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# FILTERS OF RESIDUATED LATTICES BASED ON SOFT SET THEORY

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Abstract. Strong uni-soft filters and divisible uni-soft filters in residuated lattices are introduced, and several properties are investigated. Characterizations of a strong and divisible uni-soft filter are discussed. Conditions for a uni-soft filter to be divisible are established. Relations between a divisible uni-soft filter and a strong uni-soft filter are considered.

## 1. Introduction

Uncertainty is an attribute of information. There are three major theories dealing with uncertainty viz. theory of probability, theory of fuzzy sets and interval mathematics. But these theories have their own difficulties. There are other mathematical tools available which deal with uncertainty, such as intuitionistic fuzzy sets, vague sets, and rough sets but these theories also have difficulties as mentioned by Maji et al. [8]. As a new mathematical tool for dealing with uncertainties, Molodtsov [9] introduced the concept of soft sets. Since then several authors studied (fuzzy) algebraic structures based on soft set theory in several algebraic structures. Non-classical logic has become a formal and useful tool in dealing with fuzzy and uncertain informations. Various logical algebras have been proposed as the semantical systems of non-classical logic systems. Residuated lattices are important algebraic structures which are basic of $BL$-algebras, $MV$-algebras, $MTL$-algebras, Gödel algebras, $R_0$-algebras, lattice implication algebras, and so forth. Filter theory, which is an important notion, in residuated lattices is studied by Shen and Zhang [10] and Zhu and Xu [14]. In [6], Jun and Song applied soft sets to residuated lattices. They introduced uni-soft filters and uni-soft $G$-filters in residuated lattices, and investigated their properties. They considered characterizations of uni-soft filters and uni-soft $G$-filters. They also provided conditions for a uni-soft filter to be a uni-soft $G$-filter.

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In this paper, we introduce the notions of strong uni-soft filters and divisible uni-soft filters, and investigate several properties. We consider characterizations of a strong and divisible uni-soft filter. We provide conditions for a uni-soft filter to be divisible. We establish relations between a divisible uni-soft filter and a strong uni-soft filter.

## 2. Preliminaries

We display basic definitions and properties of a residuated lattice. For more details, we refer to the papers [1], [3], [4], [7], [11], and [12].

**Definition 2.1.** A *residuated lattice* is an algebra $\mathcal{L} := (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ such that

(L1) $(L, \vee, \wedge, 0, 1)$ is a bounded lattice,
(L2) $(L, \otimes, 1)$ is a commutative monoid,
(L3) $\otimes$ and $\rightarrow$ form an adjoint pair, that is,

$$(\forall x, y, z \in L)\,(x \leq y \rightarrow z \;\Leftrightarrow\; x \otimes y \leq z)\,.$$

In a residuated lattice $\mathcal{L}$, the ordering $\leq$ and negation $\neg$ are defined as follows:

$$(\forall x, y \in L)\,(x \leq y \;\Leftrightarrow\; x \wedge y = x \;\Leftrightarrow\; x \vee y = y \;\Leftrightarrow\; x \rightarrow y = 1)$$

and $\neg x = x \rightarrow 0$ for all $x \in L$.

**Proposition 2.1.** *In a residuated lattice $\mathcal{L}$, the following properties are valid.*

(1) $1 \rightarrow x = x,\ x \rightarrow 1 = 1,\ x \rightarrow x = 1,\ 0 \rightarrow x = 1,\ x \rightarrow (y \rightarrow x) = 1.$
(2) $x \rightarrow (y \rightarrow z) = (x \otimes y) \rightarrow z = y \rightarrow (x \rightarrow z).$
(3) $x \leq y \;\Rightarrow\; z \rightarrow x \leq z \rightarrow y,\ y \rightarrow z \leq x \rightarrow z.$
(4) $z \rightarrow y \leq (x \rightarrow z) \rightarrow (x \rightarrow y),\ z \rightarrow y \leq (y \rightarrow x) \rightarrow (z \rightarrow x).$
(5) $(x \rightarrow y) \otimes (y \rightarrow z) \leq x \rightarrow z.$
(6) $\neg x = \neg\neg\neg x,\ x \leq \neg\neg x,\ \neg 1 = 0,\ \neg 0 = 1.$
(7) $x \otimes y \leq x \otimes (x \rightarrow y) \leq x \wedge y \leq x \wedge (x \rightarrow y) \leq x.$
(8) $x \leq y \;\Rightarrow\; x \otimes z \leq y \otimes z.$
(9) $x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z),\ x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z).$
(10) $x \rightarrow y \leq (x \otimes z) \rightarrow (y \otimes z).$
(11) $\neg\neg(x \rightarrow y) \leq \neg\neg x \rightarrow \neg\neg y.$
(12) $x \rightarrow (x \wedge y) = x \rightarrow y.$

**Definition 2.2** ([10])**.** A nonempty subset $F$ of a residuated lattice $\mathcal{L}$ is called a *filter* of $\mathcal{L}$ if it satisfies the conditions: for all $x, y \in L$,

(1) $x, y \in F \;\Rightarrow\; x \otimes y \in F.$
(2) $x \in F,\ x \leq y \;\Rightarrow\; y \in F.$

**Proposition 2.2** ([10])**.** *A nonempty subset $F$ of a residuated lattice $\mathcal{L}$ is a filter of $\mathcal{L}$ if and only if it satisfies*:

(1) $1 \in F.$
(2) *For all $x, y \in L$, if $x \in F$ and $x \rightarrow y \in F$, then $y \in F$.*

A soft set theory is introduced by Molodtsov [9], and Çağman et al. [2] provided new definitions and various results on soft set theory.

In what follows, let $U$ be an initial universe set and $E$ be a set of parameters. Let $\mathcal{P}(U)$ denotes the power set of $U$ and $A, B, C, \ldots \subseteq E$.

**Definition 2.3** ([2, 9])**.** A soft set $(\tilde{f}, A)$ over $U$ is defined to be the set of ordered pairs

$$(\tilde{f}, A) := \left\{(x, \tilde{f}_A(x)) : x \in E,\ \tilde{f}_A(x) \in \mathcal{P}(U)\right\},$$

where $\tilde{f}_A : E \to \mathcal{P}(U)$ such that $\tilde{f}(x) = \emptyset$ if $x \notin A$. The soft set $(\tilde{f}, A)$ is simply denoted by $\tilde{f}_A$.

For a soft set $\tilde{f}_A$ over $U$ and a subset $\tau$ of $U$, the $\tau$-exclusive set of $\tilde{f}_A$, denoted by $e(\tilde{f}_A; \tau)$, is defined to be the set

$$e(\tilde{f}_A; \tau) := \left\{x \in A \mid \tilde{f}_A(x) \subseteq \tau\right\}.$$

**Definition 2.4** ([6])**.** A soft set $\tilde{f}_{\mathcal{L}}$ over $U$ in a residuated lattice $\mathcal{L}$ is called a *uni-soft filter* of $\mathcal{L}$ over $U$ if it satisfies: for all $x, y \in L$,

(1) $\tilde{f}_{\mathcal{L}}(x \otimes y) \subseteq \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(y)$.
(2) $x \leq y \ \Rightarrow\ \tilde{f}_{\mathcal{L}}(x) \supseteq \tilde{f}_{\mathcal{L}}(y)$.

**Theorem 2.3** ([6])**.** *A soft set $\tilde{f}_{\mathcal{L}}$ over $U$ in a residuated lattice $\mathcal{L}$ is a uni-soft filter of $\mathcal{L}$ over $U$ if and only if the following assertions are valid*:

(1) $\tilde{f}_{\mathcal{L}}(1) \subseteq \tilde{f}_{\mathcal{L}}(x)$,
(2) $\tilde{f}_{\mathcal{L}}(y) \subseteq \tilde{f}_{\mathcal{L}}(x \to y) \cup \tilde{f}_{\mathcal{L}}(x)$

*for all $x, y \in L$.*

## 3. Strong uni-soft filters

In what follows let $\mathcal{L}$ denote a residuated lattice unless otherwise specified.

**Definition 3.1** ([7])**.** A filter $F$ of $\mathcal{L}$ is said to be *strong* if it satisfies:

$$\neg\neg(\neg\neg x \to x) \in F \tag{1}$$

for all $x \in L$.

**Definition 3.2.** A uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ is said to be *strong* if it satisfies:

$$\tilde{f}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{f}_{\mathcal{L}}(1) \tag{2}$$

for all $x \in L$.

**Example 3.3.** Consider a residuated lattice $L := \{0, a, b, c, d, 1\}$ with the following Hasse diagram (Figure 3.1) and Cayley tables (see Table 1 and Table 2).

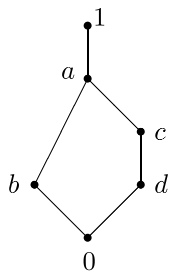

Figure 3.1

Table 1. Cayley table for the "$\otimes$"-operation

| $\otimes$ | 0 | $a$ | $b$ | $c$ | $d$ | 1 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | $a$ | $b$ | $d$ | $d$ | $a$ |
| $b$ | $c$ | $b$ | $b$ | 0 | 0 | $b$ |
| $c$ | $b$ | $d$ | 0 | $d$ | $d$ | $c$ |
| $d$ | $b$ | $d$ | 0 | $d$ | $d$ | $d$ |
| 1 | 0 | $a$ | $b$ | $c$ | $d$ | 1 |

Table 2. Cayley table for the "$\rightarrow$"-operation

| $\rightarrow$ | 0 | $a$ | $b$ | $c$ | $d$ | 1 |
|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| $a$ | 0 | 1 | $b$ | $c$ | $c$ | 1 |
| $b$ | $c$ | $a$ | 1 | $c$ | $c$ | 1 |
| $c$ | $b$ | $a$ | $b$ | 1 | $a$ | 1 |
| $d$ | $b$ | $a$ | $b$ | $a$ | 1 | 1 |
| 1 | 0 | $a$ | $b$ | $c$ | $d$ | 1 |

Define a soft set $\tilde{f}_{\mathcal{L}}$ in $\mathcal{L}$ over $U = \mathbb{Z}$ by $\tilde{f}_{\mathcal{L}}(1) = 3\mathbb{Z}$ and $\tilde{f}_{\mathcal{L}}(x) = 6\mathbb{N}$ for all $x(\neq 1) \in L$. It is routine to check that $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U = \mathbb{Z}$.

We provide characterizations of a strong uni-soft filter.

**Theorem 3.1.** *Given a soft set $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$, the following assertions are equivalent.*

(i) *$\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$.*

(ii) $\tilde{f}_{\mathcal{L}}$ *is a uni-soft filter of* $\mathcal{L}$ *over* $U$ *that satisfies*

(3) $$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}((y \to \neg\neg x) \to \neg\neg(y \to x) = \tilde{f}_{\mathcal{L}}(1)\right).$$

(iii) $\tilde{f}_{\mathcal{L}}$ *is a uni-soft filter of* $\mathcal{L}$ *over* $U$ *that satisfies*

(4) $$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}((\neg x \to y) \to \neg\neg(\neg y \to x)) = \tilde{f}_{\mathcal{L}}(1)\right).$$

*Proof.* Assume that $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$. Then $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$ over $U$. Note that

$$\begin{aligned}\neg\neg(\neg\neg x \to x) &\leq \neg\neg((y \to \neg\neg x) \to (y \to x))\\ &\leq \neg\neg((y \to \neg\neg x) \to \neg\neg(y \to x))\\ &= (y \to \neg\neg x) \to \neg\neg(y \to x)\end{aligned}$$

and

$$\begin{aligned}\neg\neg(\neg\neg x \to x) &\leq \neg\neg(((\neg x \to y) \otimes \neg y) \to x)\\ &= \neg\neg((\neg x \to y) \to (\neg y \to x))\\ &\leq \neg\neg((\neg x \to y) \to \neg\neg(\neg y \to x))\\ &= (\neg x \to y) \to \neg\neg(\neg y \to x)\end{aligned}$$

for all $x, y \in L$. If follows from (2) and Definition 2.4(2) that

(5) $$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(\neg\neg(\neg\neg x \to x)) \supseteq \tilde{f}_{\mathcal{L}}((y \to \neg\neg x) \to \neg\neg(y \to x))$$

and

(6) $$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(\neg\neg(\neg\neg x \to x)) \supseteq \tilde{f}_{\mathcal{L}}((\neg x \to y) \to \neg\neg(\neg y \to x)).$$

Combining Theorem 2.3(1), (5) and (6), we have

$$\tilde{f}_{\mathcal{L}}((y \to \neg\neg x) \to \neg\neg(y \to x)) = \tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}((\neg x \to y) \to \neg\neg(\neg y \to x))$$

for all $x, y \in L$. Therefore (ii) and (iii) are valid. Let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter of $\mathcal{L}$ over $U$ that satisfies the condition (3). If we take $y := \neg\neg x$ in (3) and use Proposition 2.1(1), then we can induce the condition (2) and so $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$. Let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter of $\mathcal{L}$ over $U$ that satisfies the condition (4). Taking $y := \neg x$ in (4) and using Proposition 2.1(1) induces the condition (2). Hence $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$. □

**Lemma 3.2** ([6]). *A soft set* $\tilde{f}_{\mathcal{L}}$ *over* $U$ *is a uni-soft filter of* $\mathcal{L}$ *if and only if the nonempty* $\tau$*-exclusive set of* $\tilde{f}_{\mathcal{L}}$ *is a filter of* $\mathcal{L}$ *for all* $\tau \in \mathscr{P}(U)$.

**Theorem 3.3.** *If a soft set* $\tilde{f}_{\mathcal{L}}$ *over* $U$ *is a strong uni-soft filter of* $\mathcal{L}$*, then the nonempty* $\tau$*-exclusive set of* $\tilde{f}_{\mathcal{L}}$ *is a strong filter of* $\mathcal{L}$ *for all* $\tau \in \mathscr{P}(U)$.

*Proof.* If a uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ is strong, then

$$\tilde{f}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{f}_{\mathcal{L}}(1) \subseteq \tau$$

for all $x \in L$, and so $\neg\neg(\neg\neg x \to x) \in e(\tilde{f}_{\mathcal{L}}; \tau)$ for all $x \in L$. Combining this with Lemma 3.2 shows that $e(\tilde{f}_{\mathcal{L}}; \tau)$ is a strong filter of $\mathcal{L}$. □

**Lemma 3.4** ([6]). *If $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$, then the set*

$$\mathcal{L}_a := \{x \in L \mid \tilde{f}_{\mathcal{L}}(a) \supseteq \tilde{f}_{\mathcal{L}}(x)\}$$

*is a filter of $\mathcal{L}$ for every $a \in L$.*

**Theorem 3.5.** *If $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$, then the set*

$$\mathcal{L}_a := \{x \in L \mid \tilde{f}_{\mathcal{L}}(a) \supseteq \tilde{f}_{\mathcal{L}}(x)\}$$

*is a strong filter of $\mathcal{L}$ for every $a \in L$.*

*Proof.* Assume that $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$. Then $\mathcal{L}_a$ is a filter of $\mathcal{L}$ for every $a \in L$ by Lemma 3.4. Since $\tilde{f}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{f}_{\mathcal{L}}(1) \subseteq \tilde{f}(a)$ for all $a, x \in L$, we have $\neg\neg(\neg\neg x \to x) \in \mathcal{L}_a$. Hence $\mathcal{L}_a$ is a strong filter of $\mathcal{L}$. □

**Theorem 3.6.** *Let $\tilde{f}_{\mathcal{L}}$ and $\tilde{g}_{\mathcal{L}}$ be uni-soft filters of $\mathcal{L}$ such that $\tilde{f}_{\mathcal{L}}(1) = \tilde{g}(1)$ and $\tilde{f}_{\mathcal{L}} \supseteq \tilde{g}_{\mathcal{L}}$, i.e., $\tilde{f}_{\mathcal{L}}(x) \supseteq \tilde{g}_{\mathcal{L}}(x)$ for all $x \in L$. If $\tilde{f}_{\mathcal{L}}$ is strong, then so is $\tilde{g}_{\mathcal{L}}$.*

*Proof.* Suppose that $\tilde{f}_{\mathcal{L}}$ is strong uni-soft filter of $\mathcal{L}$. Then

$$\tilde{g}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) \subseteq \tilde{f}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{f}_{\mathcal{L}}(1) = \tilde{g}(1),$$

and so $\tilde{g}_{\mathcal{L}}\big(\neg\neg(\neg\neg x \to x)\big) = \tilde{g}_{\mathcal{L}}(1)$ for all $x \in L$. Therefore $\tilde{g}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$. □

## 4. Divisible uni-soft filters

**Definition 4.1** ([7]). A filter $F$ of $\mathcal{L}$ is said to be *divisible* if it satisfies:

(7) $$(x \wedge y) \to [x \otimes (x \to y)] \in F$$

for all $x, y \in L$.

**Definition 4.2.** A uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ is said to be *divisible* if it satisfies:

(8) $$\tilde{f}\big((x \wedge y) \to [x \otimes (x \to y)]\big) = \tilde{f}_{\mathcal{L}}(1)$$

for all $x, y \in L$.

**Example 4.3.** Let $L = \{0, a, b, 1\}$ be a chain with Cayley tables which are given in Tables 3 and 4.

TABLE 3. Cayley table for the "⊗"-operation

| $\otimes$ | 0 | $a$ | $b$ | 1 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| $a$ | 0 | 0 | $a$ | $a$ |
| $b$ | 0 | $a$ | $b$ | $b$ |
| 1 | 0 | $a$ | $b$ | 1 |

TABLE 4. Cayley table for the "→"-operation

| → | 0 | $a$ | $b$ | 1 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 1 |
| $a$ | $a$ | 1 | 1 | 1 |
| $b$ | 0 | $a$ | 1 | 1 |
| 1 | 0 | $a$ | $b$ | 1 |

Then $\mathcal{L} := (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is a residuated lattice. Define a soft set $\tilde{f}_{\mathcal{L}}$ over $U = \mathbb{Z}$ in $\mathcal{L}$ by $\tilde{f}_{\mathcal{L}}(1) = 2\mathbb{Z}$ and $\tilde{f}_{\mathcal{L}}(x) = 2\mathbb{N}$ for all $x(\neq 1) \in L$. It is routine to verify that $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U = \mathbb{Z}$.

**Example 4.4.** Let "⊗" and "→" be two operations on $L = [0, 1]$ defined as follows:

$$x \otimes y = \begin{cases} 0 & \text{if } x + y \leq \frac{1}{2}, \\ x \wedge y & \text{otherwise.} \end{cases}$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y, \\ \left(\frac{1}{2} - x\right) \vee y & \text{otherwise.} \end{cases}$$

Then $\mathcal{L} := (L, \vee, \wedge, \otimes, \rightarrow, 0, 1)$ is a residuated lattice. The soft set $\tilde{f}_{\mathcal{L}}$ over $U = \mathbb{N}$ in $\mathcal{L}$ given by $\tilde{f}_{\mathcal{L}}(1) = 3\mathbb{N}$ and $\tilde{f}_{\mathcal{L}}(x) = 6\mathbb{N}$ for all $x(\neq 1) \in L$ is a uni-soft filter of $\mathcal{L}$. But it is not divisible since

$$\tilde{f}_{\mathcal{L}}((0.3 \wedge 0.2) \rightarrow (0.3 \otimes (0.3 \rightarrow 0.2)) = \tilde{f}_{\mathcal{L}}(0.3) \neq \tilde{f}_{\mathcal{L}}(1).$$

**Proposition 4.1.** *Every divisible uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ satisfies the following identity.*

$$\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \rightarrow (x \otimes (y \wedge z))) = \tilde{f}_{\mathcal{L}}(1) \tag{9}$$

*for all $x, y, z \in L$.*

*Proof.* Let $x, y, z \in L$. If we let $x := x \otimes y$ and $y := x \otimes z$ in (8), then

$$\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \rightarrow ((x \otimes y) \otimes ((x \otimes y) \rightarrow (x \otimes z)))) = \tilde{f}_{\mathcal{L}}(1). \tag{10}$$

Using (2) and (7) in Proposition 2.1, we have

$$(x \otimes y) \otimes ((x \otimes y) \rightarrow (x \otimes z)) \leq x \otimes (y \wedge (x \rightarrow (x \otimes z))),$$

and so

$$\begin{aligned} &((x \otimes y) \wedge (x \otimes z)) \rightarrow ((x \otimes y) \otimes ((x \otimes y) \rightarrow (x \otimes z))) \\ &\leq ((x \otimes y) \wedge (x \otimes z)) \rightarrow (x \otimes (y \wedge (x \rightarrow (x \otimes z)))) \end{aligned}$$

by Proposition 2.1(3). It follows from (10) and Definition 2.4(2) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \rightarrow ((x \otimes y) \otimes ((x \otimes y) \rightarrow (x \otimes z)))) \\ &\supseteq \tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \rightarrow (x \otimes (y \wedge (x \rightarrow (x \otimes z))))) \end{aligned}$$

and so that

$$\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z))))) = \tilde{f}_{\mathcal{L}}(1). \tag{11}$$

On the other hand, if we take $x := x \to (x \otimes z)$ in (8) then

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}((y \wedge (x \to (x \otimes z))) \to ((x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ &\supseteq \tilde{f}_{\mathcal{L}}((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &\quad (x \otimes ((x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ &= \tilde{f}_{\mathcal{L}}((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &\quad (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))\end{aligned}$$

by using Proposition 2.1(10), Definition 2.4(2) and the commutativity and associativity of $\otimes$. Hence

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))).\end{aligned} \tag{12}$$

Using Proposition 2.1(5), we get

$$\begin{aligned}&(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))))\otimes \\ &((x \otimes (y \wedge (x \to (x \otimes z)))) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ \leq\ &((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)).\end{aligned}$$

It follows from (1) and (2) of Definition 2.4, (11) and (12) that

$$\begin{aligned}&\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ \subseteq\ &\tilde{f}_{\mathcal{L}}((((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z)))))\otimes \\ &((x \otimes (y \wedge (x \to (x \otimes z)))) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ \subseteq\ &\tilde{f}_{\mathcal{L}}((((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge (x \to (x \otimes z))))))\cup \\ &\tilde{f}_{\mathcal{L}}(((x \otimes (y \wedge (x \to (x \otimes z)))) \to \\ &(x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ =\ &\tilde{f}_{\mathcal{L}}(1).\end{aligned}$$

Thus

$$\tilde{f}_{\mathcal{L}}(((x\otimes y)\wedge(x\otimes z)) \to (x\otimes(x \to (x\otimes z))\otimes((x \to (x\otimes z)) \to y))) = \tilde{f}_{\mathcal{L}}(1). \tag{13}$$

Since

$$x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)) \leq x \otimes z \otimes (z \to y) \leq x \otimes (y \wedge z),$$

we obtain

$$\begin{aligned}&((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y))) \\ \leq\ &((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z)).\end{aligned}$$

It follows that

$$\begin{aligned} &\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z))) \\ \subseteq\ &\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (x \to (x \otimes z)) \otimes ((x \to (x \otimes z)) \to y)))) \\ =\ &\tilde{f}_{\mathcal{L}}(1) \end{aligned}$$

and that $\tilde{f}_{\mathcal{L}}(((x \otimes y) \wedge (x \otimes z)) \to (x \otimes (y \wedge z))) = \tilde{f}_{\mathcal{L}}(1)$. □

We consider characterizations of a divisible uni-soft filter.

**Theorem 4.2.** *A uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ is divisible if and only if the following assertion is valid*:

$$\tilde{f}\big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\big) = \tilde{f}_{\mathcal{L}}(1) \tag{14}$$

*for all $x, y, z \in L$.*

*Proof.* Assume that $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$. If we take $x := x \to y$ and $y := x \to z$ in (8) and use (2) and (9) of Proposition 2.1, then

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(1) &= \tilde{f}\left([(x \to y) \wedge (x \to z)] \to [(x \to y) \otimes ((x \to y) \to (x \to z))]\right) \\ &= \tilde{f}\left([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\right). \end{aligned}$$

Using (4) and (10) of Proposition 2.1, we have

$$\begin{aligned} &(x \wedge y) \to [x \otimes (x \to y)] \leq [(x \otimes (x \to y)) \to z] \to [(x \wedge y) \to z] \\ \leq\ &[(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)] \end{aligned}$$

for all $x, y, z \in L$. Since $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$, it follows from (8) and Definition 2.4(2) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}((x \wedge y) \to [x \otimes (x \to y)]) \\ &\supseteq \tilde{f}_{\mathcal{L}}([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]) \end{aligned}$$

and so from Theorem 2.3(1) that

$$\tilde{f}_{\mathcal{L}}([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]) = \tilde{f}_{\mathcal{L}}(1)$$

for all $x, y, z \in L$. Using Proposition 2.1(5), we get

$$\begin{aligned} &\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\Big) \otimes \\ &\Big([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big) \\ \leq\ &[x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)], \end{aligned}$$

and so

$$\begin{aligned} &\tilde{f}\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big) \\ \subseteq\ &\tilde{f}\Big(([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]) \otimes \\ &([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)])\Big) \end{aligned}$$

$$\subseteq \tilde{f}\big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \otimes (x \to y)) \to z)]\big) \cup$$
$$\tilde{f}\big([(x \to y) \otimes ((x \otimes (x \to y)) \to z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\big)$$
$$= \tilde{f}_{\mathcal{L}}(1).$$

Therefore

$$\tilde{f}\Big([x \to (y \wedge z)] \to [(x \to y) \otimes ((x \wedge y) \to z)]\Big) = \tilde{f}_{\mathcal{L}}(1)$$

for all $x, y, z \in L$.

Conversely, let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter that satisfies the condition (14). If we take $x := 1$ in (14) and use Proposition 2.1(1), then we obtain (8). □

**Theorem 4.3.** *A uni-soft filter* $\tilde{f}_{\mathcal{L}}$ *of* $\mathcal{L}$ *over* $U$ *is divisible if and only if it satisfies*:

(15) $$\tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right) = \tilde{f}_{\mathcal{L}}(1)$$

*for all* $x, y \in L$.

*Proof.* Suppose that $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$. Note that

$$(x \wedge y) \to [x \otimes (x \to y)] \leq [y \otimes (y \to x)] \to [x \otimes (x \to y)]$$

for all $x, y \in L$. It follows from (8) and Definition 2.4(2) that

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}\left((x \wedge y) \to [x \otimes (x \to y)]\right)$$
$$\supseteq \tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right)$$

and that $\tilde{f}\left([y \otimes (y \to x)] \to [x \otimes (x \to y)]\right) = \tilde{f}_{\mathcal{L}}(1)$.

Conversely, let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter of $\mathcal{L}$ over $U$ that satisfies the condition (15). Since

$$y \to x = y \to (y \wedge x) \text{ for all } x, y \in L,$$

the condition (15) implies that

(16) $$\tilde{f}\left([y \otimes (y \to (x \wedge y))] \to [x \otimes (x \to (x \wedge y))]\right) = \tilde{f}_{\mathcal{L}}(1).$$

If we take $y := x \wedge z$ in (16), then

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}\left([(x \wedge z) \otimes ((x \wedge z) \to (x \wedge (x \wedge z)))] \to [x \otimes (x \to (x \wedge (x \wedge z)))]\right)$$
$$= \tilde{f}\left((x \wedge z) \to [x \otimes (x \to z)]\right).$$

Therefore $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$. □

We discuss conditions for a uni-soft filter to be divisible.

**Theorem 4.4.** *If a uni-soft filter* $\tilde{f}_{\mathcal{L}}$ *of* $\mathcal{L}$ *over* $U$ *satisfies the following assertion*:

(17) $$\tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes y)) = \tilde{f}_{\mathcal{L}}(1)$$

*for all* $x, y \in L$, *then* $\tilde{f}_{\mathcal{L}}$ *is divisible.*

*Proof.* Note that $x \otimes y \leq x \otimes (x \to y)$ for all $x, y \in L$. It follows from Proposition 2.1(3) that

$$(x \wedge y) \to (x \otimes y) \leq (x \wedge y) \to (x \otimes (x \to y)).$$

Hence, by (17) and Definition 2.4(2), we have

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes y)) \supseteq \tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes (x \to y))),$$

and so $\tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes (x \to y))) = \tilde{f}_{\mathcal{L}}(1)$ for all $x, y \in L$. Therefore $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$. □

**Theorem 4.5.** *If a uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ satisfies the following assertion:*

$$\tilde{f}_{\mathcal{L}}((x \wedge (x \to y)) \to y) = \tilde{f}_{\mathcal{L}}(1) \tag{18}$$

*for all $x, y \in L$ then $\tilde{f}_{\mathcal{L}}$ is divisible.*

*Proof.* Taking $y := x \otimes y$ in (18) implies that

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}((x \wedge (x \to (x \otimes y))) \to (x \otimes y)) \\ &\supseteq \tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes y))\end{aligned}$$

and so $\tilde{f}_{\mathcal{L}}((x \wedge y) \to (x \otimes y)) = \tilde{f}_{\mathcal{L}}(1)$ for all $x, y \in L$. It follows from Theorem 4.4 that $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$. □

**Theorem 4.6.** *If a uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ satisfies the following assertion:*

$$\tilde{f}_{\mathcal{L}}(x \to z) \subseteq \tilde{f}_{\mathcal{L}}((x \otimes y) \to z) \cup \tilde{f}_{\mathcal{L}}(x \to y) \tag{19}$$

*for all $x, y, z \in L$, then $\tilde{f}_{\mathcal{L}}$ is divisible.*

*Proof.* If we take $x := x \wedge (x \to y)$, $y := x$ and $z := y$ in (19), then

$$\begin{aligned}&\tilde{f}_{\mathcal{L}}((x \wedge (x \to y)) \to y) \\ &\subseteq \tilde{f}_{\mathcal{L}}(((x \wedge (x \to y)) \otimes x) \to y) \cup \tilde{f}_{\mathcal{L}}((x \wedge (x \to y)) \to x) \\ &= \tilde{f}_{\mathcal{L}}(1).\end{aligned}$$

Thus $\tilde{f}_{\mathcal{L}}((x \wedge (x \to y)) \to y) = \tilde{f}_{\mathcal{L}}(1)$ for all $x, y \in L$, and so $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$ by Theorem 4.5. □

**Theorem 4.7.** *If a uni-soft filter $\tilde{f}$ of $\mathcal{L}$ over $U$ satisfies the following assertion:*

$$\tilde{f}_{\mathcal{L}}(x \to (x \otimes x)) = \tilde{f}_{\mathcal{L}}(1) \tag{20}$$

*for all $x \in L$, then $\tilde{f}_{\mathcal{L}}$ is divisible.*

*Proof.* Let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter of $\mathcal{L}$ over $U$ that satisfies the condition (20). Using Proposition 2.1(10) and the commutativity of $\otimes$, we have

$$x \to y \leq (x \otimes x) \to (x \otimes y),$$

and so

$$(x \to (x \otimes x)) \otimes (x \to y) \leq (x \to (x \otimes x)) \otimes ((x \otimes x) \to (x \otimes y))$$

for all $x, y \in L$ by Proposition 2.1(8) and the commutativity of $\otimes$. It follows from (5) and (8) of Proposition 2.1 and the commutativity of $\otimes$ that

$$\begin{aligned} &((x \to (x \otimes x)) \otimes (x \to y)) \otimes ((x \otimes y) \to z) \\ \leq\ &((x \to (x \otimes x)) \otimes ((x \otimes x) \to (x \otimes y))) \otimes ((x \otimes y) \to z) \\ \leq\ &(x \to (x \otimes y)) \otimes ((x \otimes y) \to z) \\ \leq\ &x \to z \end{aligned}$$

and so from Definition 2.4, Theorem 2.3(1) and (20) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(x \to z) &\subseteq \tilde{f}_{\mathcal{L}}(((x \to (x \otimes x)) \otimes (x \to y)) \otimes ((x \otimes y) \to z)) \\ &\subseteq \tilde{f}_{\mathcal{L}}((x \to (x \otimes x)) \otimes (x \to y)) \cup \tilde{f}_{\mathcal{L}}((x \otimes y) \to z) \\ &\subseteq \tilde{f}_{\mathcal{L}}(x \to (x \otimes x)) \cup \tilde{f}_{\mathcal{L}}(x \to y) \cup \tilde{f}_{\mathcal{L}}((x \otimes y) \to z) \\ &= \tilde{f}_{\mathcal{L}}(1) \cap \tilde{f}_{\mathcal{L}}(x \to y) \cup \tilde{f}_{\mathcal{L}}((x \otimes y) \to z) \\ &= \tilde{f}_{\mathcal{L}}((x \otimes y) \to z) \cup \tilde{f}_{\mathcal{L}}(x \to y) \end{aligned}$$

for all $x, y, z \in L$. Therefore $\tilde{f}_{\mathcal{L}}$ is a divisible uni-soft filter of $\mathcal{L}$ over $U$ by Theorem 4.6. □

We investigate relationship between a divisible uni-soft filter and a strong uni-soft filter.

**Theorem 4.8.** *Every divisible uni-soft filter is a strong uni-soft filter.*

*Proof.* Let $\tilde{f}_{\mathcal{L}}$ be a divisible uni-soft filter of $\mathcal{L}$ over $U$. If we put $x := \neg\neg x$ and $y := x$ in (8), then we have

$$\tilde{f}_{\mathcal{L}}((\neg\neg x \wedge x) \to (\neg\neg x \otimes (\neg\neg x \to x))) = \tilde{f}_{\mathcal{L}}(1). \tag{21}$$

Using (4) and (8) of Proposition 2.1, we get

$$\begin{aligned} &(\neg\neg x \wedge x) \to (\neg\neg x \otimes (\neg\neg x \to x)) \leq \neg(\neg\neg x \otimes (\neg\neg x \to x)) \to \neg(\neg\neg x \wedge x) \\ \leq\ &(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x))) \to (\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \\ \leq\ &\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x))) \end{aligned}$$

for all $x \in L$. It follows from (21) and Definition 2.4(2) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}((\neg\neg x \wedge x) \to (\neg\neg \otimes (\neg\neg x \to x))) \\ &\supseteq \tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))). \end{aligned} \tag{22}$$

Combining (22) with Theorem 2.3(1), we have

$$\tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) = \tilde{f}_{\mathcal{L}}(1) \tag{23}$$

for all $x \in L$. Using (2), (6), (11) and (12) of Proposition 2.1, we get

$$\begin{aligned}\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) &= \neg\neg x \to \neg\neg(\neg\neg x \wedge x) \\ &\geq \neg\neg(x \to (\neg\neg x \wedge x)) \\ &= \neg\neg(x \to (x \wedge \neg\neg x)) \\ &= \neg\neg(x \to \neg\neg x) = \neg\neg 1 = 1\end{aligned}$$

and so $\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) = 1$ for all $x \in L$. It follows from (23) and Theorem 2.3(2) that

$$\begin{aligned}&\tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) \\ \subseteq\ &\tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x)) \to \neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) \cup \\ &\tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \wedge x))) \\ =\ &\tilde{f}_{\mathcal{L}}(1)\end{aligned}$$

and so that

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes \neg(\neg\neg x \otimes (\neg\neg x \to x)))) \\ &= \tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))))).\end{aligned} \tag{24}$$

Taking $x := \neg\neg x$ and $y := \neg(\neg\neg x \to x)$ in (8) induces

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}((\neg\neg x \wedge \neg(\neg\neg x \to x)) \to (\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x)))) \\ &\supseteq \tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))) \to \neg(\neg\neg x \wedge \neg(\neg\neg x \to x)))\end{aligned}$$

by using Proposition 2.1(3) and Definition 2.4(2). Thus

$$\tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \otimes (\neg\neg x \to \neg(\neg\neg x \to x))) \to \neg(\neg\neg x \wedge \neg(\neg\neg x \to x))) = \tilde{f}_{\mathcal{L}}(1). \tag{25}$$

Since $\neg(\neg\neg x \to x) \leq \neg\neg x$ for all $x \in L$, it follows from Theorem 2.3(1) Theorem 2.3(2), (24) and (25) that

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(\neg(\neg\neg x \wedge \neg(\neg\neg x \to x))) = \tilde{f}_{\mathcal{L}}(\neg\neg(\neg\neg x \to x))$$

for all $x \in L$. Therefore $\tilde{f}_{\mathcal{L}}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$. □

**Corollary 4.9.** *If a uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ over $U$ satisfies one of conditions* (14), (15), (17), (18), (19) *and* (20), *then $\tilde{f}$ is a strong uni-soft filter of $\mathcal{L}$ over $U$.*

The converse of Theorem 4.8 may not be true in general. In fact, the strong uni-soft filter $\tilde{f}$ of $\mathcal{L}$ over $U$ which is given in Example 3.3 is not a divisible uni-soft filter of $\mathcal{L}$ over $U$ since $\tilde{f}_{\mathcal{L}}((a \wedge c) \to (a \otimes (a \to c))) = \tilde{f}_{\mathcal{L}}(a) \neq \tilde{f}_{\mathcal{L}}(1)$.

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