East Asian Math. J.
Vol. 31 (2015), No. 4, pp. 609–625
http://dx.doi.org/10.7858/eamj.2015.042

THE PRE-SERVICE SECONDARY TEACHERS' PRESCRIPTION FOR THE MIDDLE SCHOOL STUDENTS' ERRORS IN LINEAR FUNCTIONS

Huijin Kim, Sung-Soo Pyo, and JongKyum Kwon

Abstract. This study was subjected to 9th graders after making a conformity analysis about errors in function from a selected linear function domain learned in 8th grade, and using this we analyzed some errors learners have in the linear function domain. Learners showed the most deficiency in mastery of prerequisite facts · concepts out of errors in linear functions and lack of skill in interpreting the content of the questions and technical errors occurred often as well. How the pre-service secondary school teachers prescribed these errors of linear function was analyzed from the point of problem solving strategies, accessing methods and whether or not the learner's error was used. Looking into the pre-service secondary teachers' prescription of the learners' errors in 3 fields, for the problem solving strategy a procedural strategy was used more than a conceptual strategy, and as for the accessing methods over 90% gave teacher led type explanations to the students. Also over 90% of pre-service secondary teachers did not use the learner's errors that turned up in problems.

1. Introduction

In the beginning of the 20th century in Germany, after advocating the reform of math education, Klein emphasized introducing function concepts to school math, thus emphasizing the importance in the education of thinking functionally. 'Functional thinking' education's importance was emphasized, and after the 'Meran curriculum' was established in Germany, function became a great branch in school math. Klein said "the concept of function is not just simply a mathematical method, it is the heart, the spirit of mathematical thinking" thus contending that function concepts should be a secondary notion of school math ([6]). Functional thinking is an important tool for connecting algebra and geometry, and because it is a basic key point of view placed in a background of all mathematical thinking including applied mathematics, and the concept of

Received August 3, 2015; Revised August 24, 2015; Accepted August 31, 2015.
2010 *Mathematics Subject Classification.* 97B50, 97I20.
Key words and phrases. Pre-service secondary teachers, students' errors in linear functions, Prescription of the learners' errors .

Correspondence should be addressed to Jongkyum Kwon, mathkjk26@hanmail.net.

(pISSN 1226-6973, eISSN 2287-2833)

function should be taught to students in math class like enzyme soaking in so that it can be a living asset to students ([22]).

Along with the side of understanding the concept of function, understanding the expression system of function and the side of utilization are factors that should be importantly considered in learning. Expressing of a function's graph is a powerful tool in understating function and is often utilized in function learning. It is a tricky concept to teach and the prejudice of visual aids by students often brings many misconceptions ([11]). Introducing the concept of function and method of expressions vary, and because they continuously change, the misconceptions able to be formed depending on the method of introducing the concept, the process of students' understanding function concepts varies widely per student.

The themes math teachers have a hard time teaching are mostly of function domain and geometry domain, and because there is a lot of new vocabulary for the students, the teachers have difficulty explaining them ([2]). Especially for function concepts, there are many related subordinate concepts and because the hierarchy structure of the concept is complex, it is the most difficult concept to master of all school math content, and it is a difficult concept for teachers to teach ([21]). Therefore, because many students have a variety of errors of function and epistemic hinders, teachers should predict some errors often shown by students in the process of planning a lesson, and utilize them appropriately in teaching · learning.

If current teachers can predict the errors learners have about function and appropriately prescribe these, pre-service teachers should have this skill as well. However pre-service teachers do not have much opportunity to be in contact with students' errors in reality compared to current teachers. By having pre-service teachers analyze students' errors about function, there is a need to provide the opportunity to fully understand the knowledge and understanding students have about function. By analyzing how pre-service teachers prescribe errors about function that students have, it will help the pre-service teacher find direction related to students' knowledge and learning.

Functional thinking is important to mathematical thinking and also is helpful for living in the real world. So studying functions are important. However many students feel the difficulty of learning functions and make errors when they learn functions. So we searched to find what kind of errors students made in linear functions and how pre-service secondary teachers prescribed for these errors, to suggest to math teachers how students understand functions better. So in this study, targeting 9th grade students, the linear function domain that the students learned in 8th grade was selected as the object of analysis and through preceding research, a conformity of analysis was written about the errors found in linear function domain, and we hope to use this in analyzing errors students have in the function domain. Also to find how pre-service secondary teachers prescribe the errors of linear function that students have, the types of errors were sorted from learners'exam papers and then exam tools were made for pre-service secondary teachers to find out of their prescriptions.

2. Review of related literatures

2.1. An observation of function errors

The reason why students make mistakes or errors could be because the knowledge they have is incomplete or inaccurate. During a class, the teacher may have moved along too quickly by just explaining it or it could have been conducted abstractly or stiffly, and so the students mechanically memorized the information. As a result the information may have seemed meaningless, or they may not have accurately understood the information being taught, consequently the students would not own accurate knowledge or have incorrect concepts and processes. Learning errors for math is the result of a very complex process. Definite separation of the possible reasons for the given errors is very difficult because between the reasons there is close interaction. However by researching these errors, the students' perception skills were comprehended and because it offers more effective education for teaching · learning, there has been lots of research about the errors in math education.

Shin([15]) investigated secondary school students' types of errors, making three classifications. A type of error occurring in the solving-for-the-answer type for function problems were first of all, arising from misuse of theorems and definitions, which means the students did not correctly understand the concepts or characteristics of function. Secondly there were many errors because of students' lack of interpreting skills between function graphs and algebraic formulas. Especially, when they had correctly solved the algebra, but often showed error in providing a graph contradicting with the functional formula. Thirdly, there were many errors due to lack of basic calculation skills, thus not being able to understand the characteristics of an equation. For secondary school students' functional thinking growth and development of functional skill, the mathematical errors of secondary school students in the function domain should be researched and actually used as reference materials when teaching · learning.

Movshovitz Hadar & Orit Zaslavsky ([13]) analyzed errors in an Israeli high school students' graduating test, and provided six error models. The groups being incorrectly used materials, incorrect interpretation of the problem's content, logically invalid inference, distorted meanings or theorems, answers that had not been double checked, and technical errors. For the high school students' case, a large part was from technical errors and inappropriate use of meanings and theorems. Oh([14]) added errors occurring while solving-for-the-answer to the 6 models Movshovitz Hadar & Orit Zaslavsky ([13]) had provided to classify the errors made by secondary school students in the function domain to get 7 categories. It was shown that being deficient of mastery of prerequisite facts

· concepts took up a large part. Sung ([20]) targeted 9th grade students to classify the errors made in the domain of quadratic function graphs into a 4 category model, also analyzing students' errors based on their level of learning abilities.

It was found that there was a difference in the type of error made depending on their level of learning abilities. By researching these errors in the function domain, the teaching method for these errors might be improved and a teaching method appropriate for these students might be designed. Therefore this study hopes to research the errors made by secondary school students in the linear function domain.

2.2. Teachers' PCK

As the importance of a teacher's knowledge became realized in the 1980s, Schulman ([16]) proposed Pedagogical Content Knowledge(PCK) as a base domain for teachers' knowledge. In the process of improving teachers' statuses PCK, which centers around existing information, science, knowledge and pedagogy knowledge, then includes reflection of teachers' education about this, and it is perceived to act as a temporary bridge between information knowledge and actual teachings, was brought to attention ([2]).

After Shulman ([16]) many studies on PCK have been conducted, and most of the researchers studied the elements of PCK shown by teachers in actual class time with Shulman's concepts of PCK in the background. Grossman ([5]) added the concept of situation knowledge to the elements of teachers' knowledge and emphasized the importance of PCK as an element influential to a teacher's behavior in the classroom. Marks([9]) considered using elements of PCK as a medium in class situation. When considering the fact that there are various perspectives about PCK in an actual class situation, PCK shown in actual class can change depending on the students participating in class and the field's environmental element, and the property of the knowledge made in the field in a school classroom causes PCK to be an important element in showing a teacher's professionalism.

Ball([1]) stated that the elements of understanding subject matter are loosely defined, thus, with Shulman's PCK concepts in the background, Content Knowledge for Teaching was studied from a special domain standpoint. The domain of Mathematical Knowledge of Teaching (MKT) of teaching was classified into subject matter knowledge and PCK, and to conceptualize the mathematical knowledge and techniques needed for a teacher. The subject matter knowledge and PCK characteristics were inspected on the background of elementary students' understanding of fractions. Many preceding studies showed the importance of composing PCK to understand students' misconceptions and errors ([16],[5],[9]).

Choi([2]) thought knowledge of students' misconceptions and difficult concepts importantly as an element, in the domain of student understanding of Mathematics Pedagogical Content Knowledge (MPCK) out of the domains of class professionalism in an analysis frame of MPCK, and made 7 categories of teachers' teaching methods. Of these, the teaching method that is to overcome a learner's misconception and difficult concepts is when the teacher thinks of the misconceptions, errors, and incorrect answers students can have ahead of time and find a way to directly fix this. Misconceptions and errors exist because of incorrect knowledge in the learner's cognitive structure and since it could be the reason hindering following learning, it is important to perceive and try to arrange precautions for learner's errors.

Although it is important for pre-service secondary teachers to have knowledge of learners' errors, there aren't many studies about the reactions and strategies for learners' errors([18]). Thus Son([17]) studied the reactions of pre-service teachers of learners' errors related to ratio of similar rectangles and proportional expression. The frame of analysis was provided to analyze pre-service teachers' knowledge and reactions of learners' errors, and the domain of the frame of analysis are comprised of mathematical matter, the focus of class, methods of approach, didactic behavior, whether or not the learner has used error, and obstacle in communication.

Song & Pang([19]) said that although there are many studies about learners there are comparatively little studies focusing on a student's understanding knowledge or learners' error. If mathematics teachers know students' errors in linear functions, they can help students not to have misconception and to understand functions better. Therefore this study is to research how pre-service secondary teachers deal with learners' errors regarding linear function.

3. Method

3.1. Participants

We targeted 9th grade students to analyze errors in linear function and analyzed how pre-service secondary teachers prescribed the errors learners had about functions. To find out what sort of errors secondary school students had of linear functions, the K secondary school located in Daegu, which has the middle level for the test of the city 61 students of 9th grade was selected as research subjects. Also to analyze pre-service secondary teachers' prescriptions of learners' errors, we selected 46 pre-service secondary teachers of J College of Education as research subjects. The junior of pre-service teachers took the course, "How to teach Math." The background variables of these test subjects are organized in Table1.

Table 1. Background variables of the test subjects

Test Subjects	Grade	Gender	Number of People	Total
Secondary School	3	M F	28(45.9) 33(54.1)	61
Pre-service Teachers	2	M F	9(40.9) 13(59.1)	22
	3	M F	10(41.6) 14(58.4)	24

3.2. Instrument

For this study examination tools have been used for each 9th grade students and pre-service secondary teachers. One is to analyze secondary school students' errors in linear function, and another one is to find how pre-service secondary school teachers prescribed the learners' errors.

3.2.1. *Examination tool targeting secondary school students.*

An examination tool was made to find out the errors in linear function secondary school students have. We selected common problems in linear function of 8th grade among 12 mathematics textbooks. This consisted of 4 areas for linear function based on content titles in the text books. In 8th grade there are largely 4 domains of the linear function domain. Therefore dividing the linear function domain into 4 domains, the definition of function, function related graphs, the algebraic expression of function, and application of function, a total of 9 questions were made. The examination tool used to analyze the errors in linear function are as shown in Table 2.

3.2.2. *Examination tool targeting pre-service secondary teachers.*

To find out how pre-service secondary teachers prescribed learners' errors of function, we created an examination tool based on actual answers given by secondary school students. The examination tool targeting pre-service secondary teachers was comprised of a method providing a scenario related to a teaching situation. First of all in order to introduce the situation of the given scenario to the pre-service secondary teachers, we provided the directions, 'Imagine you are currently teaching 8th grade students, and answer accordingly to the following

TABLE 2. Examination tool targeting secondary school students

Dividing Domains	Number	Question
The definition of function	1	The definition of function and examples
	2	Judging whether or not it is a linear function
Function related graphs	3	judging whether the given graph is a function or not
	4	Drawing a graph for the given function formula
The algebraic epression of function	5	Finding the function formula for the given graph
	6	Finding the function formula for the given conditions
	7	Finding the function formula for the given conditions
Application of function	8	Applied function questions
	9	Applied function questions

situations' and then we provided the questions of the errors largely found of the 4 domains.

In order to analyze how the pre-service secondary teachers reacted to the learners' errors of function, we comprised 6 questions. The questions were based on secondary school students' answers' error of the definition of function, judging whether or not the graph is showing a function, questions of expressing the linear function formula into a graph, questions of expressing algebraic functions into linear function formulas, and applied function. Of all the questions, the same 2 questions were given for subordinate questions. The question '(1) Is Cheol-su's solving-for-the-answer correct? If it is incorrect, find the error in Cheol-su's thought and explain why you think this is an error.' was used to see if pre-service secondary teachers correctly diagnosed the learners' error. Also, the question '(2) How would you teach Cheol-su? Please explain as specifically as possible' was to see how pre-service secondary teachers revised learners' errors in linear function.

3.3. Collecting Data

To conduct the exam targeting secondary school students, we explained the purpose of our study to the corresponding school's math teacher, and requested them to guide the students to write their solving-for-the-answer and their answers. To analyze the errors in linear function of secondary school students, our study was conducted after the secondary school students had learned the linear function chapter and in March after they had advanced a grade. It was conducted under the math teacher's guidance in each classroom for 40 minutes.

To conduct the exam targeting pre-service secondary teachers, we explained the purpose of our study to the professors of the College of Education. The examination targeting pre-service secondary teachers was conducted after the examination targeting secondary school students was conducted and the analysis finished. We requested that the pre-service secondary teachers were given enough time to fill out their answers, and the examination of pre-service secondary teachers was finished within 60 minutes.

3.4. Data Analysis

To analyze the errors of linear function made by secondary school students, we categorized the examination questions' solving-for-the-answers that the students solved themselves. The cases where a whole answer was not given for the given question on the examination and the cases where they did not provide any process of solving-for-the-answer or the answer, we marked it as unanswered and exempted it, and for the cases where multiple errors occurred in solving-for-the-answer, we only used the error that occurred first as a subject of analysis. The analysis frame for the learners' error in the function domain was created through modifying the preceding study ([13], [14]) and the specific information is given in Table 3.

TABLE 3. Analysis frame for learners' errors in the function domain

Subject	Analysis code
Misused Data	MD
Lack of skill in interpreting the content of the questions	MP
Deficient mastery of prerequisite facts and concepts	DC
Logically invalid inference	LF
Unmatched solution	RS
Technical errors	TE

To find out how pre-service teachers prescribe secondary school students' errors in function, we selected types of errors in the answer sheets they filled out themselves, made examination tests and looked to see how pre-service teachers prescribed these problems. The analysis frame for the reaction of pre-service teachers towards learners' errors was used by selecting three domains, problem solving strategy, method of approach, the degree of learners using errors, directly related to this study from Son([17])'s analysis frame. The analysis frame for pre-service secondary teachers' reactions is as shown in Table 4.

Table 4. Analysis frame for pre-service teachers for learners' error

Domain	Subject
Problem solving strategy	Conceptual, procedural
Method of approach	Teacher led types, student led types
The degree of learners using error	Used, not used

4. Results

Correctly analyzing the possible causes for the mathematical subjects for the given error is very difficult. This is because between the reasons are closely connected interconnections. However for teachers guiding students in an actual classroom, studies and results about the types of errors in a cognitive model about the reasons for errors are helpful in understanding students and appropriately composing a teaching lesson plan and therefore is necessarily required. So first of all we will take a look at the types of errors learners have about linear function and then we will find out how pre-service secondary teachers prescribe learners' errors.

4.1. The types of errors in linear function

Out of the 61 secondary school student subjects, a total of 298 errors in function had been selected. The result of the types of errors made by learners in function are as shown in Table 5.

Table 5. Result of types of errors in functions made by learners

Types of errors	Number of errors(Percentage)
Misused Data	9(3.0)
Lack of skill in interpreting the content of the questions	87(29.2)
Deficient mastery of prerequisite facts and concepts	125(42.0)
Logically invalid inference	10(3.3)
Unmatched solution	12(4.0)
Technical errors	55(18.5)

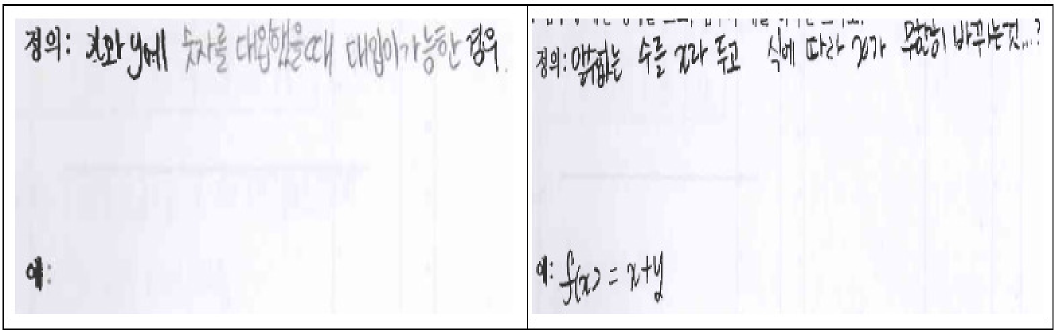

FIGURE 1. Errors of the definition of function answered by students

As seen in the result in Table 5, like the preceding studies ([13], [14], [20], [10]), related to learners' function, deficient mastery of prerequisite facts · concepts showed up the most. However there was a slight difference in the preceding study results of the frequency of errors in function. In Movshovitz Hadar & Orit Zaslavsky([13]), Oh ([14]), and Lee([10]) research results technical errors was the second highest, and in Sung's([20]) research results incorrectly translating the language was second largest. In this study lack of skill in interpreting function problems was second highest. Also there were a lot of technical errors. However misused data, logically invalid inference, and answers that had not been double checked barely appeared. A deeper look at the most appeared deficient mastery of prerequisite facts · concepts is as below.

Firstly, many learners showed errors in the first question asking about the definition of function. The result of the errors made by many learners are shown below in Figure 1.

As seen in the result of Figure 1 many learners could not answer things about the definition of function and this was also the case for examples of function. This means that the learners who had learned about functions could not correctly remember the definition and because in the process of acquiring the concept learners mostly depend on largely concept images for the concept of functions, so they could not properly express the definition in words. Secondly in the domain of function graphs, for the question judging whether or not the given graph was a function or not, students made many errors. The results of the errors made by many students are as shown below in Figure 2.

As seen in Figure 2 it is not an easy question for learners to judge if a graph is a function or not. Eisenberg ([3]) pointed out that understanding function graphs is essential and made a good analysis of the reason why students avoid graph type forms. Although for people who have knowledge of graphs, graphs show definite information about a function that can be easily accessed to, however in algebraic formulas the information is implied and it is hard to get

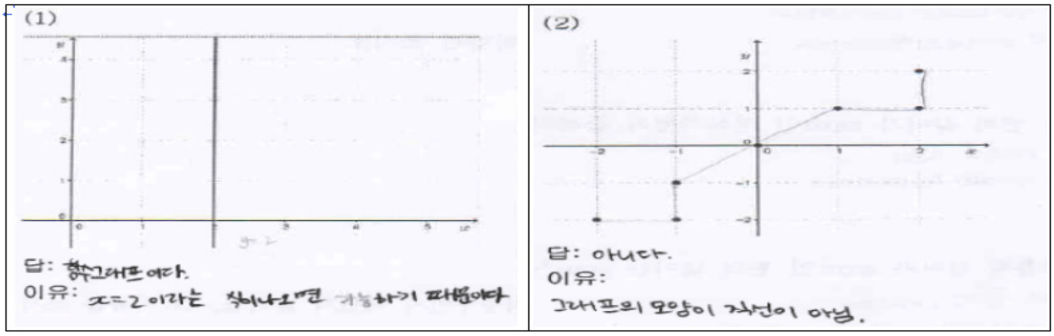

FIGURE 2. Errors answered by students about judging functions

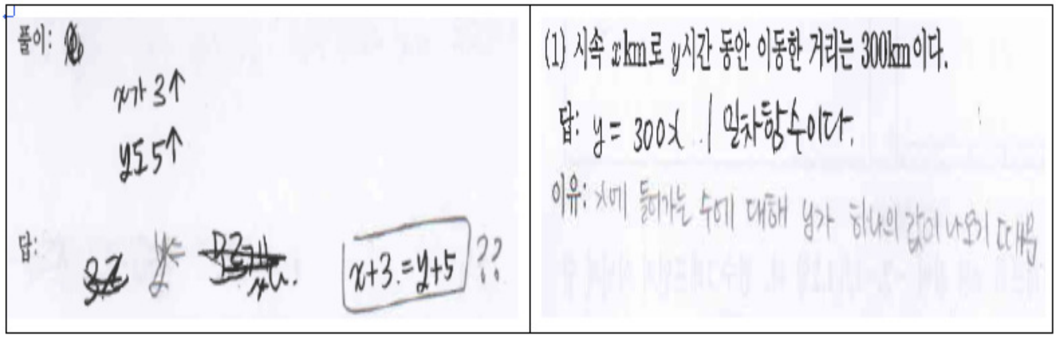

FIGURE 3. Errors made by learners due to lack of skill in interpreting the content of the question

information from this. To learners a function's graph must be easily recognizable and in a pretty shape, be continuous, and be in a line and this seems to be their prejudice. Also for a graph that is made up of one dot or a few dots, they seem to have a prejudice that this is not a function graph and if it were to be made up of many dots they seem to want to make it into a line.

After deficient mastery of prerequisite facts · concepts, the next error that learners seem to make is lack of skill in interpreting the content of the question, and a closer look is as below.

In question 2 it was a question asking the students to judge whether or not the simple sentence was expressing a linear function or not, and many students made errors. There were many cases where they could not express the given sentence problem into a formula. Also the domain that many students have trouble with, applied function was question 8 and 9, they also could not perceive the content of the question and made errors in expressing the formula. The result of errors made by many students is shown below as in Figure 3.

As seen in the result of Figure 3, it shows that they cannot properly express the formula provided in the question. Although it was known that it is a hard question for learners to understand mathematical sentences and expressing them in algebraic formulas, they were continuously confronted in many domains with difficulty in even basic questions, and in applied equation, applied inequalities and so on. In math problems composed of sentences there is a need for more skill in gaining mathematical information and the process of regenerating it into an algebraic formula.

4.2. Reactions and Strategies of Pre-service Teachers

22 second grade pre-service secondary teachers and 24 third grade pre-service secondary teachers making a total of 46 people, were the subjects of this study on finding out the reactions and strategies for prescribing errors of function by learners. The result of pre-service secondary school teacher's strategy for prescribing errors of function by learners is as shown below in Table 6.

TABLE 6. Result of reactions and strategies of pre-service teachers of errors of function by learners

Domains	Subject	Percentage
Problem solving strategy	Conceptual strategies	29.5
	Procedural strategies	63.5
	No answer	6.9
Method of approach	Teacher led types	91.3
	Student led types	1.8
	No answer	6.9
The degree of learners using error	Used	2.2
	Not used	90.9
	No answer	6.9

As seen in the result of Table 6 the reactions and strategies of pre-service secondary teachers about learners' error in function, use mostly procedural strategies, the method of approach is usually the teacher led type, and they rarely use the learners' error. Taking a closer look at problem solving strategy is as follows.

Pre-service secondary teachers use procedural strategies more than conceptual strategies for problem solving strategy. For the development of a student's

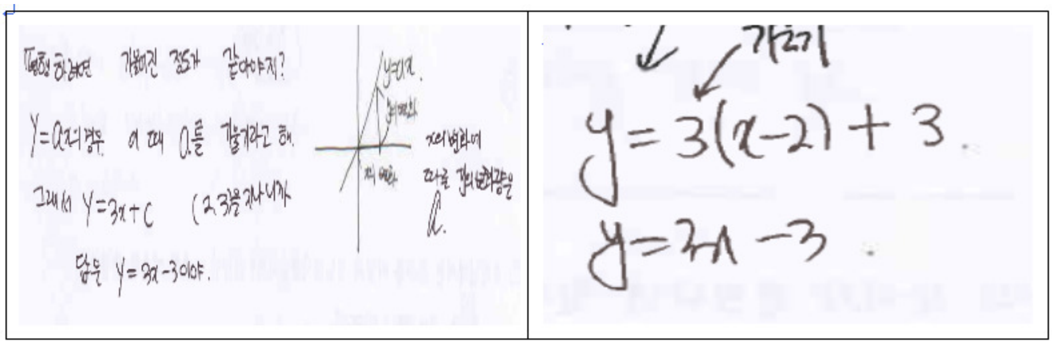

FIGURE 4. Examples of conceptual strategy and procedural strategy used by pre-service secondary teachers

mathematical knowledge, instead of only using a teaching method of procedures, better understanding is brought by a teaching method including a conceptual reason for the procedure ([4], [7]). An example of conceptual strategy and procedural strategy used by pre-service secondary teachers is shown in Figure 4 below.

As seen in Figure 4 the conceptual strategy used by the pre-service teacher's explanation is relatively more specific than the procedural strategy used explanation. Therefore to help the learner's understanding of linear function, it would be good for pre-service secondary teachers to use conceptual strategies along with a procedural approach.

Pre-service secondary teachers' method of approach to errors in function by learners was carried out through unilaterally teacher led type. Instead of helping students lead the way of solving the problem, most are approaching in a method of unilaterally teacher leading, solving the problem and explaining it. Examples of pre-service secondary teachers' usual use of teacher led approach method types and students led approach method types are shown in Figure 5 below.

As seen in the result of Figure 5 pre-service secondary teachers immediately prescribe teacher led type for learners' error in linear function, and in the approach method of student led type they listened to the students' thought on the error and then they would make their prescription. About the error a student has, it would be more helpful to learn about linear functions by first understanding what error the student has and then to prescribe it. In prescribing the errors of linear functions made by the learners, the pre-service secondary teachers rarely used the students' errors. By using the learner's error, it is important to help them realize what the error they had was and to not make that error again. Examples of learners' errors being used and not being used by pre-service secondary teachers are as shown below in Figure 6.

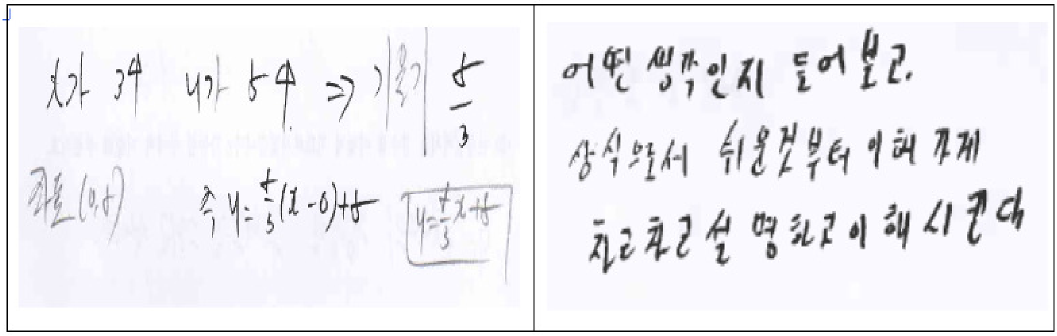

FIGURE 5. Examples of teacher led type and student led type approach methods used by pre-service secondary teachers

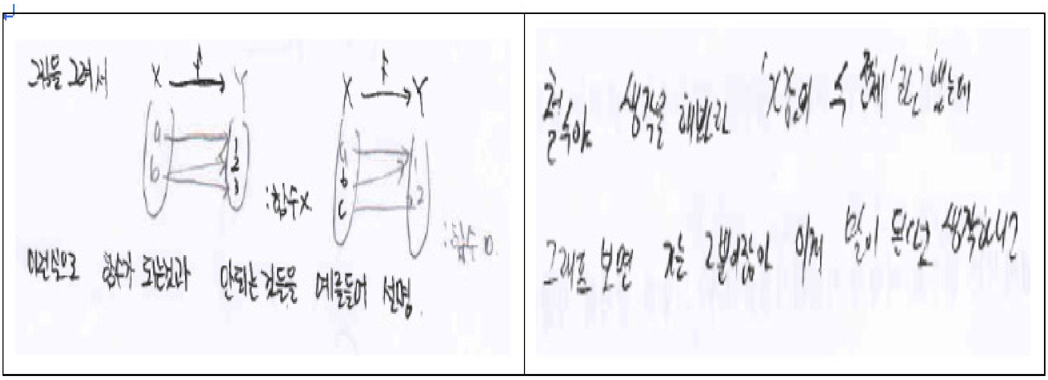

FIGURE 6. Examples of learners' errors being used and not being used by pre-service secondary teachers

As seen in the result of Figure 6 one was when pre-service secondary teachers didn't use the learner's error and immediately corrected it, and in the other example it used the learner's error explaining why the learner got the wrong answer to the question. By using the learner's error, it is induced that they think about their errors themselves and thus not allowing the error to happen again.

5. CONCLUSIONS AND DISCUSSION

This study was to analyze secondary school students' errors that occur in the domain of linear functions, and find out pre-service secondary school teacher's prescription to this.

Firstly, we found that secondary school students are deficient in mastery of prerequisite facts · concepts. Also in judging whether a graph is a function or not, secondary school students have many errors. Because the definition of function is expressed in a formal and abstract language, much difficulty follows in secondary school students' accurately understanding it. Therefore when current teachers guide the concept of function, if they were to provide various representations and explain these representations relating them to the definition of function, then not only would it help in understanding the concept, it would also decrease errors of a function's graph or formula. Also by guiding them to find examples of the definition of function in real life, it would very much help the learners understand the concept.

Secondly, secondary school students lacked in skill of interpreting the content of the linear function questions and there were many technical errors. In the math curriculum the domain of application is a domain secondary school students have trouble with and much practice is needed for the students to learn this domain, and technical errors also need much practice by the students.

Thirdly, when pre-service secondary teachers made prescriptions for secondary school students' errors of linear function, for problem solving strategies they mostly used procedural strategies rather than conceptual strategies. For the secondary school students to understand the mathematical matter well, it is helpful for a better understanding to use a teaching method that includes conceptual strategy for the procedure rather than a teaching method that only uses procedural strategy.

Fourthly, the method of approaching errors of linear functions made by secondary school students, pre-service secondary teachers usually used teacher led types. Also because they used teacher led types, they rarely used the secondary school students' errors. Instead of understanding why the secondary school students made that error, they mostly gave prescriptions of immediate correction of the error. Instead of immediately prescribing the error, it is more important to know the reason for the learner's error and they should make a bigger effort to understand the conceptual thinking behind the learner's errors.

References

[1] Ball, D. L., Thames, M. H. & Phelps, G.(2008). *Content Knowledge for Teaching: What Makes It Special?*, Journal of Teacher education, 59(5), 389-407.

[2] Choi, S. H., (2007) *Mathematics pedagogical content knowledge in accordance with the curriculum revision(PCK) research*, Technical report 2007-3-2, Seoul: Korea Institute for Curriculum and evaluation

[3] Eisenberg, T. (1992). *On the development of a sense for function.*, In E. Dubinsky & Harel(Eds.), The concept of function: Aspects of epistemology and pedagogy(pp. 153-154). MAA Notes and Report Series. U.S.A.: Mathematical Association of America.

[4] Fuson, K. C., Briars, D. J. (1990). *Base-ten blocks as a first- and second-grade learning/teaching approach for multidigit addition and subtraction and place-value concepts*, Journal for Research in Mathematics Education, 21, 180-206.

[5] Grossman, P.L.(1990). *The making of a teacher: Teacher knowledge and teacher education*, New York: Teachers College Press.

[6] Hamely, H.R.,(1934). *Relational and functional thinking in mathematics*, The National Council of Teachers of mathematics,

[7] Hiebert, J., & Wearne, D. (1996). *Instruction, understanding, and skill in multidigit addition and subtraction*, Cognition and Instruction, 14, 251-283.

[8] Joo, S. H., Lee, S. Y., Kim, H. U., Lee, G. H., Lee, M. H. (1999). *Observing Classroom Lesson and Analysis*, Seoul:Wonmisa

[9] Marks, R. (1990). *Pedagogical content knowledge : From a mathematical case to a modified conception*, Journal of teacher education, 41(3), 3-11.

[10] Lee, K. D. (2003). *A Study on the analysis of errors on the graphic of quadratic function and the correction applying Excel*, Major in Mathematics Education Graduate School of Education, Korea National University of Education.

[11] Lee, Y. H., and Park, J. H., (2011). *A study of Mathematics Teacher's PCK with Respect to Students' Misconceptions and Errors*, Journal of research in curriculum institution Vol. 15 No. 1, 223-242.

[12] Marks, R. (1990). *Pedagogical content knowledge : From a mathematical case to a modified conception*, Journal of teacher education, 41(3), 3-11.

[13] Nitsa Movschovitz -Hardar & Orit Zaslavsky. (1987). *An Empirical Calssification Model for errors in high school mathematics*, Journal for Research in Mathematics Education. Vol 18, No 1, 3-14

[14] Oh, J. H., (1996). *The Study on Mathematical errors in the function in the secondary school*, Mathematics Education Major, The Graduate School of Education, Ewha Women University.

[15] Shin, I. S., (1996).*A Study on Misconceptions an Errors of Function in Secondary School Students*, Major in Mathematics Education Graduate School of Education, Korea National University of Education.

[16] Shulman, L. S.(1986).*Those who understand: Knowledge growth in teaching* , Educational Researcher, 15(2), 4-14.

[17] Son, J. (2013). *How preservice teachers interpret and respond to student errors: ratio and proportion in similar rectangles*, Educational Studies in Mathematics, 84(1), 49-70.

[18] Son, J. & Sinclair, N. (2010). *How preservice teachers interpret and respond to student geometric errors*, School Science and Mathematics, 110(1), 31-46.

[19] Song, K. Y. & Pang, J. S.(2012). *Novice elementary teachers' knowledge of students' errors on plane figures*, Journal of the Korean school mathematics society, Vol 15, No 3, 429-451.

[20] Sung, J.G.(2000).*A Study on the analysis of errors on the graph of quadratic function*, Major in Mathematics Education Graduate School of Education, Korea National University of Education.

[21] Tall, D. (2003). *Advanced mathematical thinking*,(translate by Lew, H. C and Cho, W. Y and Kim I. S), Seoul: Kyungmonsa. (Original work: Advanced mathematical thinking. published in 1991)

[22] Woo, J. H.(2011).*Educational basics for the school mathematics*,Seoul: Seoul national university publisher.

Huijin Kim
University of Maryland
E-mail address: huijin.kim@faculty.umuc.edu

Sung-Soo Pyo
Kyungpook National University
E-mail address: ssoopyo@knu.ac.kr

JongKyum Kwon
Kyungpook National University
E-mail address: mathkjk26@hanmail.net