_1, u_2x_2 \in M$. Suppose that u_3 is uncovered by M. Note that the edge u_1x_1 lies in no triangles of G. We remove the edge u_1x_1 from M and add u_1u_3 to M. The resulting matching is still maximum and it covers more triangles of G than the primitive M, a contradiction. Thus, each vertex of T is covered by M, so $u_ix_i \in M$ for $1 \leq i \leq 3$. Since G contains no isolated triangles, S is not independent. Without loss of generality, assume that $x_1x_2 \in E(G)$. Clearly each u_ix_i lies in no triangles of G. Let $M' = (M \setminus \{u_1x_1, u_2x_2\}) \cup \{x_1x_2, u_1u_2\}$. Then M' is a maximum matching of G and it covers more triangles of G than M, which is a contradiction. □

Remark. The above assertion is still true when the cubic graph is reduced to a graph with maximum degree at most three.

Lemma 3 ([7]). *Every cubic graph with at most two bridges has a perfect matching.*

Theorem 4. *Every cubic graph with at most two bridges has a perfect matching that contains exactly one edge of each triangle in the graph.*

Proof. If G contains no isolated triangles, then the result follows from Lemma 2 and Lemma 3. Otherwise, we let G^* be the graph obtained from G by contracting each isolated triangle of G to a single vertex. Then G^* is a cubic graph with at most two bridges and it contains no isolated triangles. Hence, by Lemmas 2 and 3, G^* has a perfect matching M^* that contains exactly one edge of each triangle in G^*.

Let $T_j = x_{j1}x_{j2}x_{j3}$ $(j = 1, 2, \ldots, l)$ be all the isolated triangles of G. For each $T_j = x_{j1}x_{j2}x_{j3}$ of G, let v_{T_j} be the vertex of G^* obtained by contracting the triangle T_j. Since M^* is a perfect matching of G^*, each v_{T_j} is covered by M^*, and so precisely one edge incident with v_{T_j} is in M^*. Without loss of generality, let $v_{T_j}u \in M^*$ and $x_{j1}u \in E(G)$. If u is not the vertex of G^* obtained by contracting a triangle of G, i.e., $u \neq v_{T_i}$ for any $1 \leq i \leq l$, then we remove the edge $v_{T_j}u$ from M^* and add to M^* the edges $x_{j2}x_{j3}$ and $x_{j1}u$. Suppose that $u = v_{T_k}$ for some $k \neq j$, $1 \leq k \leq l$, i.e., u is obtained by contracting the other triangle T_k of G. Since T_j and T_k are isolated, there is exactly one edge between $V(T_j)$ and $V(T_k)$. Let $x_{j1}x_{k1} \in E(G)$. Clearly, the edge $x_{j1}x_{k1}$ of G corresponds to the edge $v_{T_j}v_{T_k}$ of G^*. In G, we add the edges $x_{j1}x_{k1}$, $x_{j2}x_{j3}$ and $x_{k2}x_{k3}$ to M^* and remove the edge $v_{T_j}v_{T_k}$ from M^*. The updated M^* is a set of edges of G. Obviously, M^* is a perfect matching of G and it contains exactly one edge of each triangle of G, the assertion follows. □

Remark. The conditions in Lemma 2 and Lemma 3 can not be dropped. Let B_1 be the graph, called a *balloon*, obtained from a complete graph K_4 on four vertices by subdividing an edge of K_4. Now let F be the cubic graph obtained from disjoint union of three copies of B_1 and a triangle C_3 by joining three vertices of the triangle to the vertices of degree 2 in the three copies of B_1, respectively. By our construction, F is a cubic graph on 18 vertices with a isolated triangle C_3 and three bridges. It is easily seen that F has a perfecting matching and $\alpha'(F) = 9$. But there is not a perfecting matching of F that contains edges of the triangle C_3.

Let G be a connected graph with at least two edges. Clearly, $L(G)$ has just two kinds of cliques, namely the 'star-cliques' and the 'triangle-cliques': the former are the sets $S(x) = \{e \in E(G) : e \text{ is incident with } x\}$, where x is either a vertex of degree ≥ 3 or a vertex of degree 2 which is not contained in a triangle of G, while the latter are the edge sets of the triangles of G. Clearly, for any graph G with minimum degree at least two, each clique-transversal set of $L(G)$ must be an edge cover of G.

Theorem 5. *For every cubic graph G with at most two bridges, $\tau_C(L(G)) = \frac{|E(G)|}{3}$.*

Proof. Let D be a minimum clique-transversal set of $L(G)$. Then D is also an edge cover of G. To cover all vertices of G, each edge cover of G contains at least $|V(G)|/2$ edges of G. Thus $\tau_C(L(G)) = |D| \geq |V(G)|/2 = |E(G)|/3$. On the other hand, by Theorem 4, we know that G has a perfect matching M that contains exactly one edge of its each triangle. Hence M is a clique-transversal set of $L(G)$, and thus $\tau_C(L(G)) \leq |M| = |E(G)|/3$. □

In [6] a sharp lower bound on the matching number $\alpha'(G)$ is obtained for a cubic graph G. Further, O and West [12] found the family $\mathcal{H}_1$ of extremal graphs achieving the bound. The family $\mathcal{H}_1$ is constructed as follows. Let B_1 be the graph as mentioned earlier. In fact, this is the smallest graph in which one vertex has degree 2 and the others have degree 3. Let T_1 be the family of trees such that every non-leaf vertex has degree 3 and all leaves have the same color in a proper 2-coloring. Let $\mathcal{H}_1$ be the family of cubic graphs obtained from trees in T_1 by identifying each leaf of such a tree with the vertex of degree 2 in a copy of B_1.

For characterizing the extremal graphs achieving the upper bound, we need the following lemma.

Lemma 6 ([12])**.** *If G is a connected cubic graph on n vertices, then $\alpha'(G) \geq (4n-1)/9$, and equality holds if and only if $G \in \mathcal{H}_1$.*

Lemma 7 ([9])**.** *For each graph G of order n, $\beta'(G) + \alpha'(G) = n$.*

Theorem 8. *If G is a connected cubic graph of order n, then*

$$\frac{|E(G)|}{3} \leq \tau_C(L(G)) \leq \frac{10|E(G)|+3}{27}.$$

Moreover, this left equality holds if and only if G has a perfect matching that contains exactly one edge of each triangle in G, while this right equality holds if and only if $G \in \mathcal{H}_1$.

Proof. Without loss of generality we may assume that G is connected. Note that each clique-transversal set of $L(G)$ is an edge cover of G while each edge cover of G contains at least $n/2$ edges of G. Hence $\tau_C(L(G)) \geq |E(G)|/3$. This equality holds if and only if G has an edge cover of size $n/2$ that contains exactly one edge of each triangle in G. Obviously, such an edge cover of size $n/2$ is also a perfecting matching of G, as desired.

In order to get the upper bound, we distinguish the following two cases.

Case 1. G contains no isolated triangles.

By Lemma 2, G has a maximum matching that contains exactly one edge of each triangle in G. Let M be such a matching. For each vertex uncovered by M, take an edge incident with the vertex. Denote by M_0 the set of all such edges. Then $|M_0| = n-2|M|$. Clearly, $M \cup M_0$ is a clique-transversal set of $L(G)$. On the other hand, since $|M \cup M_0| = n - |M| = n - \alpha'(G)$, $M \cup M_0$ is a minimum edge cover of G. This implies that $M \cup M_0$ is a minimum clique-transversal set of $L(G)$. By Lemma 6, we have

$$(1) \qquad \tau_C(L(G)) = |M \cup M_0| = n - |M| \le n - \frac{4n-1}{9} = \frac{10|E(G)|+3}{27},$$

the assertion follows.

Case 2. G contains isolated triangles.

As in the proof of Theorem 4, let G^* be the graph obtained from G by contracting each isolated triangle of G to a single vertex. Then G^* is a connected cubic graph that contains no isolated triangles. Let S^* be a minimum clique-transversal set of $L(G^*)$. Then, by Case 1, we have $\tau_C(L(G^*)) \le (10|E(G^*)|+3)/27$.

Let T_j $(j = 1, 2, \ldots, l)$ be all the isolated triangles of G. Then $|E(G^*)| = |E(G)| - 3l$. For each T_j, let v_{T_j} be the 'contracting vertex' of G^* corresponding to T_j. Clearly, at least an edge incident with each 'contracting vertex' of G^* belongs to S^*. Let $v_{T_j}u \in S^*$ and $x_ju \in E(G)$ where $x_j \in V(T_j)$. If $u \ne v_{T_i}$ for any $1 \le i \le l$, then we remove the edge $v_{T_j}u$ from S^* and add to S^* the edge x_ju and the edge of T_j not incident with x_j. Suppose that $u = v_{T_k}$ for some $k \ne j$, $1 \le k \le l$, i.e., u is obtained by contracting the other triangle T_k of G. Since T_j and T_k are isolated, there is exactly one edge between $V(T_j)$ and $V(T_k)$. Let $x_jx_k \in E(G)$ where $x_k \in V(T_k)$. Clearly, the edge x_jx_k of G corresponds to the edge $v_{T_j}v_{T_k}$ of G^*. In G, we replace the edge $v_{T_j}v_{T_k}$ of S^* by the edge x_jx_k and add the edges of T_j and T_k not incident with x_j and x_k respectively, to S^*. The updated S^* is a set of edges of G. Obviously, S is a clique-transversal set of $L(G)$. Hence,

$$\begin{aligned}\tau_C(L(G)) \le |S| \le |S^*| + l &= \frac{10|E(G^*)|+3}{27} + l \\ &< \frac{10(|E(G^*)|+3l)+3}{27} \\ &= \frac{10|E(G)|+3}{27}.\end{aligned}$$

We next show that $\tau_C(L(G)) = (10|E(G)|+3)/27$ if and only if $G \in \mathcal{H}_1$ for a connected cubic graph G of order n.

Suppose $G \in \mathcal{H}_1$. By Lemma 6, $\alpha'(G) = (4n-1)/9$. The vertices set in $L(G)$ corresponding to three edges incident with each vertex in G induces a 3-clique. The edges set of G corresponding to each clique-transversal set in $L(G)$ is an edge cover set of G. According to Lemma 7, we have

$$\tau_C(L(G)) = \beta'(G) = n - \alpha'(G) = n - (4n-1)/9 = (10|E(G)|+3)/27.$$

Hence $\tau_C(L(G)) = (10|E(G)|+3)/27$.

Conversely, suppose that $\tau_C(L(G)) = (10|E(G)|+3)/27$ for a connected cubic graph G of order n. By the above proof, the Case 2 cannot appear and the inequality in equation (1) is equality. Thus, $\alpha'(G) = (4n-1)/9$. We have $G \in \mathcal{H}_1$ by Lemma 6.

This completes the proof of Theorem 8. □

Now we are ready to present another upper bound on $\tau_C(L(G))$ for the line graph $L(G)$ of a cubic graph G. Moreover, we also give a characterization of the extremal graphs attaining the upper bound. For this purpose, we define the graph B and the cubic graph $H(g)$ as follows. First, let B be a graph with girth r in which one vertex has degree two and all others have degree 3 and such that the order g of B is as small as possible. For example, if $r = 3$, B is the balloon B_1 with $g = 5$. If $r = 4$, then B is the complete bipartite graph $K_{3,3}$ with one edge subdivided, and $g = 7$. The existence of B having girth r is shown by observing that an r-cage with one edge subdivided will serve (see [13], for a proof that r-cages exist for all $r \geq 3$). Now let $H(g)$ be the cubic graph obtained from disjoint union of three copies of B by adding a new vertex and joining it with the vertex of degree 2 in each copy of B, respectively.

Lemma 9 ([10])**.** *If G is a cubic graph of order n with girth $r \geq 3$, then $\alpha'(G) \geq \left(\frac{3g-1}{3g+1}\right)\frac{n}{2}$, where g is the number of vertices in B. Moreover, the equality holds if and only if G is the graph $H(g)$.*

For a cubic graph G with girth $r \geq 4$, By Lemma 9, we can slightly improve the result in Theorem 8.

Theorem 10. *If G is a cubic graph of order n with girth $r \geq 3$, then $\tau_C(L(G)) \leq \frac{g+1}{3g+1}|E(G)|$, where g is the number of vertices in B. Moreover, the equality holds if and only if G is the graph $H(g)$.*

Proof. We may assume G is connected. If $r = 3$, then $g = 5$. By Theorem 8, we have

$$\tau_C(L(G)) \leq \frac{10|E(G)|+3}{27} \leq \frac{3}{8}|E(G)| = \frac{g+1}{3g+1}|E(G)|,$$

the assertion holds. So we may assume that $r \geq 4$. Since G contains no triangles, all cliques of $L(G)$ are stars of G. Let M be a maximum matching of G. For each vertex uncovered by M, take an edge incident with the vertex. Denote by M_0 the set of all chosen edges. Then $|M_0| = n - 2|M|$. Clearly, $M \cup M_0$ is a clique-transversal set of $L(G)$ as well a minimum edge cover of G. By Lemma 9, we have

$$\tau_C(L(G)) = |M \cup M_0| = n - |M| \leq n - \left(\frac{3g-1}{3g+1}\right)\frac{n}{2} = \frac{g+1}{3g+1}|E(G)|.$$

That the equality holds in Theorem 10 implies that the equality in Lemma 9 holds. Therefore, the equality holds if and only if G is the graph $H(g)$. □

Remark. If G is a cubic graph with girth $r = 3$, then $g = 5$. By above result, we have $\tau_C(L(G)) \leq 3|E(G)|/8$. In this case, the result in Theorem 8 is better than the result $3|E(G)|/8$. However, when girth $r \geq 4$, since $g \geq 7$, we have $\tau_C(L(G)) \leq (g+1)|E(G)|/(3g+1) \leq 4|E(G)|/11 \leq (10|E(G)|+3)/27$.

For general regular graphs, we propose the following problem.

Problem. Find a sharp upper bound on the clique-transversal number for line graphs of k-regular graphs, where $k \geq 4$.

3. Line graphs of triangle-free graphs

Let $\chi(G)$ denote the *chromatic number* of a graph G. The *complement* $\overline{G}$ of G is the graph with the same vertex set but whose edge set consists of the edges not present in G.

For a general graph G with girth at least 4, we can obtain an upper bound on $\tau_C(L(G))$ in terms of $\chi(\overline{G})$.

Theorem 11. *For a triangle-free graph G without isolated vertices, $\tau_C(L(G)) \leq \chi(\overline{G})$; and if G has minimum degree at least two, then the equality holds.*

Proof. Since G is triangle-free, the line graph $L(G)$ of G contains only the "star-cliques", so each edge cover of G is a clique-transversal set of $L(G)$. Hence $\tau_C(L(G)) \leq \beta'(G)$. Note that $\alpha'(G) + \beta'(G) = |V(G)|$. To obtain our result, it is sufficient to show that $|V(G)| - \alpha'(G) = \chi(\overline{G})$. Let $k = \chi(\overline{G})$ and let $\{V_1, V_2, \ldots, V_k\}$ be the partition of $V(G)$, where V_i denotes the set of vertices assigned colour i in a proper k-colouring of $\overline{G}$. The triangle-freeness of G implies that $1 \leq |V_i| \leq 2$. Clearly, the set of edges induced by the colour classes V_i with $|V_i| = 2$ in G is a matching of G. So $\alpha'(G) \geq |V(G)| - \chi(\overline{G})$, that is, $\chi(\overline{G}) \geq |V(G)| - \alpha'(G)$. On the other hand, let $M = \{u_i v_i \mid i = 1, 2, \ldots, \alpha'(G)\}$ be a maximum matching of G. We colour the vertices of $\overline{G}$ with the colours $1, 2, \ldots, |V(G)| - \alpha'(G)$ as follows: assign the colours $1, 2, \ldots, |V(G)| - 2\alpha'(G)$ to each vertex $V(G) - V(M)$ respectively and the colours $|V(G)| - 2\alpha'(G) + 1, \ldots, |V(G)| - \alpha'(G)$ to each pair $\{u_i, v_i\}$ respectively. It is easy to see that the colouring is a proper colouring of $\overline{G}$. So $\chi(\overline{G}) \leq |V(G)| - \alpha'(G)$. Thus $\chi(\overline{G}) = |V(G)| - \alpha'(G)$.

If G has minimum degree at least two, then the set of edges incident with any vertex of G induces a clique of $L(G)$. Note that the line graph $L(G)$ of G contains only the "star-cliques". Thus each minimum clique-transversal set of $L(G)$ corresponds to a minimum edge cover of G. Hence $\tau_C(L(G)) = \beta'(G)$, that is, $\tau_C(L(G)) = \chi(\overline{G})$. □

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