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Robust Delay-dependent Stability Criteria for Takagi-Sugeno Fuzzy Systems with Time-varying Delay

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Abstract - This paper presents the robust stability condition of uncertain Takagi-Sugeno(T-S) fuzzy systems with time-varying delay. New augmented Lyapunov-Krasovskii function is constructed to ensure that the system with time-varying delay is globally asymptotically stable. Also, less conservative delay-dependent stability criteria are obtained by employing some integral inequality, reciprocally convex approach and new delay-partitioning method. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

Key Words : Asymptotic stability, Fuzzy systems, Time-varying delay, LMI.

1. Introduction

Since Takagi-Sugeno(T-S) fuzzy model was first introduced in [1], the stability and design conditions for T-S fuzzy systems have been paid much attention. The main advantage of T-S fuzzy model is that it can combine the exibility of fuzzy logic theory and rigorous mathematical theory of linear system into a unified framework to approximate complex nonlinear systems [2-4]. On the other hand, time delays often appears in many dynamical systems such as metallurgical processes, biological systems, neural networks, networked control systems and so on. The existence of time delay may cause poor performance or instability. Hence, the stability of T-S fuzzy systems with time delay has been studied by many researchers [5-24,30,31].

It is well known that the delay-dependent stability criteria are less conservative that delay-independent ones especially that the time-delay is small. The main issue of delay-dependent stability criteria is to find a maximum delay bounds to guarantee the asymptotic stability of the considered systems. Therefore, the study of increasing the maximum delay bounds in delay-dependent stability criteria for fuzzy systems is an important topic and have been investigated by many researchers. In [5], the delay dependent stability problem for T-S fuzzy systems with time varying delay was investigated. Some stability criteria or stabilization of delayed T-S fuzzy systems were derived by employing free-weighting matrix [6,9,12]. Furthermore, the results was further studied by using delay-partitioning-based approach [13,17,19,23]. Recently, in [18], an augmented Lyapunov-Krasovskii functional approach that introduces a triple integral and some augmented vectors was employed to investigate the stability problem of T-S fuzzy systems with time-varying delay. In [20], the improved results was obtained by quadratically convex approach. The results was further improved in [24] by employing the delay-partitioning method and reciprocally convex approach. However, though these results and analytic methods are elegant, there still exist some rooms for further improvements. First, in [8,11,20,18], Jensen's inequality, free-weighting matrix and quadratically convex combination approach are used to derive the stability condition. However, reciprocally convex approach [25], which can play an important role in reducing conservatism of the stability condition, is not used in [8,11,20,18]. Second, though the

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reciprocal convex approach, delay−partitioning and integral inequalities method are combined to obtain some less conservative results in [24], it still needs some improvements since it only used the improved inequality in constant delay, not employed in time−varying delay. Furthermore, it can be predicted that delay−partitioning approach can provide tighter upper bounds than the results without delay−partitioning approach. However, as delay−partitioning number increases, matrix formulation becomes complex and time consuming and computational burden grow bigger. Therefore, there are rooms for further improvement in stability analysis of T−S fuzzy systems with time−varying delay.

In this paper, the stability analysis conditions for uncertain T−S fuzzy systems with time−varying delay are proposed. By construction of a modified augmented Lyapunov−Krasovskii functional approach, an improved stability criterion for guaranteeing the asymptotically stable is derived by using Wirtinger−based integral inequality [26], reciprocally convex approach [25], and new delay−partitioning method. It should be pointed out that different with delay−partitioning method used in [24], we only divide the time interval into two sub−intervals, and consider two different cases of delay−partitioning method. Moreover, some robust stability criteria of uncertain T−S systems with time varying delay is provided. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Notation: Throughout the paper, R^n denotes the n −dimensional Euclidean space, $\mathrm{R}^{m\times n}$ denotes the set of m by n real matrix. For symmetric matrices X, $X>0$ and $X<0$, mean that X is a positive/negative definite symmetric matrix, respectively. I and 0 denote the identity matrix and zero matrix with appropriate dimension. ★ represents the elements below the main diagonal of a symmetric matrix. $diag\ldots$ denotes the diagonal matrix.

2. Problem Statements

Consider the following nonlinear system which can be modeled as T−F fuzzy model type subject to time−varying delay:

Rule i: If $\theta_1(t)$ is M_{i1} and ... and if $\theta_n(t)$ is M_{in}

$$\begin{aligned}&\dot{x}(t)=(A_i+\Delta A_i)x(t)+(A_{di}+\Delta A_{di})x(t-h(t)),\\&x(t)=\phi(t),t\in[-h_M,0],i=1,2,\ldots,r\end{aligned}\tag{1}$$

where $\theta_1(t),\theta_2(t),\ldots,\theta_n(t)$ are the premise variables, M_{ij} is fuzzy set, $i=1,2,\ldots,r,j=1,2,\ldots,n$ r is the index number of fuzzy rules, and $x(t)\in R^n$ denotes the state of the system. A_i and A_{di} are the known system matrices and delayed−state matrices with appropriate dimensions, respectively. $\phi(t)$ is a continuously real−valued initial function vector. we assume that $h(t)$ is a time−varying delay satisfying

$$0\le h(t)\le h_M,\dot{h}(t)\le\mu,\tag{2}$$

where h_M,μ are known constants.

The uncertainties satisfy the following condition:

$$[\Delta A_i\quad \Delta A_{di}]=DF(t)[E_i\;\; E_{di}],\tag{3}$$

where D,E_i,E_{di} are known constant matrices; $F(t)\in R^{n\times n}$ is the unknown real time−varying matrices with Lebesgue measurable elements bounded by

$$F^T(t)F(t)\le I,\;\; t\ge 0\tag{4}$$

Using singleton fuzzifier, product inference, and center−average defuzzifier, the global dynamics of the delayed T−S system (1) is described by the convex sum form

$$\dot{x}(t)=\sum_{i=1}^{r}p_i(\theta(t))[(A_i+\Delta A_i)x(t)+(A_{di}+\Delta A_{di})x(t-h(t))]\tag{5}$$

where $p_i(\theta(t))$ denotes the normalized membership function satisfying

$$p_i(\theta(t))=\frac{w_i(\theta(t))}{\sum_{i=1}^{r}w_i(\theta(t))},\;\; w_i(\theta(t))=\prod_{j=1}^{n}M_{ij}(\theta_i(t))\tag{6}$$

where $M_{ij}(\theta_i(t))$ is the grade of membership of $\theta_i(t)$ in M_{ij} It is assumed that

$$w_i(\theta(t))\ge 0,\;\sum_{i=1}^{r}w_i(\theta(t))>0,t\ge 0\tag{7}$$

Then, we have the following condition

$$p_i(\theta(t))\ge 0,\;\sum_{i=1}^{r}p_i(\theta(t))=1,t\ge 0\tag{8}$$

For the sake of simplicity, let us define

$$\overline{A}=\sum_{i=1}^{r}p_i(\theta(t))A_i,\ \overline{A_d}=\sum_{i=1}^{r}p_i(\theta(t))A_{di},\qquad (9)$$
$$\overline{E}=\sum_{i=1}^{r}p_i(\theta(t))E_i,\ \overline{E_d}=\sum_{i=1}^{r}p_i(\theta(t))E_{di}.$$

Now, the system (5) can be rewritten as

$$\dot{x}(t)=\overline{A}x(t)+\overline{A_d}x(t-h(t))+Dp(t),\qquad (10)$$
$$p(t)=F(t)q(t),$$
$$q(t)=\overline{E}(t)+\overline{E_d}x(t-h(t)).$$

In what follows, some essential lemmas are introduced.

Lemma 1 [26] For a given matrix $R>0$, the following inequality holds for all continuously differentiable function $x(t)$ in $[a,b]\in R^n$:

$$-(b-a)\int_a^b \dot{x}^T(s)R\dot{x}(s)ds\le -[x(b)-x(a)]^TR[x(b)-x(a)]-3\Omega^TR\Omega$$

where $\Omega=x(b)+x(a)-\frac{2}{b-a}\int_a^b x(s)ds$.

Lemma 2 [28] For a given matrix $M>0$, $h_m\le h(t)\le h_M$, and any appropriate dimension matrix X, which satisfies $\begin{bmatrix}\overline{M} & T\\ \star & \overline{M}\end{bmatrix}\ge 0$. Then, the following inequality holds for all continuously differentiable function $x(t)$

$$-(h_M-h_m)\int_{t-h_M}^{t-h_m}\dot{x}^T(s)M\dot{x}(s)ds\le -\alpha^T(t)\begin{bmatrix}\overline{M} & T\\ \star & \overline{M}\end{bmatrix}\alpha(t)$$

where

$$\alpha(t)=[\alpha_1^T(t),\alpha_2^T(t),\alpha_3^T(t),\alpha_4^T(t)]^T,$$
$$alplha_1(t)=x(t-h(t))-x(t-h_M),$$
$$\alpha_2(t)=x(t-h(t))+x(t-h_M)-\frac{2}{h_M-h(t)}\int_{t-h_M}^{t-h(t)}x(s)ds,$$
$$\alpha_3(t)=x(t-h_m)-x(t-h(t)),$$
$$\alpha_4(t)=x(t-h_m)+x(t-h(t))-\frac{2}{h(t)-h_m}\int_{t-h(t)}^{t-h_m}x(s)ds,$$
$$\overline{M}=\begin{bmatrix}M & 0\\ \star & 3M\end{bmatrix}.$$

Lemma 3 (Fisher's Lemma [27]) Let $\xi\in R^n, \Phi=\Phi^T\in R^{n\times n}$, and $B\in R^{m\times n}$ such that $rank(B)\le n$. The following statements are equivalent

(i) $\xi^TB\xi<0, \forall B\xi=0, \xi\ne 0$,

(ii) $B^{\perp T}\Phi B^{\perp}<0$, where $B^{\perp}$ is a right orthogonal complement of B.

(iii) $\exists X\in R^{n\times m}:\Phi+XB+(XB)^T<0$.

3. Main Results

In this section, we first propose a stability criterion for delayed T-S fuzzy systems without uncertainties, and the following nominal system will be considered:

$$\dot{x}(t)=\overline{A}x(t)+\overline{A_h}x(t-h(t)),\qquad (11)$$

For the sake of simplicity of matrix and vector representations, $e_i\in R^{8n\times n}$ $(i=1,2,...,8)$ are defined as block entry matrices (for example $(e_4=[0\ 0\ 0\ I\ 0\ 0\ 0\ 0]^T)$. The other notations are defined as :

$$\xi^T(t)=\Big[x^T(t)\ x^T(t-\alpha h_M)\ x^T(t-h(t))\ x^T(t-h_M)$$
$$\left(\frac{1}{\alpha h_M-h(t)}\int_{t-\alpha h_M}^{t-h(t)}x(s)ds\right)^T\left(\frac{1}{h(t)}\int_{t-h(t)}^{t}x(s)ds\right)^T$$
$$\left(\frac{1}{h_M-\alpha h_M}\int_{t-h_M}^{t-\alpha h_M}x(s)ds\right)^T\ \dot{x}(t)\Big],$$
$$\overline{\xi^T}(t)=\Big[x^T(t)\ x^T(t-\alpha h_M)\ x^T(t-h(t))\ x^T(t-h_M)$$
$$\left(\frac{1}{h_M-h(t)}\int_{t-h_M}^{t-h(t)}x(s)ds\right)^T\left(\frac{1}{h(t)-\alpha h_M}\int_{t-h(t)}^{t-\alpha h_M}x(s)ds\right)^T$$
$$\left(\frac{1}{\alpha h_M}\int_{t-\alpha h_M}^{t}x(s)ds\right)^T\ \dot{x}(t)\Big],$$
$$\Pi_1^1=[e_1\ (\alpha h_M-h(t))e_5+h(t)e_6\ (h_M-\alpha h_M)e_7],$$
$$\Pi_1^2=[e_1\ \alpha h_Me_7\ (h_M-h(t))e_5+(h(t)-\alpha h_M)e_6],$$
$$\Pi_2^1=[e_3-e_2\ e_3+e_2-2e_5\ e_1-e_3\ e_1+e_3-2e_6],$$
$$\Pi_2^2=[e_3-e_4\ e_3+e_4-2e_5\ e_2-e_3\ e_2+e_3-2e_6],$$
$$\Sigma_1^1=\Pi_1^1P[e_8\ e_1-e_2\ e_2-e_4]^T+[e_8\ e_1-e_2\ e_2-e_4]^TP\Pi_1^{1^T},$$
$$\Sigma_1^2=\Pi_1^2P[e_8\ e_1-e_2\ e_2-e_4]^T+[e_8\ e_1-e_2\ e_2-e_4]^TP\Pi_1^{2^T},$$
$$\Sigma_2=e_1Q_1e_1^T-(1-\mu)e_3Q_1e_3^T,$$
$$\Sigma_3=e_1Q_2e_1-e_2Q_2e_2^T+e_2Q_3e_2^T-e_4Q_3e_4^T,$$
$$\Sigma_4^1=(\alpha h_M)^2e_8R_1e_8^T-\Pi_2^1\begin{vmatrix}\overline{R_1} & S_1\\ \star & \overline{R_1}\end{vmatrix}\Pi_2^{1^T},$$
$$\Sigma_4^2=(\alpha h_M)^2e_8R_1e_8^T-[e_1-e_2]R_1[e_1-e_2]^T$$
$$-3[e_1+e_2-2e_7]R_1[e_1+e_2-2e_7]^T,$$
$$\Sigma_5^1=((1-\alpha)h_M)^2e_8R_2e_8^T-[e_2-e_4]R_2[e_2-e_4]^T$$
$$-3[e_2+e_4-2e_7]R_2[e_2+e_4-2e_7]^T,$$
$$\Sigma_5^2=((1-\alpha)h_M)^2e_8R_2e_8^T-\Pi_2^2\begin{vmatrix}\overline{R_2} & S_2\\ \star & \overline{R_2}\end{vmatrix}\Pi_2^{2^T},$$
$$Y_1=\Sigma_1^1+\Sigma_2+\Sigma_3+\Sigma_4^1+\Sigma_5^1,$$
$$Y_2=\Sigma_1^2+\Sigma_2+\Sigma_3+\Sigma_4^2+\Sigma_5^2$$
$$\overline{\Gamma}=[\overline{A}\ 0\ \overline{A_d}\ 0\ 0\ 0\ 0\ -I],$$
$$\Gamma_i=[A_i\ 0\ A_{di}\ 0\ 0\ 0\ 0\ -I],$$

Now we have the following Theorem.

Theorem 1 For given scalars $h_M>0, 0<\alpha<1,\mu$, the system (11) is globally asymptotically stable if there exist symmetric positive matrices $P \in R^{3n \times 3n}, Q_1, Q_2, Q_3, R_1, R_2$, and any matrix $S_j (j=1,2) \in R^{2n \times 2n}$ such that the following LMIs hold

$$\Gamma_i^{\perp T} [Y_1]_{h(t) \in [0, \alpha h_M]} \Gamma_i^{\perp} < 0, \tag{12}$$

$$\Gamma_i^{\perp T} [Y_2]_{h(t) \in [\alpha h_M, h_M]} \Gamma_i^{\perp} < 0, \tag{13}$$

$$\begin{bmatrix} \overline{R_j} & S_j \\ \star & \overline{R_j} \end{bmatrix} \geq 0, j=1,2, \tag{14}$$

where $\overline{R_j} = \begin{bmatrix} R_j & 0 \\ \star & 3R_j \end{bmatrix}$.

Proof: Let us consider the following Lyapunov-Krasovskii functional candidate as

$$V(t) = \sum_{i=1}^{5} V_i \tag{15}$$

where

$$V_1 = \begin{bmatrix} x(t) \\ \int_{t-\alpha h_M}^{t} x(s)ds \\ \int_{t-h_M}^{t-\alpha h_M} x(s)ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-\alpha h_M}^{t} x(s)ds \\ \int_{t-h_M}^{t-\alpha h_M} x(s)ds \end{bmatrix},$$

$$V_2 = \int_{t-h(t)}^{t} x(s)ds,$$

$$V_3 = \int_{t-\alpha h_M}^{t} x(s)ds + \int_{t-h_M}^{t-\alpha h_M} x(s)ds,$$

$$V_4 = \alpha h_M \int_{-\alpha h_M}^{0} \int_{t+\alpha}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds,$$

$$V_5 = (1-\alpha) h_M \int_{-h_M}^{-\alpha h_M} \dot{x}^T(s) R_2 \dot{x}(s) ds.$$

Depending on whether the time-varying delay $h(t)$ belongs the interval $0 \leq h(t) \leq h_M$ or $\alpha h_M \leq h(t) \leq h_M$, different upper bound of the $V_i (i=1,4,5)$ can be obtained as two cases:

When $0 \leq h(t) \leq \alpha h_M$, the time-derivative of $V_i (i=1,2,3)$ can be calculated as

$$\dot{V}_1 = 2 \begin{bmatrix} x(t) \\ \int_{t-\alpha h_M}^{t-h(t)} x(s)ds + \int_{t-h(t)}^{t} x(s)ds \\ \int_{t-h_M}^{t-\alpha h_M} x(s)ds \end{bmatrix}^T P \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-\alpha h_M) \\ x(t-\alpha h_M) - x(t-h_M) \end{bmatrix}$$

$$= \xi^T(t) \Sigma_1^1 \xi(t), \tag{16}$$

$$\dot{V}_2 \leq x^T(t) Q_1 x(t) - (1-\mu) x^T(t-h(t)) Q_1 x(t-h(t))$$

$$= \xi^T(t) \Sigma_2 \xi(t), \tag{17}$$

$$\dot{V}_3 \leq x^T(t) Q_2 x(t) - x^T(t-\alpha h_M) Q_2 x(t-\alpha h_M) + x^T(t-\alpha h_M) Q_3 x(t-\alpha h_M) - x^T(t-h_M) Q_3 x(t-h_M)$$

$$= \xi^T(t) \Sigma_3 \xi(t), \tag{18}$$

By applying Lemma 2, an upper bound of $\dot{V}_4$ is obtained as

$$\begin{aligned} \dot{V}_4 &= (\alpha h_M)^2 \dot{x}^T(t) R_1 \dot{x}(t) - \alpha h_M \int_{t-\alpha h_M}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds \\ &= (\alpha h_M)^2 \dot{x}^T(t) R_1 \dot{x}(t) - \alpha h_M \int_{t-\alpha h_M}^{t-h(t)} \dot{x}^T(s) R_1 \dot{x}(s) ds \\ &\quad - \alpha h_M \int_{t-h(t)}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds \\ &\leq (\alpha h_M)^2 \dot{x}^T(t) R_1 \dot{x}(t) - \beta^T(t) \begin{bmatrix} \overline{R_1} & S_1 \\ \star & \overline{R_1} \end{bmatrix} \beta(t) \\ &= \xi^T(t) \Sigma_4^1 \xi(t), \end{aligned} \tag{19}$$

where

$\beta(t) = [\beta_1^T(t), \beta_2^T(t), \beta_3^T(t), \beta_4^T(t)]^T$,

$\beta_1(t) = x(t-h(t)) - x(t-\alpha h_M)$,

$\beta_2(t) = x(t-h(t)) + x(t-\alpha h_M) - \dfrac{2}{\alpha h_M - h(t)} \int_{t-\alpha h_M}^{t-h(t)} x(s)ds$,

$\beta_3(t) = x(t) - x(t-h(t))$,

$\beta_4(t) = x(t) + x(t-h(t)) - \dfrac{2}{h(t)} \int_{t-h(t)}^{t} x(s)ds$,.

Note that when $h(t)=0$ or $h(t)=h_M$, we have $\beta_1(t) = \beta_2(t) = 0$ or $\beta_3(t) = \beta_4(t) = 0$ Then (19) still holds.

Next, an upper bound of $\dot{V}_5$ can be derived by Lemma 1,

$$\begin{aligned} \dot{V}_5 &= ((1-\alpha)h_M)^2 \dot{x}^T(t) R_2 \dot{x}(t) - (1-\alpha) h_M \int_{t-h_M}^{t-\alpha h_M} \dot{x}^T(s) R_2 \dot{x}(s) ds \\ &\leq ((1-\alpha)h_M)^2 \dot{x}^T(t) R_2 \dot{x}(t) - 3\Omega_1^T R_2 \Omega_1 \\ &\quad - [x(t-\alpha h_M) - x(t-h_M)]^T R_2 [x(t-\alpha h_M) - x(t-h_M)] \\ &= \xi^T(t) \Sigma_5^1 \xi(t), \end{aligned} \tag{20}$$

where

$$\Omega_1 = x(t-\alpha h_M) + x(t-h_M) - \frac{2}{h_M - \alpha h_M} \int_{t-h_M}^{t-\alpha h_M} x(s)ds$$

Therefore, in the case of $0 \leq h(t) \leq h_M$, form Eqs. (16)-(20), an upper bound of $\dot{V}(t)$ can be given as

$$\dot{V}(t) \leq \xi^T(t) Y_1 \xi(t). \tag{21}$$

Based on Lemma 3, $\xi^T(t) Y_1 \xi(t) < 0$ with $\overline{\Gamma} \xi(t) = \sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i \xi(t) = 0$ is equivalent to

$\sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i^{\perp T} Y_1 \Gamma_i^{\perp} < 0$. Furthermore, the above condition is affinely dependent on $h(t)$. Hence, (12) and (14) imply $\sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i^{\perp T} Y_1 \Gamma_i^{\perp} < 0$.

Next, when $\alpha h_M \le h(t) \le h_M$, the time-derivative of V_1 is

$$\dot{V}_1 = 2\begin{bmatrix} x(t) \\ \int_{t-\alpha h_M}^{t} x(s)ds \\ \int_{t-h_M}^{t-h(t)} x(s)ds + \int_{t-h(t)}^{t-\alpha h_M} x(s)ds \end{bmatrix}^T P \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t-\alpha h_M) \\ x(t-\alpha h_M) - x(t-h_M) \end{bmatrix}$$
$$= \overline{\xi^T}(t) \Sigma_1^2 \bar{\xi}(t), \quad (22)$$

Based on Eq. (17) and (18), an derivative of $V_i (i=2,3)$ can be calculated as

$$\dot{V}_i \le \overline{\xi^T}(t) \Sigma_i \bar{\xi}(t), i=2,3. \quad (23)$$

By Lemma 1,

$$\dot{V}_4 = (\alpha h_M)^2 \dot{x}^T(t) R_1 \dot{x}(t) - \alpha h_M \int_{t-\alpha h_M}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds$$
$$\le (\alpha h_M)^2 \dot{x}^T(t) R_1 \dot{x}(t) - 3\Omega_2^T R_1 \Omega_2$$
$$-[x(t)-x(t-\alpha h_M)]^T R_1 [x(t)-x(t-\alpha h_M)]$$
$$= \overline{\xi^T}(t) \Sigma_4^2 \bar{\xi}(t), \quad (24)$$

where $\Omega_2 = x(t) + x(t-\alpha h_M) - \frac{2}{\alpha h_M} \int_{t-\alpha h_M}^{t} x(s) ds$.

Also, an upper bound of $\dot{V}_5$ can be obtained by utilizing Lemma 2

$$\dot{V}_5 = ((1-\alpha)h_M)^2 \dot{x}^T(t) R_2 \dot{x}(t) - (1-\alpha)h_M \int_{t-h_M}^{t-\alpha h_M} \dot{x}^T(s) R_2 \dot{x}(s) ds$$
$$\le ((1-\alpha)h_M)^2 \dot{x}^T(t) R_2 \dot{x}(t) - (1-\alpha)h_M \int_{t-h_M}^{t-h(t)} \dot{x}^T(s) R_2 \dot{x}(s) ds$$
$$-(1-\alpha)h_M \int_{t-h(t)}^{t-\alpha h_M} \dot{x}^T(s) R_2 \dot{x}(s) ds$$
$$\le ((1-\alpha)h_M)^2 \dot{x}^T(t) R_2 \dot{x}(t) - \gamma^T(t) \begin{bmatrix} \overline{R_2} & S_2 \\ \star & \overline{R_2} \end{bmatrix} \gamma(t)$$
$$= \overline{\xi^T}(t) \Sigma_5^2 \bar{\xi}(t), \quad (25)$$

where

$\gamma(t) = [\gamma_1^T(t), \gamma_2^{gT}(t), \gamma_3^T(t), \gamma_4^T(t)]^T$,

$\gamma_1(t) = x(t-h(t)) - x(t-h_M)$,

$\gamma_2(t) = x(t-h(t)) + x(t-h_M) - \frac{2}{h_M - h(t)} \int_{t-h_M}^{t-h(t)} x(s) ds$,

$\gamma_3(t) = x(t-\alpha h_M) - x(t-h(t))$,

$\gamma_4(t) = x(t-\alpha h_M) + x(t-h(t)) - \frac{2}{h(t)-\alpha h_M} \int_{t-h(t)}^{t-\alpha h_M} x(s) ds$,.

Note that when $h(t) = \alpha h_M$ or $h(t) = h_M$, we have $\gamma_1(t) = \gamma_2(t) = 0$ or $\gamma_3(t) = \gamma_4(t) = 0$, $\beta(t) = 0$. Thus, Eq. (25) still holds.

Therefore, from Eqs. (22)-(25), an upper bound of $\dot{V}(t)$ in the case of $\alpha h_M \le h(t) \le h_M$ can be given as

$$\dot{V}(t) \le \overline{\xi^T}(t) Y_2 \bar{\xi}(t). \quad (26)$$

Based on Lemma 3, $\overline{\xi^T}(t) Y_2 \bar{\xi}(t) < 0$ with $\bar{\Gamma} \bar{\xi}(t) = \sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i \bar{\xi}(t) = 0$ is equivalent to $\sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i^{\perp T} Y_2 \Gamma_i^{\perp} < 0$. Furthermore, the above condition is affinely dependent on $h(t)$. Hence, (13) and (14) imply $\sum_{i=1}^{r} p_i(\theta(t)) \Gamma_i^{\perp T} Y_2 \xi(t) \Gamma_i^{\perp} < 0$. This completes the proof. ■

Remark 1. Unlike in [24], the proposed Lyapunov-Krasovskii functional in (15) are divided the time delay interval $[0,h]$ into different size because of introducing parameter α. When $\alpha = 0.5$, it can be reduced to the ones employed in [24], which divides the time delay interval into the same size, that is, $[0,h_M] = [0, \frac{h_M}{2}] \cup [\frac{h_M}{2}, h_M]$. In other words, based on two delay decomposing approach, the Lyapunov-Krasovskii functional constructed in this paper is more general than the ones used in [24]. When $\alpha = 0.5$, constructing the following Lyapunov functional candidate as

$$V(t) = \sum_{i=1}^{5} V_i \quad (27)$$

where

$$V_3 = \int_{t-\alpha h_M}^{t} \begin{bmatrix} x(s) \\ x(s-\alpha h_M) \end{bmatrix}^T Q \begin{bmatrix} x(s) \\ x(s-\alpha h_M) \end{bmatrix} ds,$$

the others are the same with the ones in (15).

Remark 2. It should be pointed out that the proposed delay-partitioning method is different from existing ones [24,29]. In [29], by using nonuniform decomposition method that the whole delay interval is nonuniformly decomposed into multiple subintervals. In [24], uniform decomposition method is used, which divides the delay interval into the same size. While the conventional method use pre-known constant value to divide the delay interval, a new nonuniform delay-partitioning method is proposed by introducing parameter α, that is, delay interval is divided as $[0,h_M]=[0,\alpha h_M]\cup[\alpha h_M,h_M]$.

Based on Eq. (27) with $\alpha=0.5$, the following Corollary can be obtained from Theorem 1.

Corollary 1. For given scalars $h_M>0,\alpha=0.5,\mu$, the system (11) is globally asymptotically stable if there exist symmetric positive matrices $P\in R^{3n\times 3n}, Q\in R^{2n\times 2n}\, Q_1,R_1,R_2$, and any matrix $S_j(j=1,2)\in R^{2n\times 2n}$ such that the following LMIs hold

$$\Gamma_i^{\perp\,T}[\breve{Y}_1]_{h(t)\in[0,\alpha h_M]}\Gamma_i^{\perp}<0, \tag{28}$$

$$\Gamma_i^{\perp\,T}[\breve{Y}_2]_{h(t)\in[\alpha h_M,h_M]}\Gamma_i^{\perp}<0, \tag{29}$$

$$\begin{bmatrix}\overline{R_j} & S_j\\ \star & \overline{R_j}\end{bmatrix}\geq 0, j=1,2, \tag{30}$$

where

$$\breve{Y}_1=\Sigma_1^1+\Sigma_2+\breve{\Sigma}_3+\Sigma_4^1+\Sigma_5^1,$$
$$\breve{Y}_2=\Sigma_1^2+\Sigma_2+\breve{\Sigma}_3+\Sigma_4^2+\Sigma_5^2,.$$
$$\breve{\Sigma}_3=[e_1\ e_2]Q[e_1\ e_2]^T-[e_2\ e_4]Q[e_2\ e_4]^T.$$

Remark 3. Unlike the constructed Lyapunov-Krasovskii functional in (15), the cross term of the state $x(t)$ and $x(t-\alpha h_M)$ in (15) are considered, which may provide improved stability condition.

For uncertain T-S fuzzy system (10), since $p^T(t)p(t)\leq q^T(t)q(t)$, there exists a positive scalar ϵ satisfying the following inequality: $\epsilon[q^T(t)q(t)-p^T(t)p(t)]\geq 0$. Define $\widetilde{\xi}^T(t)=[\xi^T(t),p^T(t)]$ and $\widehat{\xi}^T(t)=[\overline{\xi^T}(t),p^T(t)]$, $\widetilde{e}_i\in R^{pn\times n}$ $(i=1,2,...,9)$, and the other notations are given as follows:

$$\Psi_i=[E_i\ 0\ E_{di}\ 0\ 0\ 0\ 0\ 0\ 0],$$
$$\widetilde{\Pi}_1^1=[\widetilde{e}_1\ (\alpha h_M-h(t))\widetilde{e}_5+h(t)\widetilde{e}_6\ (h_M-\alpha h_M)\widetilde{e}_7],$$
$$\Pi_1^2=[\widetilde{e}_1\ \alpha h_M\widetilde{e}_7\ (h_M-h(t))\widetilde{e}_5+(h(t)-\alpha h_M)\widetilde{e}_6],$$
$$\Pi_2^1=[\widetilde{e}_3-\widetilde{e}_2\ \widetilde{e}_3+\widetilde{e}_2-2\widetilde{e}_5\ \widetilde{e}_1-\widetilde{e}_3\ \widetilde{e}_1+\widetilde{e}_3-2\widetilde{e}_6],$$
$$\Pi_2^2=[\widetilde{e}_3-\widetilde{e}_4\ \widetilde{e}_3+\widetilde{e}_4-2\widetilde{e}_5\ \widetilde{e}_2-\widetilde{e}_3\ \widetilde{e}_2+\widetilde{e}_3-2\widetilde{e}_6],$$
$$\widetilde{\Sigma_1^1}=\widetilde{\Pi_1^1}P[\widetilde{e}_8\ \widetilde{e}_1-\widetilde{e}_2\ \widetilde{e}_2-\widetilde{e}_4]^T+[\widetilde{e}_8\ \widetilde{e}_1-\widetilde{e}_2\ \widetilde{e}_2-\widetilde{e}_4]^TP\widetilde{\Pi_1^1}^T,$$
$$\widetilde{\Sigma_1^2}=\widetilde{\Pi_1^2}P[\widetilde{e}_8\ \widetilde{e}_1-\widetilde{e}_2\ \widetilde{e}_2-\widetilde{e}_4]^T+[\widetilde{e}_8\ \widetilde{e}_1-\widetilde{e}_2\ \widetilde{e}_2-\widetilde{e}_4]^TP\widetilde{\Pi_1^2}^T,$$
$$\widetilde{\Sigma_2}=\widetilde{e}_1Q_1\widetilde{e}_1^T-(1-\mu)\widetilde{e}_3Q_1\widetilde{e}_3^T,$$
$$\widetilde{\Sigma_3}=\widetilde{e}_1Q_2\widetilde{e}_1-\widetilde{e}_2Q_2\widetilde{e}_2^T+\widetilde{e}_2Q_3\widetilde{e}_2^T-\widetilde{e}_4Q_3\widetilde{e}_4^T,$$
$$\widetilde{\Sigma_4^1}=(\alpha h_M)^2\widetilde{e}_8R_1\widetilde{e}_8^T-\widetilde{\Pi_2^1}\begin{vmatrix}\overline{R_1} & S_1\\ \star & \overline{R_1}\end{vmatrix}\widetilde{\Pi_2^1}^T,$$
$$\widetilde{\Sigma_4^2}=(\alpha h_M)^2\widetilde{e}_8R_1\widetilde{e}_8^T-[\widetilde{e}_1-\widetilde{e}_2]R_1[\widetilde{e}_1-\widetilde{e}_2]^T-3[\widetilde{e}_1+\widetilde{e}_2-2\widetilde{e}_7]R_1[\widetilde{e}_1+\widetilde{e}_2-2\widetilde{e}_7]^T,$$
$$\widetilde{\Sigma_5^1}=((1-\alpha)h_M)^2\widetilde{e}_8R_2\widetilde{e}_8^T-[\widetilde{e}_2-\widetilde{e}_4]R_2[\widetilde{e}_2-\widetilde{e}_4]^T-3[\widetilde{e}_2+\widetilde{e}_4-2\widetilde{e}_7]R_2[\widetilde{e}_2+\widetilde{e}_4-2\widetilde{e}_7]^T,$$
$$\widetilde{\Sigma_5^2}=((1-\alpha)h_M)^2\widetilde{e}_8R_2\widetilde{e}_8^T-\widetilde{\Pi_2^2}\begin{vmatrix}\overline{R_2} & S_2\\ \star & \overline{R_2}\end{vmatrix}\widetilde{\Pi_2^2}^T,$$
$$\widetilde{Y}_1=\widetilde{\Sigma_1^1}+\widetilde{\Sigma_2}+\widetilde{\Sigma_3}+\widetilde{\Sigma_4^1}+\widetilde{\Sigma_5^1}-\widetilde{e}_9\widetilde{e}_9^T-\Psi_i^T\Psi_i,$$
$$\widetilde{Y}_2=\widetilde{\Sigma_1^2}+\widetilde{\Sigma_2}+\widetilde{\Sigma_3}+\widetilde{\Sigma_4^2}+\Sigma_5^2-\widetilde{\widetilde{e}_9\widetilde{e}_9^T}-\Psi_i^T\Psi_i$$
$$\widetilde{\Gamma}_i=[\Gamma_i,D].$$

Now we have the following Corollary 2 and Corollary 3.

Corollary 2. For given scalars $h_M>0, 0<\alpha<1,\mu$, the system (10) is globally asymptotically stable if there exist symmetric positive matrices $P\in R^{3n\times 3n}, Q_1,Q_2,Q_3,R_1,R_2$, and any matrix $S_j(j=1,2)\in R^{2n\times 2n}$, and a positive scalar ϵ such that the following LMIs hold

$$\widetilde{\Gamma_i^{\perp}}^T[\widetilde{Y}_1]_{h(t)\in[0,\alpha h_M]}\widetilde{\Gamma_i^{\perp}}<0, \tag{31}$$

$$\widetilde{\Gamma_i^{\perp}}^T[\widetilde{Y}_2]_{h(t)\in[\alpha h_M,h_M]}\widetilde{\Gamma_i^{\perp}}<0, \tag{32}$$

$$\begin{bmatrix}\overline{R_j} & S_j\\ \star & \overline{R_j}\end{bmatrix}\geq 0, j=1,2, \tag{33}$$

Corollary 3. For given scalars $h_M>0,\alpha=0.5,\mu$, the system (10) is globally asymptotically stable if there exist symmetric positive matrices $P\in R^{3n\times 3n}, Q\in R^{2n\times 2n}\, Q_1,R_1,R_2$, and any matrix $S_j(j=1,2)\in R^{2n\times 2n}$, and a positive scalar ϵ such that the following LMIs hold

$$\widetilde{\Gamma_i^{\perp}}^T[\widehat{Y}_1]_{h(t)\in[0,\alpha h_M]}\widetilde{\Gamma_i^{\perp}}<0, \tag{34}$$

$$\widetilde{\Gamma_i^{\perp}}^T[\widehat{Y}_2]_{h(t)\in[\alpha h_M,h_M]}\widetilde{\Gamma_i^{\perp}}<0, \tag{35}$$

$$\begin{bmatrix}\overline{R_j} & S_j\\ \star & \overline{R_j}\end{bmatrix}\geq 0, j=1,2, \tag{36}$$

where

$$\widehat{Y}_1=\widetilde{\Sigma_1^1}+\widetilde{\Sigma_2}+\widehat{\Sigma}_3+\widetilde{\Sigma_4^1}+\widetilde{\Sigma_5^1}-\widetilde{e}_9\widetilde{e}_9^T-\Psi_i^T\Psi_i,$$
$$\widehat{Y}_2=\widetilde{\Sigma_1^2}+\widetilde{\Sigma_2}+\widehat{\Sigma}_3+\widetilde{\Sigma_4^2}+\Sigma_5^2-\widetilde{\widetilde{e}_9\widetilde{e}_9^T}-\Psi_i^T\Psi_i$$

$$\breve{Y}_1 = \Sigma_1^1 + \Sigma_2 + \breve{\Sigma}_3 + \Sigma_4^1 + \Sigma_5^1,$$
$$\breve{Y}_2 = \Sigma_1^2 + \Sigma_2 + \breve{\Sigma}_3 + \Sigma_4^2 + \Sigma_5^2,.$$
$$\hat{\Sigma}_3 = [\widetilde{e_1}\ \widetilde{e_2}]Q[\widetilde{e_1}\ \widetilde{e_2}]^T - [\widetilde{e_2}\ \tilde{}e_4]Q[\widetilde{e_2}\ \widetilde{e_4}]^T.$$

4. Numerical Examples

In this section, two numerical examples are given to show the effectiveness of the proposed method.

Example 1 Consider the system with the following parameters:

$$A_1 = \begin{bmatrix} -3.2 & 0.6 \\ 0 & -2.1 \end{bmatrix}, A_{d1} = \begin{bmatrix} 1 & 0.9 \\ 0 & 2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.$$

For different μ, the upper bounds of the time-varying delay computed by the proposed method and those in [8,11,20,18,24] are listed in Table 1. It is easy to know that the proposed method in this paper is less conservative than those in the existing results.

표 1 다른 μ값에 대한 상한유계지연 h_M

Table 1 Upper delay bound h_M for different μ

μ	0.03	0.1	0.5	0.9
[8]	0.5423	0.4809	0.4752	0.4455
[11]	0.5456	0.5030	0.4995	0.4988
[20]	0.7806	0.5906	0.5392	0.5268
[18]	0.8369	0.7236	0.7154	0.7014
[24]	0.8771	0.7687	0.7584	0.7524
Theorem 1 ($\alpha=0.5$)	1.5835	1.2444	1.2216	1.1686
Theorem 1 ($\alpha=0.6$)	1.5906	1.2698	1.2445	1.1852
Corollary 1 ($\alpha=0.5$)	1.6840	1.3925	1.3566	1.2771

Example 2 Consider the system with the following parameters:

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.5 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix}, E_1 = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, E_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} 1.6 & 0 \\ 0 & -0.05 \end{bmatrix}, E_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, D = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix}.$$

For different μ, the upper bounds of the time-varying delay computed by the proposed method and those in [5,9,12,24] are listed in Table 2. It can be concluded that the result proposed in this paper is better than the existing ones.

표 2 다른 μ값에 대한 상한유계지연 h_M

Table 2 Upper delay bound h_M for different μ

μ	0.01	0.1	0.5	unknown
[5]	0.944	0.892	0.637	−
[9]	1.163	1.122	0.934	0.499
[12]	1.187	1.155	1.100	1.050
[24]	1.382	1.318	1.132	1.127
Theorem 1 ($\alpha=0.5$)	1.379	1.323	1.151	1.148
Theorem 1 ($\alpha=0.4$)	1.382	1.326	1.154	1.149
Corollary 1 ($\alpha=0.5$)	1.382	1.325	1.152	1.149

5. Conclusions

The robust stability for uncertain T-S fuzzy systems with time-varying delay has been investigated. Based on a modified Lyapunov-Krasovskii functional, some less conservative criteria have been obtained by employing new delay-partitioning technique, integral inequality and reciprocally convex approach. It should be worthwhile pointed out that different case of delay-partitioning method is used in this paper, that is, the delay interval is divided into even and not even. Two numerical examples have been given to demonstrate the effectiveness of the proposed method.

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