

## Load Modeling based on System Identification with Kalman Filtering of Electrical Energy Consumption of Residential Air-Conditioning

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### Abstract

*This paper is proposed mathematical load modelling based on system identification approach of energy consumption of residential air conditioning. Due to air conditioning is one of the significant equipment which consumes high energy and cause the peak load of power system especially in the summer time. The demand response is one of the solutions to decrease the load consumption and cutting peak load to avoid the reservation of power supply from power plant. In order to operate this solution, mathematical modelling of air conditioning which explains the behaviour is essential tool. The four type of linear model is selected for explanation the behaviour of this system. In order to obtain model, the experimental setup are performed by collecting input and output data every minute of 9,385 BTU/h air-conditioning split type with 25 °C thermostat setting of one sample house. The input data are composed of solar radiation ( $W/m^2$ ) and ambient temperature (°C). The output data are power and energy consumption of air conditioning. Both data are divided into two groups follow as training data and validation data for getting the exact model. The model is also verified with the other similar type of air condition by feed solar radiation and ambient temperature input data and compare the output energy consumption data. The best model in term of accuracy and model order is output error model with 70.78% accuracy and 17<sup>th</sup> order. The model order reduction technique is used to reduce order of model to seven order for less complexity, then Kalman filtering technique is applied for remove white Gaussian noise for improve accuracy of model to be 72.66%. The obtained model can be also used for electrical load forecasting and designs the optimal size of renewable energy such photovoltaic system for supply the air conditioning.*

**Keywords:** load modelling, System identification, Kalman filter, Air-conditioning

## 1. INTRODUCTION

In the present, electrical energy demand is gradually increasing, the load forecasting is predicted that the

power demand will be increased 5-6% each year upon to the expanding of economy situation. From this situation the supply side management are concerned by energy reserve and power plant construction planning for support this peak demand. Otherwise the power system stability will be occur when the load demand is higher that power supply. However to solve this problem by this way, is use high cost of investment as well as the facing of environmental pollution protest from community are the problem. In opposite way, the demand side management is one of the solution to decrease the peak demand of utility customer by avoid utilization the load when the peak time. There are various methods to solve the demand side management (DSM) such as load management, identification and promotion of new uses, strategic energy management& conservation, electrification, customer generation, cost of electricity on demand, and demand response [1]. Particularly the latter case is one of the significant methods for smart grid power system. Because of the peak demand occur in the summer time from the temperature dependent electrical equipment such as air-conditioning is necessary for planning and management. From this reason, the behaviour and modelling of air-conditioning is vital for demand side management and demand response strategy. The mathematical modelling of load modelling can be represented by static model [2] which is no complex but the limitation is some error from the system change may occur. The dynamic model has more accuracy and fast response but contain high technique and complex mathematical [3]. In practical, there are two way for receive model composed of analytical method and experimental method. The analytical method is need the equation and theory for interpret the system which is difficult to know the status. The advantage of experimental or system identification technique is high accuracy and no need inside information from the system to get the model [4]. Generally, the system contain nonlinear behaviour, then nonlinear model is appropriate for explain nonlinear system however nonlinear model has less tool for analyze system compare with linear model. The linear model can be acceptable for explain nonlinear system as long as it operate at equilibrium and linear model has high order and more noise [5-6]. The model order reduction technique is used for reduce order of model in the next step [7] and Kalman filtering is applied for remove Gaussian white noise for improve accuracy [8]. The obtained model can be utilized in such application as load forecasting, demand response planning. In this paper, the air-conditioning model with system identification method is proposed. The next section, the theory of system identification, model order reduction and Kalman filtering are introduced. The third section, the experimental of air conditioner data collector is illustrated. The analysis and discussion of model is discussed.

## **2. MODELING BASED ON SYSTEM IDENTIFICATION**

In this section, modeling based on system identification theory is introduced follow as linear system identification, model order reduction and Kalman filtering respectively.

### **2.1 System identification**

System identification is the process for modelling dynamical systems by measuring the input/output from system as shown in Figure 1. Experiments are designed and implemented for data collection. After that, the structure of model will be chosen. System identification can be divided into linear and nonlinear system identification based on linear property. Linear system identifications are used to describe linear systems and nonlinear systems as long as the systems operate in linear range. As the next step, iterative simulation output data and experimented have been compared until the best model performance is derived by results include a quantitative measure of the model quality in terms of goodness of fit to estimation data.

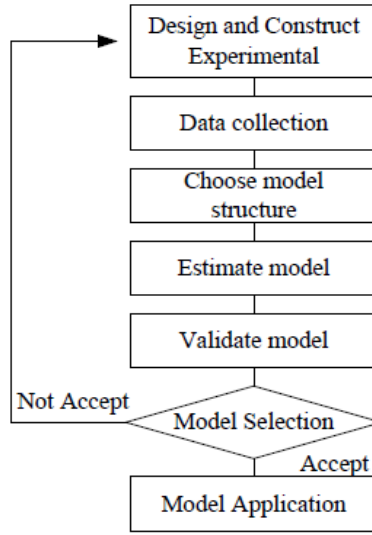


Figure 1. System Identification Principle [9]

The percentage of best fit accuracy in Eq. 1 is obtained from comparison between experimental waveform and simulation modelling waveform, where  $y^*$  is simulated output,  $y$  is measured output and  $\bar{y}$  is mean of output.

$$Best\ fit = 100 \times \left(1 - \frac{|y^* - y|}{|y - \bar{y}|}\right) \tag{1}$$

When the Model Output plot does not show a good fit, the next different type of model and model order is trial. System identification is largely a trial-and-error process when selecting model structure and model order. Ideally, the lowest-order model that adequately captures the system dynamics is preferred and High-order models are more expensive to compute and result in greater parameter uncertainty. A linear dynamic block is a Linear Time Invariant (LTI) system which can be modelled by using polynomial model structure as shown in Fig. 2. The polynomial structure can be classified into a continuous and a discrete time model. Here, we are focusing on a discrete time model according to a data logger of temperature and electrical power data.

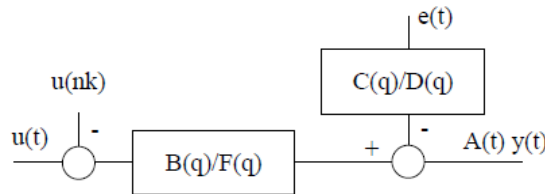


Figure 2. Linear Model Block Diagram

The general form of discrete time polynomial model structures can be represented as follows, as Eq. (2)

$$A(q)y(t) = \sum_{i=1}^{nu} \frac{B_i(q)}{F_i(q)} u_i(t - k_n) + \frac{C(q)}{D(q)} e(t) \tag{2}$$

The polynomials  $A, B, C, D$  and  $F$  contain the time-shift operator  $q$ , essentially the z-transform.  $A, B, C, D$  and  $F$  which be expanded as in Eq. (3).  $u_i$  is the  $i^{\text{th}}$  input,  $nu$  is the total number of inputs,  $y(t)$  is the output.

$$\begin{aligned}
 A(q) &= 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a} \\
 B(q) &= b_1 + b_2 q^{-1} + \dots + b_{n_b} q^{-n_b+1} \\
 C(q) &= 1 + c_1 q^{-1} + \dots + c_{n_c} q^{-n_c} \\
 D(q) &= 1 + d_1 q^{-1} + \dots + d_{n_d} q^{-n_d} \\
 F(q) &= 1 + f_1 q^{-1} + \dots + f_{n_f} q^{-n_f}
 \end{aligned} \tag{3}$$

$a_n$  is an output coefficient,  $b_n$  and  $f_n$  are input coefficients,  $c_n$  and  $d_n$  are error coefficients.  $k_n$  is the  $n^{\text{th}}$  input delay that characterizes the delay response time and  $e(t)$  is the error signal. The order of the model is the sum  $b_n$  and  $f_n$ . This should be minimum for the best model. The general equation shown can be reduced to various linear models as follows

- (i) An auto regressive with exogenous model (ARX), the polynomials  $F(q)$ ,  $C(q)$ , and  $D(q)$  are zero.
- (ii) An auto regressive moving average with exogenous model (ARMAX), the polynomials  $F(q)$  and  $D(q)$  are zero.
- (iii) A Box-Jenkins (BJ) model, the polynomial  $A(q)$  and  $D(q)$  are zero.
- (iv) An output– error (OE) model, the polynomial  $A(q)$ ,  $C(q)$ , and  $D(q)$  are zero.

## 2.2 Model Order Reduction

Model Order Reduction (MOR) is a branch of systems and control theory, which studies properties of dynamical systems in application for reducing their complexity, while preserving their input-output behavior. For linear time invariant system, state space in original model is reduced to low dimensional as shown in Fig.3 a). The most frequently employed model reduction algorithms for linear, stable, continuous- or discrete-time systems are the balancing related absolute error methods like the balanced truncation approximation (BTA), singular perturbation approximation (SPA), and Hankel-norm approximation (HNA) [10]. These so-called balancing-free square-root (BFSR) methods rely on computing well-conditioned truncation matrices using exclusively square-root information. In this paper balancing-free square-root technique (BFSR) is used to computes a reduced-order approximation of linear Time Invariant model of system. The desired order (number of states) is specified by ORDERS which obtain from Hankel singular values by pick an adequate approximation order from high energy state and discard low energy state [11-12]. In state coordinates that equalize the input-to-state and state-to-output energy transfers, Hankel singular values measure the contribution of each state to the input/output behavior. Hankel singular values are to model order what singular values are to matrix rank. In particular, small Hankel singular values signal states that can be discarded to simplify the model. The principle of balancing-free square-root (BFSR) algorithm is described in Fig.3 b)..

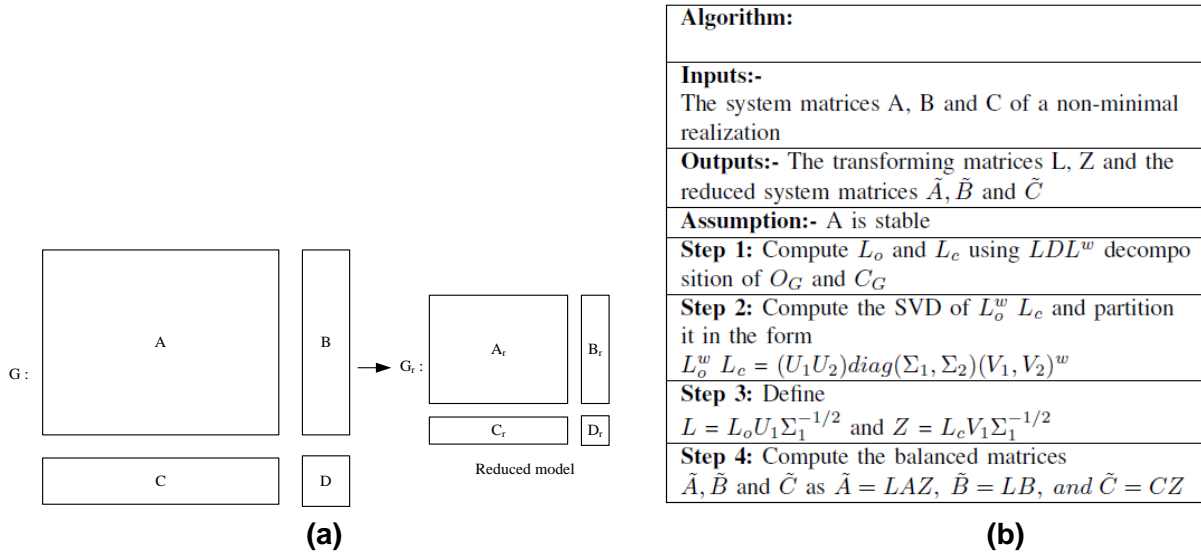


Figure 3. a) Model Order Reduction concept, b) balancing free square root Algorithm [13]

### 2.3 Kalman Filter

The Kalman filter is an estimator for what is called the linear-quadratic-Gaussian problem, which is the problem of estimating the instantaneous “state” of a linear dynamic system perturbed by Gaussian white noise by using measurements linearly related to the state but corrupted by the noise. The resulting estimator is statistically optimal with respect to any quadratic function of estimation error. Assume that the equations that describe the behaviour of a dynamical system is known, and that there are some noisy observations of the system, and the current state of the system or predict the state of the system in the future is required. The Kalman filter equations are as follows

Prediction Step:

$$\text{The state} \quad \hat{x}_k^- = A\hat{x}_{k-1} + w_{k-1} \quad (4)$$

$$\text{The error covariance} \quad \hat{P}_k^- = A\hat{P}_{k-1}A^T + Q \quad (5)$$

Update Step:

$$\text{Kalman gain} \quad K_k = \hat{P}_k^- H^T (H\hat{P}_k^- H^T + R)^{-1} \quad (6)$$

$$\text{Update with new measurement} \quad \hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-) \quad (7)$$

$$\text{Update with new error covariance} \quad \hat{P}_k = (I - K_k H)\hat{P}_k^- \quad (8)$$

Where  $\hat{x}_k$  is the state estimate of the system at time  $k$

$\hat{x}_{k-1}$  is the state estimate of the system at time  $k-1$

$\hat{P}_k$  is the estimation-error covariance matrix

$w_{k-1}$  is the random noise affecting the system at time  $k-1$

$z_k$  is the vector of measurements at time  $k$

Assume that

$w_{k-1}$  has a multivariate normal distribution with mean 0 and covariance matrix  $Q$ .

$v_k$  is normally distributed with mean 0 and covariance matrix  $R$ .

The matrices A, H, Q, and R are all assumed to be known.

The sequence of computational steps for the Kalman filter estimator is showed in Fig. 4

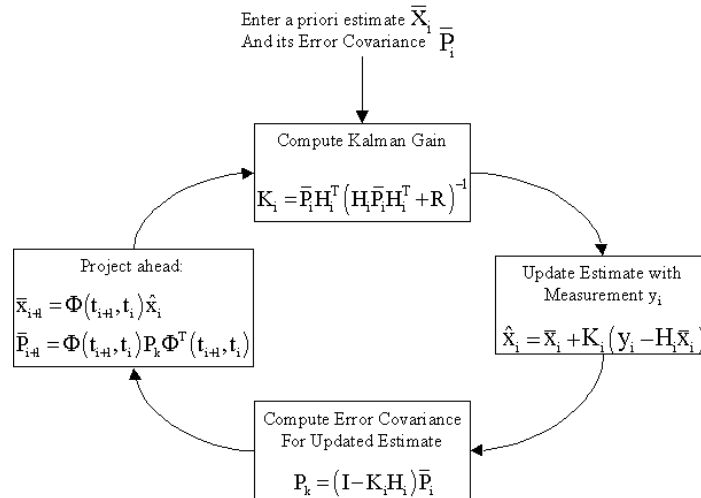


Figure 4. The sequence of computational steps for the Kalman filter estimator [14]

### 3. EXPERIMENTAL RESULTS

To receive load modelling of residential air condition, the experimental setup is performed as shown in Fig.5. The air-conditioner with cooling capacity 9,538 BTU/h which use single phase electrical 220 V 50 Hz installed in the area 16 sq.m. of the room in one sample residential house is operated. The data logger with thermocouple sensor for measure the temperature such ambient, evaporator, condenser, room temperature and also solar radiation are installed. The power and energy meter is use to measure electrical value such voltage, current, power, power factor and energy. The sampling data of mention parameter are collected in every minute for 24 hour.

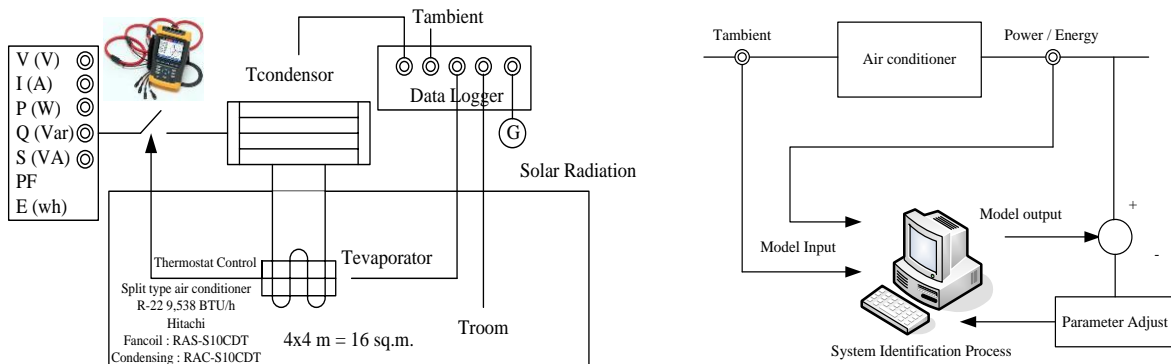


Figure 5. Experimental setup for air conditioning load modelling for system identification.

After collected the data from experimental, the significant data such ambient temperature and power consumption of air conditioning are selected to be input-output of model by system identification. The collected data are divided into two group follow as estimate data and validate data. The program with system

identification algorithm is devolved to estimate and check the accuracy of model and experimental for getting the best model of air conditioner. In this paper, four types of linear model are selected to represent the system.

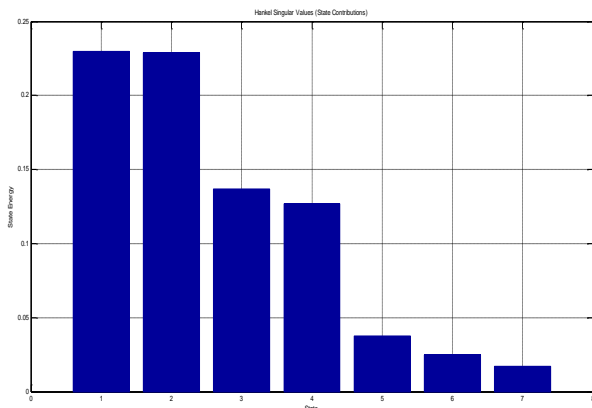
### 4. DISCUSSION

From the experimental and data collection process, the two significant data such ambient temperature and power consumption of air condition in 24 hours duration are plotted. The peak demand of has occur in afternoon around 2.00 PM which has high temperature. The input-output data of air-conditioning system are collected and transmitted to the computer. The developed program generates model output to compare with the experimental. The software program is developed by system identification technique. There are four type of linear models used in this experimental. The parameter of linear model from system identification is shown in Table I.

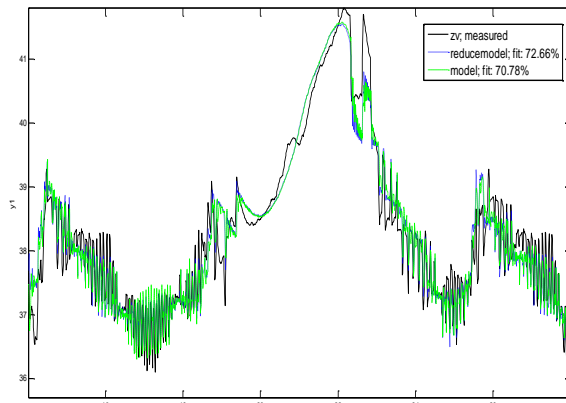
**Table 1. Linear model properties**

No	Type of model	Linear parameter						% accuracy	Order
		na	nb	nc	nd	nf	nk		
1	ARX	11	2	-	-	-	2	36.15	14
2	ARMAX	4	11	2	-	-	2	56	15
3	Box Jenkin	-	1	1	2	2	14	60.05	3
4	Output Error	-	5			12	4	70.78	17

From the table, the best model which concern from accuracy model is output error which has accuracy 70.78%. In order to reduce the complexity of model, Hankel singular value is used to show the energy of each state are determined as shown in Fig 6. From Hankel singular value, keep the state that contain high energy than 7 and discard state have low energy than 7 then model order reduction is used the seven order to reduce by balance balancing-free square-root technique and Kalman filter is used to remove white gaussian noise from data then model has accuracy increase to 72.66% as shown in Fig. 7.



**Figure 6. Hankel singular value**



**Figure 7. Model accuracy compare from Kalman filter**

## 5. CONCLUSION

This paper is proposed mathematical load modelling based on system identification approach of energy consumption of residential air conditioning. Due to air conditioning is one of the significant equipment which consumes high energy and cause the peak load of power system especially in the summer time. The demand response is one of the solutions to decrease the load consumption and cutting peak load to avoid the reservation of power supply from power plant. In order to operate this solution, mathematical modelling of air conditioning which explains the behaviour is essential tool. The four type of linear model is selected for explanation the behaviour of this system. In order to obtain model, the experimental setup are performed by collecting input and output data every minute of 9,385 BTU/h air-conditioning split type with 25 °C thermostat setting of one sample house. The input data are composed of solar radiation ( $\text{W/m}^2$ ) and ambient temperature ( $^{\circ}\text{C}$ ). The output data are power and energy consumption of air conditioning. Both data are divided into two groups follow as training data and validation data for getting the exact model. The model is also verified with the other similar type of air condition by feed solar radiation and ambient temperature input data and compare the output energy consumption data. The best model in term of accuracy and model order is output error model with 70.78% accuracy and 17<sup>th</sup> order. The model order reduction technique is used to reduce order of model to 7<sup>th</sup> order for less complexity, then Kalman filtering technique is applied for remove white Gaussian noise for improve accuracy of model with 72.66%.

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