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Inelastic Out-of-plane Design of Parabolic Arches

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Abstract

In this paper, improved out-of-plane design of parabolic arches was proposed based on the current design code. The arches resist general loading by a combination of axial compression and bending actions, and the interaction formula between two extreme cases of axial and bending actions is generally used for the design. Firstly, the out-of-plane buckling strength of arches in a pure axial compression and a pure bending were studied. Then, out-of-plane design of parabolic aches under general transverse loading was investigated. From the results, it can be found that the proposed design equations provided good prediction of out-of-plane strength for parabolic arches which satisfy the thresholds for deep arches, while proposed design equations overestimated the buckling load of shallow arches

Keywords: *Arches, Out-of-plane buckling, Stability analysis, Inelastic behavior*

1. Introduction

In this paper, out-of-plane design of parabolic arches was investigated. Arches are supported at both ends, and the end deformations are prevented. In this case, the arches resist general loading by a combination of axial compression and bending actions. Thus, for the design of out-of-plane buckling of the arches, the interaction formula between two extreme cases of axial and bending actions is generally adopted. In this paper, firstly, the out-of-plane buckling strength of arches in a pure axial compression or a pure bending were studied. The elastic out-of-pane buckling strength of arches under a pure axial compression or a pure bending were studied in previous work by the author (Moon et al, 2009a). Based on this elastic out-of-plane strength of the parabolic arches, the inelastic out-of-plane buckling strength of arches under a pure axial compression or a pure bending action were evaluated. Then, out-of-plane design of parabolic aches under general loading was investigated.

Researches on the inelastic out-of-plane buckling of aches were reported by several researchers. Komatu and Sakimoto (1977) and Sakimoto and Komatu (1983) investigated the inelastic lateral-buckling of parabolic pin-ended arches, where box section is used so that warping effects are ignored. They concluded that the effects of the rise to span ratio on the ultimate lateral strength are not important and the design rules for a column can be apply to determine the ultimate strength of through-type steel box arches. Pi and Trahair (1998) studied inelastic lateral buckling of circular arches either in pure axial compression or in a pure bending by finite element analyses. However, study on arches under a combination of axial and bending actions is limited. In most case, arches are experienced general combined axial compression and bending action. A few studies (Pi and Trahair, 2000) give the results of behavior for the circular arches under general transverse loading.

In this study, the numerical solutions developed in previous work by author (Moon et al., 2009a), which is limited to the elastic out-of-plane buckling of parabolic arches under a pure axial compression or a pure bending, were extended to design of out-of-plane buckling of parabolic arches under general combined axial compression and bending action.

Out-of-plane design based on the AASHTO LRFD (2004) was proposed followed by description of finite element model. Then, the results of finite element analysis were compared to the proposed design criteria. From the

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results, it was found that the proposed design criteria base on AASHTO LRFD (2004) provided good predictions of out-of-plane strength of parabolic arches when the arch satisfy the deep arch category described in previous work by author (Moon et al., 2009b).

2. Out-of-plane Design Based on the Current Design Code

The arches design formula was proposed based on the interactive curves between axial and flexural strength in AASHTO LRFD (2004) and is given by

$$\frac{P_u}{\phi_c P_n}+\frac{8.0}{9.0}\frac{M_u}{\phi_f M_n}\leq 1 \text{ for } \frac{P_u}{\phi_c P_n}\geq 0.2 \text{ (a)} \tag{1}$$

$$\frac{P_u}{2.0\phi_c P_n}+\frac{M_u}{\phi_f M_n}\leq 1 \text{ for } \frac{P_u}{\phi_c P_n}<0.2 \text{ (b)}$$

where P_u is the axial compression, P_n is the nominal compression resistance, M_u is the flexural moment, M_n is the nominal flexural resistance, and ϕ_c and ϕ_f are the resistance factor for compression and flexure, respectively. Flexural moment M_u can either be calculated from the second-order elastic analysis that accounts for the magnification of moment caused by axial compression, or from the simple approximate equation described in AASHTO LRFD (2004) design code.

In Eq. (1) P_n and M_n can be determined from the column buckling curve. P_n and M_n shall be taken as

$$P_n or M_n = 0.66^{(\lambda_{oc} or \lambda_{oc})} f_y A \text{ for } \lambda_{oc} \text{ or } \lambda_{ob}\leq 2.25 \text{ (a)} \tag{2}$$

$$P_n or M_n = \frac{0.88 f_y A}{(\lambda_{oc}\ or\ \lambda_{ob})} \text{ for } \lambda_{oc} \text{ or } \lambda_{ob}>2.25 \text{ (b)}$$

where

$$\lambda_{oc}=\frac{P_y}{P_{cr,e}} \text{ or } \lambda_{ob}=\frac{M_p}{M_{cr,e}}. \tag{3}$$

In Eq. (3), λ_{oc} and λ_{ob} is the buckling parameter for axial compression and flexural moment, respectively. $P_{cr,e}$ and $M_{cr,e}$ is the elastic critical axial compression and moment, respectively. A is the area of the arch section, and f_y is the yield stress of the material. It is noted that M_n is calculated from un-braced length according to AASHTO LRFD (2004). However, in this study, column buckling curves are used to determine M_n for the consistency of the proposed equations. This methodology is also used in Eurocode 3 (2003). It should be noted that the sections are compact so that all local instabilities are ignored for the calculation of P_n and M_n in this study.

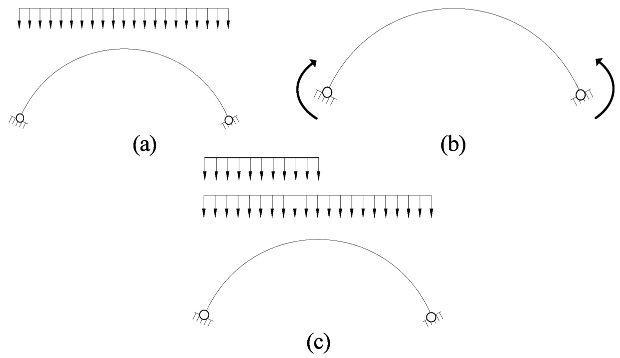

Fig. 1. 1 Loading conditions: (a) LC1 (vertically distributed load); (b) LC2 (equal end moments); (c) LC3 (vertically distributed load over whole span with vertically distributed load over 0.5*l*)

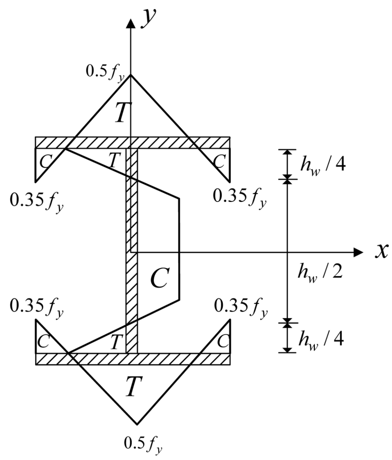

Fig. 2. Distribution of residual stresses for I-section.

3. Description of Finite Element Model

To evaluate the out-of-plane strengths of the parabolic arches under general transverse loadings, finite element analyses are conducted by using the ABAQUS (2009). The out-of-plane strength of the parabolic arches is determined by taking into account the effect of large deformation, material inelasticity, initial imperfection, and residual stresses.

Load cases used in this study are shown in Fig. 1. LC1 is the vertically distributed load and produce the pure axial compression to the parabolic arch cross section. LC1 is used to investigate the inelastic out-of-plane buckling strength of parabolic arches under pure axial compression. LC2 is equal end moment and is used for investigation of arches under pure bending moment. LC3 is vertically distributed load over whole span with vertically distributed load over 0.5*l*. LC3 is used to verify the interactive formula shown in Eq. (1).

The rise span ratio h/l varies from 0.1 to 0.3. I cross-sections was used for the analysis, where assumed residual stresses are same as Fig. 2.

The tri-linear elastic-plastic stress-strain relationship was used for steel material, where the yield stress f_y was 250 MPa, the ultimate stress f_u was 400 MPa, and the modulus of elasticity E was 210,000 MPa. It is assumed that arch had a initial imperfection along the length, and it is given by

$$e_{out} = e_{out,0}\sin\left(\frac{\pi}{l}x\right) \tag{4}$$

where e_{out} is the amplitude of the out-of-plane initial imperfection, and $e_{out,o}$ is the maximum out-of-plane amplitude of the initial imperfection. $e_{out,o}$ was equal to $(l/2)/1500$.

4. Verification of Proposed Design

4.1 Parabolic arches under pure axial compression or pure bending action

Out-of-plane buckling strengths of parabolic arches under pure axial compression or pure bending are evaluated and compared to those from proposed design equations [Eq (1)-(3)]. Fig. 3 show the variations of non-dimensional buckling load M_n/M_p with buckling parameter for flexural moment λ_{ob}. All analyzed models included the effects of material inelasticity, initial imperfection, and residual stresses. It can be found that the most of analysis results matched well with results from proposed design criteria. However, for the analysis model which have low rise to span ratio and low buckling parameters, proposed design equations overestimated the results from finite element analyses. This is because such arches don't satisfy the thresholds for the deep arches (Moon et al, 2009b). Thus, such arches show the shallow arches responses.

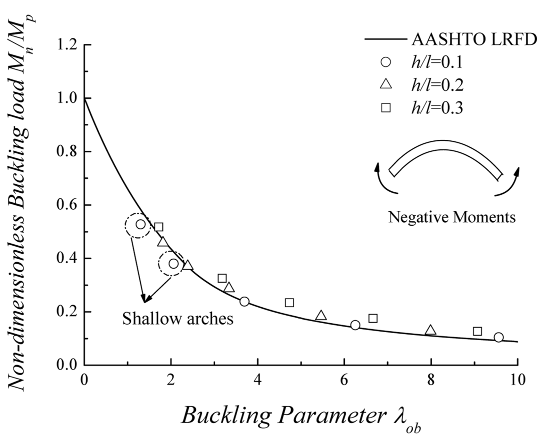

Fig. 3 Out-of-plane strength of parabolic arches under pure bending moment.

Fig. 4 shows the out-of-plane strength of parabolic arches under pure axial compression. In Fig. 4, x and y axes denote buckling parameter for axial compression λ_{oc} and non-dimensional buckling load P_n/P_y, respectively. Similar observations are made with Fig. 3. The out-of-plane strengths of the parabolic arches from the finite element analyses are compared to those from Eq. (2). The comparison shows that Eq. (2) agreed well with the results obtained from the finite element analyses that satisfy the threshold for the deep parabolic arches, while proposed design criteria overestimated the buckling strength of shallow arches. Thus, it is concluded that the proposed design criteria only applicable to arches, which satisfy the thresholds for deep arches described in previous work conducted by author (Moon et al., 2009b).

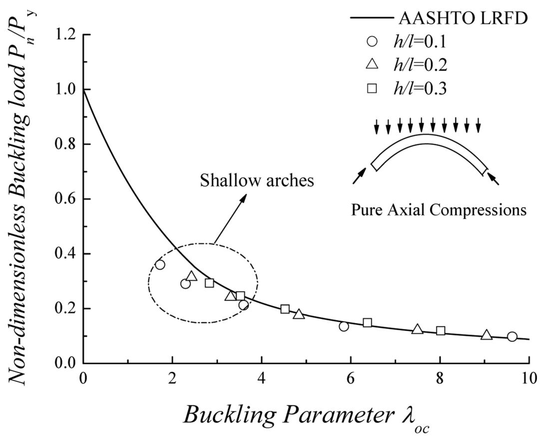

Fig. 4 Out-of-plane strength of parabolic arches under pure axial compression moment.

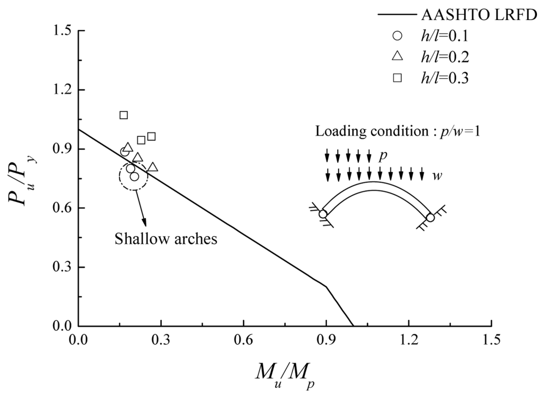

Fig. 5 Out-of-plane strength of parabolic arches under a combination of axial and bending action.

4.2 Parabolic arches under combined axial compression and in-plane bending

The out-of-plane strength of parabolic arches under a combination of axial compression and bending were evaluated. The load case shown in Fig. 1(c) was used to produce combinations of axial compression and bending action, where p/w=1.

The comparison results are shown in Fig. 5. It can be seen that Eq. (1) provided good prediction of the out-of-plane strengths of the arches under a combination of axial compression and bending, except arches have low rise to span ratio. From the observation, it is found that the analysis results which located below proposed design curve don't satisfy the thresholds for deep arches. Thus, proposed design curve overestimated the buckling loads.

5. Conclusions

The out-of-plane design of parabolic arches under general transverse loading was investigated in this study. For this, out-of-plane design equations based on the column buckling curves in AASHTO LRFD for parabolic arches under a pure axial compression or a pure bending were proposed. Then, the interactive formula in AASHTO LRFD was applied to design of parabolic arches under a combination of axial and bending action. Form the comparison results, it can be found that the proposed design criteria provided good prediction of out-of-plane strength for parabolic arches which satisfy the thresholds for deep arches, while proposed design equations overestimated the buckling load of shallow arches. It is noted that the study for out-of-plane design of parabolic arches is limited to simply supported arches. Thus, investigations of the interaction design formula for out-of-plane buckling of the parabolic arches for various loading and boundary conditions are recommended for rigorous design of the parabolic arches.

Acknowledgements

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