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Developing the Accurate Method of Test Data Assessment with Changing Reliability Growth Rate and the Effect Evaluation for Complex and Repairable Products

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Reliability growth rate (or reliability growth curve slope) have the two cases of trend as a constant or changing one during the reliability growth testing. The changing case is very common situation. The reasons of reliability growth rate changing are that the failures to follow the NHPP (None-Homogeneous Poisson Process), and the solutions implemented during test to break out other problems or not to take out all of the root cause permanently. If the changing were big, the “Goodness of Fit (GOF)” of reliability growth curve to test data would be very low and then reduce the accuracy of assessing result with test data. In this research, we are using Duane model and AMSAA model for assessing test data and projecting the reliability level of complex and repairable system as like construction equipment and vehicle. In case of no changing in reliability growth rate, it is reasonable for reliability engineer to implement the original Duane model (1964) and Crow-AMSAA model (1975) for the assessment and projection activity. However, in case of reliability growth rate changing, it is necessary to find the method to increase the “GOF” of reliability growth curves to test data. To increase GOF of reliability growth curves, it is necessary to find the proper parameter calculation method of interesting reliability growth models that are applicable to the situation of reliability growth rate changing. Since the Duane and AMSAA models have a characteristic to get more strong influence from the initial test (or failure) data than the latest one, the both models have a limitation to contain the latest test data information that is more important and better to assess test data in view of accuracy, especially when the reliability growth rate changing. The main objective of this research is to find the parameter calculation method to reflect the latest test data in the case of reliability growth rate changing. According to my experience in vehicle and construction equipment developments over 18 years, over the 90% in the total development cases are with such changing during the developing test. The objective of this research was to develop the newly assessing method and the process for GOF level increasing in case of reliability growth rate changing that would contribute to achieve more accurate assessing and projecting result. We also developed the new evaluation method for GOF that are applicable to the both models as Duane and AMSAA, so it is possible to compare it between models and check the effectiveness of new parameter calculation methods in any interesting situation. These research results can reduce the decision error for development process and business control with the accurately assessing and projecting result.

Keywords: Reliability Growth Test, Reliability Growth Rate, Regression, Least Square Method, Duane Model, AMSAA Model, Maximum Likelihood Estimation

1. Introduction

Reliability growth is the improvement in the reliability of developing products over a period due to the changes in the product’s design and (or) the manufacturing process with the deficiency in the required reliability target. Reliability growth occurs from the corrective and (or) preventive actions based on the experience gained from the failure information and analysis result of equipment, design, production and operation processes. In this research, the reliability measurement of interest is the Mean-Time-Between-Failure (MTBF) metric for complex and repairable products as like construction equipment (Excavator / Wheel-loader / Articular-hauler) and vehicle. The reliability growth “Test-Analyze-Fix” concept in design is meaning to uncover the weaknesses dur-

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ing reliability growth testing stages and performing the appropriate corrective actions before full-scale production. Reliability growth management concerns itself with the planning and management of reliability growth process in a new product development to achieve the planned reliability growth expressed as a function of time and resources. It composite with (1) the developing process from design and fabrication of the proto / pilot products; (2) reliability growth planning; (3) reliability growth test and solving process; (4) developing activities monitoring; (5) assessing the test data and projecting the current reliability level; (6) controlling the developing process and test result to compensate the deficiency developing part. Reliability growth test is providing opportunities to identify the weaknesses and failure modes in the design and manufacturing process. The main roles of reliability growth test in the management process are to make the plan for plotting the anticipated system reliability target versus test duration during the development program and to assess (or monitor) periodically reliability growth test result. Finally, it is to project the current reliability level of developing product also including the function to compare with the planned reliability target. More information is in MIL-STD-1635 (1978), MIL-HDBK-189A (1981), RADC-TR- 84-20 (1984).

When the project team uses the reliability-growth management process to set-up or updates the product developing direction during product developing, it is very important to assess the test data with the accurate method and evaluate the result with the reliable GOF. There should be an accurate reliability growth assessing method to evaluate the test data for the high level of GOF achievement under considering a product characteristic and failure types. There are many reliability growth models for test data assessment and projection of reliability level, each model usually make the different results even with the same test data because of their unique specifications and limitation. In this research, we have an interesting about the complex (and repairable) system as like vehicle and construction equipment which are using the distance (kilometer) and operation hour as a measuring unit. The industry use MTBF as reliability measurement for such products. Duane model and AMSAA model are historically representative ones for such products as mentioned by Crow (1975). We also used these models for this research to develop the new assessing methodology and new GOF evaluation method.

Kumaraswarny (2002) used Duane model to check the reliability growth existence for helicopter development with the result of positively constant reliability growth rate. According to Codier research (1968), it is general to have a changing in reliability growth rate during reliability growth test because of the imperfect solutions and design changes implemented during the test. He also said the latter test data to have the latest information that is more important than the initial test data. Donovan and Murphy (1999) pointed out that Duane model has a tendency to get too much influence with the initial test data (or failures). MIL-STD-1635 (1978) also pointed out the issue and suggested to use the moving average method. Demko (1993) used Duane model and AMSAA model to assess many test cases as the constant and changing reliability growth rate. There have been many researches to show the limitation of conventional assessing methodology at the growth rate changing condition and suggested many ideas to solve the problem after Codier (1968), but all of them were also showing the another limitations in implementation to general cases. Related with GOF, there are only limited researches. Dwyer (2009) said if the plotted points are not independent, then proportionally weighting the cumulative number of failures at each point is a reasonable way to improve accuracy of these estimates. The technique assigns greater weight to the preceding data point (the most recent one). They do this by giving each point a weight according to its order in the cumulative statistic except for the last point, and find the resulting "Center of Gravity" of those points. They also want to go through the last point. The calculation method for GOF is getting too much influence with the multiple and big changing in the reliability-growth rate to check the effectiveness of it. When there is the multiple or big changing in the reliability-growth rate. Demko (1993) pointed out that the least squares regression is performed on all data points (i.e. the cumulative MTBF versus the cumulative test hours). The earliest MTBF data point is omitted for the next regression calculation. This data compression process is repeated again until only the three latest MTBF data points remain. Only the MTBF data point is decremented for the regression iterations, not the test hours, or the number of failures. With the each regression iteration, even it is the one of good solutions, but there are also concerns as follow: should be at least 20 failures data to use the suggested method, the big influence with the multiples times of reliability-growth rate changing, no common GOF evaluation method for Duane and AMSAA models.

Based on our experience in SUV (Sport Utility Vehicle), Bus, Truck, Construction Equipment developments and other researches as like Codier (1968) and Demko (1993), it is very natural that the test data are showing the growth rate changing during the test. In addition, the later test data in any test program have more important and latest information than the initial one, so it is absolutely necessary for reliability engineer to weight on the later ones. Since the assessed result with test data regardless of growth rate changing should be a baseline to control the development process and business decision, reliability engineer must have an accurately assessing method.

As the research results, there are three major information. The first one is the mathematical formulas to calculate the parameters for Duane and AMSAA models for the case of reliability growth rate changing to make more accurate assessment, the second is the common evaluation method of GOF for both models to measure the improvement effectiveness with the new assessing method. The last one is the total process from detecting growth rate change to evaluating the improved GOF, which never ever be touched by any researches in detail as our one. Chapter 2 treats the conventional assessing method with Dune model and AMSAA model with some of sample test cases. Chapter 3 shows the research results of the new mathematical formulas development to calculate the reliability-growth model parameters that will increase the GOF of both models in case of growth rate changing. There is also the new evaluation method introduction of GOF for both models. Chapter 4 treats the sample cases from Monte Carlo Simulation and Demko research for new methodology implementation.

2. Background

There are at least 30 reliability growth models that can be classified to some of group according to their purpose as planning / assessing / projecting and product specifications, the more detail classification informations are in Hall (2008) and MIL-STD-189C (2009). The reliability growth assessing and projecting models may be different with the developing target product specification as like repairable or none repairable, failures following HPP or NHPP, one shot system or continuous operation systems. We used Duane model and AMSAA model for this research because of interesting at complex and repairable products. We wanted to find more accurate method and process to increase the assessing accuracy with the test data, especially in case of the reliability growth rate changing during the testing. Also we mainly tried to find the reliable GOF evaluation method.

2.1 Reliability Growth Models for Test Data Assessment

We focus on finding the accurate assessing method and process with the test data breaking out the growth rate changing during the test, especially with the Duane model and AMSAA model for repairable and complex products. Duane (1964) at GE published the report with his observations on failure data for five divergent types of systems during development programs, the systems (or products) were the complex hydro-mechanical system, aircraft generators and aircraft jet engine. Duane assumed the probability distribution of failures to follow Homogenous Poisson Process. His research result for Duane model showed that the observed cumulative failure rate (or Mean Time between Failures) versus cumulative operating hours fell close to a straight line when plotted on log-log paper. The mathematical expression for Duane model is as Equation 1.

$$\mathrm{M_c(T)} = \mathrm{bT}^{\alpha} \tag{1}$$

Where Mc, T, b, α are the cumulative MTBF and test time, constant and growth rate. MIL-STD-1635 (1978) and O'Connor (2007) described that the growth rate range from 0.2 to 0.6 is reasonable level as a proper reliability growth existing during the product development time. The equation of the line can be expressed as $Y = \alpha X + Z$. Let $Y = \ln M_c$, $X = \ln T$ and $Z = \ln b$. It yields:

$$\ln \mathrm{M_c(T)} = \ln b + \alpha \ln T \tag{2}$$

The instantaneous MTBF, M_i, as shown in Equation 3 is showing the current reliability level at a specific instant or after a specific test and development time.

$$\mathrm{M_i(T)} = \mathrm{M_c(T)}/(1-\alpha) \tag{3}$$

The Duane model have two parameter as growth rate (α), constant (b). Therefore, to use the model as a basis for assessing and projecting reliability level that could be expected in an product development program, these parameters must be defined as a function of product characteristics. These parameters can be estimated for a given data set using the curve-fitting methods and the least square method that was suggested by Duane. These parameters are:

$$\hat{\alpha} = \left[\sum_{i=1}^{n}\{\ln(T_i)\ln(M_{ci})\} - \left\{\sum_{i=1}^{n}\ln(T_i)\sum_{i=1}^{n}\ln(M_{ci})\right\}/n\right] \Big/ \left[\sum_{i=1}^{n}\ln(T_i)^2 - \left\{\sum_{i=1}^{n}\ln(T_i)\right\}^2/n\right] \tag{4}$$

$$\hat{\mathrm{b}} = \mathrm{e}^{\left[(1/\mathrm{n})\left\{\sum_{i=1}^{n}\ln(M_{ci}) - \hat{\alpha}\sum_{i=1}^{n}\ln(T_i)\right\}\right]} \tag{5}$$

Where n, Mci and Ti are the number of failures, cumulative MTBF and test times for each failures. The detail information can be founded at Duane (1964). The reliability analysis of test data during product development would involve many data generated by multiple systems, which was the initiation of AMSAA model development. Crow (1974) who worked at Army Materiel Systems Analysis Activity (AMSAA) proposed AMSAA model which is assuming the failures to follow the Non-Homogeneous

Poisson Process (NHPP) for a complex and repairable system under the customer use of analysis. He also propsed the appropriate statistical procedures for Maximum Likelihood Estimation as calculating parameters. He suggested the cumulative and instantaneous AMSAA model as:

$$\mathrm{M_c(T)} = \mathrm{T}^{(1-\beta)}/\lambda \tag{6}$$

$$\mathrm{M_i(T)} = \mathrm{T}^{(1-\beta)}/(\lambda\beta) \tag{7}$$

Where λ and β are the scale and shape parameter for AMSSA model. The probability density function (pdf) of the i^{th} event given that the $(i-1)^{th}$ event occurred at T_{i-1} is:

$$f(T_i|T_{i-1}) = \frac{\beta}{\eta}\left(\frac{T_i}{\eta}\right)^{\beta-1} e^{-\frac{1}{\eta^{\beta}}(T_i^{\beta} - T_{i-1}^{\beta})} \tag{8}$$

Where λ is equal to $\eta^{-\beta}$.
The λ is:

$$\hat{\lambda} = \mathrm{n}/\mathrm{T}^{\beta} \tag{9}$$

Then the β is:

$$\hat{\beta} = \mathrm{n}/\left[\mathrm{n}\ln(\mathrm{T}) - \sum_{\mathrm{i}=1}^{\mathrm{n}} \ln(T_i)\right] \tag{10}$$

According to MIL-HDBK-189C (2009), for the case where the individual failure times are known, a Cramer-von Mises statistic is used to test the null hypothesis that the NHPP properly describes the reliability growth of the system. That means that there are different methods to measure GOF of Duane and AMSAA curves to test data. But as mentioned by Crow (1975), AMSAA model is the extension of Duane model and straight line at log-log graph for cumulative MTBF and test time, he also mentioned the relation of both models'parameter as $1-\beta=\alpha$ and $1/\lambda$ = b. So it may be possible to apply the same evaluation method of GOP for both models. One of the research results is to show the common evalation method applicable to both models.

2.2 Case of Constant Reliability Growth Rate

The constant reliability growth rate was a representative case for reliability-growth test during Duane introduced his model (1964). The sample case with the constant one is made for repairable system with Monte Carlo Simulation as shown <Table 1>, it is simulated with the 1000 hours testing time with 15 failures. With the test data of <Table 1> and Equation (4), (5), (9), (10), it is possible to get the parameters for Duane model and AMSAA model. After then it is necessary to put theses parameters to Equation (1), (3), (6), (7) to calculate a cumulative and instantaneous MTBF for the models. The calculation results are on the <Table 2>.

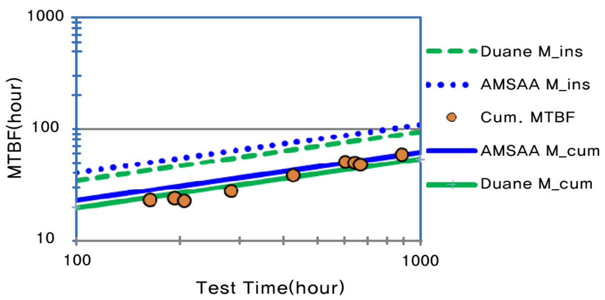
〈Table 2〉 Analysis Result for Constant Growth Rate

Duane Model				AMSAA Model			
α	b	M_c	M_i	β	λ	M_c	M_i
0.43	2.75	54	94	0.57	0.31	62	109

〈Table 1〉 Test Data for Constant Growth Rate

Failure Number	Test Time	Cumulative Failure Number	ln(T_c)	ln(Tc)^2	Mc	ln(M_c)	ln(T_c)×ln(M_c)	Duane Value	AMSAA Value
1	33	1	3	12.1	32.6	3.5	12.1	19.9	23.9
1	88	2	4	20.0	44.0	3.8	16.9	35.5	42.3
1	92	3	5	20.5	30.7	3.4	15.5	36.5	43.4
1	193	4	5	27.7	48.2	3.9	20.4	56.2	66.4
1	261	5	6	31.0	52.3	4.0	22.0	67.2	79.1
1	449	6	6	37.3	74.9	4.3	26.4	92.2	108.1
1	838	7	7	45.3	119.7	4.8	32.2	132.7	154.6
1	916	8	7	46.5	114.5	4.7	32.3	139.8	162.7
1	1,408	9	7	52.6	156.4	5.1	36.6	179.7	208.2
1	2,445	10	8	60.9	244.5	5.5	42.9	248.1	286.0
1	3,490	11	8	66.5	317.2	5.8	47.0	305.5	351.0
1	4,805	12	8	71.9	400.4	6.0	50.8	368.2	421.8
1	6,122	13	9	76.0	470.9	6.2	53.7	424.2	484.8
1	8,860	14	9	82.6	632.9	6.5	58.6	526.5	599.6
1	9,178	15	9	83.3	611.9	6.4	58.5	537.5	611.9
Sum	9,178	15	101.6	734.2	3,350.9	73.7	526.0		

<Figure 1> shows the cumulative test data and the reliability growth curves plotting with the data of <Table 2>, the reliability growth curves based on the Duane model and AMSAA model shows the very good GOF with the test data. During the research, we felt the necessity to have one GOF measuring standard for the models to compare and evaluate the effectiveness of new assessing method with the test data.



〈Figure 1〉 Reliability Growth Curves with Constant Growth Rate

2.3 Case of Changing Reliability Growth Rate

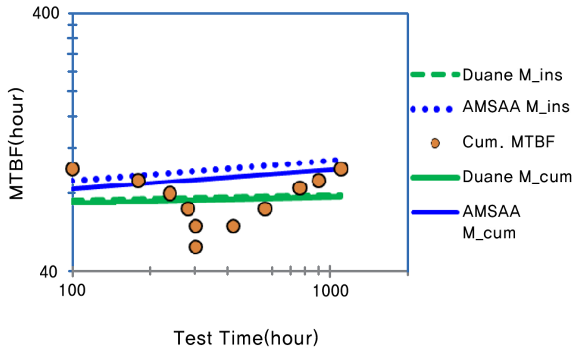
According to the research result as like Codier (1968), Demko (1993), Dwyer (2009), many practical cases in reliability growth tests to develop the new product development in the variety areas have been showing the growth rate changing during reliability growth test.

The 90% in the developing projects that we have involved for vehicle and construction equipment had such a reliability growth rate changing during testing.

<Table 3> is the typical example of reliability growth rate changing taken from the research by Demko (1993) which are showing the almost same pattern in the current industrial area. <Table 4> is the analysis result with the test data to show the parameters for models and the calculated cumulative (and instantaneous) MTBF for them. <Figure 2> shows the cumulative test data and the reliability growth curves with the data of <Table 4>; there seems a big changing in the slope at 300-hour test time.

〈Table 4〉 Assessment Result with Test Data

Duane Model				AMSAA Model			
α	b	M_c	M_i	β	λ	M_c	M_i
0.02	66.45	54	94	0.92	0.02	100	108



〈Figure 2〉 Reliability Growth Curves with Changing Growth Rate

3. New Gof (Goodness Of Fitness) Evaluation Method

Demko (1993) used Correlation Coefficient (R) for Duane model and Critical Value for AMSAA model that are the input for Cramer-von-Mises test to decide whether the model is acceptable to the test data or not. With these results, it is impos-

〈Table 3〉 Test Data for Growth Rate Changing

Failure Number	Test Time	Cumulative Failure Number	$\ln(T_c)$	ln(Tc)^2	Mc	$\ln(M_c)$	$\ln(T_c)\times\ln(M_c)$	Duane Value	AMSAA Value
1	100	1	4.6	21.2	100.0	4.6	21.2	73.7	83.3
1	180	2	5.2	27.0	90.0	4.5	23.4	74.7	87.1
1	240	3	5.5	30.0	80.0	4.4	24.0	75.2	89.1
1	280	4	5.6	31.8	70.0	4.2	23.9	75.4	90.1
1	300	5	5.7	32.5	60.0	4.1	23.4	75.5	90.6
1	300	6	5.7	32.5	50.0	3.9	22.3	75.5	90.6
1	420	7	6.0	36.5	60.0	4.1	24.7	76.1	92.9
1	560	8	6.3	40.0	70.0	4.2	26.9	76.6	95.0
1	760	9	6.6	44.0	84.4	4.4	29.4	77.1	97.2
1	900	10	6.8	46.3	90.0	4.5	30.6	77.4	98.5
1	1,100	11	7.0	49.0	100.0	4.6	32.3	77.8	100.0
Sum	11,000	11	65.1	390.9	854.4	47.6	282.1		

sible to compare the GOF of both models. There is limitation to check the GOF for both models, so it is necessary to find the new one for both models. Even the R-value is reliable in case of constant growth rate, the value under changing growth rate is unreasonable as 7.1% for the data in <Table 3>.

The main objective of this research was to develop the new assessing method to make the reliability growth curves of Duane model and AMSAA model having the high GOF in case of the growth rate changing. Therefor we should have develop the new concept of GOF for both models with the newly assessing method, then it was possible to compare the GOF values for models and check the effectiveness of new method.

During this research we developed the new one to measure the level of GOF between the both model curves and test data. It compose with two measuring values, the first value is Determination Coefficient (R^2: DC) showing how well the curve is matching with the test data and 100% means the curve perfectly representing the actual data. Since the reliability growth curves of both models should be the straight lines at the log-log chart expressed with cumulative MTBF and test time, the concept of DC must be applicable to them. The second value is Average MTBF Gap (or AMG) which is developed for this research, and then showing the average gap between the actual cumulative MTBF and the calculated MTBF of growth curves at each failure times. The mathematical expression for Duane and AMSAA models are as like Equation (11) and (12).

$$AMG_{DUANE} = \left[\sqrt{\sum_{n=1}^{n} \left\{ \left(\frac{T_i}{n_i} \right) - (b T_i^{\alpha}) \right\}^2} \right] / n \tag{11}$$

$$AMG_{AMSAA} = \left[\sqrt{\sum_{n=1}^{n} \left\{ \left(\frac{T_i}{n_i} \right) - (\frac{1}{\lambda}) T_i^{1-\beta} \right\}^2} \right] / n \tag{12}$$

<Table 5> is showing the new GOF analysis result for the case with the constant growth rate as shown in <Table 1>.

〈Table 5〉 New GOF Evaluation Result with Constant Growth Rate

Duane Model		AMSAA Model	
R^2	AVG MTBF Gap	R^2	AVG MTBF Gap
94.2%	1.0	90.6%	1.3

<Table 6> is showing the GOF evaluation result with test data and reliability growth models made with the data in <Table 4>. The average DC values for both models are very low, and the graphical checking is showing the big slope changing.

〈Table 6〉 New GOF Evaluation Result with Changing Growth Rate

Duane Model		AMSAA Model	
R^2	AVG MTBF Gap	R^2	AVG MTBF Gap
1.0%	4.9	85.3%	6.6

4. New Test Data Assessing Process and Method For Reliability Growth Rate Changing Method

The reliability-growth test result is used for projection of currenttly developing product reliability level and market quality level, which is based on the instantaneous MTBF estimated from the cumulative MTBF. The MTBF is calculated with the assessed result with the test data that is strongly got the influence from the GOF of reliability growth curves to the test data. Therefore, it is very important to find the best assessing method to make the reliability growth curve with high GOF to test data. Even many trials have been in there to develop the new method to make the better GOF with the test data breaking out the reliability growth changing, they are showing another limitation in the generally implementing cases. In additional, there have not been the researches to show the process how to handle such a case. In this chapter there is the research result to show the better assessment method and process in case of reliability growth rate changing during testing.

4.1 What is the Reliability Growth Rate?

The reliability growth rate for the Duane model and AMSAA model is the average slope of cumulative MTBF on the test points plotted on the log-log chart for MTBF versus test time. When a systematic and deliberate reliability improvement effort is being made, the range of reasonable reliability growth rate is from 0.2 to 0.6 expressed at MIL-STD-1635 (1978). The value is getting the influence from the many factors as; the speed and effectiveness of the test failure solving effort; the speed of accumulated test time; the amount of effort required to solve failure effectively; the systematic and permanent removal of failure mode through implementing corrective actions. The positively higher value means that reliability level is increasing positively.

4.2 Factors Changing Reliability Growth Rate (or Slope)

MIL-STD-1635 (1978) and Dwyer (2010) described the factors to break out the growth rate changing during the reliability growth test as:

〈Table 7〉 Modifying Test Data for Growth Rate Changing

	Failure Number	Test Time	Cumulative Failure Number	$\ln(T_c)$	$\ln(T_c)$^2	M_c	$\ln(M_c)$	$\ln(T_c)$ $\times\ln(M_c)$	Duane Value	AMSAA Value	Duane Error	AMSAA Error
Section 1	1	100	1	4.6	21.2	100.0	4.6	21.2	108.9	533.7	-8.9	-433.7
	1	180	2	5.2	27.0	90.0	4.5	23.4	80.6	150.4	9.4	-60.4
	1	240	3	5.5	30.0	80.0	4.4	24.0	69.6	80.9	10.4	-0.9
	1	280	4	5.6	31.8	70.0	4.2	23.9	64.3	58.0	5.7	12.0
	1	300	5	5.7	32.5	60.0	4.1	23.4	62.1	50.0	-2.1	10.0
	1	300	6	5.7	32.5	50.0	3.9	22.3	62.1	50.0	-12.1	0.0
	Sum 1	300	6	32.3	175.0	450.0	26.7	138.2		Average 1	0.4	-78.8
Section 2	1	420	7	6.0	36.5	60.0	4.1	24.7	60.3	61.8	-0.3	-1.8
	1	560	8	6.3	40.0	70.0	4.2	26.9	70.3	71.3	-0.3	-1.3
	1	760	9	6.6	44.0	84.4	4.4	29.4	82.7	83.1	1.7	1.3
	1	900	10	6.8	46.3	90.0	4.5	30.6	90.5	90.4	-0.6	-0.4
	1	1,100	11	7.0	49.0	100.0	4.6	32.3	100.7	100.0	-0.7	0.0
	Sum 2	11,000	11	32.8	215.8	404.4	21.9	143.9		Average 2	0.0	-0.4

(1) unsystematic failure distributions and imperfect solution implemented during test so failure modes are not removed uniformly.
(2) failures not happened at the same time for all test units, really not the same cycle time for corrective action of each failure modes
(3) multiple units in the test of varying degrees of maturity time to determine corrective actions for field returns may be long compared to the MTBF hours
(4) design or part changing during the test to break out the non-uniform affect to reliability level of multiple units.

There are many reasons to happen the reliability growth rate changing during test. Nevertheless, regardless any reason, the reliability engineer should have an ability to assess the latest test data for projecting the current MTBF level with a high accuracy.

4.3 New Assessing Process and Method for Test Data with Reliability Growth Rate Changing

When there is the reliability growth rate changing during testing test, the latest test data usually have more recent and important information than the initial one. It is better for reliability engineer should know the method to analyze the trend of only later test data in such case. The Duane model and AMSAA model are historically representative ones for repairable system's reliability growth test planning, assessing and projecting a current reliability level, but they have also theoretically weak point as getting too much influence from the initial test data. If there is only the constant reliability growth rate, the engineers do not need to care about the assessing test data location because of the same reliability growth rate regardless of the data location. But when a growth rate is changing during testing, it is absolutely necessary for engineers to have the ability to define the latest data trend and assess them. Also it is necessary for them to make the growth curve with the high GOF to the latest test data. In this Chapter there is the research result with the assessing process and the method to increase the GOF of Duane and AMSAA models curves especially in case of reliability growth changing as like below;

[1] Plotting test data and reliability growth curves for Duane model and AMSAA model without considering a growth rate changing: Let calculate the parameters for both models and the cumulative / instantaneous MTBF using the Equation from (1) to (10), then plotting test data and reliability growth curves on the log-log chart for MTBF versus test time. Let us graphically check the deviation between test data and curves. If there is the noticeably big changing poing of reliability growth rate as like <Figure 2>, then there will be a big gap between them and low GOF of reliabiliy growth curve to test data.
[2] Calculate the GOF of reliability growth curves to test data and select the slope changing point: Calculate the DC for reliability growth curves with test data and estimate AMG with Equation (11), (12). The easiest way to select the reliability growth changing point is to use the reliability growth chart as like <Figure 1> and <Figure 2>. The growth changing point is a kind of inflection one, so there are two

different lines with the different slop before and after of the point. In this research, we took the example for changing growth rate with one time slope changing, but it can be extended to the other cases with more many changing points. Since the most important part in test data assessing is to find the reliability growth curve with the higher GOF to the latest test data, it is possible to consider the test data block before the last changing point as just one. It is necessary to divide the data block into the section 1 for test data before the growth rate changing and the section 2 after the point as like <Table 7> which is modified from <Table 3>, so there will be two curves for the both sections.

[3] Calculate the parameters and cumulative / instantaneous MTBF for the section 1: The parameters for Duane model and AMSAA model at the section 1 are same as Equation (4), (5), (9), (10) for the case without the growth rate changing and as follows;

$$\widehat{\alpha_1}=\left[\sum_{1i=1}^{n_1}\{\ln(T_{1i})\ln(M_{c1i})\} - \left\{\sum_{1i=1}^{n_1}\ln(T_{1i})\sum_{1i=1}^{n_1}\ln(M_{c1i})\right\}/n_1\right] \Big/ \left[\sum_{1i=1}^{n_1}\ln(T_{1i})^2-\left\{\sum_{1i=1}^{n_1}\ln(T_{1i})\right\}^2/n_1\right] \quad (13)$$

$$\widehat{b_1}=e^{\left[(1/n_1)\left\{\sum_{1i=1}^{n_1}\ln(M_{c1i})-\widehat{\alpha_1}\sum_{1i=1}^{n_1}\ln(T_{1i})\right\}\right]} \quad (14)$$

$$\widehat{\beta_1}=n_1\Big/\left[n_1\ln(T_1)-\sum_{1i=1}^{n_1}\ln(T_{1i})\right] \quad (15)$$

$$\widehat{\lambda_1}=n_1/T_1^{\beta_1} \quad (16)$$

Where;

- T_1 is the cumulative test time before the slope changing time
- T_{1i} is the each failure time before T_1 time
- M_{C1i} is the cumulative MTBF before T_1 time
- n_1 is the number of failures observed up to time T_1.

It is possible to calculate the cumulative / instantaneous MTBF for the models with inputting the calculated parameters into Equation (1), (3), (6), (7), and then calculate the GOF with Equation (11), (12). The information at this area is just reference.

[4] Calculate the parameters and cumulative / instantaneous MTBF for the section 2: To calculate the parameters for the reliability growth models at the section 2, it is also necessary to use the information from the section 1. According to the research result, the parameters for Duane model can be calculated with the replacement of the test number and test time value with the data at the section 1, the result is like Equation (17) and (18). The parameters for AMSAA models can be calculated with Equation (19) and (20) by the replacement of the test time with the test time proportional value between the section 2 and 1.

$$\widehat{\alpha_2}=\left[\sum_{2i=1}^{n_2-n_1}\{\ln(T_{2i})\ln(M_{c2i})\} - \left\{\sum_{2i=1}^{n_2-n_1}\ln(T_{2i})\sum_{2i=1}^{n_2-n_1}\ln(M_{c2i})\right\}/(n_2-n_1)\right] \Big/ \left[\sum_{2i=1}^{n_2-n_1}\ln(T_{2i})^2-\left\{\sum_{2i=1}^{n_2-n_1}\ln(T_{2i})\right\}^2/(n_2-n_1)\right] \quad (17)$$

$$\widehat{b_2}=e^{\left[\{1/(n_2-n_1)\}\left\{\sum_{2i=1}^{n_2-n_1}\ln(M_{c2i})-\widehat{\alpha_2}\sum_{2i=1}^{n_2-n_1}\ln(T_{2i})\right\}\right]} \quad (18)$$

$$\widehat{\beta_2}=(n_2-n_1)\Big/\left[n_1\ln(T_2/T_1)+\sum_{2i=1}^{n_2-n_1}\ln(T_2/T_{2i})\right] \quad (19)$$

$$\widehat{\lambda_2}=n_2/T_2^{\beta_2} \quad (20)$$

Where:

- T_2 is the end time of the test
- T_{2i} is the each failure time after T_1
- M_{C2i} is the cumulative MTBF at each T_{2i} before T_2 time
- n_2 is the number of failures observed up to time T_2.

After then it is possible to calculate the cumulative / instantaneous MTBF for the models with inputting the parameters into Equation (1), (3), (6), (7).

[5] Calculate the GOF to evaluate the effectiveness of new assessing method for the reliability growth rate changing: Next is to measure the GOF with DC and AMG calculation and compare how much there is an improvement in the GOF with the new assessing method. If the DC and AMG with implementing the new method are separately decreased and increased at the Section 2, then the other slope point should be chosen. It is also important that there should be at least 3 test data keeping the constant slope for Section 2 area as described by MIL-STD-1635 (1978).

5. Application Case and Result With New Assessing Process and Metho

5.1 Application Case

As a sample case to validate the new assessing process and method as the researching result, we took the example of Demko

(1993) with 11 failures during 1100 hours test times as shown at the <Table 3>.

[1] Plotting test data and reliability growth curves of Duane model and AMSAA model without considering a growth rate changing: After getting parameters for both models as shown in <Table 4> and plotting curves as shown in <Figure 2>, there is a big gap between the test data and the curves from models.

[2] Calculate the GOF and select the slope point: The DC and AMG calculation results of reliability growth curve with test data and Equation (11), (12) is at <Table 6>. Since the DC is very low and high value for AMG, so there must be a reliability growth changing. With the <Figure 2> it is possible to see the big gap between the curves and test data plotted on the chart, the slope changing point seems to be happened at the test time of 300 hours.

[3] Calculate the parameters and cumulative / instantaneous MTBF at the section 1: With Equation (13), (14), (15) and (16), the calculation result for the parameters and instantaneous / cumulative MTBF at the test ending time of 300 hours is as like <Table 8>:

<Table 8> Calculation Result of Test Data at Section 1

Duane Model				AMSAA Model			
α	b	M_c	M_i	β	λ	M_c	M_i
-0.51	1148	62	41	3.16	9.165E-08	50	16

The analysis result is not important for the reliability level projection of the current developing product, but it contribute to the section 2 analysis for the consistency in the analysis result.

[4] Calculate the parameters and cumulative / instantaneous MTBF at the section 2: With Equation (17), (18), (19) and (20), the calculation result for the parameters and instantaneous / cumulative MTBF at 1100 hours test time is as like below <Table 9>:

<Table 9> Calculation Result of Test Data at Section 2

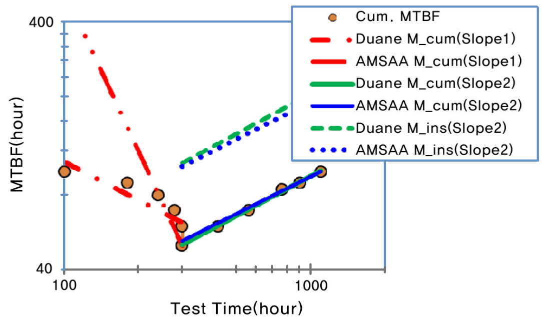
Duane Model				AMSAA Model			
α	b	M_c	M_i	β	λ	M_c	M_i
0.53	2.40	101	216	0.50	0.33	100	200

[5] Calculate the GOF of the reliability growth curves at the section 2: To evaluate the effectiveness in the GOF improvement implemented with the new test data assessing methodology, it is necessary to measure of DC and AMG for the reliability growth curves newly plotted with Equation (11), (12). The result is at <Table 10>:

<Table 10> New GOF Evaluation Result at Section 2

Duane Model		AMSAA Model	
R^2	AVG MTBF Gap	R^2	AVG MTBF Gap
99.7%	0.40	99.5%	0.53

The result is showing that the average values of DC is increasing with 130.8% and the average value of AMG decreasing with 91.9% from the conventional assessing method result at <Table 6>. The newly introduced methodology was dramatically improving the GOF and the effectiveness would contribute to improve in the assessing result with the test data. Next, it is graphically possible to check how much the new curve is matching well with test data at <Figure 3>:



<Figure 3> Redrawing Reliability Growth Curves with Changing Growth Rate

5.2 More Cases from Monte Carlo Simulation

To prove the effectiveness of the research with more cases, we prepared the 20 cases with the Monte Carlo Simulation. The simulation cases were focused on the typical reliability growth condition as like $0 < \alpha \leq 0.6$ for the growth rate of Duane model and $0 < \beta < 1$ for the shape parameter for AMSAA model as shown in <Table 11>. The test time is 10000 hours and the failure numbers are limited by the cumulative MTBF level.

<Table 11> Cases from Monte Carlo Simulation

Data set	α	β	λ	M_c	M_i
1	0.318	0.682	0.131	210	293
2	0.095	0.905	0.017	53	75
3	0.177	0.823	0.043	143	243
4	0.147	0.853	0.027	170	238
5	0.202	0.798	0.054	151	211

Data set	α	β	λ	M_c	M_i
6	0.124	0.876	0.019	190	266
7	0.320	0.680	0.126	223	312
8	0.429	0.571	0.364	250	350
9	0.128	0.872	0.023	161	274
10	0.278	0.722	0.097	185	258
11	0.739	0.261	2.722	1,279	1,791
12	0.507	0.493	0.319	675	946
13	0.280	0.720	0.046	397	793
14	0.161	0.839	0.011	458	642
15	0.106	0.894	0.009	350	489
16	0.407	0.593	0.038	1,874	2,624
17	0.000	1.000	0.004	227	318
18	0.051	0.928	0.010	182	291
19	0.115	0.885	0.014	231	323
20	0.084	0.916	0.021	113	158
AVG.	0.233	0.766	0.205	376	545

According to the analysis result with the simulated test data the 90% in total 20 cases are having the reliability growth rate changing during reliability growth test, the level of changing in the slope are high and required to implement the new methodology. <Table 12> is showing the result of new GOF evaluation implementing to these cases, it shows that the method is very good to increase the GOF and it will contribute to increase the accuracy of assessment with the test data.

〈Table 12〉 Summary of Simulation Data Analysis Result

Model	Average DC Value (Determine Coefficient)			Average AME Value (Average MTBF Error)		
	Before	After	Increasing Ratio	Before	After	Reduction Ratio
Duane	62.6%	89.8%	43%	8.1	2.3	-96%
AMSAA	65.1%	79.3%	22%	7.1	3.5	-101%

6. Conclusion

Duane (1964) showed that the curves between cumulative MTBF and test time on log-log chart are keeping s constant reliability growth rate during the developing test, but Codiner (1968) and Demko (1993) were showing us that many cases in real world are undergoing the changing in it because of many reasons. In addition, the construction equipment and vehicle developing results that we have involved for 20 years have those changing during the developing time over 90% in all cases.

Stewart (1985) and Dwyer (2009) mentioned the excellence of AMSAA model and Duane model for complex-repairable system and also pointed out their mathematical limitation getting too much influence at the initial failure trends. In case of reliability growth rate changing the limitation can make reliability engineer difficult to perform an accurate assessment with test data and it also make the error in the projection of current reliability level.

As a summary this research are presenting the method to define the existence of reliability growth changing, the mathematical formulas to calculate the parameters of Duane model and AMSAA model at the separated regions before and after the slope changing point, the new evaluation method of GOF applicable to all interesting models. The new process and method can contribute to increase the GOF and the accuracy of assessing test result, it will also reduce the gap to project the current reliability level and market one.

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