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Fuzzy system reliability using intuitionistic fuzzy Weibull lifetime distribution

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Abstract. Present study investigates the fuzzy reliability of some systems using intuitionistic fuzzy Weibull lifetime distribution, in which the lifetime parameters are assumed to be fuzzy parameter due to uncertainty and inaccuracy of data. Expressions for fuzzy reliability, fuzzy mean time to failure, fuzzy hazard function and their α-cut have been discussed when systems follow intuitionistic fuzzy Weibull lifetime distribution. A numerical example is also taken to illustrate the methodology to calculate the fuzzy reliability characteristics of systems.

Key Words: *α-cut set, fuzzy hazard function, fuzzy mean time to failure, fuzzy reliability, trapezoidal intuitionistic fuzzy number, Weibull distribution*

1. INTRODUCTION

The reliability is one of the most important engineering tasks in design of engineering systems. There are different types of functions to analyse the different aspects of the reliability measure of the systems. The frequently used function in lifetime data analysis and reliability engineering is the reliability function. Thus, if the random variable (*rv*) X denotes the lifetime of an item, then $R(t) = P(X > t)$. It is worth mentioning that the Weibull distribution has been found to be useful for describing lifetimes in reliability applications. The Weibull distribution generalizes the exponential and Rayleigh distribution and it may have increasing, decreasing or constant failure rate. This is one of the widely applied distributions used in the studies of reliability modelling.
In classical reliability theory lifetimes of components have crisp parameters. But in real world, many times the precise values of parameters are difficult to determine due to uncertainty and inaccuracy of data. Under such circumstances the lifetimes of components are assumed to have fuzzy parameters. The concept of fuzzy set theory introduced by

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Zadeh (1965) is based on the assumption that the membership degree is equal to one minus non-membership degree. In real life situation an object may or may not be in a set A to a certain degree. In other words, some hesitation about the degree of belongingness may exist. This hesitation in the membership degree may be modeled by intuitionistic fuzzy sets. Atanassov (1986) proposed the idea of intuitionistic fuzzy sets by introducing the intuitionistic index which exists because of the uncertainty of information. An intuitionistic fuzzy set (IFS) is one of the generalisations of the fuzzy sets theory. In these sets the non-membership of an element x of the universe need not be one minus membership degree. Rather, it might be any number lying between 0 and 1. The degree of membership and the degree of non-membership hold the condition $0 \le \xi(x) + \eta(x) \le 1$.

Cai et al. (1995) discussed the system reliability for coherent system based on the fuzzy state assumption and probability assumption. Baloui Jamkhaneh (2011, 2012 and 2013) discussed reliability estimation using fuzzy lifetime distribution, fuzzy Weibull lifetime distribution and fuzzy environments. Singer (1990) presented a fuzzy approach for fault tree and reliability of both series and parallel system. Sharma (2012) presented the reliability of a system using intuitionistic fuzzy sets. Mahapatra(2009) investigate the reliability using triangular intuitionistic fuzzy number. Chen (1994) discussed a method for fuzzy system reliability analysis using fuzzy number arithmetic operations. Chen (2003) developed a new method to analyse the fuzzy system reliability based on vague sets. Utkin (1999) proposed imprecise reliability models for the general lifetime distribution classes. In above mentioned studies researchers have done good work but they did not give emphasis on the importance of intuitionistic fuzzy lifetime rate.

Keeping above facts in view, in this study we will investigate the fuzzy reliability of systems using intuitionistic fuzzy Weibull lifetime distribution. The lifetime rate of system is represented by trapezoidal intuitionistic fuzzy number. The paper is organised as follows: Section 2 introduces the fuzzy Weibull distribution; Section 3 discuss the fuzzy hazard function, Section 4 presents the fuzzy survival function, Sectionv5 describes fuzzy mean time to failure and Section 6 discuss the fuzzy reliability function of series, parallel, series-parallel and parallel-series systems along with an example and the last section discuss the conclusion of the study.

2. FUZZY WEIBULL DISTRIBUTION

The Weibull distribution is widely used distribution function in reliability analysis. Its probability density function is defined as

$$f(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^{\beta}\right), \quad x > 0, \theta > 0, \beta > 0, \tag{1}$$

where θ is scale parameter, β is shape parameter and δ is location parameter.

By changing the value of the shape parameter, this distribution can model a wide variety of lifetime data. If $\beta = 1$ the Weibull distribution is reduced to exponential distribution; if $\beta = 2$, the Weibull distribution is identical to Rayleigh distribution.

Due to the uncertainty and inaccuracy of data, the estimation of precise value of lifetime parameters becomes very difficult. To tackle this issue here the lifetime of Weibull distribution θ is replaced by intuitionistic fuzzy number $\tilde{\theta}$. In this case, the fuzzy probability of event $X \epsilon [c, d], c \geq 0$ $\tilde{P}(c \leq X \leq d)$ and its α-cut set is given by:

$$\tilde{P}(c \leq X \leq d)[\alpha] = \int_c^d \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1} \exp(-(\frac{x}{\theta})^{\beta})dx \mid \theta \in \tilde{\theta}[\alpha]$$
$$= [P^L[\alpha], P^U[\alpha]], \tag{2}$$

where $\tilde{\theta}[\alpha]$ is the α-cut of trapezoidal intuitionistic fuzzy numberand

$$P^L[\alpha] = \min\left\{\int_c^d \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1} \exp(-(\frac{x}{\theta})^{\beta})dx \mid \theta \in \tilde{\theta}[\alpha]\right\}\forall\alpha$$
$$P^U[\alpha] = \max\left\{\int_c^d \frac{\beta}{\theta}(\frac{x}{\theta})^{\beta-1} \exp(-(\frac{x}{\theta})^{\beta})dx \mid \theta \in \tilde{\theta}[\alpha]\right\}\forall\alpha \tag{3}$$

3. FUZZY HAZARD FUNCTION

The fuzzy hazard function $\tilde{h}(t)$ is the fuzzy conditional probability of an item failing in the interval t to $(t + dt)$ given that it has not failed by time t. Hazard function is also known as the failure rate. Mathematically, the fuzzy hazard function is defined as

$$\tilde{h}(t)[\alpha] = \lim_{\Delta t \to 0} \frac{\tilde{P}(t < x < t + \Delta t \mid X > t)}{\Delta t} = \{\frac{f(t)}{S(t)} \mid \theta \in \tilde{\theta}[\alpha]\} \tag{4}$$

If the lifetime follows fuzzy Weibull distribution, then the fuzzy hazard function is given by

$$\tilde{h}(t)[\alpha] = \{\frac{\beta}{\theta}(\frac{t}{\theta})^{\beta-1} \mid \theta \in \tilde{\theta}[\alpha]\}$$
$$= [\frac{\beta\, t^{\beta-1}}{(a_1 + (a_2 - a_1)\alpha)^{\beta}}, \frac{\beta\, t^{\beta-1}}{(a_4 - (a_4 - a_3)\alpha)^{\beta}};$$
$$\frac{\beta\, t^{\beta-1}}{(a_2 - (1-\alpha)(a_2 - a'_1))^{\beta}}, \frac{\beta\, t^{\beta-1}}{(a_3 + (1-\alpha)(a'_4 - a_3))^{\beta}}] \tag{5}$$

It is to be noted that $\tilde{h}(t)[\alpha]$ is a two dimensional function in terms of α and t ($0 \leq \alpha \leq 1$ and $t > 0$). In this method, for every α-cut, hazard rate curve is like a band.

4. FUZZY RELIABILITY FUNCTION

Fuzzy reliability or fuzzy survival function $\tilde{S}(t)$ is the fuzzy probability in which a unit survives beyond time t. Let the random variable X denote lifetime of a system components, and it has density function and cumulative distribution function $f(x, \theta)$ and $F_X(t) = P(X \leq t)$ respectively, then the fuzzy reliability function at time t is defined as

$$\tilde{S}(t) = \tilde{P}(X > t) = 1 - F_X(t), \quad t > 0 \tag{6}$$

The unreliability function $\tilde{Q}(t)$ is the probability of failure or the probability of an item failing in the time interval [0, t], is given by

$$\tilde{Q}(t) = \tilde{P}(X \le t) = F_X(t), \; t > 0 \tag{7}$$

Let the lifetime parameter of a component has intuitionistic fuzzy Weibull distribution and lifetime parameter $\tilde{\theta}$ represent a trapezoidal intuitionistic fuzzy number as $\tilde{\theta} = \{a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4\}$ then we can describe a membership function $\xi_{\tilde{\theta}}(x)$ and a non-membership function $\eta_{\tilde{\theta}}(x)$ in the following manner:

$$\xi_{\tilde{\theta}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

and

$$\eta_{\tilde{\theta}}(x) = \begin{cases} \dfrac{a_4 - x}{a_2 - a'_1}, & a'_1 \le x \le a_2 \\ 0, & a_2 \le x \le a_3 \\ \dfrac{x - a_3}{a'_4 - a_3}, & a_3 \le x \le a'_4 \\ 1, & \text{otherwise} \end{cases} \tag{9}$$

where $(a'_1, a_2, a_3, a_4, a'_4)$ are real numbers.

The α-cut set of trapezoidal intuitionistic fuzzy lifetime parameter $\tilde{\theta}$ is given by

$$\tilde{\theta}[\alpha] = [\{a_1 + (a_2 - a_1)\alpha, a_4 - (a_4 - a_3)\alpha\}; \{a_2 - (1-\alpha)(a_2 - a'_1), a_3 + (1-\alpha)(a'_4 - a_3)\}] \tag{10}$$

So the fuzzy reliability function of a component is obtained as

$$\tilde{S}(t)[\alpha] = \int_t^\infty \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{x}{\theta}\right)^{\beta}\right) dx \mid \theta \in \tilde{\theta}[\alpha]$$

$$= \exp\left(-\left(\frac{x}{\theta}\right)^{\beta}\right) \mid \theta \in \tilde{\theta}[\alpha] \tag{11}$$

The α-cut of intuitionistic fuzzy reliability function is given by

$$\begin{aligned} \tilde{S}(t)[\alpha] = [&\exp\left(-\left(\frac{t}{a_1 + (a_2 - a_1)\alpha}\right)^{\beta}\right), \exp\left(-\left(\frac{t}{a_4 - (a_4 - a_3)\alpha}\right)^{\beta}\right); \\ &\exp\left(-\left(\frac{t}{a_2 - (1-\alpha)(a_2 - a'_1)}\right)^{\beta}\right), \exp\left(-\left(\frac{t}{a_3 + (1-\alpha)(a'_4 - a_3)}\right)^{\beta}\right)] \end{aligned} \tag{12}$$

It is worth mentioning that $\tilde{S}(t)[\alpha]$ is a two dimensional function in terms of α and t ($0 \le \alpha \le 1$ and $t > 0$). Let any particular time t_0, fuzzy reliability is a trapezoidal intuitionistic fuzzy number and its membership function and non-membership function are $\xi_{t_0}(x)$ and $\eta_{t_0}(x)$ respectively, then

$$\xi_{t_0}(x)=\begin{cases}\left(\dfrac{x-e^{-(t_0/a_1)^{\beta}}}{e^{-(t_0/a_2)^{\beta}}-e^{-(t_0/a_1)^{\beta}}}\right), & \text{if } e^{-(t_0/a_1)^{\beta}}\le x\le e^{-(t_0/a_2)^{\beta}}\\ 1, & \text{if } e^{-(t_0/a_2)^{\beta}}\le x\le e^{-(t_0/a_3)^{\beta}}\\ \left(\dfrac{e^{-(t_0/a_4)^{\beta}}-x}{e^{-(t_0/a_4)^{\beta}}-e^{-(t_0/a_3)^{\beta}}}\right), & \text{if } e^{-(t_0/a_3)^{\beta}}\le x\le e^{-(t_0/a_4)^{\beta}}\end{cases} \tag{13}$$

and

$$\eta_{t_0}(x)=\begin{cases}\left(\dfrac{e^{-(t_0/a_2)^{\beta}}-x}{e^{-(t_0/a_2)^{\beta}}-e^{-(t_0/a'_1)^{\beta}}}\right), & \text{if } e^{-(t_0/a'_1)^{\beta}}\le x\le e^{-(t_0/a_2)^{\beta}}\\ 0, & \text{if } e^{-(t_0/a_2)^{\beta}}\le x\le e^{-(t_0/a_3)^{\beta}}\\ \left(\dfrac{x-e^{-(t_0/a_3)^{\beta}}}{e^{-(t_0/a'_4)^{\beta}}-e^{-(t_0/a_3)^{\beta}}}\right), & \text{if } e^{-(t_0/a_3)^{\beta}}\le x\le e^{-(t_0/a'_4)^{\beta}}\end{cases} \tag{14}$$

For every α-cut, the reliability curve is like a band whose width depends on the ambiguity parameter θ. This reliability curve has following properties:

- $\tilde{S}(0)[\alpha]=\tilde{1}$, i.e. no one starts off dead,
- $\tilde{S}(\infty)[\alpha]=\tilde{0}$, i.e. everyone dies eventually,
- $\tilde{S}(t_1)[\alpha]\ge\tilde{S}(t_2)[\alpha]$ if and only if $t_1\le t_2$, i.e. $\tilde{S}(t)[\alpha]$ band of declines monotonically,
- For any fixed α and $\beta\le 1$ reliability band have convex functions.

5. FUZZY MEAN TIME TO FAILURE

Fuzzy mean time to failure (*FMTTF*) is the expected time to failure. It is defined as

$$F\tilde{MTTF}[\alpha]=\int_0^{\infty}xf(x)dx\mid\theta\in\tilde{\theta}[\alpha]$$

$$=\int_0^{\infty}S(t)dt\mid\theta\in\tilde{\theta}[\alpha]=[P^L[\alpha],P^U[\alpha]] \tag{15}$$

where

$$P^{L}[\alpha]=\min\{\int_{0}^{\infty}S(t)dt \mid \theta\in\tilde{\theta}[\alpha]\}$$

$$P^{U}[\alpha]=\max\{\int_{0}^{\infty}S(t)dt \mid \theta\in\tilde{\theta}[\alpha]\}$$

When the lifetime follows intuitionistic fuzzy Weibull distribution, then

$$\begin{aligned}FM\tilde{T}TF &=\{\theta\,\Gamma(1+\beta^{-1}) \mid \theta\in\tilde{\theta}[\alpha]\}\\ &=[\{(a_1+(a_2-a_1)\alpha)\Gamma(1+\beta^{-1}),(a_4-(a_4-a_3)\alpha)\Gamma(1+\beta^{-1})\};\\ &\quad\{(a_2-(1-\alpha)(a_2-a'_1))\Gamma(1+\beta^{-1}),(a_2-(1-\alpha)(a_2-a'_1))\Gamma(1+\beta^{-1})\}]\end{aligned} \tag{16}$$

The membership and non-membership function of *FMTTF* is given by

$$\xi_{M\tilde{T}TF}(x)=\begin{cases}\dfrac{x-a_1\Gamma(1+\beta^{-1})}{(a_2-a_1)\Gamma(1+\beta^{-1})}, & \text{if } a_1\Gamma(1+\beta^{-1})\le x\le a_2\Gamma(1+\beta^{-1})\\ 1, & \text{if } a_2\Gamma(1+\beta^{-1})\le x\le a_3\Gamma(1+\beta^{-1})\\ \dfrac{a_4\Gamma(1+\beta^{-1})-x}{(a_4-a_3)\Gamma(1+\beta^{-1})}, & \text{if } a_3\Gamma(1+\beta^{-1})\le x\le a_4\Gamma(1+\beta^{-1})\end{cases} \tag{17}$$

and

$$\eta_{M\tilde{T}TF}(x)=\begin{cases}\dfrac{a_2\Gamma(1+\beta^{-1})-x}{(a_2-a'_1)\Gamma(1+\beta^{-1})}, & \text{if } a'_1\Gamma(1+\beta^{-1})\le x\le a_2\Gamma(1+\beta^{-1})\\ 0, & \text{if } a_2\Gamma(1+\beta^{-1})\le x\le a_3\Gamma(1+\beta^{-1})\\ \dfrac{x-a_3\Gamma(1+\beta^{-1})}{(a'_4-a_3)\Gamma(1+\beta^{-1})}, & \text{if } a_3\Gamma(1+\beta^{-1})\le x\le a'_4\Gamma(1+\beta^{-1})\end{cases} \tag{18}$$

6. RELIABILITY FUNCTION OF DIFFERENT SYSTEMS

(a) Series system: If *n*-components are connected in series (see Figure 1), then the α-cut of fuzzy reliability function $\tilde{R}_S(t)[\alpha]$ with intuitionistic fuzzy Weibull distribution is given by

$$\begin{aligned}\tilde{R}_S(t)[\alpha]=S(t)^n[\alpha]=[\{&\exp(-n(\frac{t}{a_1+(a_2-a_1)\alpha})^{\beta}),\exp(-n(\frac{t}{a_4-(a_4-a_3)\alpha})^{\beta})\};\\ &\{\exp(-n(\frac{t}{a_2-(1-\alpha)(a_2-a'_1)})^{\beta}),\exp(-n(\frac{t}{a_3+(1-\alpha)(a'_4-a_3)})^{\beta})\}]\end{aligned}$$

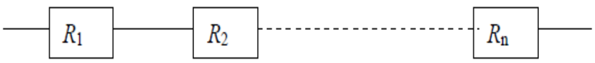

Figure 1. Series system

Numerical example: If four components of an electronic system are connected in series and lifetime of the electronic component is modeled by Weibull distribution with intuitionistic fuzzy parameter $\tilde{\theta}$. If $\tilde{\theta} = (1.34, 1.36, 1.38, 1.40; 1.32, 1.36, 1.38, 1.42)$ then α-cut of fuzzy reliability of one component is given by:

$$\tilde{\theta}[\alpha] = [1.34 + .02\alpha, 1.40 - .02\alpha; 1.36 - .04(1-\alpha), 1.38 + .04(1-\alpha)]$$

$$\tilde{S}(t)[\alpha] = [\exp(-(\frac{t}{1.34 + .02\alpha})^{\beta}), \exp(-(\frac{t}{1.40 - .02\alpha})^{\beta});$$
$$\exp(-(\frac{t}{1.36 - .04(1-\alpha)})^{\beta}), \exp(-(\frac{t}{1.38 - .04(1-\alpha)})^{\beta})]$$

for all α.

The α-cut of fuzzy reliability of series system $\tilde{R}_S(t)[\alpha]$ at particular time t = 10 and β = 0.5 is given by (19) and shown in Figure 2.

$$\tilde{R}_S(t)[\alpha] = S(t)^n[\alpha] = [\exp\{-4(t/(1.34 + 0.02\alpha))^{\beta}\}, \exp\{-4(t/(1.40 - 0.02\alpha))^{\beta}\};$$
$$\exp\{-4(t/(1.36 - 0.04(1-\alpha)))^{\beta}\}, \exp\{-4(t/(1.38 + 0.04(1-\alpha)))^{\beta}\}] \quad (19)$$

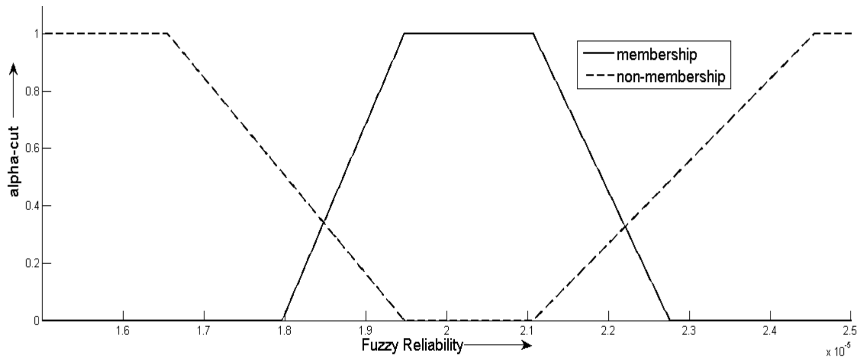

Figure 2. α-cut of fuzzy reliability in series system

(b) Parallel system: Let *n*- components are connected in parallel (see Figure 3), and then the α-cut of fuzzy reliability function $\tilde{R}_p(t)[\alpha]$ with intuitionistic fuzzy Weibull distribution is given by

$$\tilde{R}_p(t)[\alpha] = \{1 - (1 - \tilde{S}(t))^n\}$$
$$= [1 - \{1 - \exp(-(\frac{t}{a_1 + (a_2 - a_1)\alpha})^{\beta})\}^n, 1 - \{1 - \exp(-(\frac{t}{a_4 - (a_4 - a_3)\alpha})^{\beta})\}^n;$$
$$1 - \{1 - \exp(-(\frac{t}{a_2 - (1-\alpha)(a_2 - a'_1)})^{\beta})\}^n, 1 - \{1 - \exp(-(\frac{t}{a_3 + (1-\alpha)(a'_4 - a_3)})^{\beta})\}^n]$$

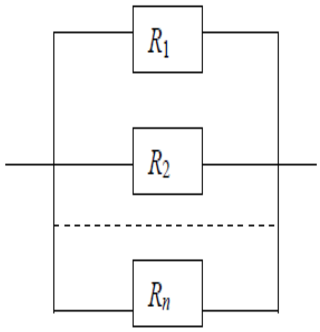

Figure 3. Parallel system

Numerical example: Consider the lifetime of the electronic component is modeled by Weibull distribution with intuitionistic fuzzy parameter $\tilde{\theta}$. Let four components of the considered system are connected in parallel.

If $\tilde{\theta} = (1.24, 1.26, 1.28, 1.30; 1.22, 1.26, 1.28, 1.32)$ then α-cut of fuzzy reliability of one component is given by:

$$\tilde{\theta}[\alpha] = [1.24 + 0.02\alpha, 1.30 - 0.02\alpha; 1.26 - 0.04(1-\alpha), 1.28 + 0.04(1-\alpha)]$$

$$\tilde{S}(t)[\alpha] = [\exp(-(\frac{t}{1.24 + 0.02\alpha})^{\beta}), \exp(-(\frac{t}{1.30 - 0.02\alpha})^{\beta});$$
$$\exp(-(\frac{t}{1.26 - 0.04(1-\alpha)})^{\beta}), \exp(-(\frac{t}{1.28 + 0.04(1-\alpha)})^{\beta})]$$

for all α.

The α-cut of fuzzy reliability of parallel system $\tilde{R}_p(t)[\alpha]$ at particular time t = 10 and β = 0.5 is given by (20) and shown in Figure 4.

$$\tilde{R}_p(t)[\alpha] = [1 - \{1 - \exp(-(t/(1.24 + 0.02\alpha))^{\beta})\}^4, 1 - \{1 - \exp(-(t/(1.30 - 0.02\alpha))^{\beta})\}^4;$$
$$1 - \{1 - \exp(-(t/(1.26 - 0.04(1-\alpha)))^{\beta})\}^4, 1 - \{1 - \exp(-(t/(1.28 + 0.04(1-\alpha)))^{\beta})\}^4] \quad (20)$$

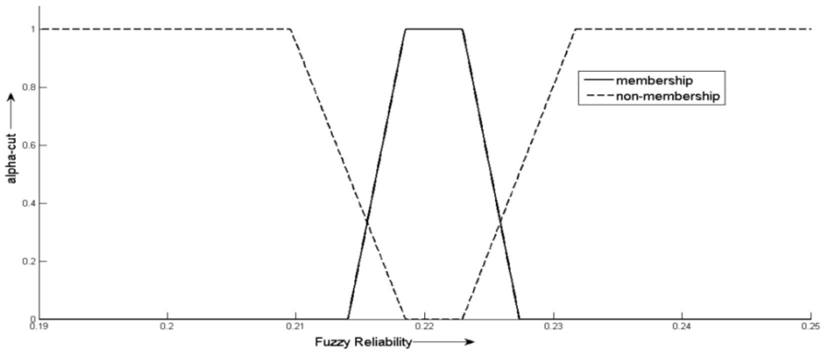

Figure 4. α-cut of fuzzy reliability in parallel system

(c) Series-Parallel configuration: Let n-components are in series and m-components are in parallel as shown in Figure 5, then the α-cut of fuzzy reliability function $\tilde{R}_{sp}(t)[\alpha]$ with intuitionistic fuzzy Weibull distribution is given by

$$\tilde{R}_{sp}(t)[\alpha] = \prod_{i=1}^{n}[1-\prod_{j=1}^{m}(1-S(t))]$$

$$= \prod_{i=1}^{n}[1-\{1-\exp(-(\frac{t}{a_1+(a_2-a_1)\alpha})^{\beta})\}^{m}, 1-\{1-\exp(-(\frac{t}{a_4-(a_4-a_3)\alpha})^{\beta})\}^{m};$$

$$1-\{1-\exp(-(\frac{t}{a_2-(1-\alpha)(a_2-a'_1)})^{\beta})\}^{m}, 1-\{1-\exp(-(\frac{t}{a_3+(1-\alpha)(a'_4-a_3)})^{\beta})\}^{m}]$$

$$= [(1-\{1-\exp(-(\frac{t}{a_1+(a_2-a_1)\alpha})^{\beta})\}^{m})^{n}, (1-\{1-\exp(-(\frac{t}{a_4-(a_4-a_3)\alpha})^{\beta})\}^{m})^{n};$$

$$(1-\{1-\exp(-(\frac{t}{a_2-(1-\alpha)(a_2-a_1)})^{\beta})\}^{m})^{n}, (1-\{1-\exp(-(\frac{t}{a_3+(1-\alpha)(a_2-a_1)})^{\beta})\}^{m})^{n}]$$

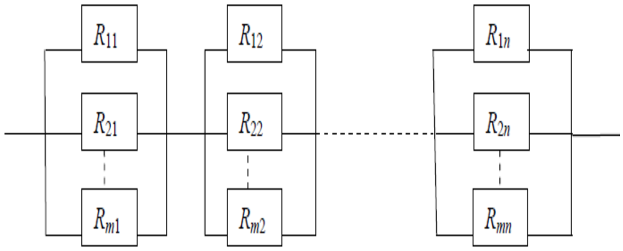

Figure 5. Series-parallel system

Numerical example: If four components of the electronic system are connected in series-parallel and lifetime of the electronic component is modeled by Weibull distribution with intuitionistic fuzzy parameter $\tilde{\theta}$. If $\tilde{\theta} = (1.30, 1.34, 1.38, 1.42; 1.28, 1.34, 1.38, 1.44)$ then α-cut of fuzzy reliability of a component is given by:

$$\tilde{\theta}[\alpha] = [1.30 + 0.04\alpha, 1.42 - 0.04\alpha; 1.34 - 0.06(1-\alpha), 1.38 + 0.06(1-\alpha)]$$

$$\tilde{S}(t)[\alpha] = [\exp(-(\frac{t}{1.30+0.04\alpha})^{\beta}), \exp(-(\frac{t}{1.42-0.04\alpha})^{\beta});$$

$$\exp(-(\frac{t}{1.34-0.06(1-\alpha)})^{\beta}), \exp(-(\frac{t}{1.38+0.06(1-\alpha)})^{\beta})]$$

for all α.

The α-cut of fuzzy reliability of series-parallel system $\tilde{R}_{sp}(t)[\alpha]$ at particular time t = 10 and β = 0.5 is given by (21) and shown in Figure 6.

$$\tilde{R}_{sp}(t)[\alpha] = [\{1-(1-\exp(-(t/(1.30+0.04\alpha))^{\beta}))^{2}\}^{2}, \{1-(1-\exp(-(t/(1.42-0.04\alpha))^{\beta}))^{2}\}^{2};$$

$$\{1-(1-\exp(-(t/(1.34-0.06(1-\alpha)))^{\beta}))^{2}\}^{2}, \{1-(1-\exp(-(t/(1.38+0.06(1-\alpha)))^{\beta}))^{2}\}^{2}] \quad (21)$$

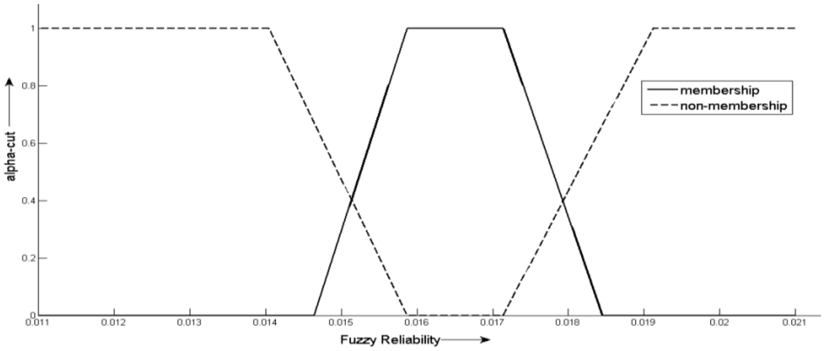

Figure 6. α-cut of fuzzy reliability in series-parallel system

(d) Parallel-Series configuration: If n-components are connected in series and m-components are in parallel as shown Figure 7, then the α-cut of fuzzy reliability function $\tilde{R}_{ps}(t)[\alpha]$ with intuitionistic fuzzy Weibull distribution is given by

$$\tilde{R}_{PS}(t)[\alpha] = 1 - \prod_{j=1}^{m}[1 - \prod_{i=1}^{n} S(t)]$$

$$= 1 - \prod_{j=1}^{m}[1 - \exp(-n(\frac{t}{a_1 + (a_2 - a_1)\alpha})^{\beta}), 1 - \exp(-n(\frac{t}{a_4 - (a_4 - a_3)\alpha})^{\beta});$$

$$1 - \exp(-n(\frac{t}{a_2 - (1-\alpha)(a_2 - a'_1)})^{\beta}), 1 - \exp(-n(\frac{t}{a_3 + (1-\alpha)(a'_4 - a_3)})^{\beta})]$$

$$= [1 - \{1 - \exp(-n(\frac{t}{a_1 + (a_2 - a_1)\alpha})^{\beta})\}^{m}, 1 - \{1 - \exp(-n(\frac{t}{a_4 - (a_4 - a_3)\alpha})^{\beta})\}^{m};$$

$$1 - \{1 - \exp(-n(\frac{t}{a_2 - (1-\alpha)(a_2 - a'_1)})^{\beta})\}^{m}, 1 - \{1 - \exp(-n(\frac{t}{a_3 + (1-\alpha)(a'_4 - a_3)})^{\beta})\}^{m}]$$

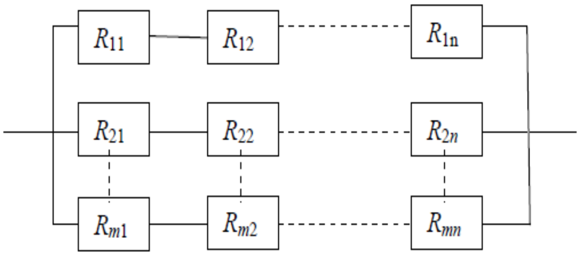

Figure 7. parallel-series system

Numerical example: Consider the lifetime of the electronic component is modeled by Weibull distribution with intuitionistic fuzzy parameter $\tilde{\theta}$ and four components of the considered system are connected in parallel-series.

If $\tilde{\theta} = (1.20, 1.24, 1.28, 1.32; 1.18, 1.24, 1.28, 1.34)$ then α-cut of fuzzy reliability of a component is given by:

$$\tilde{\theta}[\alpha] = [1.20 + 0.04\alpha, 1.32 - 0.04\alpha; 1.24 - 0.06(1-\alpha), 1.28 + 0.06(1-\alpha)]$$

$$\tilde{S}(t)[\alpha] = [\exp(-(\frac{t}{1.20 + 0.04\alpha})^{\beta}), \exp(-(\frac{t}{1.32 - 0.04\alpha})^{\beta}); \exp(-(\frac{t}{1.24 - 0.06(1-\alpha)})^{\beta}), \exp(-(\frac{t}{1.28 + 0.06(1-\alpha)})^{\beta})]$$

for all α.

The α-cut of fuzzy reliability of parallel-series system $\tilde{R}_{ps}(t)[\alpha]$ at particular time t = 10 and β = 0.5 is given by (22) and shown in Figure 8.

$$\tilde{R}_{ps}(t)[\alpha] = [1 - \{1 - \exp(-2(t/(1.20 + 0.04\alpha))^{\beta})\}^2, 1 - \{1 - \exp(-2(t/(1.32 - 0.04\alpha))^{\beta})\}^2; 1 - \{1 - \exp(-2(t/(1.24 - 0.06(1-\alpha)))^{\beta})\}^2, 1 - \{1 - \exp(-2(t/(1.28 + 0.06(1-\alpha)))^{\beta})\}^2] \quad (22)$$

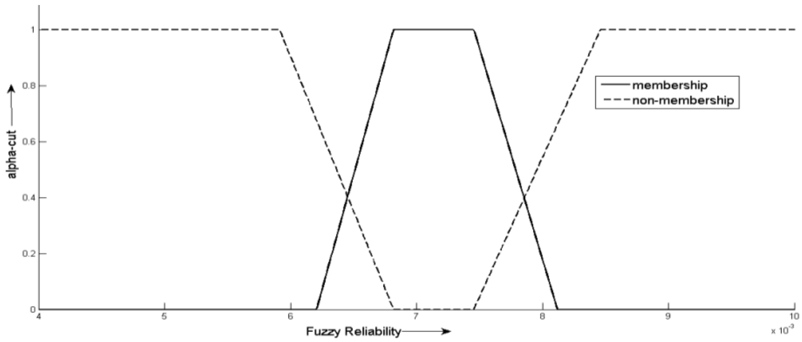

Figure 8. α-cut of fuzzy reliability in parallel-series system

7. CONCLUSION

In this paper we have evaluated the fuzzy reliability using intuitionistic fuzzy Weibull lifetime rate where the lifetime rate of the components of systems is considered as a trapezoidal intuitionistic fuzzy number. Fuzzy mean time to failure (FMTTF), fuzzy hazard function and their α-cut set have been successfully investigated by intuitionistic fuzzy Weibull lifetime distribution. The membership and non-membership function of fuzzy reliability and fuzzy mean time to failure (FMTTF) have also been computed. Using this method the fuzzy reliability of different type of systems (series, parallel, series-parallel and parallel-series) have been evaluated using lifetime rate of the components is considered as a trapezoidal intuitionistic fuzzy number. Numerical examples have also been taken to illustrate the methodology. In numerical examples we observed that the fuzzy reliability of the systems is in the form of trapezoidal intuitionistic fuzzy number. The reliability is maximum in the case when the components are connected in parallel configuration and it is minimum in the case of series configuration.

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