Analysis of Body Circumference Measures in Predicting Percentage of Body Fat

Sung Ha Park[†]

Department of Industrial and Management Engineering, Hannam University

인체둘레치수를 활용한 체지방율 예측 다중회귀모델 개발

박성하

한남대학교 산업경영공학과

As a measure of health, the percentage of body fat has been utilized for many ergonomist, physician, athletic trainers, and work physiologists. Underwater weighing procedure for measuring the percentage of body fat is popular and accurate. However, it is relatively expensive, difficult to perform and requires large space. Anthropometric techniques can be utilized to predict the percentage of body fat in the field setting because they are easy to implement and require little space. In this concern, the purpose of this study was to find a regression model to easily predict the percentage of body fat using the anthropometric circumference measurements as predictor variables. In this study, the data for 10 anthropometric circumference measurements for 252 men were analyzed. A full model with ten predictor variables was constructed based on subjective knowledge and literature. The linear regression modeling consists of variable selection and various assumptions regarding the anticipated model. All possible regression models and the assumptions are evaluated using various statistical methods. Based on the evaluation, a reduced model was selected with five predictor variables to predict the percentage of body fat. The model is : % Body Fat = 2.704-0.601 (Neck Circumference) + 0.974 (Abdominal Circumference) - 0.332 (Hip Circumference) + 0.409 (Arm Circumference) - 1.618 (Wrist Circumference) + ϵ . This model can be used to estimate the percentage of body fat using only a tape measure.

Keywords : Body Fat, Anthropometric Circumference, Underwater Weighing, Regression Model

1. Introduction

Measuring the percentage of body fat has become a popular and standard practice for many ergonomist, physician, athletic trainers, and work physiologists. Evidence supports that obesity (excessive fat) is closely related to musculoskeletal injury, reduced motor performance, and many health problems in industry [1, 3]. Overweight individuals have a higher risk of some musculoskeletal disorders, specifically lower back [4]. Craig et al. [2] demonstrated a close relationship between the rate of handlers' injuries at work and the high percentage of body fat. In another study performed at an aluminum manufacturing company, approximately 85% of the employees who had sustained at least one injury were classified as overweight or obese [9]. In addition, many researchers showed that the indirect medical costs are also higher for obese workers than non-obese-workers [7, 14].

Hydrostatic or underwater weighing is the most widely used laboratory procedure for measuring body density. This method uses Archimedes' principle that a body immersed in a fluid

Received 2 February 2015; Finally Revised 11 May 2015; Accepted 11 May 2015

^{*} Corresponding Author : shpark@hnu.kr

is acted on by a buoyancy force that is evidenced by a loss of weight equal to the weight of the displaced fluid. The body density measured through the underwater weighing can then be used to calculate the percentage of body fat using Siri's equation [13]. The hydrostatic technique has been shown to be highly reliable when measurements were made over time intervals ranging from 30 minutes to a couple of days. A standard error of measurement (less than 0.002g/cc) has also been observed [10]. However, difficulties associated with implementing the underwater weighing procedure are that it is relatively expensive, difficult to perform and requires large space.

Anthropometry deals with the measurement of size, weight, and proportion of the human body. Anthropometric techniques are popular for predicting body composition in the field setting because they are cheap to implement, require little space, and are easy to perform. In addition, anthropometric procedures are noninvasive, and training can be provided without prerequisite courses. Consequently, anthropometric methods are applicable to large samples [11].

Fitting the percentage of body fat (measured through the underwater weighing procedure) to the other anthropometric measurements using the multiple regression analysis provides a convenient way of estimating body fat. In this concern, the objective of present study was to examine what anthropometric variables and how they were related to the percentage of body fat. For this study, the percentage of body fat obtained using Siri's equation and ten anthropometric variables (i.e., body circumference measures) for 252 men were analyzed.

In general, the objective of a regression analysis is to control, describe, and predict response variables in relation to predictor variables. The present study was aimed to achieve last two purposes. For the description purpose, this study tried to explain what body circumference variables and how they are related to the percentage of body fat. For the prediction purpose, a statistical regression model developed by analyzing the relationship between the percentage of body fat and the body circumference variables can be used to predict the body fat. This regression model is easy to use, inexpensive, and convenient, as compared to the underwater weighing techniques. To meet these purposes, a multiple regression analysis was performed.

2. Source and Characteristics of Data

The data for the Percentage of body fat and body circum-

ference were gathered from the web site http://math.arizona. edu/~jwatkins/505d/body.htm. The data were originally supplied by Dr. A. Garth Fisher, Human performance Research Center, Brigham Young University, Provo, Utah 84602, who gave permission to freely distribute the data and use them for non-commercial purposes [8].

Percentage of body fat and ten body circumference measurements were recorded for 252 men (cross-sectional data). Body fat was estimated through an underwater weighing technique. The percentage of body fat was treated as a dependent variable. Independent variables consisted of 10 body circumferences as follows :

Dependent (response) variable: The percentage of body fat (*PCTFAT*) is a measure of health. The data were estimated through two steps. The body density (D_b) was measured through the underwater weighing. The percentage of body fat then was calculated based on Siri's equation [13] (i.e., *PCTFAT* = 495/Db-450).

Independent (predictor) variables : 10 independent variables include, neck circumference (*NECKCIR*, cm), chest circumference (*CHESTCIR*, cm), abdomen circumference (*ABDOCIR*, cm), hip circumference (*HIPCIR*, cm), thigh circumference (*THIGHCIR*, cm), knee circumference (*KNEECIR*, cm), ankle circumference (*ANKLECIR*, cm), extended biceps circumference (*BICEPCIR*, cm), forearm circumference (*ARMCIR*, cm), and wrist circumference (*WRISTCIR*, cm). In taking of circumference measurements the tape measure was positioned in a horizontal plane or perpendicular to the length of the segment being measured [10].

3. Development of Prediction Model

3.1 Full Model(Initial Model with 10 Predictor Variables)

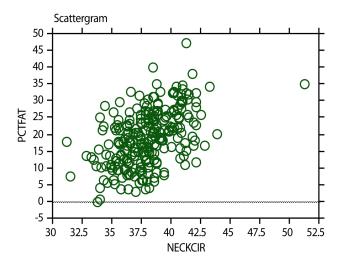
It was expected that 10 predictor variables positively relate to the percentage of body fat (PCTFAT). Therefore, a model was chosen with 10 predictor variables as an initial model :

 $PCTFAT = \beta_0 + \beta_1 NECKCIR + \beta_2 CHESTCIR + \beta_3 ABDOCIR + \beta_4 HIPCIR + \beta_5 THIGHCIR + \beta_6 KNEECIR + \beta_7 ANKLECIR + \beta_8 BICEPCIR + \beta_9 ARMCIR + \beta_{10} WRISTCIR + \epsilon$

All signs of regression coefficients are anticipated to be positive because all anthropometric circumferences seem to increase as the percentage of body fat increases. For some cases such as a body builder, the increase in *BICEPCIR* or *THIGHCIR* may result decrease in % body fat. However, this will not affect entire positive relation to body fat. Interaction terms are not included in the initial model. If other variables such as sex or a history of physical exercise is included, we can suspect some interaction terms. For example, it is easy to suspect some interaction between the sex and the *HIPCIR* (due to the difference in *HIPCIR* between male and female). In the initial model, however, there are no known pairs of variables that interact with each other. Therefore, no interaction terms are included.

Scatter plots of the response variable against each predictor variable can aid in determining the nature and strength of bivariate relationships between each of the predictor variable and the response variable. A compliment to the scatter plot matrix that may be useful at times is the correlation matrix [6]. To get preliminary information about variables, scatter plots of the response variable against each predictor variable and the correlation matrix were generated. An example of the scatter plot and the correlation matrix are presented in <Figure 1> and <Table 1>, respectively. The plots show positive linear relationships between the response variable and each predictor variable. The correlation matrix shows that there exist several values greater than 0.7 implying some multicollinearity (MC) between the predictor variables. These findings were subjected to further analysis. Variance inflation factor (VIF) was analyzed to check the MC problems. The VIF is often used as a measure of the severity of MC and a maximum VIF greater than 10 is generally taken as an indication of MC between the predictor variables [5]. The largest VIF was 9.868 for *HIPCIR*. Even though MC exists, the degree of MC was not significant.

As shown in <Figure 1>, there may be some outliers showing distinct increases or decreases in *PCTFAT*. For example, extremely short person or extremely well trained (e.g. body builder) person might be presented by outliers. Some expected outliers, if they exist, will make the normality assumption violated. However, the sample size is relatively high (n = 252), therefore, it is anticipated that one or two outliers, even possibly exist, will not affect the entire relationship between the dependent variable and the independent variables.



<Figure 1> Scatter Plot of PCTFAT vs. NECKCIR

<table 1=""> Correlation Matrix for PCTFAT a</table>	and 10 Predictor Variables
------------------------------------------------------	----------------------------

Correlation M	Correlation Matrix										
	PCTFAT	NECKCIR	CHESTCIR	ABDOCIR	HIPCIR	THIGHCIR	KNEECIR	ANKLECIR	BICEPCIR	ARMCIR	WRISTCIR
PCTFAT	1	0.491	0.703	0.813	0.625	0.56	0.509	0.266	0.493	0.361	0.347
NECKCIR	0.491	1	0.785	0.754	0.735	0.696	0.672	0.478	0.731	0.624	0.745
CHESTCIR	0.703	0.785	1	0.916	0.829	0.73	0.719	0.483	0.728	0.58	0.66
ABDOCIR	0.813	0.754	0.916	1	0.874	0.767	0.737	0.453	0.685	0.503	0.62
HIPCIR	0.625	0.735	8.29E-01	0.874	1	0.896	0.823	0.558	0.739	0.545	0.63
THIGHCIR	0.56	0.696	0.73	0.767	0.896	1	0.799	0.54	0.761	0.567	0.559
KNEECIR	0.509	0.672	7.19E-01	0.737	0.823	0.799	1	0.612	0.679	0.556	0.665
ANKLECIR	0.266	0.478	0.483	0.453	0.558	0.54	0.612	1	0.485	0.419	0.566
BICEPCIR	0.493	0.731	0.728	0.685	0.739	0.761	0.679	0.485	1	0.678	0.632
ARMCIR	0.361	0.624	0.58	0.503	0.545	0.567	0.556	0.419	0.678	1	0.586
WRISTCIR	0.347	0.745	0.66	0.62	0.63	0.559	0.665	0.566	0.632	0.586	1

Note) 252 observations were used in this computation.

3.1.1 Analysis of the Full Model

All 10 predictor variables were included in the full model. The regression coefficients of intercept and 10 predictor variables are shown in <Table 2>. The coefficient of multiple determination (R^2) and mean squared error (*MSE*) for the full model are 0.7347 and 19.3462, respectively.

<Table 2> Regression Coefficients for the Full Model

Regression	n C	oeff	ficients
PCTFAT	VS.	10	Independents

	Coefficient	Std. Error	Std. Coeff.	t-Value	P-Value
Intercept	6.643	6.714	6.643	0.99	0.3234
NECKCIR	-0.629	0.225	-0.183	-2.792	0.0057
CHESTCIR	-0.098	0.092	-0.099	-1.061	0.2899
ABDOCIR	1.036	0.077	1.335	13.401	<.0001
HIPCIR	-0.419	0.122	-0.359	-3.446	0.0007
THIGHCIR	0.128	0.135	0.081	0.952	0.3422
KNEECIR	-0.075	0.229	-0.022	-0.328	0.7435
ANKLECIR	-0.001	0.219	-1.35E-04	-0.003	0.9976
BICEPCIR	0.136	0.172	0.049	0.79	0.4303
ARMCIR	0.358	0.2	0.086	1.786	0.0753
WRISTCIR	-1.486	0.509	-0.166	-2.918	0.0039

3.1.2 Variable Selection

Roche indicated that there were no known pairs of body circumference measures that were good predictor of total body composition [11]. Therefore all possible regression models were investigated. RSQUARE procedure of SAS 9.2 performs all possible regressions for a collection of independent variables (SAS Institute Inc., 2009). Using the RSQUARE procedure and options, R^2 , *Adjusted* R^2 , C_p , and *MSE* statistics were obtained for all possible models and data for 5, 6, and 7 variable models with high R^2 are presented in <Table 3>. Here, C_p was introduced by Mallows as a criterion for selecting a regression model [6]. The model with little bias tends to be near the line $C_p = p$. The first five predictor variable model (i.e., the model with *neckcir, abdocir, hipcir, armcir, and wristcir*) was selected based on the parsimony principle. This model was subjected to the further analysis presented in next section.

3.2 Reduced Model (Model with 5 Predictor Variables)

The model with 5 predictor variables (*NECKCIR, ABDOCIR, HIPCIR, ARMCIR, AND WRISTCIR*) selected from the initial model was subjected to further analysis. As shown in the <Table 4>, all 5 variables were significant at significance level of 0.05. The coefficient of multiple determination (R^2) and mean squared error (*MSE*) for the reduced model are 0.7312 and 19.2103, respectively. The model is summarized as follows :

PCTFAT = 2.704-0.601NECKCIR+0.974ABDOCIR -0.332HIPCIR+0.409ARMCIR -1.618WRISTCIR+ ε

<Table 4> Regression Coefficients for the Reduced Model

Regression	Coefficients	

PCTFAT	VS.	5	Independents
--------	-----	---	--------------

	Coefficient	Std. Error	Std. Coeff.	t-Value	P-Value
Intercept	2.704	6.231	2.704	0.434	0.6647
NECKCIR	-0.601	0.215	-0.175	-2.798	0.0056
ABDOCIR	0.974	0.056	1.255	17.301	< .0001
HIPCIR	-0.332	0.083	-0.284	-3.977	< .0001
ARMCIR	0.409	0.182	0.099	2.249	0.0254
WRISTCIR	-1.618	0.462	-0.18	-3.503	0.0005

<Table 3> R^2 , Adjusted R^2 , Cp, MSE Statistics for 5, 6, and 7 Variable Models (Only Two Models of Each Size that Produced Highest R^2 are Included. Other Subsets of Predictor Variables with Low R^2 are not Included.)

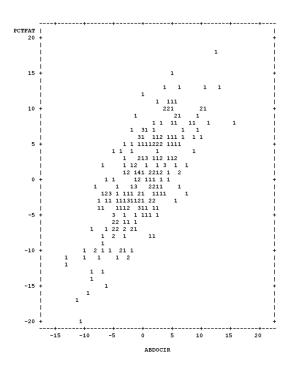
Number of variables in Model	R ²	Adjusted R ²	Ср	MSE	Variables in Model			
5	0.7312	0.7257	4.2715	19.21026	NECKCIR ABDOCIR HIPCIR ARMCIR WRISTCIR			
5	0.7289	0.7234	6.3156	19.37102	NECKCIR ABDOCIR HIPCIR BICEPCIR WRISTCIR			
6	0.7330	0.7264	4.6505	19.16067	NECKCIR ABDOCIR HIPCIR THIGHCIR ARMCIR WRISTCIR			
6	0.7324	0.7258	5.1541	19.20044	NECKCIR CHESTCIR ABDOCIR HIPCIR ARMCIR WRISTCIR			
7	0.7339	0.7263	5.7501	19.16781	NECKCIR CHESTCIR ABDOCIR HIPCIR THIGHCIR ARMCIR WRISTCIR			
7	0.7338	0.7261	5.9188	19.18118	NECKCIR CHESTCIR ABDOCIR HIPCIR BICEPCIR ARMCIR WRISTCIR			

3.2.1 Evaluation of Assumptions for the Reduced Model

In general, a linear regression modeling consists of various assumptions regarding the anticipated model. These include assumptions for linearity, constant variance, and normality. In order to see whether a particular variable should enter linearly or not, partial residual plots were examined. An example plot for *PCTFAT* vs. *ABDOCIR* is shown in <Figure 2>. No visible curvature supports that linear terms are adequate. The linearity assumption for the reduced model is not violated.

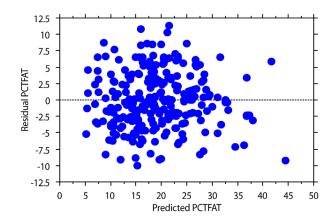
To check the constant variance assumption, the plot of Residual vs. Predicted Value was generated as presented in <Figure 3>. No visible systematic pattern indicates that the constant variance assumption is not violated.

To identify unusual outliers, the studentized residual and Cook's D statistics were investigated. Here, Cook's D_i is an overall measure of influence of the ith observation on the estimated regression coefficients [6]. All the studentized residual were less than 3. The high absolute values of studentized residual were 2.527, 2.482, and 2.613 for obs 39, 82, and 207 respectively. The Cook's D values were less than 1. The highest values among them were 0.450, 0.026, and 0.026 for obs 39, 82, and 207 respectively. Based on this analysis, no outliers were found. To support this finding, a normal probability plot was investigated and presented in



<Figure 2> Partial Residual Plot for PCTFAT vs. ABDOCIR (Reduced Model)

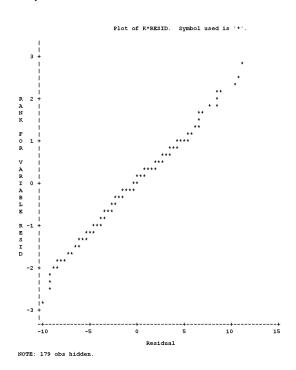
<Figure 4>. The linear relationship on the figure shows the normality.



<Figure 3> Plot of Residual vs. Predicted Value of PCTFAT (Reduced Model)

3.2.2 Multicollinearity

As introduced in the full model, Variance inflation factors (VIFs) were investigated to check the possible multicollinearity (MC) problems in the reduced model (see $\langle Table 5 \rangle$). The highest VIF was 4.81776 for *ABDOCIR* (which is less than 10). Even though MC exists, the degree of MC was not significant. As compared to the full model, the VIFs were relatively small.



<Figure 4> Normal Probability Plot (Reduced Model)

Variable	Variance Inflation Factor (VIF)
NECKCIR	3.56499
ABDOCIR	4.81776
HIPCIR	4.66234
ARMCIR	1.76908
WRISTCIR	2.43006

<Table 5> Variance Inflation Factors (VIFs) of Each Predictor Variable in the Reduced Model

3.2.3 Interactions

As mentioned in the initial model, since there are no known pairs of variables that interact with each other, no interaction terms are included. Although no interaction term was anticipated, all possible two way interaction terms of the reduced model were investigated as shown in <Table 6>. No interaction term was significant at 5% significance level although *abdocir×armcir* (p = 0.0517) was marginal.

4. Discussion and Conclusion

Roche (1996) revealed that relatively accurate estimates of body composition for men were found with bicep circumference, hip circumference, abdomen circumference, and arm circumference. However, there were no known pairs of variables that were good predictor of total body composition [11]. Based on the data analysis in the present study, 5 variables selected were hip circumference, abdomen circumference, arm circumference, neck circumference, and wrist circumference. The bicep circumference did not turn out to be a good predictor of body fat percentage. This may be due to the high variability between individuals in developing biceps muscles (e.g. different physical training among individuals).

It was also anticipated that all signs of regression coefficients were to be positive because all anthropometric circumferences seemed to increase as the percentage of body fat increases. However, the signs of NECKCIR, HIPCIR, and WRISTCIR were negative. I suspected some wrong signs due to the multicollinearity (MC) between predictor variables. However, the results of VIF analysis showed no MC problems. Therefore, I conclude that partial relationships are different from marginal relationships.

The fitted prediction model is : % Body Fat = 2.704 - 0.601 (Neck Circumference)+0.974 (Abdominal Circumference) - 0.332 (Hip Circumference) + 0.409 (Arm Circumference) - 1.618 (Wrist Circumference) + ε . This model can now be used to estimate the percentage of body fat simply using a scale and a measuring tape. The units are percent (%) for the percentage of body fat and cm for the body circumference. In taking of circumference measurements, the tape measure should be placed as follows [10] :

- Neck circumference : The tape measure is placed in a horizontal plane at the level of the widest part of the neck as seen from the front aspect.
- (2) Abdomen circumference : The tape measure is positioned horizontally at the level of the greatest anterior extension of the abdomen.
- (3) Hip circumference : The tape measure is placed in a horizontal plane at the level of maximum extension of the buttocks.
- (4) Forearm circumference : The tape measure is placed

<Table 6> ANOVA Table for the Reduced Model (5 Predictor Model) with Two-Way Interaction Terms

Source	DF	Type III SS	Mean Square	F Value	Pr > F
NECKCIR	1	3.6830151	3.6830151	0.20	0.6545
ABDOCIR	1	36.5549002	36.5549002	1.99	0.1594
HIPCIR	1	117.9348047	117.9348047	6.43	0.0119
ARMCIR	1	10.5863355	10.5863355	0.58	0.4482
WRISTCIR	1	23.3122957	23.3122957	1.27	0.2608
NECKCIR×ABDOCIR	1	4.0495828	4.0495828	0.22	0.6389
NECKCIR×HIPCIR	1	2.2193700	2.2193700	0.12	0.7283
NECKCIR×ARMCIR	1	3.1197088	3.1197088	0.17	0.6804
NECKCIR×WRISTCIR	1	0.0004446	0.0004446	0.00	0.9961
ABDOCIR×HIPCIR	1	13.6636701	13.6636701	0.74	0.3890
ABDOCIR×ARMCIR	1	9.2251404	9.2251404	0.50	0.4790
ABDOCIR×WRISTCIR	1	70.1320448	70.1320448	3.82	0.0517
HIPCIR×ARMCIR	1	4.7462449	4.7462449	0.26	0.6115
HIPCIR×WRISTCIR	1	59.0806426	59.0806426	3.22	0.0740
ARMCIR×WRISTCIR	1	13.7633926	13.7633926	0.75	0.3873

around the proximal part of the forearm, perpendicular to its long axis, at the level of maximum circumference.

(5) Wrist circumference : The tape measure is placed perpendicular to the long axis of the forearm and in the same plane on the anterior and posterior aspects of the wrist.

Some limitations of the present study should be addressed in future work. The underwater weighing technique is not free from measurement error. The measurement errors associated with the underwater weighing technique are mainly due to the errors in residual volume in the lung. Consequently, the errors associated with residual volume can have a considerable effect on body density [9, 10]. Another possible error results from the conversion of body density to percent fat. Although universally accepted, Siri's equation is based on the results of direct compositional analysis of human cadavers, but only a few cadavers were used and they did not represent a distribution of the normal population. Measuring anthropometric circumferences is also not free from errors associated with measurement devices and techniques. In addition, the model did not consider age and sex. The anthropometric circumferences may vary with the age, sex, and the race. The data collected were for only men within the unspecified races. For future research, it would be interesting to investigate the effect of age, sex, and the race in predicting the percentage of body fat.

Acknowledgement

This paper has been supported by 2014 Hannam University Research Fund.

References

- [1] Bary, G.A., Complications of Obesity. Annals of Internal Medicine, 1985, Vol. 103, pp. 1052-1062.
- [2] Craig, B.N., Congleton, J.J., Kerk, C.J., Amendola, A. A., and Gaines, W.G., Personal and non-occupational risk factors and occupational injury/illness. *Am J Ind Med*, 2006, Vol. 49, No. 4, pp. 249-260.
- [3] Hamilton, M.A., Strawdermana, L., Babski-Reevesa, K., and Hale, B., Effects of BMI and task parameters on joint angles during simulated small parts assembly. *International Journal of Industrial Ergonomics*, 2013, Vol. 43, No. 5, pp. 417-424.
- [4] Kerr, M.S., Frank, J.W., Shannon, H.S., Norman, R.W., Wells, R.P., Neumann, P., and Bombardier, C., Biomechanical and psychosocial risk factors for low back pain

at work. *American Journal of Public Health*, 2001, Vol. 91, No. 7, pp. 1069-1075.

- [5] Lohman, T.G., Boileau, R.A., aand Massey, B.H., Prediction of Lean Body Weight in Young Boys from Skinfold Thickness and Body Weight. *Human Biology*, 1975, Vol. 47, pp. 245-262s.
- [6] Neter, J., Kuter, M.H., Nachtsheim, C.J., and Wasserman, W., *Applied Linear Regression Models*. 3rd ed. Burr Ridge, Illinois : Irwin, 1996.
- [7] Ostbye, T., Dement, J.M., and Krause, K.M., Obesity and workers' compensation : results from the Duke Health and Safety Surveillance System. *Arch Intern Med*, 2007, Vol. 167, No. 8, pp. 766-773.
- [8] Penrose, K., Nelson, A., and Fisher, A., Generalized Body Composition Prediction Equation for Men Using Simple Measurement Techniques (abstract). *Medicine* and Science in Sports and Exercise, 1985, Vol. 17, No. 2, p. 189.
- [9] Pollack, K.M., Sorock, G.S., Slade, M.D., Cantley, L., Sircar, K., Taiwo, O., and Cullen, M.R., Association between body mass index and acute traumatic workplace injury in hourly manufacturing employees. *Am J Epidemiol*, 2007, Vol. 166, No. 2, pp. 204-211.
- [10] Pollock, M.L., The Measurement of Body Composition. In P. J. Maud and C. Foster (Eds.), *Physiological Assessment of Human Fitness*, Champaign, IL : Human Kinetics, 1995, pp. 167-204.
- [11] Roche, A.F., Anthropometry and Ultrasound. In A.F. Roche, S.B. Heymsfield, and T.G. Lohman (Eds.). Human Body Composition, Champaign, IL : Human Kinetics, 1996, pp. 167-189.
- [12] SAS Institute, Inc., SAS/STAT9.2 User's Guide, Cary, NC : Author; 2009.
- [13] Siri, W.E., Body Composition from Fluid Space and Density. In J. Brozek and A. Henschel (Eds.), Techniques for measuring body composition Washington, DC : *National Academy of Science*, 1961, pp. 223-224.
- [14] Trogdon, J.G., Finkelstein, E.A., Hylands, T., Dellea, P.S., and Kamal-Bahl, S.J., Indirect costs of obesity : a review of the current literature. *Obes Rev*, Vol. 9, No. 5, 2008, pp. 489-500.
- [15] University of Arizona, Mathematics Department homepage. http://math.arizona.edu/~jwatkins/505d/body.htm. 2014.

ORCID

Sung Ha Park | http://orcid.org/0000-0002-9983-9951