

EMPLOYING COMMON LIMIT RANGE PROPERTY WITH VARIANTS OF R -WEAKLY COMMUTING MAPPINGS IN METRIC SPACES

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ABSTRACT. The object of this paper is to emphasize the role of ‘common limit range property’ and utilize the same with variants of R -weakly commuting mappings for the existence of common fixed point under strict contractive conditions in metric spaces. We also furnish some interesting examples to validate our main result. Our results improve a host of previously known results including the ones contained in Pant [Contractive conditions and common fixed points, *Acta Math. Acad. Paedagog. Nyházi.* (N.S.) 24(2) (2008), 257–266 MR2461637 (2009h:54061)]. In the process, we also derive a fixed point result satisfying ϕ -contractive condition.

1. INTRODUCTION AND PRELIMINARIES

The celebrated Banach fixed point theorem also known as Banach Contraction Principle appeared in its explicit form in the thesis of Banach [4]. Owing to its simplicity and usefulness, it became a very powerful tool in solving existence problems in pure and applied sciences which include biology, medicine, physics, computer science etc. This theorem asserts that every contraction mappings defined on a complete metric space has a unique fixed point and that fixed point can be explicitly obtained as limit of repeated iteration of the mapping at any point of the underlying space. Evidently, every contraction mapping is a continuous but not conversely. Some recent development in fixed point theory can be easily seen in [2, 3, 6, 9, 10, 13, 21].

In 1976, Jungck [7] proved a fixed point theorem for a pair of commuting mappings in complete metric space. The first ever attempt to improve commutativity

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conditions in common fixed point theorems is due to Sessa [23] wherein he introduced the notion of weakly commuting mappings. Later on, Jungck [8] improved the notion of weak commutativity due to Sessa [23] by introducing the concept of compatible mappings. In the study of common fixed points of compatible mappings we often require assumptions on the completeness of the underlying space and continuity of the involved mappings. In an interesting note, Kannan [11] showed that there exists mappings that have a discontinuity in the domain but which have fixed points. Moreover, the involved mappings in every case were continuous at the fixed point. The study of common fixed points theorems for non-compatible mappings is firstly initiated by Pant [15] with the introduction of the notion of R -weakly commuting mappings in metric spaces. Many mathematicians have contributed towards the vigorous development of fixed point theory (e.g. [5, 16, 17, 18]). Further, Pathak et al. introduced the notion of R -weakly commuting mappings of types (A_g) and (A_f) and generalized the result of Pant [15].

A result on the existence and uniqueness of common fixed point in metric spaces, generally involves conditions on commutativity, continuity and contraction along with a suitable condition on the containment of range of one mapping into the range of other. Hence, one is always required to improve one or more of these conditions to prove a new fixed point theorem. In 2002, Amari and Moutawakil [1] introduced the notion of property (E.A) which generalized the concept of non-compatible mappings. The fixed point results proved under property (E.A) always require the closedness of the underlying subspaces for the existence of common fixed point. In 2011, Sintunavarat and Kumam [27] coined the idea of “common limit range property” (also see [25, 28, 29, 30, 31]).

In 2008, Pant [19] utilized the notion of property (E.A) with R -weakly commuting mappings of type (A_g) for the existence of common fixed point under strict contractive condition which improved and extended the results of Singh and Kumar [24]. Since then, Kumar [12] proved a common fixed point theorem for a pair of weakly compatible mappings along with property (E.A) in metric spaces which improves and generalizes the result of Jungck [7] without any continuity requirement besides relaxing the containment of the range of one mapping into the range of other mapping. In [12], he also introduced the notion of R -weakly commuting of type (P) in metric space and obtained some fixed point theorems for variants of R -weakly commuting mappings with property (E.A).

In this paper, utilizing the notion of common limit range property due to Sintunavarat and Kumam [27], we prove some common fixed point theorems for a pair of mappings under variants of R -weakly commuting mappings. In process, many known results (especially the ones contained in Pant [19]) are enriched and improved. Some related results are also derived besides furnishing illustrative examples.

Throughout this paper, let Y be an arbitrary non-empty set and (X, d) a metric space.

Definition 1.1. Let $f, g : X \rightarrow X$ be two self mappings of a metric space (X, d) . Then the pair (f, g) is said to be

- (1) *commuting* if $fgx = gfx$, for all $x \in X$.
- (2) *weakly commuting* [23] if $d(fgx, gfx) \leq d(fx, gx)$, for all $x \in X$.
- (3) *R -weakly commuting* [15] if there exists some real number $R > 0$ such that $d(fgx, gfx) \leq Rd(fx, gx)$ for all $x \in X$.
- (4) *pointwise R -weakly commuting* [15] if given $x \in X$ there exists some real number $R > 0$ such that $d(fgx, gfx) \leq Rd(fx, gx)$.
- (5) *R -weakly commuting of type (A_g)* [20] if there exists some real number $R > 0$ such that $d(ffx, gfx) \leq Rd(fx, gx)$ for all $x \in X$.
- (6) *R -weakly commuting of type (A_f)* [20] if there exists some real number $R > 0$ such that $d(fgx, ggx) \leq Rd(fx, gx)$ for all $x \in X$.
- (7) *R -weakly commuting of type (P)* [12] if there exists some real number $R > 0$ such that $d(ffx, ggx) \leq Rd(fx, gx)$ for all $x \in X$.
- (8) *compatible* [8] if $\lim_{n \rightarrow \infty} d(fgx_n, gfx_n) = 0$ for each sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n$.
- (9) *non-compatible* [15] if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n$ but $\lim_{n \rightarrow \infty} d(fgx_n, gfx_n)$ is either nonzero or nonexistent.

For more details on systematic comparisons and illustrations of earlier described notions, we refer to Singh and Tomar [26], Murthy [14] and Kumar [12].

Definition 1.2. Let f and g be mappings on Y with values in X . Then f and g are said to satisfy the

- (1) *property (E.A)* [22] if there exists a sequence $\{x_n\}$ in Y such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = t,$$

for some $t \in X$.

(2) the *common limit range property* [27] with respect to mapping g , denoted by (CLRg) in short, if there exists a sequence $\{x_n\}$ in Y such that

$$\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = g u,$$

for some $u \in Y$.

Remark 1.3. If we take $Y = X$ then we get the definition of property (E.A) for two self mappings of X studied by Aamri and Moutawakil [1]. In this case, t is called a tangent point by Sastry and Murthy [22].

Example 1.4. Consider $Y = X = [0, 14]$ and d be the usual metric on X . Define the self mappings f and g on X as

$$f x = \begin{cases} 3, & \text{if } 0 \leq x \leq 11; \\ \frac{x+4}{5}, & \text{if } 11 < x \leq 14. \end{cases} \quad g x = \begin{cases} 2, & \text{if } 0 \leq x \leq 11; \\ 14 - x, & \text{if } 11 < x \leq 14. \end{cases}$$

Then $f(X) = [3, \frac{18}{5}]$ and $g(X) = [0, 3]$. Now consider a sequence $\{x_n\} = \{11 + \frac{1}{n}\}$ in X . Then clearly,

$$\begin{aligned} \lim_{n \rightarrow \infty} f x_n &= \lim_{n \rightarrow \infty} \left(\frac{11 + \frac{1}{n} + 4}{5} \right) = \lim_{n \rightarrow \infty} \left(3 + \frac{1}{5n} \right) \\ &= 3 \in X = \lim_{n \rightarrow \infty} \left(3 - \frac{1}{n} \right) = \lim_{n \rightarrow \infty} g x_n. \end{aligned}$$

Here it is pointed out that $3 \notin g(X)$ which shows that the pair (f, g) does not satisfy the (CLRg) property while it enjoys the property (E.A).

Example 1.5. In the setting of Example 1.4, replace the self mapping g by the following besides retaining the rest:

$$g x = \begin{cases} 3, & \text{if } 0 \leq x \leq 11; \\ 14 - x, & \text{if } 11 < x \leq 14. \end{cases}$$

Then $f(X) = [3, \frac{18}{5}]$ and $g(X) = [0, 3]$. Consider a sequence similar as Example 1.4, one can see that,

$$\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} \left(3 + \frac{1}{5n} \right) = g(3) = \lim_{n \rightarrow \infty} \left(3 - \frac{1}{n} \right) = \lim_{n \rightarrow \infty} g x_n.$$

Hence the pair (f, g) enjoys the (CLRg) property.

Remark 1.6. Thus, one can infer that a pair (f, g) satisfying the property (E.A) along with closedness of the underlying subspace $g(X)$ always enjoys the (CLRg) property with respect to the mapping g .

2. MAIN RESULTS

In 2008, Pant [19] proved the following result for a pair of R -weakly commuting mappings employing property (E.A).

Theorem 2.1 ([19, Theorem 1]). *Let f and g be self mappings of a complete metric space (X, d) such that*

$$(2.1) \quad \overline{f(X)} \subset g(X),$$

where $\overline{f(X)}$ denotes the closure of range of the mapping f ,

$$(2.2) \quad d(fx, fy) < \max \left\{ d(gx, gy), \frac{k}{2}[d(fx, gx) + d(fy, gy)], \frac{1}{2}[d(fy, gx) + d(fx, gy)] \right\},$$

whenever the right hand side is positive and $1 \leq k < 2$. If f and g be R -weakly commuting of type of type (A_g) satisfying the property (E.A), then f and g have a unique common fixed point.

Now we prove a more general result by using the notion of common limit range property with variants of R -weakly commuting mappings.

Theorem 2.2. *Let (X, d) be a metric space and $f, g : Y \rightarrow X$. Suppose that the following hypotheses hold:*

- (1) the pair (f, g) enjoys the (CLRg) property,
- (2) for all $x \neq y \in X$ and $0 \leq k < 2$,

$$(2.3) \quad d(fx, fy) < \max \left\{ d(gx, gy), \frac{k}{2}[d(fx, gx) + d(fy, gy)], \frac{k}{2}[d(fy, gx) + d(fx, gy)] \right\},$$

whenever the right hand side of the above inequality is positive. Then the pair (f, g) has a coincidence point.

Moreover, if $Y = X$ and $0 \leq k < 1$, then f and g have a unique common fixed point provided the pair (f, g) is R -weakly commuting or R -weakly commuting of type (A_g) or R -weakly commuting of type (A_f) or R -weakly commuting of type (P) .

Proof. Since the pair (f, g) satisfies the (CLRg) property, there exists a sequence $\{x_n\}$ in Y such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = gu,$$

for some $u \in Y$. First we show that $fu = gu$. If not, then using inequality (2.3) with $x = x_n, y = u$, we get

$$d(fx_n, fu) < \max \left\{ d(gx_n, gu), \frac{k}{2}[d(fx_n, gx_n) + d(fu, gu)], \frac{k}{2}[d(fu, gx_n) + d(fx_n, gu)] \right\},$$

which on making $n \rightarrow \infty$, reduces to

$$\begin{aligned} d(gu, fu) &< \max \left\{ d(gu, gu), \frac{k}{2}[d(gu, gu) + d(fu, gu)], \frac{k}{2}[d(fu, gu) + d(gu, gu)] \right\} \\ &= \max \left\{ 0, \frac{k}{2}d(fu, gu), \frac{k}{2}d(fu, gu) \right\} \\ &= \frac{k}{2}d(fu, gu) \\ &< d(fu, gu), \end{aligned}$$

which is a contradiction. Hence $fu = gu$ which shows that f and g have a coincidence point.

Now consider $Y = X$ and $0 \leq k < 1$.

Case I: If the pair (f, g) is R -weakly commuting, then we have

$$d(fgu, gfu) \leq Rd(fu, gu) = 0,$$

that is, $fgu = gfu$. Therefore, we obtain $ffu = fgu = gfu = ggu$.

Case II: Suppose that the pair (f, g) is R -weakly commuting of type (A_g) , we obtain

$$d(ffu, gfu) \leq Rd(fu, gu) = 0,$$

that is, $ffu = gfu$ and so we get $fgu = ffu = gfu = ggu$.

Case III: Assume that the pair (f, g) is R -weakly commuting of type (A_f) , we have

$$d(fgu, ggu) \leq Rd(fu, gu) = 0,$$

which implies that $fgu = ggu$. Hence $ffu = fgu = ggu = gfu$.

Case IV: Finally, if we consider the pair (f, g) is R -weakly commuting of type (P) , then we have

$$d(ffu, ggu) \leq Rd(fu, gu) = 0,$$

that is, $ffu = ggu$. Therefore, we get $fgu = ffu = ggu = gfu$.

Now we assert that fu is a common fixed point of the mappings f and g . Suppose that $fu \neq ffu$, using inequality (2.3) with $x = u$, $y = fu$, we get

$$\begin{aligned} d(fu, ffu) &< \max \left\{ d(gu, gfu), \frac{k}{2}[d(fu, gu) + d(ffu, gfu)], \frac{k}{2}[d(ffu, gu) + d(fu, gfu)] \right\} \\ &= \max \{d(fu, ffu), 0, kd(fu, ffu)\} \\ &= d(fu, ffu), \end{aligned}$$

which is a contradiction, we have $fu = ffu$. Therefore $fu = ffu = gfu$ which shows that fu is a common fixed point of the pair (f, g) .

Uniqueness of common fixed point is an easy consequence of inequality (2.3). This concludes the proof. \square

Theorem 2.3. *Let (X, d) be a metric space and $f, g : Y \rightarrow X$ satisfying inequality (2.3). Suppose that the pair (f, g) enjoys the property (E.A) whereas $g(X)$ is a closed subspace of X . Then the pair (f, g) has a coincidence point.*

Moreover, if $Y = X$ and $0 \leq k < 1$, then f and g have a unique common fixed point provided the pair (f, g) is either R -weakly commuting or R -weakly commuting of type (A_g) or R -weakly commuting of type (A_f) or R -weakly commuting of type (P) .

Proof. If the pair (f, g) satisfies the property (E.A), then there exists a sequence $\{x_n\}$ in Y such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = t,$$

for some $t \in X$. Since it is assumed that $g(X)$ is a closed subspace of X , there exists a point $u \in X$ such that $gu = t$. Hence in view of Remark 1.6, the pair (f, g) also enjoys the (CLRg) property. By Theorem 2.2, we can obtain the mappings f and g have a unique common fixed point. \square

Since the pair of non-compatible mappings implies to the pair satisfying property (E.A), we get the following corollary.

Corollary 2.4. *Let (X, d) be a metric space and $f, g : Y \rightarrow X$ satisfying inequality (2.3). Suppose that the pair (f, g) is non-compatible whereas $g(X)$ is a closed subspace of X . Then the pair (f, g) has a coincidence point.*

Moreover, if $Y = X$ and $0 \leq k < 1$, then f and g have a unique common fixed point provided the pair (f, g) is either R -weakly commuting or R -weakly commuting of type (A_g) or R -weakly commuting of type (A_f) or R -weakly commuting of type (P) .

Our next theorems involve a function $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ which satisfies the following conditions:

- (1) ϕ is upper semi-continuous on \mathbb{R}^+ ,
- (2) $0 < \phi(s) < s$ for each $s \in \mathbb{R}^+$.

Theorem 2.5. *Let (X, d) be a metric space and $f, g : Y \rightarrow X$. Suppose that the following hypotheses hold:*

- (1) *the pair (f, g) enjoys the (CLRg) property,*
- (2) *for all $x, y \in Y$,*

$$(2.4) \quad d(fx, fy) \leq \phi(\max\{d(gx, gy), d(fx, gx), d(fy, gy), d(fy, gx), d(fx, gy)\}).$$

Then the pair (f, g) has a coincidence point.

Moreover, if $Y = X$, then f and g have a unique common fixed point provided the pair (f, g) is either R -weakly commuting or R -weakly commuting of type (A_g) or R -weakly commuting of type (A_f) or R -weakly commuting of type (P) .

Proof. Suppose that the pair (f, g) enjoys the (CLRg) property, there exists a sequence $\{x_n\}$ in Y such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = gu,$$

for some $u \in Y$. We assert that $fu = gu$. Let on the contrary $fu \neq gu$, then using inequality (2.4) with $x = x_n, y = u$, we get

$$d(fx_n, fu) \leq \phi \left(\max \left\{ \begin{array}{l} d(gx_n, gu), \frac{k}{2}[d(fx_n, gx_n) + d(fu, gu)], \\ \frac{k}{2}[d(fu, gx_n) + d(fx_n, gu)] \end{array} \right\} \right).$$

Taking limit as $n \rightarrow \infty$, we have

$$\begin{aligned} d(gu, fu) &\leq \phi(\max\{d(gu, gu), d(gu, gu), d(fu, gu), d(fu, gu), d(gu, gu)\}) \\ &= \phi(\max\{0, 0, d(fu, gu), d(fu, gu), 0\}) \\ &= \phi(d(fu, gu)) \\ &< d(fu, gu), \end{aligned}$$

which is a contradiction, we obtain $fu = gu$ which shows that f and g have a coincidence point. The rest of the proof can be completed on the lines of the proof of Theorem 2.2, hence the details are omitted. \square

Now, we introduce some example to support the useability of our results.

Example 2.6. Let $X = [0, +\infty)$. Define $d : X \times X \rightarrow X$ by

$$d(x, y) = \begin{cases} 0, & \text{if } x = y; \\ \max\{x, y\}, & \text{if } x \neq y, \end{cases}$$

Also, define $f, g : X \rightarrow X$ by $fx = \frac{1}{4}x$ and $gx = 2x$. Then

- (1) The pair (f, g) enjoys the (CLRg) property.
- (2) The pair (f, g) is R -weakly commuting.

(3) $d(gx, gy) > 0$ for all $x, y \in X$ with $x \neq y$.

(4) for all $x, y \in X$ with $x \neq y$, we have

$$d(fx, fy) < \max \left\{ d(gx, gy), \frac{k}{2}[d(fx, gx) + d(fy, gy)], \frac{k}{2}[d(fy, gx) + d(fx, gy)] \right\}.$$

Proof. The proof of (1), (2) and (3) are clear. To prove (4), given $x, y \in X$ with $x \neq y$. Without loss of generality, we may assume that $x > y$. Thus

$$\begin{aligned} d(fx, fy) &= \frac{1}{4}x \\ &\leq 2x = d(gx, gy) \\ &< \max \left\{ d(gx, gy), \frac{k}{2}[d(fx, gx) + d(fy, gy)], \frac{k}{2}[d(fy, gx) + d(fx, gy)] \right\}. \end{aligned}$$

Thus f and g satisfy all the hypotheses of Theorem 2.2. So f and g have a unique common fixed point. Here, 0 is the unique common fixed point of f and g . \square

Example 2.7. Let $X = [0, 1]$. Define $d : X \times X \rightarrow X$ by

$$d(x, y) = \begin{cases} 0, & \text{if } x = y; \\ \max\{x, y\}, & \text{if } x \neq y, \end{cases}$$

Also, define $f, g : X \rightarrow X$ by $fx = \frac{x}{1+x}$ and $gx = x$. Also, define $\phi : [0, +\infty) \rightarrow [0, +\infty)$ by $\phi(t) = \frac{t}{1+t}$. Then

- (1) The pair (f, g) enjoys the (CLRg) property.
- (2) The pair (f, g) is R -weakly commuting.
- (3) $d(gx, gy) > 0$ for all $x, y \in X$ with $x \neq y$.
- (4) for all $x, y \in X$ with $x \neq y$, we have

$$d(fx, fy) \leq \phi(\max \{d(gx, gy), d(fx, gx), d(fy, gy), d(fy, gx), d(fx, gy)\}).$$

Proof. The proof of (1), (2) and (3) are clear. To prove (4), given $x, y \in X$ with $x \neq y$. Without loss of generality, we may assume that $x > y$. Thus

$$\begin{aligned} d(fx, fy) &= \frac{x}{1+x} \\ &\leq \phi(x) = \phi(d(gx, gy)) \\ &\leq \phi(\max \{d(gx, gy), d(fx, gx), d(fy, gy), d(fy, gx), d(fx, gy)\}). \end{aligned}$$

Thus f and g satisfy all the hypotheses of Theorem 2.5. So f and g have a unique common fixed point. Here, 0 is the unique common fixed point of f and g . \square

Remark 2.8. It may be noticed that the earlier proved results namely; Theorem 2.2, Theorem 2.3 and Theorem 2.5 (also Corollary 2.4) remain valid in symmetric space (X, d) whenever d is continuous.

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REFERENCES

1. M. Aamri & D.El Moutawakil: Some new common fixed point theorems under strict contractive conditions. *J. Math. Anal. Appl.* **270**(1) (2002), 181-188. MR1911759 (2003d:54057)
2. M. Abbas, T. Nazir & S. Radenović: Common fixed point of power contraction mappings satisfying (E.A) property in generalized metric spaces. *Appl. Math. Comput.* **219** (2013), 7663-7670.
3. M.A. Alghamdi, S. Radenović & N. Shahzad: On some generalizations of commuting mappings. *Abstr. Appl. Anal.* **2012**, Article ID 952052.
4. S. Banach: Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund. Math.* **3** (1922), 133–181.
5. Lj.B. Ćirić: Generalized contractions and fixed point theorems. *Publ. Inst. Math. (Beograd) (N.S.)* **12** (1971), no. 26, 19-26. MR0309092 (46 #8203)
6. D. Đorić, Z. Kadelburg & S. Radenović: A note on occasionally weakly compatible mappings and common fixed points. *Fixed Point Theory* **13** (2012), no. 2, 475-480.
7. G. Jungck: Commuting mappings and fixed point. *Amer. Math. Monthly* **83** (1976), no. 4, 261-263. MR0400196 (53 #4031)
8. G. Jungck: Compatible mappings and common fixed points. *Internat. J. Math. Math. Sci.* **9** (1986), no. 4, 771-779. MR0870534 (87m:54122)
9. Z. Kadelburg, S. Radenović & B. Rasić: Strict contractive conditions and common fixed point theorems in cone metric spaces. *Fixed Point Theory Appl.* **2009**, Article ID 173838, 14 pages.
10. Z. Kadelburg, S. Radenović & N. Shahzad: A note on various classes of compatible type pairs of mappings and common fixed point theorems. *Abstr. Appl. Anal.* **2013**, Article ID 697151, 6 pages.
11. R. Kannan: Some results on fixed points. *Bull. Calcutta Math. Soc.* **60** (1968), 71-76. MR0257837 (41 #2486)
12. S. Kumar: A note on Jungck's fixed point theorem. *Fasciculi Math.* **45** (2010), 59–69.
13. W. Long, M. Abbas, T. Nazir & S. Radenović: Common fixed point for two pairs of mappings satisfying (E.A) property in generalized metric spaces. *Abstr. Appl. Anal.* **2012**, Article ID 394830, 15 pages.

14. P.P. Murthy: Important tools and possible applications of metric fixed point theory. Proceedings of the Third World Congress of Nonlinear Analysts, Part 5 (Catania, 2000), *Nonlinear Anal.* **47** (2001), no. 5, 3479-3490. MR1979244 (2004d:54038)
15. R.P. Pant: Common fixed points of noncommuting mappings. *J. Math. Anal. Appl.* **188** (1994), no. 2, 436-440. MR1305460 (95j:54037)
16. R.P. Pant: Common fixed point theorems for contractive maps. *J. Math. Anal. Appl.* **226** (1998), no. 1, 251-258. MR1646430 (99h:54063)
17. R.P. Pant: Discontinuity and fixed points. *J. Math. Anal. Appl.* **240** (1999), no. 1, 280-289. MR1728194 (2000j:54048)
18. R.P. Pant: Noncompatible mappings and common fixed points. *Soochow J. Math.* **26** (2000), no. 1, 29-35. MR1755133 (2000m:54048)
19. V. Pant: Contractive conditions and common fixed points. *Acta Math. Acad. Paedagog. Nyházi. (N.S.)* **24** (2008), no. 2, 257-266. MR2461637 (2009h:54061)
20. H.K. Pathak, Y.J. Cho & S.M. Kang: Remarks on R -weakly commuting mappings and common fixed point theorems. *Bull. Korean Math. Soc.* **34** (1997), no. 2, 247-257.
21. S. Radenović, Z. Kadelburg, D. Jandrlić & A. Jandrlić: Some results on weakly contractive maps. *Bull. Iran. Math. Soc.* **38** (2012), no. 3, 625-645.
22. K.P.R. Sastry & I.S.R. Krishna Murthy: Common fixed points of two partially commuting tangential selfmaps on a metric space. *J. Math. Anal. Appl.* **250** (2000), no. 2, 731-734. MR1786095
23. S. Sessa: On a weak commutativity condition in fixed point considerations. *Publ. Inst. Math. (Beograd) (N.S.)* **32** (1982), no. 46, 149-153. MR0710984 (85f:54107)
24. S.L. Singh & A. Kumar: Common fixed point theorems for contractive maps. *Mat. Vesnik* **58** (2006), no. 3-4, 85-90.
25. S.L. Singh, B.D. Pant & S. Chauhan: Fixed point theorems in Non-Archimedean Menger PM-spaces. *J. Nonlinear Anal. Optim.* **3** (2012), no. 2, 153-160. MR2982403
26. S.L. Singh & A. Tomar: Weaker forms of commuting maps and existence of fixed points. *J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math.* **10** (2003), no. 3, 145-161. MR2011365 (2004h:54039)
27. W. Sintunavarat & P. Kumam: Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces. *J. Appl. Math.* **Vol. 2011**, Article ID 637958, 14 pp. MR2822403
28. W. Sintunavarat & P. Kumam: Common fixed points for R -weakly commuting in fuzzy metric spaces. *Ann. Univ. Ferrara Sez. VII Sci. Mat.* **58** (2012), no. 2, 389-406.
29. W. Sintunavarat & P. Kumam: Coincidence and common fixed points for hybrid strict contractions without weakly commuting condition. *Appl. Math. Lett.* **22** (2009), 1877-1881.

30. W. Sintunavarat, P. Kumam & P. Patthanangkoor: Common random fixed points for multivalued random operators without S and T -weakly commuting random. *Random Oper. Stoch. Equ.* **17** (2009), no. 4, 381-388.
31. N. Wairojjana, W. Sintunavarat & P. Kumam: Common tripled fixed points for W -compatible mappings along with CLR_g property in abstract metric spaces. *J. Inequal. Appl.* **2014**, 2014:133. DOI: 10.1186/1029-242X-2014-133

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