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Optimal Ordering Policy in Dual-Sourcing Supply Chain Considering Supply Disruptions and Demand Information

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ABSTRACT

It is necessary for retailers to determine the optimal ordering policy of products considering supply disruptions due to a natural disaster and a production process failure as quality and machine breakdowns. Under the situation, a dual-sourcing supply chain (DSSC) is one of effective SC for retailers to order products reliably. This paper proposes the optimal ordering policy of a product in a DSSC with a retailer and two manufacturers. Two manufacturers may face supply disruptions due to a natural disater and a production process failure after they received the retailer's order of products. Here, two scenarios of demand information of products are assumed: (i) the demand distribution is known (ii) mean and variance of the demand are known. Under above situations, two types of DSSC are discussed. Under a decentralized DSSC (DSC), a retailer determines the optimal ordering policy to maximize his/her total expected profit. Under the integrated DSSC (ISC), the optimal ordering policy is determined to maximize the whole system's total expected profit. Numerical analysis investigates how demand information and supply disruptions affect the optimal decisions under DSC and ISC. Besides, profitability of supply chain coordination adjusting the wholesale price is evaluated to encourage the optimal decision under ISC.

Keywords: Supply Disruptions, Dual-Sourcing, Ordering Policy, Supply Chain Coordination, Game Theory

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1. INTRODUCTION

An 8.9-magunitude earthquake hits off the east coast of Japan at 11th March, 2011. The earthquake triggered a large tsunami that battered many buildings there. Specifically, when the massive earthquake hits industrial plants, plants for production and distribution chains may be damaged and this may cause supply disruptions. Thus, recent catastrophic events, such as the recent earthquake and tsunami in Japan, underline the dramatic effect of supply disruptions in global supply chain (Zeiler, 2011; Whipp, 2011; MacKenzie *et al*., 2012; Jabbarzadeh *et al*., 2012; Xanthopoulos *et al*., 2012). If a great natural disaster occurs, it will be possible to occur the following things during several months after the earthquake:

(i) the shortage of products such as materials, components and final products due to reduction of the production quantity,
(ii) delay of due date of products due to traffic disturbance,
(iii) delay of product sales.

Such a great natural disaster has ever occurred and in not only Japan but also other foreign countries such as the United States, Turkey, Indonesia, Philippines, etc. The earthquake will occur in many countries where a natural disaster is likely to occur in the future. So, it is important for decision makers to consider the impacts of disasters on the optimal inventory policy of a variety of products.

Also, supply chain management assumes a major role in the logistics activities associated with responding to disasters caused by hazards such as major hurricanes, earthquakes and so on. Regarding the effect of natural disasters on the supply disruptions, there are some previous papers (Ozbay and Ozguven, 2007; Lodree Jr. and Taskin, 2008; Dillon and Mazzola, 2010; Li *et al*., 2012). Ozbay and Ozguven (2007) focused on the effects of natural disasters on the delivery and consumption processes in an efficient and quick-response humanitarian inventory management model. Both delivery and consumption are modeled as stochastic processes. This previous study determined the minimal safety stock level of an inventory to prevent the disruption. The Hungarian inventory Control Model was used to solve this optimal decision. Lodree Jr. and Taskin (2008) pointed out that supply chain and logistics response to disaster could be very costly. So, this previous paper discussed emergency response supply chain decisions with hazards that created the need for disaster-relief operations. The optimal inventory level were determined in consideration of the uncertainty in product demands due to hazards.

In additions to supply disruptions due to natural disasters, there are supply uncertainty in supply chains due to various failures such as machine breakdowns in production processes of suppliers and manufacturers. Here, supply uncertainty is a major issue in both the industrial and academic worlds. Most of the papers considering supply uncertainty focus on the production planning problems of uncertain production capacity, random manufacturing yield, or unreliable supplier (Gerchak *et al*., 1988; Parlar and Perry, 1996; Zimmer, 2002; Gupta and Cooper, 2005; Chopra *et al*., 2007; Serel, 2008; He and Zhang, 2008; Keren, 2009; Pac *et al*., 2009; Xu, 2010; He and Zhang, 2010; Li *et al*., 2010; Giri, 2011; He and Zhao, 2012; Li *et al*., 2012; Tang *et al*., 2012a, 2012b; Xanthopoulos *et al*., 2012; Li *et al*., 2013). Giri (2011) focused on an inventory model for a single type of products and a single period in which the retailer could source from two suppliers. This paper assumed that the primary supplier was cheaper but unreliable in the sense that it generated supply yield uncertainty, whereas the secondary supplier was perfectly reliable but more expensive. The reliable supplier's capacity was fixed and the retailer could place an order more than the quantity reserved in advance. He and Zhao (2012) studied the supplier's raw-material production-planning decision, the retailer's replenishment decision, and the choice of contract for a three-tier supply chain with uncertainty about both demand and raw-material yield. This previous paper investigated contracts terms that coordinated raw-material planning and replenishment decisions and achieved the most efficient performance of the entire supply chain. The production-decision problem was discussed for a risk-averse supplier who faced supply uncertainty.

Also, it has been a significant problem to design coordination schemes under supply uncertainty so as to increase supply chain performance and properly allocate the supply risk between channel members (Gurnani and Gerchak, 2007; Guler and Bilgic, 2009; Yan *et a*l., 2010; He and Zhao, 2012; Li *et al*., 2013; Xu and Lu, 2013; Dillon and Mazzola, 2010). Li *et al*. (2013) discussed a generalized supply chain model subject to supply uncertainty after the supplier chose the production input level. This previous paper focused on the interface between supply uncertainty and supply chain coordination. The underlying double marginalization of a supply chain was explored, considering supply uncertainty and the related problem of coordination contract design. Xu and Lu (2013) considered a price-setting newsvendor model in which a firm needed to make joint inventory and pricing decisions before the selling season. Here, the supply process was uncertain since the received quantity was considered from both the product of the order quantity and a random yield rate. Dillon and Mazzola (2010) focused on the situation where many retailers diversified their supply disruption risk by sourcing from multiple suppliers. This previous paper discussed a global supply chain (GSC) with supply disruptions due to a natural disaster or a human-induced event. The concept of a supply-risk network was introduced in order to capture potential disruptions. The optimal selection regarding suppliers in GSC was determined.

The previous studies mentioned above incorporated supply disruptions into both production processes and distribution processes in a supply chain model. However, they did not discuss concretely processes of supply disruptions which occurred in a SC/GSC. Furthermore, suppliers and manufacturers may face supply disruptions including natural disasters and the failures in production processes. It is necessary for retailers to consider the effects of supply disruptions on their optimal ordering policy as to the relevant process where supply disruptions may occur. Xanthopoulos *et al*. (2012) tackled jointly supply chain disruption management and risk aversion issues for supply chain procurement. Generic single period (newsvendor-type) inventory models was proposed in order to capture the trade-off between inventory policies and disruption risks in a dual-sourcing supply chain network under both no constraint and service level constraints. It was considered that supply disruptions occurred due to natural disasters and machine breakdowns in the dual-sourcing supply chain network. Here, it was assumed that the individual event probability of a supply disruption in each supply channel was different in the dual-sourcing supply chain. However, in this previous paper, the event probability of the supply disruption in each supply channel was not distinguished between a natural disaster and a failure in a production process. Also, this previous paper did not compare the optimal inventory management with supply disruptions under a centralized supply chain with those under a decentralized supply chain. Li *et al*. (2010) investigated the sourcing strategy of a retailer and the pricing strategies of two suppliers in a supply chain under an environment of supply disruption. This previous paper characterized the sourcing strategies of the retailer in a centralized supply chain (CSC) and a decentralized supply chain (DSC). In a decentralized supply chain, it was assumed that two suppliers were either competitive or cooperative. The optimal order quantities and the optimal wholesale prices were found in both CSC and DSC. It was assumed that any supplier whose location faced a natural disaster could not produce products and a retailer could not order products from the relevant supplier there, and then the retailer procured the shortage of the required quantity of product from a spot market. However, this previous paper assumed that the event probabilities of not only natural disasters in locations of two suppliers, but also failures of their production processes were same probability. Under the assumption, it was impossible to identify the event probabilities of supply disruptions which each supplier might face. It was hard for a retailer to determine the optimal ordering policies to multiple suppliers whose event probabilities are different. Also, the previous papers of Xanthopoulos *et al*. (2012) and Li *et al*. (2010) did not discuss any fixed cost including the equipment machine, the employment cost and the electricity charges, etc in production cost of productions. Also, supply chain coordination (SCC) was not considered and these previous papers did not consider the limitation of demand information of products. A demand of a single type of products is assumed a random variable, and the probability distribution of the demand is known. This implies that it is possible to know the full information of the product demand. In a real situation for SCM, it may be possible to know the limited information such as mean and variance of the product demand. Under such a situation, Gallego and Moon (1993), Moon and Gallego (1994), Moon and Choi (1995), Alfares and Elmorra (2005) applied the distribution-free approach (DFA) into the newsboy problem for a single type of products in a single period.

Differing from the previous papers above, this paper verifies theoretically both the optimal ordering policy and supply chain coordination in a dual sourcing supply chain (DSSC) with a retailer and two manufacturers in consideration of the uncertainty in product demand and supply disruptions after two manufacturers received the retailer's order of a single type of products. Concretely, this paper tries to provide the following contributions for academic researchers and real-world policymakers regarding operations in a DSSC with the uncertain demand and supply disruptions: Presentations of theoretical analysis to

- verify how the uncertainty in product demand affect the optimal ordering policy in a DSSC by considering two scenarios of the demand information of products under supply disruptions: (scenario 1): the demand distribution is known and (scenario 2): only both mean and variance of the demand are known.
- verify the lower limit of the total expected profits of a retailer and the whole system which face the uncertainty in the product demand and supply disruptions in a DSSC by using DFA.
- verify how the different event probabilities of supply disruptions regarding a natural disaster and a failure in the production process affect the optimal ordering policy in a DSSC as to the demand information of products.
- verify how the separation between variable cost and fixed cost in production cost as to a situation without/with supply disruptions affects the optimal ordering policy in a DSSC as to the demand information of products.
- provide how not only supply chain coordination in a DSSC regarding the unit wholesale prices between a retailer and two manufacturers is incorporated into the optimal decision of ISC, but also it can shift to the optimal decision under ISC from that under DSC.

Concretely, this paper discusses a dual-sourcing supply chain (DSSC) which consists of a retailer and two manufacturers. The DSSC faces both the uncertainty in product demand and supply disruptions due to a natural disaster and a failure in the production process after two manufacturers received the retailer's order of a single type of products. In terms of supply disruptions, this paper considers a natural disaster which occurs in the location of each manufacturer and a failure in the production process such as quality and machine breakdowns. Here, it is assumed that the event probabilities of two manufacturers regarding supply disruptions including a natural disaster and a production process failure are different. When a natural disaster occurs to each manufacturer, it is assumed that it is impossible for relevant manufacturer to produce any product and when a failure in production process occurs to each manufacturer, it is assumed that it is possible for the relevant manufacturer to produce and supply some rate of the retailer's order quantity. Also, it is necessary for two manufacturers to separate between variable cost and fixed cost in production cost of products in the situation where they may face supply disruptions in their production processes. This is because two manufacturers always incur the fixed cost in the production cost of products despite the production quantity of products.

As to two situations: demand information of the product and supply disruptions due to a natural disaster and a failure in the production process, this paper proposes two types of the optimal decisions for a DSSC: a decentralized DSSC (DSC) and an integrated DSSC (ISC). Here, the optimal decision under DSC in this paper adopts the Stackelberg game (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Lee and Ammons (2011), Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Su, 2012). This paper considers that a retailer is a leader of the decision-making and two manufacturers are followers of the retailer's decision-making. Under DSC, the optimal ordering policy is decided to maximize the total expected profit of a retailer. Under ISC, a policy-maker under ISC decides the optimal ordering policy to maximize the total expected profit of the whole system which is the sum of the total expected profits of all members in the DSSC as to above situations.

Besides, profit sharing is discussed as supply chain coordination (SCC) between a retailer and two manufacturers in order to guarantee more profits to all members when the optimal decision under ISC is adopted. Concretely, the following two approaches of profit sharing are discussed under the optimal decision under ISC:

- Profit sharing I: coordinating the unit wholesale price between a retailer and two manufacturers under the optimal decision of ISC as Nash bargaining solution considering profit balance.
- Profit sharing II: coordinating the unit wholesale price of each manufacturer by combining Profit sharing I and the magnitude relation between the total expected profit of a retailer and the sum of the total expected profits of two manufacturers.

Using the numerical examples, the numerical analysis illustrates how four factors: (i) event probability of a natural disaster, (ii) event probability of a failure in production process, (iii) production ratio after a failure occurs in production process and (iv) demand information of the products, affect the optimal ordering policies to two manufacturers under DSC and ISC. Also, the optimal order quantities and the total expected profits under DSC are compared with those under ISC. The effects of profit sharing I and II as SCC on the total expected profits of members under the optimal decision of ISC are verified. The benefit of Profit sharing I is compared with that of Profit sharing II in aspects of the profit of a retailer who is a leader of the decision-making under DSC.

The contribution of this paper is to provide the following managerial insights from the outcomes obtained from the theoretical research and the numerical analysis to academic researchers and real-world policymakers regarding operations in a DSSC with the uncertain demand and supply disruptions:

- The optimal order quantities to two manufacturers in the scenario 2, where only both mean and variance are known, are determined as lower values than the scenario 1, where the distribution of product demand is known. This is due to the situation where the optimal ordering policy in the scenario 2 is made under the worst situation where a retailer obtains the lowest total expected profit.
- The optimal order quantities to two manufacturers under ISC can be determined as larger values than those under DSC even if demand information of products is limited and supply disruptions occur. Therefore, the optimal ordering policy under ISC can encourage the more assured procurement of product in a DSSC even under uncertainty in product demand and supply disruptions.
- It is possible to guarantee to bring more profits to all members (a retailer and two manufacturers) in a DSSC by supply chain coordination adjusting the unit wholesale prices of two manufacturers between all members under the optimal ordering policy under ISC.

The rest of this paper is organized as follows: in Section 2, operational flows of a DSSC addressed in this paper is described. In Section 3, model assumptions of the DSSC in this paper are described. In Section 4, the expected profits and the total expected profits of a retailer and two manufacturers and the whole system in the DSSC with the uncertain demand and supply disruptions are described. In Section 5 proposes the optimal ordering policy of a single type of products under DSC with the uncertain demand and supply disruptions. Also, Section 6 proposes the optimal ordering policy of a single type of products under ISC with the uncertain demand and supply disruptions. Section 7 incorporates two types of profit sharing approaches as SCC in order to encourage the optimal decision under ISC, guaranteeing the more total expected profits of all members under ISC than under DSC. Section 8 shows the results in numerical analysis to illustrate not only the optimal ordering policy in a DSSC with the uncertain demand, supply disruptions, but also the benefit of SCC in a DSSC. In Section 9, conclusions, managerial insights and future researches for this paper are summarized.

2. OPERATIONAL FLOWS OF A DSSC

(1) A dual-sourcing supply chain (DSSC) consists of a retailer and two manufacturers (M_1 and M_2). Each manufacturer produces a single type of products and a retailer sells them in a market. Two types of supply disruptions may occur to each manufacturer in the DSSC. One is a natural disaster which may occur in the location of each manufacturer. The other one is a production process failure which may occur in the production process of each manufacturer due to quality or machine breakdown.

(2) A retailer orders a single type of products to manufacturer M_i (i = 1, 2) under the uncertainty in product demand. The product order quantity to M_i is Q_i (i = 1, 2).

(3) M_i (i = 1, 2) produces the products with the unit production cost c_i (i = 1, 2)($c_1 < c_2$). When a natural disaster occurs to the location of M_i, it is impossible to produce any product. When a production process failure occurs in the production process of M_i, the production ratio of M_i is reduced to $y_i (0 \le y_i \le 1)$.

(4) M_i (i = 1, 2) sells the products to the retailer with the unit wholesale price w_i (i = 1, 2)($w_1 < w_2$).

(5) The retailer sells the products in a market with the unit sales price s of the product. The retailer sells the unsold products at the unit disposal sales price r and incurs the unit shortage penalty cost k of the unsatisfied demand.

3. MODEL ASSUMPTIONS IN DSSC

(1) Two scenarios of the demand information of a single

type of products are assumed: in scenario 1 (j = 1), the demand x follows a probabilistic distribution which is the probability density function, $f(x)(0 \le f(x) < 1)$, is known, and in scenario 2 (j = 2), only mean $\mu(>0)$ and variance $\sigma^2(>0)$ of the demand x.

(2) Manufacturer 1(M_1) and Manufacturer 2(M_2) are located in distance. It is assumed that the same natural disaster doesn't occur to both manufacturers simultaneously, that is, the event probability of a natural disaster $\alpha_i (i=1, 2)$ $(0 \le \alpha_i \le 1)$ which may occur to manufacturer M_i (i = 1, 2) is independent.

(3) The event probability of a production process failure $\beta_i (i=1, 2)$ $(0 \le \alpha_i \le 1)$ which may occur to M_i (i = 1, 2) is independent.

(4) A retailer can predict event probabilities of supply disruptions, caused by a natural disaster and a production process failure which may occur to M_i (i = 1, 2), as $\alpha_i (0 \le \alpha_i \le 1)$ and $\beta_i (0 \le \beta_i \le 1)$, respectively. Concretely, from 2. (3), nine types of event regarding supply disruptions occurring to manufacturer 1(M_1) and manufacturer 2(M_2) are considered in this paper. Here, it is assumed that the event probability of each supply disruption is independent. Table 1 shows events and event probabilities of supply disruptions and productive quantity of product of M_1 and M_2 when a natural disaster and a production process failure occur to M_i (i = 1, 2).

(5) When a natural disaster occurs in the location of M_i (i = 1, 2), it is impossible for the relevant M_i to produce products. In this case, M_i incurs no variable cost for production of products. Also, it is impossible for the retailer to procure any products from the relevant M_i who faces a natural disaster. Meanwhile, when a production process failure occurs to M_i, the production quantity of the relevant M_i reduces to y_iQ_i (i = 1, 2, $y_i (0 \le y_i \le 1)$). Here, it is assumed that M_i incurs the fixed production cost $\gamma c_i Q_i (i=1, 2,\ 0 \le \gamma \le 1)$ regardless if a natural disaster and a production process failure occurs.

(6) The condition $s \ge k > w_i > c_i \ge r\,(i=1, 2)$ is satisfied so as to guarantee profitable operations of a retailer, manufacturer M_i (i = 1, 2) and the whole system in a DSSC.

4. EXPECTED PROFITS AND TOTAL EXPECTED PROFITS IN A DSSC

From 2., 3. and Table 1, the expected profits of a retailer, two manufacturers and the whole system are formulated according to events E1-E9 of supply disruptions to manufacturer M_i (i = 1, 2) as to the demand information of a single type of products. First, the profit of a retailer is discussed. The profit of a retailer is formulated from the sales of the products, the procurement cost of the products, the disposal sales of the unsold products and the shortage penalty cost for the unsatisfied demand by considering the magnitude relation between order quantity Q_i (i = 1, 2) to manufacturer M_i (i = 1, 2) and product demand x.

In scenario 1 of demand information of the products, by taking the expectation of the demand x based on a probabilistic distribution $f(x)$ of x, the individual expected profit of a retailer as to events E1-E9 is obtained as follows:

$$E^1[\pi_R^{E1}(Q_1, Q_2)] = \int_0^{Q_1+Q_2}[sx + r(Q_1+Q_2-x)]f(x)dx + \int_{Q_1+Q_2}^{\infty}[s(Q_1+Q_2) - k(x-Q_1-Q_2)]f(x)dx - (w_1Q_1 + w_2Q_2) \tag{1}$$

$$E^1[\pi_R^{E2}(Q_1, Q_2)] = \int_0^{Q_2}[sx + r(Q_2-x)]f(x)dx + \int_{Q_2}^{\infty}[sQ_2 - k(x-Q_2)]f(x)dx - w_2Q_2 \tag{2}$$

Table 1. Events, probabilities of supply disruptions and productive quantity (PQ) when a natural disaster (ND) and a production process failure (PPF) occur to manufacturer 1 and manufacturer 2

Type of event	Relation between events of supply disruptions and production quantity of each manufacturer						Event probability $P_i (i=1, 2, \cdots, 9)$ of event E_i of supply disruptions
	Manufacturer 1 (M_1)			Manufacturer 2 (M_2)			
	ND	PPF	PQ	ND	PPF	PQ	
E1	No	No	Q_1	No	No	Q_2	$P_1=(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)$
E2	Yes	No	0	No	No	Q_2	$P_2=\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)$
E3	No	Yes	y_1Q_1	No	No	Q_2	$P_3=(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)$
E4	No	No	Q_1	Yes	No	0	$P_4=(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)$
E5	No	No	Q_1	No	Yes	y_2Q_2	$P_5=(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2$
E6	Yes	No	0	No	Yes	y_2Q_2	$P_6=\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2$
E7	No	Yes	y_1Q_1	Yes	No	0	$P_7=(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)$
E8	No	Yes	y_1Q_1	No	Yes	y_2Q_2	$P_8=(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2$
E9	Yes	No	0	Yes	No	0	$P_9=\alpha_1(1-\beta_1)\alpha_2(1-\beta_2)$

$$E^1[\pi_R^{E3}(Q_1,Q_2)] = \int_0^{y_1Q_1+Q_2}[sx + r(y_1Q_1+Q_2-x)]f(x)dx + \int_{y_1Q_1+Q_2}^{\infty}[s(y_1Q_1+Q_2) - k(x-y_1Q_1-Q_2)]f(x)dx - (w_1y_1Q_1 + w_2Q_2) \quad (3)$$

$$E^1[\pi_R^{E4}(Q_1,Q_2)] = \int_0^{Q_1}[sx + r(Q_1-x)]f(x)dx + \int_{Q_1}^{\infty}[sQ_1 - k(x-Q_1)]f(x)dx - w_1Q_1 \quad (4)$$

$$E^1[\pi_R^{E5}(Q_1,Q_2)] = \int_0^{Q_1+y_2Q_2}[sx + r(Q_1+y_2Q_2-x)]f(x)dx + \int_{Q_1+y_2Q_2}^{\infty}[s(Q_1+y_2Q_2) - k(x-Q_1-y_2Q_2)]f(x)dx - (w_1Q_1 + w_2y_2Q_2) \quad (5)$$

$$E^1[\pi_R^{E6}(Q_1,Q_2)] = \int_0^{y_2Q_2}[sx + r(y_2Q_2-x)]f(x)dx + \int_{y_2Q_2}^{\infty}[sy_2Q_2 - k(x-y_2Q_2)]f(x)dx - w_2y_2Q_2 \quad (6)$$

$$E^1[\pi_R^{E7}(Q_1,Q_2)] = \int_0^{y_1Q_1}[sx + r(y_1Q_1-x)]f(x)dx + \int_{y_1Q_1}^{\infty}[sy_1Q_1 - k(x-y_1Q_1)]f(x)dx - w_1y_1Q_1 \quad (7)$$

$$E^1[\pi_R^{E8}(Q_1,Q_2)] = \int_0^{y_1Q_1+y_2Q_2}[sx + r(y_1Q_1+y_2Q_2-x)]f(x)dx + \int_{y_1Q_1+y_2Q_2}^{\infty}[s(y_1Q_1+y_2Q_2) - k(x-y_1Q_1-y_2Q_2)]f(x)dx - (w_1y_1Q_1 + w_2y_2Q_2) \quad (8)$$

$$E^1[\pi_R^{E9}(Q_1,Q_2)] = -\int_0^{\infty}kxdx. \quad (9)$$

Here, Eq. (1) shows the expected profit of a retailer in event $E1$ where neither a natural disaster nor a production process failure occur to manufacturer 1(M_1) and manufacturer 2(M_2). Using Table 1, in Eq. (1), the first term is the sum of the expected sales of the products and the expected disposal sales of the unsold products when the demand x is satisfied with the sum of order quantities to two manufacturers, M_1 and M_2, Q_1+Q_2, the second term is the sum of the expected sales of the products and the expected shortage penalty cost for the unsatisfied demand when the demand x is unsatisfied with Q_1+Q_2, the third term is the procurement cost of the products from M_1 and M_2. Similarly, using Table 1, the individual expected profit of a retailer in Eqs. (2)-(9) as to events $E2$-$E9$ are obtained as to the situations if a natural disaster and a production process failure occur to M_1 and M_2. Here, note that the following expected quantities of M_1 and M_2 change as to events $E2$-$E9$, using Table 1: (1) the sales quantity of the products, (2) the excess inventory quantity of the products, (3) the shortage quantity of the products and (4) the procurement quantities of the products from M_1 and M_2.

In scenario 2 of the demand information of the products, only both mean μ and variance σ^2 of the demand x are known. Using the distribution-free approach (DFA) (Gallego and Moon, 1993; Moon and Gallego, 1994; Moon and Choi, 1995; Alfares and Elmorra, 2005), the upper limits of both the expected excess inventory quantity and the expected shortage quantity due to the magnitude relation between order quantities Q_i (i = 1, 2) and the demand x as to events $E1$-$E8$ can be derived by using the Cauchy-Schwarz inequality as follows:

$$E1: E[Q_1+Q_2-x]^+ \le \frac{[\sigma^2+(\mu-Q_1-Q_2)^2]^{1/2}-(\mu-Q_1-Q_2)}{2} \quad (10)$$

$$E1: E[x-Q_1-Q_2]^+ \le \frac{[\sigma^2+(Q_1+Q_2-\mu)^2]^{1/2}-(Q_1+Q_2-\mu)}{2} \quad (11)$$

$$E2: E[Q_2-x]^+ \le \frac{[\sigma^2+(\mu-Q_2)^2]^{1/2}-(\mu-Q_2)}{2} \quad (12)$$

$$E2: E[x-Q_2]^+ \le \frac{[\sigma^2+(Q_2-\mu)^2]^{1/2}-(Q_2-\mu)}{2} \quad (13)$$

$$E3: E[y_1Q_1+Q_2-x]^+ \le \frac{[\sigma^2+(\mu-y_1Q_1-Q_2)^2]^{1/2}-(\mu-y_1Q_1-Q_2)}{2} \quad (14)$$

$$E3: E[x-y_1Q_1-Q_2]^+ \le \frac{[\sigma^2+(y_1Q_1+Q_2-\mu)^2]^{1/2}-(y_1Q_1+Q_2-\mu)}{2} \quad (15)$$

$$E4: E[Q_1-x]^+ \le \frac{[\sigma^2+(\mu-Q_1)^2]^{1/2}-(\mu-Q_1)}{2} \quad (16)$$

$$E4: E[x-Q_1]^+ \le \frac{[\sigma^2+(Q_1-\mu)^2]^{1/2}-(Q_1-\mu)}{2} \quad (17)$$

$$E5: E[Q_1+y_2Q_2-x]^+ \le \frac{[\sigma^2+(\mu-Q_1-y_2Q_2)^2]^{1/2}-(\mu-Q_1-y_2Q_2)}{2} \quad (18)$$

$$E5: E[x-Q_1-y_2Q_2]^+ \le \frac{[\sigma^2+(Q_1+y_2Q_2-\mu)^2]^{1/2}-(Q_1+y_2Q_2-\mu)}{2} \quad (19)$$

$$E6: E[y_2Q_2-x]^+ \le \frac{[\sigma^2+(\mu-y_2Q_2)^2]^{1/2}-(\mu-y_2Q_2)}{2} \quad (20)$$

$$E6: E[x-y_2Q_2]^+ \le \frac{[\sigma^2+(y_2Q_2-\mu)^2]^{1/2}-(y_2Q_2-\mu)}{2} \quad (21)$$

$$E7: E[y_1Q_1-x]^+ \le \frac{[\sigma^2+(\mu-y_1Q_1)^2]^{1/2}-(\mu-y_1Q_1)}{2} \quad (22)$$

$$E7: E[x-y_1Q_1]^+ \le \frac{[\sigma^2+(y_1Q_1-\mu)^2]^{1/2}-(y_1Q_1-\mu)}{2} \quad (23)$$

$$E8: E[y_1Q_1+y_2Q_2-x]^+ \le \frac{[\sigma^2+(\mu-y_1Q_1-y_2Q_2)^2]^{1/2}-(\mu-y_1Q_1-y_2Q_2)}{2} \quad (24)$$

$$E8: E[x-y_1Q_1-y_2Q_2]^+ \le \frac{[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2]^{1/2}-(y_1Q_1+y_2Q_2-\mu)}{2}. \quad (25)$$

Appendix A shows the elicitation process of Eqs. (10) and (11) in event $E1$. In the similar way, the elicitation processes of Eqs. (12)-(25) as to events $E2$-$E8$ can be obtained.

The individual lower limit of the retailer's expected profit as to events $E1$-$E8$ can be obtained by substituting the upper limits of the expected excess inventory quantity and the expected shortage quantity as to events $E1$-$E8$ in Eqs. (11)-(25) into terms regarding the expected excess inventory quantity and the expected shortage quantity in the retailer's expected profit in Eqs. (1)-(8) as to events $E1$-E8. Appendix A shows the elicitation process of the lower limit of the retailer's expected profit in event $E1$. In Similar way, the elicitation processes of the lower limit of the retailer's expected profit as to events $E2$-$E8$ can be derived.

Next, the profits of two manufacturers, M_1 and M_2 are discussed. The profit of each manufacturer, M_i (i = 1, 2), is formulated from the wholesales of the products, the variable production cost when products can be produced and the fixed production cost of the products. The expected profit of each manufacturer, M_i (i = 1, 2), as to events $E1$-$E9$ for order quantity Q_i to M_i is formulated as follows:

$$E^1\left[\pi_{M_i}^{E1}(Q_1, Q_2)\right](i=1,2) = w_iQ_i - c_iQ_i - \gamma c_iQ_i (i=1,2) \quad (26)$$

$$E^1[\pi_{M_1}^{E2}(Q_1, Q_2)] = -\gamma c_1 Q_1$$
$$E^1[\pi_{M_2}^{E2}(Q_1, Q_2)] = w_2Q_2 - c_2Q_2 - \gamma c_2Q_2 \quad (27)$$

$$E^1[\pi_{M_1}^{E3}(Q_1, Q_2)] = w_1y_1Q_1 - c_1y_1Q_1 - \gamma c_1Q_1$$
$$E^1[\pi_{M_2}^{E3}(Q_1, Q_2)] = w_2Q_2 - c_2Q_2 - \gamma c_2Q_2 \quad (28)$$

$$E^1[\pi_{M_1}^{E4}(Q_1, Q_2)] = w_1Q_1 - c_1Q_1 - \gamma c_1Q_1$$
$$E^1[\pi_{M_2}^{E4}(Q_1, Q_2)] = -\gamma c_2Q_2 \quad (29)$$

$$E^1[\pi_{M_1}^{E5}(Q_1, Q_2)] = w_1Q_1 - c_1Q_1 - \gamma c_1Q_1$$
$$E^1[\pi_{M_2}^{E5}(Q_1, Q_2)] = w_2y_2Q_2 - c_2y_2Q_2 - \gamma c_2Q_2 \quad (30)$$

$$E^1[\pi_{M_1}^{E6}(Q_1, Q_2)] = -\gamma c_1Q_1$$
$$E^1[\pi_{M_2}^{E6}(Q_1, Q_2)] = w_2y_2Q_2 - c_2y_2Q_2 - \gamma c_2Q_2 \quad (31)$$

$$E^1[\pi_{M_1}^{E7}(Q_1, Q_2)] = w_1y_1Q_1 - c_1y_1Q_1 - \gamma c_1Q_1$$
$$E^1[\pi_{M_2}^{E7}(Q_1, Q_2)] = -\gamma c_2Q_2 \quad (32)$$

$$E^1[\pi_{M_i}^{E8}(Q_1, Q_2)](i=1,2) = w_iy_iQ_i - c_iy_iQ_i - \gamma c_iQ_i (i=1,2) \quad (33)$$

$$E^1[\pi_{M_i}^{E9}(Q_1, Q_2)](i=1,2) = -\gamma c_iQ_i (i=1,2). \quad (34)$$

Here, Eq. (26) shows the expected profit of each manufacturer, M_i (i = 1, 2), in event $E1$ where neither a natural disaster nor a production process failure occur to M_i (i = 1, 2). Using Table 1, in Eq. (26), the first term is the wholesales of the products, the second term is the variable production cost of the products and the final term is the fixed production cost of the products. Similarly, using Table 1, the individual expected profit of M_i (i = 1, 2) in Eqs. (27)-(34) as to events $E2$-$E9$ are obtained as to the situations if a natural disaster and a production process failure occur to M_1 and M_2. From Eqs. (26)-(34), it can be seen that the individual expected profit of M_i (i = 1, 2) as to events $E1$-$E9$ is unaffected by the demand of the products.

Considering the individual event probability as to events $E1$-$E9$ in Table 1, the total expected profits considering all events from $E1$ through $E9$ of a retailer, two manufacturers (M_1 and M_2) and the whole system in scenario j (= 1, 2) of the demand information of the products are calculated as the following sum of the individual expected profits for events $E1$-$E9$ of the relevant members and the whole system:

$$G_R^j(Q_1, Q_2)(j=1,2) = \sum_{\ell=1}^{9} P^\ell E^j[\pi_R^{E\ell}(Q_1, Q_2)] \quad (35)$$

$$G_{M_i}^j(Q_1, Q_2)(i=1,2,\ j=1,2) = \sum_{i=1}^{2}\sum_{\ell=1}^{9} P^\ell E^j[\pi_{M_i}^{E\ell}(Q_1, Q_2)] \quad (36)$$

$$G_S^j(Q_1, Q_2)(j=1,2) = G_R^j(Q_1, Q_2) + G_M^j(Q_1, Q_2). \quad (37)$$

5. OPTIMAL DECISION-MAKING UNDER ISC

In an integrated DSSC(ISC), a decision-maker under ISC determines the optimal order quantity $Q_i^{*Ij}(i=1, 2,\ j=1,2)$ in scenario j of the demand information of the products so as to maximize the whole system's total expected profit. From conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0 \le \alpha_i \le 1)$ in 3.(2) and $\beta_i(0 \le \beta_i \le 1)$ in 3. (3) and $y_i(0 \le y_i \le 1)$, in 3.(5), $s \ge k > r$ in 3. (6), the second-order differential equation of the whole system's total expected profit under scenario j (= 1, 2) of the demand information in Eq. (37) in terms of order quantity Q_i (i = 1, 2) to each manufacturer M_i is negative as

$$\partial^2 G_S^j(Q_1, Q_2)(j=1,2)/\partial Q_1^2 < 0 \quad (38)$$
$$\partial^2 G_S^j(Q_1, Q_2)(j=1,2)/\partial Q_2^2 < 0 \quad (39)$$

(See Appendix B). Also, it is derived that the Hessian matrix of the whole system's total expected profit is positive in terms of Q_i as

$$\left|H_S^j\right|(j=1,2) = \begin{vmatrix} \dfrac{\partial^2 G_S^j(Q_1, Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 G_S^j(Q_1, Q_2)}{\partial Q_1 Q_2} \\ \dfrac{\partial^2 G_S^j(Q_1, Q_2)}{\partial Q_2 Q_1} & \dfrac{\partial^2 G_S^j(Q_1, Q_2)}{\partial Q_2^2} \end{vmatrix}$$
$$= \{\partial^2 G_S^j(Q_1, Q_2)/\partial Q_1^2\}\{\partial^2 G_S^j(Q_1, Q_2)/\partial Q_2^2\}$$
$$-\{\partial^2 G_S^j(Q_1, Q_2)/\partial Q_1\partial Q_2\}\{\partial^2 G_S^j(Q_1, Q_2)/\partial Q_2\partial Q_1\} > 0 \quad (40)$$

(See Appendix B). Eq. (40) indicates that the whole system's total expected profit under scenario j (= 1, 2) of the demand information in Eq. (37) has the unique values regarding optimal order quantities, Q_i^{Ij} (i = 1, 2, j = 1, 2), to each manufacturer, M_i (i = 1, 2), so as to maximize Eq. (37). Therefore, the optimal order quantities, Q_i^{Ij} (i = 1, 2, j = 1, 2), can be determined as order quantities Q_i (i = 1, 2) which satisfy the conditions where the first-order differential equation of the whole system's total expected profit under scenario j (= 1, 2) of the demand information in Eq. (37) in terms of order quantity Q_i (i = 1, 2) is 0 shown in Eqs. (42) and (43) (See Appendix C). Concretely, this paper determines the optimal order quantity Q_i^{Ij} (i = 1, 2, j = 1, 2) to each manufacturer, M_i (i = 1, 2), under ISC in scenario j (= 1, 2) of the demand information of the products as integer values, satisfying Eq. (40) and the following equations:

$$Max\ G_S^j(Q_1, Q_2)(j=1,2) \tag{41}$$

$$\text{Subject to } \partial G_S^j(Q_1, Q_2)(j=1,2)\big/\partial Q_1 = 0 \tag{42}$$

$$\partial G_S^j(Q_1, Q_2)(j=1,2)\big/\partial Q_2 = 0 \tag{43}$$

using the numerical calculation and the numerical search.

6. OPTIMAL DECISION-MAKING UNDER DSC

Under a decentralized DSSC (DSC), this paper adopts the optimal decisions approach in the Stackelberg game (Aust and Buscher, 2012; Berr, 2011; Cachon and Netessine, 2004; Cai *et al*., 2009; Esmaeili and Zeephongsekul, 2010; Hu *et al*., 2011; Lee and Ammons(2011), Leng and Parlar, 2009; Liu *et al*., 2012; Mukhopadhyay *et al*., 2011; Xu *et al*., 2012; Yan and Su, 2012). Here, a retailer is the leader of the decision-making under DSC, and two manufacturers are the followers of the decision-making. This is because a retailer sells a single type of products in a market. Therefore, the retailer can earn the more total expected profit than each manufacturer M_i (i = 1, 2). The retailer determines the optimal order quantity Q_i^{*Dj} (i = 1, 2, j = 1, 2) to M_i in scenario j of the demand information of the demands so as to maximize the total expected profit of the retailer. Each manufacturer, M_i (i = 1, 2), produces the same quantity of the optimal order quantity Q_i^{*Dj} and sells the products with the unit wholesale price w_i to the retailer.

From conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0 \le \alpha_i \le 1)$ in 3.(2) and $\beta_i(0 \le \beta_i \le 1)$ in 3. (3) and $y_i(0 \le y_i \le 1)$, in 3.(5), $s \ge k > r$ in 3. (6), the second-order differential equation of the retailer's total expected profit under scenario j (= 1, 2) of the demand information in Eq. (35) in terms of order quantity Q_i (i = 1, 2) to each manufacturer M_i is negative as

$$\partial^2 G_R^j(Q_1, Q_2)(j=1,2)\big/\partial Q_1^2 < 0 \tag{44}$$

$$\partial^2 G_R^j(Q_1, Q_2)(j=1,2)\big/\partial Q_2^2 < 0 \tag{45}$$

(See Appendix B). Also, it is derived that the Hessian matrix of the retailer's total expected profit is positive in terms of Q_i (i = 1, 2) as

$$\left|H_R^j\right|(j=1,2) = \begin{vmatrix} \dfrac{\partial^2 G_R^j(Q_1,Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 G_R^j(Q_1,Q_2)}{\partial Q_1 Q_2} \\ \dfrac{\partial^2 G_R^j(Q_1,Q_2)}{\partial Q_2 Q_1} & \dfrac{\partial^2 G_R^j(Q_1,Q_2)}{\partial Q_2^2} \end{vmatrix}$$
$$= \{\partial^2 G_R^j(Q_1,Q_2)/\partial Q_1^2\}\{\partial^2 G_R^j(Q_1,Q_2)/\partial Q_2^2\}$$
$$-\{\partial^2 G_R^j(Q_1,Q_2)/\partial Q_1 \partial Q_2\}\{\partial^2 G_R^j(Q_1,Q_2)/\partial Q_2 \partial Q_1\} > 0 \tag{46}$$

(See Appendix B). Eq. (46) indicates that the retailer's total expected profit under scenario j (= 1, 2) of the demand information in Eq. (35) has unique values regarding optimal order quantities, Q_i^{Dj} (i = 1, 2, j = 1, 2), to each manufacturer, M_i (i = 1, 2), so as to maximize Eq. (35). Therefore, the optimal order quantities, Q_i^{Dj} (i = 1, 2, j = 1, 2), can be determined as order quantities Q_i (i = 1, 2) which satisfy the conditions where the first-order differential equation of the retailer's total expected profit under scenario j(= 1, 2) of the demand information in Eq. (35) in terms of order quantity Q_i (i = 1, 2) is 0 shown in Eqs. (48) and (49) (See Appendix D). Concretely, this paper determines the optimal order quantity Q_i^{Dj} (i = 1, 2, j = 1, 2) to each manufacturer, M_i (i = 1, 2), under DSC in scenario j (= 1, 2) of the demand information as integer values, satisfying Eq. (46) and the following equations:

$$Max\ G_R^j(Q_1, Q_2)(j=1,2) \tag{47}$$

$$\text{Subject to } \partial G_R^j(Q_1, Q_2)(j=1,2)\big/\partial Q_1 = 0 \tag{48}$$

$$\partial G_R^j(Q_1, Q_2)(j=1,2)\big/\partial Q_2 = 0 \tag{49}$$

using the numerical calculation and the numerical search.

7. PROFIT SHARING AS SUPPLY CHAIN COORDINATION (SCC)

Supply chain coordination (SCC) between a retailer and two manufacturers is discussed in order to guarantee the more total expected profit for all members under the optimal decision of ISC. This paper adopts the following two approaches of profit sharing as SCC under ISC:

- Profit sharing I: coordinating the unit wholesale price between a retailer and two manufacturers under the optimal decision of ISC as Nash bargaining solution considering profit balance
- Profit sharing II: coordinating the unit wholesale price of each manufacturer by combining Profit sharing I and the magnitude relation between the profit ratio of a retailer and that of two manufacturers.

This paper verifies analytically how profit sharing as SCC can bring the more total expected profits of all members under the optimal ordering policy of ISC in scenario j(= 1, 2) of the demand information of the products.

First, profit sharing I is discussed. In profit sharing I, the unit wholesale prices of two manufacturers, M_1 and M_2, are coordinated between a retailer and two manufacturers, M_1 and M_2, under the optimal ordering policy of ISC. Concretely, as to scenario j of the demand information of the products, the unit wholesale prices of two manufacturer, M_1 and M_2, are coordinated as Nash bargaining solutions (Mahesh Greys, 2008; Du *et al*., 2011), w_1^{N1j}, w_2^{N1j}, so as to maximize Eq. (50) satisfying the constrained conditions in Eqs. (51) and (52):

$$\begin{aligned}\max \Pi(w_1^{N1j}, w_2^{N1j}) &= \{G_R(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}) - G_R(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj})\} \\ &\times\{G_{M_{12}}(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}) \\ &- G_{M_{12}}(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj})\} \end{aligned} \tag{50}$$

Subject to

$$G_R(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}) - G_R(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj}) > 0 \tag{51}$$

$$G_{M_{12}}(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}) - G_{M_{12}}(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj}) > 0 \tag{52}$$

$$G_{M_{12}}(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}) = \sum_{i=1}^{2} G_{M_i}\left(w_1^{N1j}, w_2^{N1j} | Q_1^{Ij}, Q_2^{Ij}\right) \tag{53}$$

$$G_{M_{12}}(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj}) = \sum_{i=1}^{2} G_{M_i}\left(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj}\right). \tag{54}$$

Here, Eq. (50) coordinates the unit wholesale prices of two manufacturers, M_1 and M_2, as w_1^{NIj} and w_2^{NIj} under the optimal ordering policy of ISC in scenario j(= 1, 2) of the demand information. w_1^{NIj} and w_2^{NIj} are determined so as to maximize the multiplication of (the difference of the total expected profit of a retailer for w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information and that for w_1 and w_2 under the optimal decision of DSC in scenario j(=1,2) of the demand information) and (the difference of the sum of the total expected profits of two manufacturers for w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information and that for w_1 and w_2 under the optimal decision of DSC in scenario j(= 1, 2) of the demand information).

Also, Eq. (51) is the constrained condition to guarantee the situation where the total expected profit of a retailer for the coordinated wholesale prices, w_1^{NIj} and w_2^{NIj}, of two manufacturers, M_1 and M_2, under the optimal decision of ISC in scenario j(= 1, 2) of the demand information is always higher than that for the wholesale prices w_1 and w_2 provided by two manufacturers, M_1 and M_2, under the optimal decision of DSC in scenario j(= 1, 2) of the demand information. Similarly, Eq. (52) is the constrained condition to guarantee the situation where the sum of the total expected profits of two manufacturers, M_1 and M_2, for the coordinated wholesale prices, w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information is always higher than that for the wholesale prices w_1 and w_2 under the optimal decision of DSC in scenario j(= 1, 2) of the demand information. Eq. (53) indicates the sum of the total expected profits of two manufacturers for w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information. Eq. (54) indicates the sum of the total expected profits of two manufacturers for w_1 and w_2 under the optimal decision of DSC in scenario j(= 1, 2) of the demand information.

Next, Profit sharing II is discussed. In Profit sharing II, the unit wholesale prices of two manufacturers, M_1 and M_2, are coordinated as w_1^{NIj} and w_2^{NIj} by combining Profit sharing I considering the profit balance and the magnitude relation between the total expected profit of a retailer and the sum of the total expected profits of two manufacturers, M_1 and M_2. A retailer is the leader of the decision-making under DSC and earns most of the total expected profit of the whole system in DSC. This approach adds newly the following condition:

$$\begin{aligned}&G_R(w_1^{N2j}, w_2^{N2j} | Q_1^{Ij}, Q_2^{Ij}) - G_R(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj}) \\ &> G_{M_{12}}(w_1^{N2j}, w_2^{N2j} | Q_1^{Ij}, Q_2^{Ij}) - G_{M_{12}}(w_1, w_2 | Q_1^{Dj}, Q_2^{Dj})\end{aligned} \tag{55}$$

to the constrained conditions, Eqs. (51) and (52), in Profit sharing I. Here, Eq. (55) is the constrained condition to guarantee the situation where (the difference of the total expected profit of a retailer for w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information and that for w_1 and w_2 under the optimal decision of DSC in scenario j(= 1, 2) of the demand information) is higher than (the difference of the sum of the total expected profits of two manufacturers for w_1^{NIj} and w_2^{NIj} under the optimal decision of ISC in scenario j(= 1, 2) of the demand information and that for w_1 and w_2 under the optimal decision of DSC in scenario j(= 1, 2) of the demand information). Therefore, Profit sharing II coordinates the unit wholesale prices of two manufacturers, M_1 and M_2, to $w_i^{N2j} (i = 1, 2,\ j = 1, 2)$ so as to maximize Eq. (50) under the constrained conditions shown in Eqs. (51), (52) and (55) as to scenario j(= 1, 2) of the demand information.

8. NUMERICAL ANALYSIS

The analysis numerically investigates how four factors: (i) event probability of a natural disaster, (ii) event probability of a failure in production process, (iii) production ratio after a failure occurs in production process and (iv) demand information of the products, impact the optimal ordering policies to two manufacturers under DSC and ISC. Also, the optimal order quantities and the total expected profits under DSC are compared with those under ISC. Besides, the effects of Profit sharing as supply chain coordination (SCC) on the total expected profits of all members under the optimal decision of ISC are verified. The benefit of Profit sharing I is compared with that of Profit sharing II in aspects of the profit of a retailer who is a leader of the decision-making under DSC.

The following values of system parameters are provided as numerical examples: s = 280, r = 30, k = 220, γ = 0.4, c_1 = 60, c_2 = 61, w_1 = 123, w_2 = 125, $\alpha_i = \beta_i$ = [2(%), 13(%)] and y_i = 60(%). All data sources of the numerical examples in this paper above are provided so as to satisfy the following conditions:

$$s \geq k > w_i > c_i \geq r\,(i=1,2),\ 0 \leq \gamma \leq 1,\ 0 \leq y_i \leq 1\,(i=1,2),$$
$$0 \leq \alpha_i \leq 1\left(i=1,2\right) \Leftrightarrow 0(\%) \leq \alpha_i \leq 100(\%)\left(i=1,2\right),$$
$$0 \leq \beta_i \leq 1\left(i=1,2\right) \Leftrightarrow 0(\%) \leq \beta_i \leq 100(\%)\left(i=1,2\right),$$

guaranteeing profitable operations of a retailer, two manufacturers, M_1 and M_2, and the whole system.

In scenario 1 of the demand information of a single type of products, The demand x follows the normal distribution with mean μ and variance σ^2. Here, as the numerical examples, this paper sets μ and σ^2 as μ = 1000 and σ^2 = 300^2, satisfying the condition where $\mu - 3\sigma^2 \geq 0$. It is because the product demand follows the normal distribution, and the probability $P_r\left((\mu-3\sigma) \leq x \leq (\mu+3\sigma)\right)$ is known as 99.73(%). In scenario 2 of the demand information, only both mean μ and variance σ^2 of the product demand x are known. In the numerical examples, the values of mean μ and variance σ^2 of the product demand x in scenario 2 of the demand information are known as μ = 1000 and σ^2 = 300^2 which are same values as scenario 1. All data sources of the numerical examples satisfying the above conditions are modifiable if necessary for sensitive analysis.

The individual optimal order quantities to two manufacturers, M_1 and M_2, under DSC can be determined so as to satisfy Eqs. (46)-(49), using the numerical calculation and numerical search in the range where $0 \leq Q_i(i=1,2) \leq \mu + 3\sigma$. Also, the individual optimal order quantity to two manufacturers, M_1 and M_2, under ISC can be determined so as to satisfy Eqs. (40)-(43), using the numerical calculation and numerical search in the range where $0 \leq Q_i(i=1,2) \leq \mu + 3\sigma$. Here, the upper limit of the numerical search regarding the individual optimal order quantities under DSC and ISC, Q_i^{kj} ($k = D, I$; i = 1, 2; j = 1, 2), in each scenario of the demand information is set to $(\mu+3\sigma)$. It is because the product demand follows the normal distribution, the probability of the product demand $P_r\left(\{0 \leq (\mu-3\sigma)\} \leq x \leq (\mu+3\sigma)\right)$ is known as 99.73(%) and the optimal order quantities Q_i^{kj} are found in the prospective range of the product demand.

A computer programming was developed by using Visual Studio C# in Visual Studio Express 2010 for Windows Desktop in order to conduct numerical experiments and obtain the results for the optimal order quantities under DSC and ISC by the numerical calculation and the numerical search. In the development of the computer programming and implementation of the numerical experiment, the following computer: the Dell computer, Vostro 260s model, CPU: Intel(R) Core(TR) i5-2400, 3.10 GHz: Memory: 4 GB, OS: Windows 7 Professional 32 bit was used in this paper.

8.1 Effect of Supply Disruption due to A Natural Disaster to Two Manufacturers on the Optimal Ordering Policy

It is investigated how a natural disaster which may occur to two manufacturers, M_1 and M_2, affect the optimal order quantities to M_1 and M_2 and the total expected profits under DSC and ISC as to scenario j (= 1, 2) of the demand information of the products. Table 2 shows the effect of a natural disaster $\alpha_i(i=1,2)$ on the optimal ordering policy and the total expected profits under ISC and DSC as to scenario j (= 1, 2) of the demand information. In Table 2, α_1 = [2(%), 13(%)], α_2 = β_i (I = 1, 2) = 2(%) and y_i (i = 1, 2) = 60(%) were used.

From Table 2, the following results are verified:

- When the event probabilities of a natural disaster, α_1 and α_2, to Manufacturer 1 (M_1) and Manufacturer 2 (M_2) is low such as 2(%), the retailer tends to order the more quantity of the product to M_1 under DSC and ISC regardless of any scenario of the demand information. This is because the unit wholesale price and production cost of M_1 is cheaper than those of M_2, that is, $w_1 < w_2$ and $c_1 < c_2$.
- The higher α_1 under a fixed α_2 = 2(%) is, the smaller the optimal order quantity to M_1 under DSC and ISC is, meanwhile the larger that to M_2 is regardless of any scenario of the demand information. This is because a retailer tends to order the more quantity of the product to M_2 who can supply the required quantity of products reliably and safely, even if $w_1 < w_2$ and $c_1 < c_2$. Therefore, regardless of any scenario of the demand information, it can be seen that the higher α_1 under a fixed α_2 = 2(%) is, the lower the total expected profit of M_1 is, meanwhile the higher that of M_2 is.
- The total expected profits of a retailer and the whole system tend to reduce when event probability of a natural disaster α_1 increases. This is because the unit whole sale price and production cost of M_2 are higher than those of M_1 and the order quantity to M_2 increases in this case.

Table 2. The effect of natural disaster on optimal ordering policy and the total expected profits under ISC and DSC as to scenarios of the demand information of the products

Scenario	Type of DSSC	α_1 (%)	Optimal order quantity			Total expected profits			
			M_1	M_2	Total	WS	Retailer	M_1	M_2
1	ISC	2	773	617	1,390	161,221	109,103	28,787	23,331
		5	410	987	1,397	158,617	106,812	14,506	37,299
		10	230	1,164	1,394	157,181	105,812	7,425	43,944
		13	176	1,216	1,392	156,722	105,489	5,354	45,879
	DSC	2	765	511	1,276	159,218	111,407	28,489	19,322
		5	379	903	1,282	156,623	109,089	13,409	34,125
		10	218	1,065	1,283	155,238	107,995	7,037	40,206
		13	176	1,107	1,283	154,788	107,667	5,354	41,767
2	ISC	2	798	612	1,410	144,197	91,337	29,718	23,141
		5	424	997	1,421	141,801	89,122	15,002	37,677
		10	273	1,144	1,417	140,413	88,412	8,813	43,189
		13	219	1,194	1,413	139,921	88,210	6,662	45,049
	DSC	2	781	474	1,255	141,629	94,620	29,085	17,923
		5	343	919	1,262	139,241	92,376	12,136	34,729
		10	210	1,053	1,263	137,978	91,446	6,779	39,753
		13	174	1,089	1,263	137,539	91,158	5,293	41,088

(M_1: Manufacturer 1, M_2: Manufacturer 2, WS: the whole system).

Table 3. The effect of a production process failure on optimal ordering policy and the total expected profits under ISC and DSC as to scenarios of the demand information of the products

Scenario	Type of DSSC	β_1 (%)	Optimal order quantity			Total expected profits			
			M_1	M_2	Total	WS	Retailer	M_1	M_2
1	ISC	2	773	617	1,390	161,221	109,103	28,787	23,331
		5	701	699	1,400	160,530	108,518	25,597	26,415
		10	617	795	1,412	159,615	107,819	21,783	30,013
		13	578	839	1,417	159,155	107,514	19,986	31,655
	DSC	2	765	511	1,276	159,218	111,407	28,489	19,322
		5	678	606	1,284	158,487	110,829	24,757	22,901
		10	585	710	1,295	157,597	110,139	20,653	26,804
		13	547	753	1,300	157,147	109,822	18,914	28,410
2	ISC	2	798	612	1,410	144,197	91,337	29,718	23,141
		5	729	694	1,423	143,612	90,766	26,619	26,226
		10	653	783	1,436	142,830	90,216	23,054	29,560
		13	621	821	1,442	142,428	89,978	21,473	30,976
	DSC	2	781	474	1,255	141,629	94,620	29,085	17,923
		5	668	598	1,266	141,027	94,037	24,392	22,599
		10	564	713	1,277	140,251	93,422	19,912	26,917
		13	528	755	1,283	139,898	93,155	18,257	28,486

(M_1: Manufacturer 1, M_2: Manufacturer 2, WS: the whole system).

8.2 Effect of Supply Disruption Due to a Production Process Failure to Two Manufacturers on the Optimal Ordering Policy

It is investigated how a production process failure which may occur to two manufacturers, M_1 and M_2, affect the optimal order quantities to M_1 and M_2 and the total expected profits under DSC and ISC as to scenario j(= 1, 2) of the demand information of the products. Table 3 shows the effect of a production process failure $\beta_i (i = 1, 2)$ on the optimal ordering policy and the total expected profits under ISC and DSC as to scenario j(= 1,

2) of the demand information. In Table 3, $\beta_1 = [2(\%), 13(\%)]$, $\beta_2 = \alpha_i$ $(i = 1, 2) = 1(\%)$ and $y_i = 60(\%)$.

From Table 3, the following results are verified:

- When the event probability of a production process failure β_1 of M_1 is low such as 2(%), the retailer tends to order the more quantity of the product to M_1 under DSC and ISC. This is because the unit whole sale price and production cost of M_1 is cheaper than those of M_2, that is, $w_1 < w_2$ and $c_1 < c_2$.
- The higher β_1 under a fixed $\beta_2 = 2(\%)$ is, it is verified under ISC and DSC that the optimal order quantity to M_1 tends to decrease, meanwhile the optimal order quantity to M_2 tend to increase.
- The total expected profits of a retailer and the whole system tend to reduce when event probability of a production process failure β_1 increases. The reason is same as that in Table 2.

In this paper, it is assumed that when a natural disaster occurs to either M_1 or M_2, it is impossible for the relevant manufacturer to produce any product ordered from a retailer, meanwhile, when a production process failure occurs to either M_1 or M_2, it is possible for the relevant manufacturer to produce the limited quantity of products determined at a certain production ratio y_i $(i = 1, 2, 0 \le y_i \le 1)$. From Tables 2 and 3, the following results are verified: The reduction volume of order quantity to M_1 in Table 2 becomes larger than that in Table 3 as the event probabilities of a natural disaster, α_1, and a production process failure, β_1, to M_1 increase under fixed event probabilities α_2 and β_2 to M_2 regardless of any scenario of the demand information. Meanwhile, the increment of order quantity to M_2 in Table 2 becomes larger than that in Table 3 as the probabilities of a natural disaster to M_1 and a production process failure to M_1 increase regardless of any scenario of the demand information. This stems from the difference of the possible production quantity in a DSSC when a natural disaster and a production process failure occur to M_1 and M_2 regardless of any scenario of the demand information. Therefore, it is verified that the reduction volumes of the total expected profits of a retailer and two manufacturers due to a production process failure in Table 3 are less than those due to a natural disaster in Table 2.

8.3 Effect of Production Ratio to Two Manufacturers After a Production Process Failure on the Optimal Ordering Policy

It is investigated how production ratios of two manufacturers, M_1 and M_2, after a production process failure to M_1 and M_2, affect the optimal order quantities to M_1 and M_2 under DSC and ISC and the total expected profits as to scenario $j(= 1, 2)$ of the demand information of the products. Table 4 shows the results mentioned above. In Table 4, two combinations of production ratios of M_1 and M_2 are considered.

Case 1: $\beta_1 = 13.0$ (%), $\beta_2 = 2.0(\%)$, $y_1 = 90(\%)$, $y_2 = 60(\%)$, $\alpha_1 = \alpha_2 = 2.0(\%)$.
Case 2: $\beta_1 = 13.0$ (%), $\beta_2 = 2.0(\%)$, $y_1 = y_2 = 60(\%)$, $\alpha_1 = \alpha_2 = 2.0(\%)$.

From Table 4, the following results are verified:

- The optimal order quantity to M_1 in Case 1 is larger than that in Case 2 regardless of the type of the optimal decision in a DSSC and the scenario of the demand information, even if $\beta_1 > \beta_2$. This is because M_1 in Case 1 has a higher ability to produce the required quantity of the products than M_2 in Case 1 has under the situation where $y_1 > y_2$, $w_1 < w_2$ and $c_1 < c_2$ regarding M_1 and M_2. Meanwhile, the optimal order quantity to M_2 in Case 1 is smaller than that in Case 2 regardless of the type of the optimal decision in a DSSC and the scenario of the demand information, even if $y_1 > y_2$, $w_1 < w_2$ and $c_1 < c_2$. This is because M_1 in Case 2 has as same ability to produce the required

Table 4. Effect of production ratio after a production process failure on optimal ordering policy and the total expected profits under ISC and DSC as to scenario of the demand information of the products

Scenario	Case	Type of DSSC	Optimal order quantity			Total expected profits			
			M_1	M_2	Total	WS	Retailer	M_1	M_2
1	1	ISC	875	516	1,391	161,461	109,630	32,362	19,468
		DSC	888	390	1,278	159,466	111,908	32,843	14,715
	2	ISC	578	839	1,417	159,155	107,514	19,986	31,655
		DSC	547	753	1,300	157,147	109,822	18,914	28,410
2	1	ISC	904	501	1,405	144,447	92,110	33,434	18,903
		DSC	918	337	1,255	141,946	95,279	33,952	12,715
	2	ISC	621	821	1,442	142,428	89,978	21,473	30,976
		DSC	528	755	1,283	139,898	93,155	18,257	28,486

(M_1: Manufacturer 1, M_2: Manufacturer 2, WS: the whole system).

quantity of the products as M_2 in Case 1 has under the situation where $\beta_1 > \beta_2$ regarding M_1 and M_2. Therefore, the total expected profit of M_1 in Case 1 is higher than that in Case 2.

- Regardless of the type of the optimal decision in a DSSC and the scenario of the demand information, the optimal total order quantity to two manufacturers in Case 1 is smaller than that in Case 2. The total expected profits of the whole system and a retailer in Case 1 are higher than those in Case 2. It is because two manufacturers in Case 2 have lower abilities to produce the required quantity of the products than those in Case 1. Therefore, the more quantity of the products tends to be ordered to cover supply disruption due to production process failures to two manufacturers.

8.4 Effects of Type of DSSC on the Optimal Ordering Policy

First, it is investigated how type of DSSC on the optimal ordering policy.

From Tables 2-4, the following results are verified: The optimal order quantity in ISC is larger than that in DSC regardless of the demand information of the products. The reason is shown as follows: the optimal order quantities to two manufacturers, M_1 and M_2, under DSC in scenario j (= 1, 2) of the demand information, Q_1^{Dj} and Q_2^{Dj} (j = 1, 2), are affected by the unit whole sale price w_i (i = 1, 2) of each manufacturer. The order quantities under ISC in scenario j(= 1, 2) of the demand information, Q_1^{Ij} and Q_2^{Ij} (j = 1, 2), are affected by the unit production cost c_i(i = 1, 2) of each manufacturer. From model assumptions 3. (2)-(6), $w_i(i = 1, 2) > c_i(i = 1, 2)$, the value of the cumulative distribution function of product demand under ISC in Eqs. (42) and (43) are larger than that under DSC in Eqs. (48) and (49). The magnitude relation between the optimal product order quantities to two manufacturers under ISC and those under DSC in scenario 2 of the demand information are proved analytically in Appendix E. Therefore, the sum of optimal order quantities, $(Q_1^{Ij} + Q_2^{Ii})(j = 1, 2)$ under ISC are determined as a larger value than the sum of optimal order quantities, $(Q_1^{Dj} + Q_2^{Dj})\ (j = 1, 2)$ under DSC.

8.5 Effects of Demand Information of Products on the Total Expected Profits and the Optimal Ordering Policy in DSSC

It is investigated how the demand information of the products on the optimal ordering policy. From Tables 2-4, the following results are verified:

- The total expected profits of a retailer and whole system in scenario 2 of the demand information of the products are lower than those in scenario 1. This is because the available demand information is limited in scenario 2. Therefore, the optimal ordering policy in scenario 2 adopting DFA is made so as to maximize both the lowest expected profit of retailer under DSC and that of the whole system under ISC.
- Regardless of the production ratio of each manufacturer, the optimal order quantity to M_1 and the optimal total order quantity under ISC in scenario 2 of the demand information tend to be larger than those in scenario 1 of the demand information. Meanwhile, the optimal order quantity to M_2 under ISC in scenario 2 of the demand information tend to be smaller than that in scenario 1 of the demand information.

8.6 Effect of Supply Chain Coordination on Shift to Optimal Decision under ISC

The total expected profits of a retailer and two manufacturers and the whole system under the optimal ordering policy of DSC are compared with those under ISC as to the demand information of the products and supply disruptions. From Tables 2-4, the following results are verified: Regardless of a situation without/with supply disruptions, the total expected profits of the whole system and two manufacturers under the optimal ordering policy under ISC are higher than those under DSC, meanwhile the total expected profits of a retailer under the optimal ordering policy under ISC are lower than that under DSC. From the aspect of the total optimization in DSSC, the optimal ordering policy under ISC, which can enhance the total expected profit of the whole system, is recommended. However, it is difficult for the retailer, who is a leader of the decision-making under DSC, to shift the optimal ordering policy under ISC. Under the situation, any reasonable profit sharing is necessary between members under ISC so as to shift to the optimal ordering policy under ISC from that under DSC, guaranteeing the more total expected profits to members under ISC than those under DSC.

Here, it is investigated how supply chain coordination (SCC) encourage to shift to the optimal decision under ISC from that under DSC. Table 5 shows the effect of profit sharing as supply chain coordination (SCC) on the total expected profits under ISC as to the demand information. In Table 5, the following system parameters are used: α_1 = 13(%), α_2 = 2.0(%), β_i = 2.0(%), y_i =60(%) (i = 1, 2).

From Tables 5, the following results are verified:

- It can be seen that the expected profits of all members under ISC adopting Profit sharing I and II are higher than those under DSC in both scenarios of the demand information. In both Profit sharing I and II, the increment of the expected profit obtained under ISC is shared between all members by using supply chain coordination adjusting the unit whole sales prices of two manufacturers as Nash bargaining solutions.
- The total expected profit in scenario 1 of the demand information under ISC adopting profit sharing II is higher than that in Profit sharing I. It is because Profit sharing II combines Profit sharing I considering the

Table 5. Effect of profit sharing as supply chain coordination on total expected profits under ISC in each demand scenario

Scenario		Total expected profits		Total expected profits under ISC with SCC	
		DSC	ISC without SCC	Profit sharing I	Profit sharing II
1	Wholesale price	$w_1 = 123$, $w_2 = 125$	-	$w_1^{N12} = 126$, $w_2^{N12} = 122$	$w_1^{N22} = 125$, $w_2^{N22} = 122$
	R	107,667	105,489	108,570	108,722
	M_1	5,354	5,354	5,810	5,658
	M_2	41,767	45,879	42,342	42,342
	WS	154,788	156,722	156,722	156,722
2	Wholesale price	$w_1 = 123$, $w_2 = 125$	-	$w_1^{N12} = 119$, $w_2^{N12} = 122$	$w_1^{N22} = 119$, $w_2^{N22} = 122$
	R	91,158	88,210	92,439	92,439
	M_1	5,293	6,662	5,906	5,906
	M_2	41,088	45,049	41,576	41,576
	WS	137,539	139,921	139,921	139,921

(M_1: Manufacturer 1, M_2: Manufacturer 2, WS: the whole system).

profit balance and the magnitude relation between the total expected profit of a retailer and the sum of the total expected profits of two manufacturers.

For the retailer who is the leader of the decision-making under DSC, Profit sharing II is the most reasonable one to encourage all members in DSSC to shift to the optimal decisions under ISC from those under DSC.

9. CONCLUSIONS

This paper discussed a dual-sourcing supply chain (DSSC) which consisted of a retailer and two manufacturers. The DSSC faced both the uncertainty in product demand and supply disruptions due to a natural disaster and a failure in the production process after two manufacturers had received the retailer's order of a single type of products. This paper dealt with the uncertain in demand of the products as (i) full demand information and (ii) uncertain demand with known only mean and variance. Also, when a natural disaster occurs to each manufacturer, it was assumed that it was impossible for relevant manufacturer to produce any product. When a failure in production process occurred to each manufacturer, it was assumed that it was possible for the relevant manufacturer to produce and supply some rate of the retailer's order quantity. Here, it was assumed that the event probabilities of two manufacturers regarding supply disruptions including a natural disaster and a production process failure were different.

Under above situation, the optimal ordering policy to two manufacturers was proposed for a decentralized DSSC (DSC) and an integrated DSSC (ISC). The optimal ordering policy under DSC can maximize the retailer's total expected profit, meanwhile that under ISC can maximize the whole system's total expected profit.

Supply chain coordination (SCC) was discussed between a retailer and two manufacturers in order to guarantee more profits to all members when the optimal decision under ISC was adopted. Concretely, as SCC, the following two approaches of profit sharing were adopted under the optimal decision of ISC: In Profit sharing I, the unit wholesale price of each manufacturer was coordinated between a retailer and two manufacturers under the optimal decision of ISC as Nash bargaining solution considering profit balance. In Profit sharing II, the unit wholesale price of each manufacturer was coordinated by Profit sharing I and the magnitude relation between the total expected profit of a retailer and the sum of the total expected profits of two manufacturers.

Using the numerical examples, the numerical analysis illustrated how four factors: (i) event probability of a natural disaster, (ii) event probability of a failure in production process, (iii) production ratio after a failure occurs in production process and (iv) demand information of the products, affected the optimal ordering policies to two manufacturers under DSC and ISC. Also, the optimal order policy and the total expected profits under DSC are compared with those under ISC. As benefit of SCC, the total expected profits under ISC adopting Profit sharing I was compared with those adopting Profit sharing II in aspects of the profit of a retailer who was a leader of the decision-making under DSC.

This paper contributed the following managerial insights from outcomes obtained from both the theoretical research and the numerical analysis to both academic researchers and real-world policymakers regarding operations in a DSSC with the uncertain demand and supply disruptions:

- When supply disruptions affect the procurement of the required quantities of products, it is necessary for decision-makers under both the decentralized supply chain and the integrated supply chain to construct a dual-sourcing supply chain (DSSC) or a multiple-sourcing supply chain (MSSC) in order to enable to procure the required quantity of the products reliably and safely from multiple manufacturers.

- It is essential for manufacturers/suppliers to prepare the required stock, reserve of products or alternative production facility under the situations where they might face supply disruptions such as a natural disaster and a failure in production process.
- It is necessary that causes of supply disruptions are distinguished between natural disasters and failures in production processes in a DSSC.
- When the optimal ordering policy for a DSSC with supply disruptions is made, it is necessary to enable to predict the event probabilities regarding natural disasters and failures in production process in a DSSC precisely as much as possible.
- It is necessary for manufacturers/suppliers to separate between variable cost and fixed cost in production cost of products in the situation where they might face supply disruptions in their production processes.
- It is necessary to consider the magnitude relation between event probabilities of natural disasters which occur to manufacturers and the unit wholesale price of each manufacturer under the decentralized DSSC/MSSC (DSC). Meanwhile, it is necessary to consider the magnitude relation between manufacturers and the unit production cost of each manufacturer under the integrated DSSC/MSSC (ISC).
- When the demand of products is uncertain, it is necessary to investigate the demand information of products. When the demand probability is known, it is possible for decision-makers to determine the optimal ordering policy under DSC so as to maximize the total expected profits of a retailer, meanwhile that under ISC is done so as to maximize the total expected profits of the whole system. When only both mean and variance of the demand are known, it is possible for decision-makers to determine the optimal ordering policy under DSC so as to maximize the lowest total expected profit of a retailer using distribution-free approach (DFA), meanwhile that under ISC is done so as to maximize the lowest total expected profit of the whole system using DFA.
- Introduction of supply chain coordination into the optimal ordering policy under ISC can promote the shift to the optimal ordering policy under ISC from that under DSC, guaranteeing the more total expected profits of all members under ISC. Also, introduction of profit sharing II into the optimal ordering policy under ISC is recommended as the aspects of the total optimization of a DSSC and the total expected profit of a decision-maker under DSC.

Thus, the contribution of this paper can provide not only informative motivations regarding the optimal procurement planning and the optimal production planning of products in a DSSC, but also one the optimal solution to construct safely and reliably a DSSC with the uncertainty in product demand and supply disruptions due to natural disasters and failures in production process.

As future researches, it will be necessary to incorporate the following extendable topics into a DSSC:

- Recovery processes from supply disruptions over time stochastic change of supply disruptions over time
- Interaction between locations and initiation times of supply disruptions which and manufacturers may face.
- Replenishment of the required quantity of products.

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<Appendix A>

- The elicitation processes of Eqs. (10) and (11) using the Cauchy-Schwarz inequality in the distribution-free approach

The expected profit of a retailer in $E1$ in scenario1 is as follows:

$$E^1[\pi_R^{E1}(Q_1,Q_2)] = \int_0^{Q_1+Q_2}[sx + r(Q_1+Q_2-x)]f(x)dx + \int_{Q_1+Q_2}^{\infty}[s(Q_1+Q_2) - k(x-Q_1-Q_2)]f(x)dx - w_1Q_1 - w_2Q_2 \tag{A-1}$$

Eq. (A-1) is rewritten as follows:

$$\begin{aligned}
&s\int_0^{Q_1+Q_2} xf(x)dx + r\int_0^{Q_1+Q_2}(Q_1+Q_2-x)f(x)dx + s\int_{Q_1+Q_2}^{\infty}(Q_1+Q_2)f(x)dx - k\int_{Q_1+Q_2}^{\infty}(x-Q_1-Q_2)f(x)dx - w_1Q_1 - w_2Q_2\\
&= s\int_0^{Q_1+Q_2}\{(Q_1+Q_2)-(Q_1+Q_2-x)\}f(x)dx + r\int_0^{Q_1+Q_2}(Q_1+Q_2-x)f(x)dx + s(Q_1+Q_2)\int_{Q_1+Q_2}^{\infty}f(x)dx\\
&-k\int_{Q_1+Q_2}^{\infty}(x-Q_1-Q_2)f(x)dx - w_1Q_1 - w_2Q_2\\
&= s(Q_1+Q_2)\int_0^{Q_1+Q_2}f(x)dx - s\int_0^{Q_1+Q_2}(Q_1+Q_2-x)f(x)dx + r\int_0^{Q_1+Q_2}(Q_1+Q_2-x)f(x)dx + s(Q_1+Q_2)\int_{Q_1+Q_2}^{\infty}f(x)dx\\
&-k\int_{Q_1+Q_2}^{\infty}(x-Q_1-Q_2)f(x)dx - w_1Q_1 - w_2Q_2\\
&= s(Q_1+Q_2)\int_0^{\infty}f(x)dx - (s-r)\int_0^{Q_1+Q_2}(Q_1+Q_2-x)f(x)dx - k\int_{Q_1+Q_2}^{\infty}(x-Q_1-Q_2)f(x)dx - w_1Q_1 - w_2Q_2
\end{aligned} \tag{A-2}$$

Here, Eq. (A-2) is rewritten as follows:

$$E^1[\pi_R^{E1}(Q_1,Q_2)] = s(Q_1+Q_2) - (s-r)E[Q_1+Q_2-x]^+ - kE[x-Q_1-Q_2]^+ - w_1Q_1 - w_2Q_2 \tag{A-3}$$

By applying the Cauchy-Schwarz inequality into the term $E[Q_1+Q_2-x]^+$ in Eq. (A-3), it is derived that the expected excess inventory quantity in the left term in (A-4) is smaller than or equal to the upper limit of the right term in (A-4) as follows:

$$E[Q_1+Q_2-x]^+ \le \frac{\left[\sigma^2 + (\mu-Q_1-Q_2)^2\right]^{1/2} - (\mu-Q_1-Q_2)}{2}. \tag{A-4}$$

Therefore, the elicitation process of Eq. (10) can be shown as (A-4).

In the similar way, by applying the Cauchy-Schwarz inequality into the term $E[x-Q_1-Q_2]^+$ in Eq. (A-3), it is derived that the expected shortage quantity in the left term in (A-5) is smaller than or equal to the upper limit of the expected shortage quantity in the right term in (A-5) as follows:

$$E[x-Q_1-Q_2]^+ \le \frac{\left[\sigma^2 + (Q_1+Q_2-\mu)^2\right]^{1/2} - (Q_1+Q_2-\mu)}{2}. \tag{A-5}$$

Therefore, the elicitation process of Eq. (11) can be shown as (A-5).

In the similar way as the elicitation processes of Eqs. (10) and (11) corresponding to (A-4) and (A-5), the elicitation processes of Eqs. (12)-(25) as to events *E2*-*E*8 can be obtained.

- The elicitation process of the lower limit of the retailer's expected profit in event $E1$ using the Cauchy-Schwarz inequality in the distribution-free approach

Substituting the upper limits of the expected excess inventory quantity and the expected shortage quantity in (A-4) and (A-5) into the terms regarding the expected excess inventory quantity and the expected shortage quantity in the retailer's expected profit in Eqs. (1) in event $E1$, the lower limit of the retailer's expected profit in Eqs. (1) in event $E1$ can be derived as

$$E^1[\pi_R^{E1}(Q_1, Q_2)] \geq s(Q_1+Q_2)-(s-r)\frac{\left[\sigma^2+(\mu-Q_1-Q_2)^2\right]^{1/2}-(\mu-Q_1-Q_2)}{2}-k\frac{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}-(Q_1+Q_2-\mu)}{2}$$
$$-w_1Q_1-w_2Q_2=E^2[\pi_R^{E1}(Q_1, Q_2)]. \quad \text{(A-6)}$$

Eq. (A-6) is rewritten as follows:

$$\begin{aligned}
E^2[\pi_R^{E1}(Q_1, Q_2)] &= -w_1Q_1-w_2Q_2+s(Q_1+Q_2)\\
&-\frac{1}{2}\left\{(s-r)\left[\sigma^2+(\mu-Q_1-Q_2)^2\right]^{1/2}-(s-r)(\mu-Q_1-Q_2)+k\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}-k(Q_1+Q_2-\mu)\right\}\\
&=(s-w_1)Q_1+(s-w_2)Q_2-\frac{1}{2}\left\{(s-r+k)\left[\sigma^2+(\mu-Q_1-Q_2)^2\right]^{1/2}-(s-r-k)\mu+(s-r-k)(Q_1+Q_2)\right\}\\
&=\frac{1}{2}\left\{2(s-w_1)Q_1+2(s-w_2)Q_2-(s-r+k)\left[\sigma^2+(\mu-Q_1-Q_2)^2\right]^{1/2}+(s-r-k)\mu-(s-r-k)(Q_1+Q_2)\right\}\\
&=\frac{1}{2}\left\{(s-r-k)\mu+(s+r+k-2w_1)Q_1+(s+r+k-2w_2)Q_2-(s-r+k)\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}\right\}. \quad \text{(A-7)}
\end{aligned}$$

Therefore, the elicitation process of the lower limit of the retailer's expected profit in Eq. (1) in event $E1$ can be shown as (A-7).

In the similar way as the elicitation process of the lower limit of the retailer's expected profit in Eq. (1) in event $E1$ corresponding to (A-7), the individual lower limit of the retailer's expected profit as to events $E2$-$E8$ can be obtained by substituting the upper limits of the expected excess inventory quantity and the expected shortage quantity as to events $E2$-$E8$ in Eqs. (12)-(25) into terms regarding the expected excess inventory quantity and the expected shortage quantity in the retailer's expected profit in Eqs. (2)-(8) as to events $E1$-$E8$.

<Appendix B>

- Proof that the Hessian matrix of the whole system's total expected profit in Eq. (40) is positive in terms of the product order quantity Q_i (i =1, 2) to two manufacturers, M_1 and M_2 in scenario 1 of the demand information.

$$\begin{aligned}
\frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1^2} &= -(s-r+k)\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)f(Q_1+Q_2)\\
&+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)y_1^2 f(y_1Q_1+Q_2)\\
&+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)f(Q_1)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 f(Q_1+y_2Q_2)\\
&+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)y_1^2 f(y_1Q_1)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 y_1^2 f(y_1Q_1+y_2Q_2)\}. \quad \text{(B-1)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2^2} &= -(s-r+k)\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)f(Q_1, Q_2)\\
&+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)f(Q_2)\\
&+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)f(y_1Q_1+Q_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 y_2^2 f(Q_1+y_2Q_2)\\
&+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2 y_2^2 f(y_2Q_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 y_2^2 f(y_1Q_1+y_2Q_2)\}. \quad \text{(B-2)}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1 \partial Q_2} = \frac{\partial^2 G(Q_1, Q_2)}{\partial Q_2 \partial Q_1} &= -(s-r+k)\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)f(Q_1, Q_2)\\
&+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)y_1 f(y_1Q_1+Q_2)\\
&+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 y_2 f(Q_1+y_2Q_2)\\
&+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 y_1 y_2 f(y_1Q_1+y_2Q_2)\}. \quad \text{(B-3)}
\end{aligned}$$

The Hessian matrix of the whole system's total expected profit in Eq. (40) in terms of the product order quantity

Q_i (i = 1, 2) to two manufacturers, M_1 and M_2, in scenario 1 of the demand information is shown as follow:

$$\left|H_S^1\right| = \begin{pmatrix} H_1 & H_2 \\ H_3 & H_4 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1 \partial Q_2} \\ \dfrac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2 \partial Q_1} & \dfrac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2^2} \end{pmatrix}. \tag{B-4}$$

It is necessary to investigate if the Hessian matrix in Eq. (B-4) are either positive or negative to prove that the whole system 's total expected profit under scenario 1 of the demand information in Eq. (37) has the unique values regarding optimal order quantities to two manufacturer, M_1 and M_2, so as to maximize Eq. (37). These investigations use the following conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0 \le \alpha_i \le 1)$ in 3.(2) and $\beta_i(0 \le \beta_i \le 1)$ in 3.(3) and $y_i(0 \le y_i \le 1)$, in 3.(5), $s \ge k > r$ in 3. (6). Also, the following notations are used:

$$\begin{aligned}
&A = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2) f(Q_1 + Q_2), \quad B = (1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2) f(y_1 Q_1 + Q_2) \\
&C = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 f(Q_1 + y_2 Q_2) \quad \text{and} \quad D = (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 f(y_1 Q_1 + y_2 Q_2)\,. \\
&X_1 = (1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\,, \quad Z_1 = (1-\alpha_1)\beta_1\alpha_2(1-\beta_2)y_1^2\,, \quad X_2 = \alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2) \\
&Z_2 = \alpha_1(1-\beta_1)(1-\alpha_2)\beta_2 y_2^2
\end{aligned} \tag{B-5}$$

It is derived that $|H_1|$ corresponding to Eq. (B-1) is negative in terms of Q_1 as follows:

$$|H_1| = \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1^2} = -(s-r+k)\{A + y_1^2 B + Xf(Q_1) + C + Zf(y_1 Q_1) + y_1^2 D\} < 0. \tag{B-1'}$$

Therefore, the elicitation process of Eq. (38) can be shown as Eq. (B-1)'.

It is derived that $|H_4|$ corresponding to Eq. (B-2) is negative in terms of Q_2 as follows:

$$|H_4| = \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2^2} = -(s-r+k)\{A + X_2 f(Q_2) + B + y_2^2 C + Z_2 f(y_2 Q_2) + y_2^2 D\} < 0. \tag{B-2'}$$

Therefore, the elicitation process of Eq. (39) can be shown as Eq. (B-2)'.

Eq. $|H_2|$ and $|H_3|$ corresponding to Eq. (B-3) is rewritten as

$$|H_2| = \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1 \partial Q_2} = |H_3| = \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2 \partial Q_1} = -(s-r+k)\{A + y_1 B + y_2 C + y_1 y_2 D\}. \tag{B-3'}$$

Using Eqs. (B-1)'~(B-3)', the Hessian matrix in Eq. (B-4) is rewritten as

$$\left|H_S^1\right| = \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1^2} \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2^2} - \{\frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1 \partial Q_2}\}^2. \tag{B-4'}$$

The first term in Eq. (B-4)' is rewritten as

$$\begin{aligned}
&\frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_1^2} \frac{\partial^2 G_S^1(Q_1, Q_2)}{\partial Q_2^2} \\
&\quad = (s-r+k)^2 \left\{A + y_1^2 B + X_1 f(Q_1) + C + Z_1 f(y_1 Q_1) + y_1^2 D\right\}\left\{A + X_2 f(Q_2) + B + y_2^2 C + Z_2 f(y_2 Q_2) + y_2^2 D\right\} \\
&\quad \left\{A + y_1^2 B + X_1 f(Q_1) + C + Z_1 f(y_1 Q_1) + y_1^2 D\right\}\left\{A + X_2 f(Q_2) + B + y_2^2 C + Z_2 f(y_2 Q_2) + y_2^2 D\right\} \\
&\quad = A^2 + X_2 f(Q_2) A + AB + y_2^2 CA + Z_2 f(y_2 Q_2) A + y_2^2 DA + y_1^2 AB + y_1^2 X_2 f(Q_2) B
\end{aligned}$$

$$
\begin{aligned}
&+y_1^2B^2+y_1^2y_2^2BC+y_1^2Z_2f(y_2Q_2)B+y_1^2y_2^2BD\\
&+X_1f(Q_1)A+X_1f(Q_1)X_2f(Q_2)+X_1f(Q_1)B+y_2^2X_1f(Q_1)C+X_1f(Q_1)Z_2f(y_2Q_2)+y_2^2X_1f(Q_1)D\\
&+AC+X_2f(Q_2)C+BC+y_2^2C^2+Z_2f(y_2Q_2)C+y_2^2CD\\
&+Z_1f(y_1Q_1)A+Z_1f(y_1Q_1)X_2f(Q_2)+Z_1f(y_1Q_1)B+y_2^2Z_1f(y_1Q_1)C+Z_1f(y_1Q_1)Z_2f(y_2Q_2)+y_2^2Z_1f(y_1Q_1)D\\
&+y_1^2AD+y_1^2X_2f(Q_2)D+y_1^2BD+y_1^2y_2^2CD+y_1^2Z_2f(y_2Q_2)D+y_1^2y_2^2D^2\\
&=A^2+AB+y_2^2CA+y_2^2DA+y_1^2AB+y_1^2B^2+y_1^2y_2^2BC+y_1^2y_2^2BD+AC+BC+y_2^2C^2+y_2^2CD\\
&+y_1^2AD+y_1^2BD+y_1^2y_2^2CD+y_1^2y_2^2D^2+K,
\end{aligned}
\tag{B-6}
$$

where,

$$
\begin{aligned}
K=\;&X_2f(Q_2)A+Z_2f(y_2Q_2)A+y_1^2X_2f(Q_2)B+y_1^2Z_2f(y_2Q_2)B\\
&+X_1f(Q_1)A+X_1f(Q_1)B+X_1f(Q_1)X_2f(Q_2)+y_2^2X_1f(Q_1)C+X_1f(Q_1)Z_2f(y_2Q_2)+y_2^2X_1f(Q_1)D\\
&+X_2f(Q_2)C+Z_2f(y_2Q_2)C+Z_1f(y_1Q_1)A+Z_1f(y_1Q_1)B+Z_1f(y_1Q_1)X_2f(Q_2)+y_2^2Z_1f(y_1Q_1)C\\
&+Z_1f(y_1Q_1)Z_2f(y_2Q_2)+y_2^2Z_1f(y_1Q_1)D+y_1^2X_2f(Q_2)D+y_1^2Z_2f(y_2Q_2)D.
\end{aligned}
\tag{B-7}
$$

It is clear that $K>0$ in Eq. (B-7).

The second term in Eq. (B-4)' is rewritten as

$$
\left\{\frac{\partial^2 G_S^1(Q_1,Q_2)}{\partial Q_1\partial Q_2}\right\}^2=(s-r+k)^2(A+y_1B+y_2C+y_1y_2D)^2,
\tag{B-8}
$$

where

$$
\begin{aligned}
(A+y_1B+y_2C+y_1y_2D)^2=\;&A^2+y_1AB+y_2AC+y_1y_2AD+y_1AB+y_1^2B^2+y_1y_2BC+y_1^2y_2BD\\
&+y_2AC+y_1y_2BC+y_2^2C^2+y_1y_2^2CD+y_1y_2AD+y_1^2y_2BD+y_1y_2^2CD+y_1^2y_2^2D^2.
\end{aligned}
\tag{B-9}
$$

Therefore, using Eqs. (B-6)-(B-9) and, the Hessian matrix in Eq. (B-4)' is rewritten as

$$
\begin{aligned}
\left|H_S^1\right|&=|H_1||H_4|-|H_2||H_3|=\frac{\partial^2G_S^1(Q_1,Q_2)}{\partial Q_1^2}\frac{\partial^2G_S^1(Q_1,Q_2)}{\partial Q_2^2}-\left\{\frac{\partial^2G_S^1(Q_1,Q_2)}{\partial Q_1\partial Q_2}\right\}^2\\
&=(s-r+k)^2\{(1+y_1^2-2y_1)AB+(1+y_2^2-2y_2)AC+(y_1^2+y_2^2-2y_1y_2)AD\\
&+(1+y_1^2y_2^2-2y_1y_2)BC+(y_1^2+y_1^2y_2^2-2y_1^2y_2)BD+(y_2^2+y_1^2y_2^2-2y_1y_2^2)CD+K\}\\
&=(s-r+k)^2\{(y_1-1)^2AB+(y_2-1)^2AC+(y_1-y_2)^2AD\\
&+(y_1y_2-1)^2BC+y_1^2(y_2-1)^2BD+y_2^2(y_2-1)^2CD+K\}.
\end{aligned}
\tag{B-4)''}
$$

From Eq. (B-5) and the following conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0\le\alpha_i\le1)$ in 3.(2) and $\beta_i(0\le\beta_i\le1)$ in 3.(3) and $y_i(0\le y_i\le1)$, in 3.(5), $s\ge k>r$ in 3.(6), it is clear that $\left|H_S^1\right|>0$ in (B-4)'' in terms of the product order quantity Q_i (i = 1, 2) to two manufacturers, M_1 and M_2, in scenario 1 of the demand information. Therefore, the proof of Eq. (40) can be shown in scenario 1 of the demand information.

Similarly, it can be proved that the Hessian matrix of the retailer's total expected profit in Eq. (46) is positive in terms of the product order quantities Q_1 and Q_2 to two manufacturers, M_1 and M_2 in scenario 1 of the demand information.

- Proof that the Hessian matrix of the whole system's total expected profit in Eq. (40) is positive in terms of in terms of the product order quantity Q_i (i = 1, 2) to two manufacturers, M_1 and M_2 in scenario 2 of the demand information.

$$\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1^2} = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\left\{-\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\left\{-\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\left\{-\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}. \tag{B-10}$$

$$\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2^2} = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_2-\mu)^2\right]^{3/2}}\right\}+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\left\{-\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\left\{-\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\left\{-\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}. \tag{B-11}$$

$$\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1 \partial Q_2} = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\left\{-\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\left\{-\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}. \tag{B-12}$$

$$\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2 \partial Q_1} = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\left\{-\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{3/2}}\right\}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\left\{-\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}$$
$$+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\left\{-\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{3/2}}\right\}. \tag{B-13}$$

The Hessian matrix of the whole system's total expected profit in Eq. (40) in terms of the product order quantity Q_i (i = 1, 2) to two manufacturers, M_1 and M_2 in scenario 2 of the demand information is shown as follow:

$$\left|H_S^2\right| = \begin{pmatrix} H_1 & H_2 \\ H_3 & H_4 \end{pmatrix} = \begin{pmatrix} \dfrac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1^2} & \dfrac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1 \partial Q_2} \\ \dfrac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2 \partial Q_1} & \dfrac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2^2} \end{pmatrix} \tag{B-14}$$

It is necessary to investigate if the Hessian matrix in Eq. (B-14) are either positive or negative to prove that the whole system's total expected profit under scenario 2 of the demand information in Eq. (37) has the unique values regarding optimal order quantities to two manufacturer, M_1 and M_2, so as to maximize Eq. (37). These investigations use the following conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0 \le \alpha_i \le 1)$ in 3.(2) and $\beta_i(0 \le \beta_i \le 1)$ in 3.(3) and $y_i(0 \le y_i \le 1)$, in 3.(5), $s \ge k > r$ in 3. (6). Also, the following notations are used. Also, the following notations are used:

$$A = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{3/2}},\ B = (1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{3/2}}$$
$$X_1 = (1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1-\mu)^2\right]^{3/2}},\ C = (1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{3/2}}$$
$$Z_1 = (1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\frac{y_1\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{3/2}},\ D = (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{3/2}}$$
$$X_2 = \alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{\sigma^2(s-r+k)}{2\left[\sigma^2+(Q_2-\mu)^2\right]^{3/2}},\ Z_2 = \alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\frac{y_2\sigma^2(s-r+k)}{2\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{3/2}} \tag{B-15}$$

It is derived that $|H_1|$ in Eq. (B-10) is negative in terms of Q_1 as follows:

$$|H_1| = \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1^2} = -\left(A + y_1B + X_1 + C + Z_1 + y_1D\right) < 0 \tag{B-10)'}$$

Therefore, the elicitation process of Eq. (38) can be shown as (B-10)'.

It is derived that $|H_4|$ in Eq. (B-11) is negative in terms of Q_2 as follows:

$$|H_4| = \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2^2} = -\left(A + X_2 + B + y_2C + Z_2 + y_2D\right) < 0 \tag{B-11)'}$$

Therefore, the elicitation process of Eq. (39) can be shown as (B-11)'.

$|H_2|$ in Eq. (B-12) is rewritten as

$$|H_2| = \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1 \partial Q_2} = -\left(A + B + y_2C + y_2D\right). \tag{B-12)'}$$

$|H_3|$ in Eq. (B-13) is rewritten as

$$|H_3| = \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2 \partial Q_1} = -\left(A + y_1B + C + y_1D\right). \tag{B-13)'}$$

Next, using Eqs. (B-15), (B-10)'~(B-13)', it is necessary to investigate if the Hessian matrix in Eq. (B-14) are either positive or negative. The Hessian matrix in Eq. (B-14) is rewritten as

$$\left|H_S^2\right| = |H_1||H_4| - |H_2||H_3| = \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1^2}\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2^2} - \frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_1 \partial Q_2}\frac{\partial^2 G_S^2(Q_1, Q_2)}{\partial Q_2 \partial Q_1}. \tag{B-14)'}$$

The first term in Eq. (B-14)' is rewritten as

$$\begin{aligned}
\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_1^2}\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_2^2} &= \left(A+y_1B+X_1+C+Z_1+y_1D\right)\left(A+X_2+B+y_2C+Z_2+y_2D\right) \\
&= A^2+X_2A+BA+y_2CA+Z_2A+y_2DA+y_1BA+X_2y_1B+y_1B^2+y_1y_2BC+y_1Z_2B+y_1y_2BD \\
&+X_1A+X_1X_2+X_1B+X_1y_2C+X_1Z_2+X_1y_2D+CA+X_2C+BC+y_2C^2+Z_2C+y_2CD \\
&+Z_1A+Z_1X_2+Z_1B+Z_1y_2C+Z_1Z_2+Z_1y_2D+y_1DA+y_1X_2D+y_1BD+y_1y_2CD+Z_2y_1D+y_1y_2D^2.
\end{aligned} \tag{B-16}$$

The second term in Eq. (B-14)' is rewritten as

$$\begin{aligned}
\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_1\partial Q_2}\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_2\partial Q_1} &= \left(A+B+y_2C+y_2D\right)\left(A+y_1B+C+y_1D\right) \\
&= A^2+y_1AB+AC+y_1DA+AB+y_1B^2+BC+y_1BD \\
&+y_2CA+y_1y_2BC+y_2C^2+y_1y_2CDy_2DA+y_1y_2BD+y_2CD+y_1y_2D^2.
\end{aligned} \tag{B-17}$$

Therefore, using Eqs. (B-16)~(B-17), the Hessian matrix in Eq. (B-14)' is rewritten as

$$\begin{aligned}
\left|H_S^2\right| = |H_1||H_4|-|H_2||H_3| &= \frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_1^2}\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_2^2} - \frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_1\partial Q_2}\frac{\partial^2 G_S^2(Q_1,Q_2)}{\partial Q_2\partial Q_1} \\
&= X_2A+Z_2A+X_2y_1B+y_1Z_2B+X_1A+X_1X_2+X_1B+X_1y_2C+X_1Z_2+X_1y_2D \\
&+X_2C+Z_2C+Z_1A+Z_1X_2+Z_1B+Z_1y_2C+Z_1Z_2+Z_1y_2D+y_1X_2D+Z_2y_1D.
\end{aligned} \tag{B-14}''$$

From Eq. (B-15) and the following conditions: $\mu(>0)$ and $\sigma^2(>0)$ in 3.(1), $\alpha_i(0 \le \alpha_i \le 1)$ in 3.(2) and $\beta_i(0 \le \beta_i \le 1)$ in 3.(3) and $y_i(0 \le y_i \le 1)$, in 3.(5), $s \ge k > r$ in 3. (6), it is clear that $\left|H_S^2\right| > 0$ in (B-14)" in terms of the product order quantity Q_i (i = 1, 2) to two manufacturers, M_1 and M_2 in scenario 2 of the demand information. Therefore, the proof of Eq. (40) in scenario 2 of demand information can be shown.

Similarly, it can be proved that the Hessian matrix of the retailer's total expected profit in Eq. (46) is positive in terms of the product order quantities Q_1 and Q_2 to two manufacturers, M_1 and M_2 in scenario 2 of the demand information.

<Appendix C>

- Derivation of the first-order differential equations of the total expected profit of whole system in Eq. (37) in scenario 1 of the demand information to obtain the optimal order quantities, Q_1^{I1} and Q_2^{I1}, of two manufacturers, M_1 and M_2 under ISC

The total expected profit of the whole system as to event $E1$-$E9$ is calculated as the sum of the total expected profit of a retailer as to event $E1$-$E9$ in Eqs. (1)-(9) and the total expected profit of two manufacturers, M_1 and M_2 as to event $E1$-$E9$ in Eqs. (26)-(34). The first-order differential equation of the total expected profit of the whole system as to event $E1$-$E9$ in terms of Q_1 in scenario 1 of the demand information can be derived as follows:

$$\partial E^1[\pi_S^{E1}(Q_1,Q_2)]/\partial Q_1 = (r-s-k)F(Q_1+Q_2)+(s-c_1+k)-\gamma c_1 \tag{C-1}$$
$$\partial E^1[\pi_S^{E2}(Q_1,Q_2)]/\partial Q_1 = -\gamma c_1 \tag{C-2}$$
$$\partial E^1[\pi_S^{E3}(Q_1,Q_2)]/\partial Q_1 = y_1(r-s-k)F(y_1Q_1+Q_2)+y_1(s-c_1+k)-\gamma c_1 \tag{C-3}$$
$$\partial E^1[\pi_S^{E4}(Q_1,Q_2)]/\partial Q_1 = (r-s-k)F(Q_1)+y_1(s-c_1+k)-\gamma c_1 \tag{C-4}$$
$$\partial E^1[\pi_S^{E5}(Q_1,Q_2)]/\partial Q_1 = (r-s-k)F(Q_1+y_2Q_2)+(s-c_1+k)-\gamma c_1 \tag{C-5}$$
$$\partial E^1[\pi_S^{E6}(Q_1,Q_2)]/\partial Q_1 = -\gamma c_1 \tag{C-6}$$

$$\partial E^1[\pi_S^{E7}(Q_1, Q_2)]/\partial Q_1 = y_1(r-s-k)F(y_1Q_1) + (s-c_1+k) - \gamma c_1 \tag{C-7}$$
$$\partial E^1[\pi_S^{E8}(Q_1, Q_2)]/\partial Q_1 = y_1(r-s-k)F(y_1Q_1 + y_2Q_2) + (s-c_1+k) - \gamma c_1 \tag{C-8}$$
$$\partial E^1[\pi_S^{E9}(Q_1, Q_2)]/\partial Q_1 = -\gamma c_1. \tag{C-9}$$

Considering the individual event probability as to events $E1$-$E9$ in Table 1, the first-order differential equation of the total expected profit of the whole system considering all events from event $E1$ to event $E9$ in Eq. (37) in terms of Q_1 in scenario 1 of the demand information can be derived as the following sum from Eq. (C-1) to Eq. (C-9):

$$\begin{aligned}\partial G_S^1(Q_1, Q_2)/\partial Q_1 &= \sum_{\ell=1}^{9} P^\ell \left\{\partial E^1[\pi_S^{E\ell}(Q_1, Q_2)]/\partial Q_1\right\} \\ &= (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_1+Q_2) + (1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)y_1F(y_1Q_1+Q_2) \\ &\quad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)y_1F(y_1Q_1) + (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 y_1F(y_1Q_1+y_2Q_2) \\ &\quad -\frac{1+\alpha_2+\beta_2-\alpha_1\beta_1-\alpha_2\beta_2+\alpha_1\alpha_2\beta_1\beta_2}{(r-s-k)}\gamma c_1 \\ &\quad +\frac{(s-c_1+k)(1-\alpha_1-\beta_1+\alpha_1\beta_1-\alpha_2\beta_2+\alpha_1\alpha_2\beta_2+\alpha_2\beta_1\beta_2-\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} \\ &\quad +\frac{(s-c_1+k)y_1(\beta_1-\alpha_1\beta_1-\alpha_2\beta_1\beta_2+\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} = 0\end{aligned} \tag{C-10}$$

Therefore, the derivation of Eq. (42) can be derived as (C-10).

In the similar way mentioned above, the first-order differential equation of the total expected profit of the whole system as to event $E1$-$E9$ in terms of Q_2 in scenario 1 of the demand information can be derived as follows:

$$\partial E^1[\pi_S^{E1}(Q_1, Q_2)]/\partial Q_2 = (r-s-k)F(Q_1+Q_2) + (s-c_2+k) - \gamma c_2 \tag{C-11}$$
$$\partial E^1[\pi_S^{E2}(Q_1, Q_2)]/\partial Q_2 = (r-s-k)F(Q_2) + (s-c_2+k) - \gamma c_2 \tag{C-12}$$
$$\partial E^1[\pi_S^{E3}(Q_1, Q_2)]/\partial Q_2 = (r-s-k)F(y_1Q_1+Q_2) + (s-c_2+k) - \gamma c_2 \tag{C-13}$$
$$\partial E^1[\pi_S^{E4}(Q_1, Q_2)]/\partial Q_2 = -\gamma c_2 \tag{C-14}$$
$$\partial E^1[\pi_S^{E5}(Q_1, Q_2)]/\partial Q_2 = y_2(r-s-k)F(Q_1+y_2Q_2) + y_2(s-c_2+k) - \gamma c_2 \tag{C-15}$$
$$\partial E^1[\pi_S^{E6}(Q_1, Q_2)]/\partial Q_2 = y_2(r-s-k)F(y_2Q_2) + y_2(s-c_2+k) - \gamma c_2 \tag{C-16}$$
$$\partial E^1[\pi_S^{E7}(Q_1, Q_2)]/\partial Q_2 = -\gamma c_2 \tag{C-17}$$
$$\partial E^1[\pi_S^{E8}(Q_1, Q_2)]/\partial Q_2 = y_2(r-s-k)F(y_1Q_1+y_2Q_2) + y_2(s-c_2+k) - \gamma c_2 \tag{C-18}$$
$$\partial E^1[\pi_S^{E9}(Q_1, Q_2)]/\partial Q_2 = -\gamma c_2 \tag{C-19}$$

Considering the individual event probability as to events $E1$-$E9$ in Table 1, the first-order differential equation of the total expected profit of the whole system considering all events from event $E1$ to event $E9$ in Eq. (37) in terms of Q_2 in scenario 1 of the demand information can be derived as the following sum from Eq. (C-11) to Eq. (C-19):

$$\begin{aligned}\partial G_S^1(Q_1, Q_2)/\partial Q_2 &= \sum_{\ell=1}^{9} P^\ell \left\{\partial E^1[\pi_S^{E\ell}(Q_1, Q_2)]/\partial Q_2\right\} \\ &= (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_1+Q_2) + \alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_2) \\ &\quad +(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)F(y_1Q_1+Q_2) + (1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 y_2F(Q_1+y_2Q_2) \\ &\quad +\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2 y_2F(y_2Q_2) + (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2 y_2F(y_1Q_1+y_2Q_2) \\ &\quad -\frac{1+\alpha_2+\beta_2-\alpha_1\beta_1-\alpha_2\beta_2+\alpha_1\alpha_2\beta_1\beta_2}{(r-s-k)}\gamma c_2 \\ &\quad +\frac{(s-c_2+k)(1-\alpha_2-\beta_2+\alpha_2\beta_2-\alpha_1\beta_1+\alpha_1\alpha_2\beta_1+\alpha_1\beta_1\beta_2-\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} \\ &\quad +\frac{(s-c_2+k)y_2(\beta_2-\alpha_2\beta_2-\alpha_1\beta_1\beta_2+\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} = 0\end{aligned} \tag{C-20}$$

Therefore, the derivation of Eq. (43) can be derived as (C-20).

In Similar way as scenario 1 of the demand information, the first-order differential equations of the total expected profit of the whole system in Eq. (37) in scenario 2 of the demand information in terms of the order quantities Q_1 and Q_2 can be derived respectively as

$$\begin{aligned}\frac{\partial G_S^2(Q_1,Q_2)}{\partial Q_1} &= \frac{1}{2}(s+r+k-2c_1)\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\\&\quad+y_1\{(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big]\\&-\frac{1}{2}(s-r+k)\times\Bigg[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}}\\&\quad+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\frac{Q_1-\mu}{\left[\sigma^2+(Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\\&\quad+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\frac{y_1Q_1-\mu}{\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg]\\&-\gamma c_1\times\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\\&\quad+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\\&\quad+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)\alpha_2(1-\beta_2)\}=0\end{aligned}\tag{C-21}$$

$$\begin{aligned}\frac{\partial G_S^2(Q_1,Q_2)}{\partial Q_2} &= \frac{1}{2}(s+r+k-2c_2)\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\\&\quad+y_2\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big]\\&-\frac{1}{2}(s-r+k)\times\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_2-\mu}{\left[\sigma^2+(Q_2-\mu)^2\right]^{1/2}}\\&\quad+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\\&\quad+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\frac{y_2Q_2-\mu}{\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg\}\\&-\gamma c_2\times\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\\&\quad+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\\&\quad+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)\alpha_2(1-\beta_2)\}=0\end{aligned}\tag{C-22}$$

<Appendix D>

- Derivation of the first-order differential equations of the total expected profit of a retailer in Eq. (35) in scenario 1 of the demand information to obtain the optimal order quantities, Q_1^{D1} and Q_2^{D1}, of two manufacturers, M_1 and M_2 under DSC

The first-order differential equation of the total expected profit of a retailer as to event $E1$-$E9$ in Eqs. (1)-(9) in terms of Q_1 in scenario 1 of the demand information can be derived as follows:

$$\partial E^1[\pi_R^{E1}(Q_1,Q_2)]/\partial Q_1=(r-s-k)F(Q_1+Q_2)+(s-w_1+k)\tag{D-1}$$

$$\partial E^1[\pi_R^{E2}(Q_1,Q_2)]/\partial Q_1=0\tag{D-2}$$

$$\partial E^1[\pi_R^{E3}(Q_1,Q_2)]/\partial Q_1=y_1(r-s-k)F(y_1Q_1+Q_2)+y_1(s-w_1+k)\tag{D-3}$$

$$\partial E^1[\pi_R^{E4}(Q_1, Q_2)] \big/ \partial Q_1 = (r-s-k)F(Q_1) + y_1(s-w_1+k) \quad \text{(D-4)}$$
$$\partial E^1[\pi_R^{E5}(Q_1, Q_2)] \big/ \partial Q_1 = (r-s-k)F(Q_1+y_2Q_2) + (s-w_1+k) \quad \text{(D-5)}$$
$$\partial E^1[\pi_R^{E6}(Q_1, Q_2)] \big/ \partial Q_1 = 0 \quad \text{(D-6)}$$
$$\partial E^1[\pi_R^{E7}(Q_1, Q_2)] \big/ \partial Q_1 = y_1(r-s-k)F(y_1Q_1) + (s-w_1+k) \quad \text{(D-7)}$$
$$\partial E^1[\pi_R^{E8}(Q_1, Q_2)] \big/ \partial Q_1 = y_1(r-s-k)F(y_1Q_1+y_2Q_2) + (s-w_1+k) \quad \text{(D-8)}$$
$$\partial E^1[\pi_R^{E9}(Q_1,Q_2)] \big/ \partial Q_1 = 0. \quad \text{(D-9)}$$

The first-order differential equation of the total expected profit of a retailer considering all events from event $E1$ to event $E9$ in Eq. (35) in terms of Q_1 in scenario 1 of the demand information can be derived as the following sum from Eq. (D-1) to Eq. (D-9):

$$\begin{aligned}\partial G_R^1(Q_1, Q_2) \big/ \partial Q_1 &= \sum_{\ell=1}^{9} P^\ell \left\{ \partial E^1[\pi_R^{E\ell}(Q_1, Q_2)] \big/ \partial Q_1 \right\} \\ &= (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_1+Q_2) + (1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)y_1F(y_1Q_1+Q_2) \\ &+ (1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)F(Q_1) + (1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2F(Q_1+y_2Q_2) \\ &+ (1-\alpha_1)\beta_1\alpha_2(1-\beta_2)y_1F(y_1Q_1) + (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2y_1F(y_1Q_1+y_2Q_2) \\ &+ \frac{(s-w_1+k)(1-\alpha_1-\beta_1+\alpha_1\beta_1-\alpha_2\beta_2+\alpha_1\alpha_2\beta_2+\alpha_2\beta_1\beta_2-\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} \\ &+ \frac{(s-w_1+k)y_1(\beta_1-\alpha_1\beta_1-\alpha_2\beta_1\beta_2+\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} = 0 \end{aligned} \quad \text{(D-10)}$$

Therefore, the derivation of Eq. (48) can be derived as (D-10).

In the similar way mentioned above, the first-order differential equation of the total expected profit of a retailer as to event $E1$-$E9$ in Eqs. (1)-(9) in terms of Q_2 in scenario 1 of the demand information can be derived as follows:

$$E^1[\pi_R^{E1}(Q_1, Q_2)] \big/ \partial Q_2 = (r-s-k)F(Q_1+Q_2) + (s-w_2+k) \quad \text{(D-11)}$$
$$E^1[\pi_R^{E2}(Q_1, Q_2)] \big/ \partial Q_2 = (r-s-k)F(Q_2) + (s-w_2+k) \quad \text{(D-12)}$$
$$E^1[\pi_R^{E3}(Q_1, Q_2)] \big/ \partial Q_2 = (r-s-k)F(y_1Q_1+Q_2) + (s-w_2+k) \quad \text{(D-13)}$$
$$E^1[\pi_R^{E4}(Q_1, Q_2)] \big/ \partial Q_2 = 0 \quad \text{(D-14)}$$
$$E^1[\pi_R^{E5}(Q_1, Q_2)] \big/ \partial Q_2 = y_2(r-s-k)F(Q_1+y_2Q_2) + y_2(s-w_2+k) \quad \text{(D-15)}$$
$$E^1[\pi_R^{E6}(Q_1, Q_2)] \big/ \partial Q_2 = y_2(r-s-k)F(y_2Q_2) + y_2(s-w_2+k) \quad \text{(D-16)}$$
$$E^1[\pi_R^{E7}(Q_1, Q_2)] \big/ \partial Q_2 = 0 \quad \text{(D-17)}$$
$$E^1[\pi_R^{E8}(Q_1, Q_2)] \big/ \partial Q_2 = y_2(r-s-k)F(y_1Q_1+y_2Q_2) + y_2(s-w_2+k) \quad \text{(D-18)}$$
$$E^1[\pi_R^{E9}(Q_1, Q_2)] \big/ \partial Q_2 = 0 \quad \text{(D-19)}$$

The first-order differential equation of the total expected profit of a retailer considering all events from event $E1$ to event $E9$ in Eq. (35) in terms of Q_2 in scenario 1 of the demand information can be derived as the following sum from Eq. (D-11) to Eq. (D-19):

$$\begin{aligned}\partial G_R^1(Q_1,Q_2) \big/ \partial Q_2 &= \sum_{\ell=1}^{9} P^\ell \left\{ \partial E^1[\pi_R^{E\ell}(Q_1,Q_2)] \big/ \partial Q_2 \right\} \\ &= (1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_1+Q_2) + \alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)F(Q_2) \\ &+ (1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)F(y_1Q_1+Q_2) + (1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2y_2F(Q_1+y_2Q_2) \\ &+ \alpha_1(1-\beta_1)(1-\alpha_2)\beta_2y_2F(y_2Q_2) + (1-\alpha_1)\beta_1(1-\alpha_2)\beta_2y_2F(y_1Q_1+y_2Q_2) \\ &+ \frac{(s-w_2+k)(1-\alpha_2-\beta_2+\alpha_2\beta_2-\alpha_1\beta_1+\alpha_1\alpha_2\beta_1+\alpha_1\beta_1\beta_2-\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} \\ &+ \frac{(s-w_2+k)y_2(\beta_2-\alpha_2\beta_2-\alpha_1\beta_1\beta_2+\alpha_1\alpha_2\beta_1\beta_2)}{(r-s-k)} = 0 \end{aligned} \quad \text{(D-20)}$$

Therefore, the derivation of Eq. (49) can be derived as (D-20).

In Similar way as scenario 1 of the demand information, the first-order differential equations of the total expected profit of a retailer in Eq. (35) in scenario 2 of the demand information in terms of the order quantities Q_1 and Q_2 can be derived respectively as

$$\begin{aligned}\frac{\partial G_R^2(Q_1,Q_2)}{\partial Q_1} &= \frac{1}{2}(s+r+k-2w_1)\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 \\ &\quad +y_1\{(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big] \\ &\quad -\frac{1}{2}(s-r+k)\times\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}} \\ &\quad +(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\frac{Q_1-\mu}{\left[\sigma^2+(Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}} \\ &\quad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\frac{y_1Q_1-\mu}{\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg\}=0. \qquad \text{(D-21)}\end{aligned}$$

$$\begin{aligned}\frac{\partial G_R^2(Q_1,Q_2)}{\partial Q_2} &= \frac{1}{2}(s+r+k-2w_2)\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2) \\ &\quad +y_2\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big] \\ &\quad -\frac{1}{2}(s-r+k)\times\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_2-\mu}{\left[\sigma^2+(Q_2-\mu)^2\right]^{1/2}} \\ &\quad +(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}} \\ &\quad +\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\frac{y_2Q_2-\mu}{\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg\}=0. \qquad \text{(D-22)}\end{aligned}$$

<Appendix E>

- Derivation of the magnitude relation between the optimal product order quantities to two manufacturers under ISC and those under DSC in scenario 2 of the demand information.

Eq. (C-21) where $\frac{\partial G_S^2(Q_1,Q_2)}{\partial Q_1}=0$ can be rewritten as

$$\begin{aligned}&\Bigg[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}} \\ &\quad +(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\frac{Q_1-\mu}{\left[\sigma^2+(Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}} \\ &\quad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\frac{y_1Q_1-\mu}{\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg] \\ &=\frac{(s+r+k-2c_1)}{(s-r+k)}\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\end{aligned}$$

$$
\begin{aligned}
&\quad +y_1\{(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big] \\
&-\frac{2\gamma c_1}{(s-r+k)}\times\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2) \\
&\quad +(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2 \\
&\quad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)\alpha_2(1-\beta_2)\}.
\end{aligned} \tag{E-1}
$$

Eq. (C-22) where $\frac{\partial G_S^2(Q_1,Q_2)}{\partial Q_1}=0$ can be rewritten as

$$
\begin{aligned}
&\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_2-\mu}{\left[\sigma^2+(Q_2-\mu)^2\right]^{1/2}} \\
&\quad +(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}} \\
&\quad +\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\frac{y_2Q_2-\mu}{\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg\} \\
&\quad =\frac{(s+r+k-2c_2)}{(s-r+k)}\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2) \\
&\qquad +y_2\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big] \\
&\quad -\frac{2\gamma c_2}{(s-r+k)}\times\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2) \\
&\qquad +(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2 \\
&\qquad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)\alpha_2(1-\beta_2)\}.
\end{aligned} \tag{E-2}
$$

Eq. (D-21) where $\frac{\partial G_R^2(Q_1,Q_2)}{\partial Q_1}=0$ can be rewritten as

$$
\begin{aligned}
&\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}} \\
&\quad +(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)\frac{Q_1-\mu}{\left[\sigma^2+(Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}} \\
&\quad +(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)\frac{y_1Q_1-\mu}{\left[\sigma^2+(y_1Q_1-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\Bigg\} \\
&\quad =\frac{(s+r+k-2w_1)}{(s-r+k)}\times\big[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)(1-\beta_1)\alpha_2(1-\beta_2)+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2 \\
&\quad +y_1\{(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1\alpha_2(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\}\big].
\end{aligned} \tag{E-3}
$$

Eq. (D-22) where $\frac{\partial G_R^2(Q_1,Q_2)}{\partial Q_1}=0$ can be rewritten as

$$
\Bigg\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_1+Q_2-\mu}{\left[\sigma^2+(Q_1+Q_2-\mu)^2\right]^{1/2}}+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)\frac{Q_2-\mu}{\left[\sigma^2+(Q_2-\mu)^2\right]^{1/2}}
$$

$$+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\frac{y_1Q_1+Q_2-\mu}{\left[\sigma^2+(y_1Q_1+Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2\frac{Q_1+y_2Q_2-\mu}{\left[\sigma^2+(Q_1+y_2Q_2-\mu)^2\right]^{1/2}}$$

$$\left.+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2\frac{y_2Q_2-\mu}{\left[\sigma^2+(y_2Q_2-\mu)^2\right]^{1/2}}+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\frac{y_1Q_1+y_2Q_2-\mu}{\left[\sigma^2+(y_1Q_1+y_2Q_2-\mu)^2\right]^{1/2}}\right\}$$

$$=\frac{(s+r+k-2w_2)}{(s-r+k)}\times\left[(1-\alpha_1)(1-\beta_1)(1-\alpha_2)(1-\beta_2)+\alpha_1(1-\beta_1)(1-\alpha_2)(1-\beta_2)+(1-\alpha_1)\beta_1(1-\alpha_2)(1-\beta_2)\right.$$

$$\left.+y_2\left\{(1-\alpha_1)(1-\beta_1)(1-\alpha_2)\beta_2+\alpha_1(1-\beta_1)(1-\alpha_2)\beta_2+(1-\alpha_1)\beta_1(1-\alpha_2)\beta_2\right\}\right]. \quad \text{(E-4)}$$

In order to verify analytically the magnitude relation between optimal order quantity to manufacturer 1 (M_1) under DSC and that under ISC, Eq. (E-1) is compared with Eq. (E-3). It is shown that all terms in left part of Eq. (E-1) is equal to that in Eq. (E-3). Also, it can be seen that the difference between the right part of Eq. (E-1) and that of Eq. (E-3) is the magnitude relation between c_1 and w_1. Here, we can see the magnitude relation where (E-1) > (E-3) due to the condition $c_1 < w_1$. This indicates that the value of the cumulative density function of the product demand under ISC in Eq. (E-1) corresponding to Eq. (42) is larger than that under DSC in Eq. (E-3) corresponding to Eq. (48). Moreover, it can be seen that the third term in the right part of Eq. (E-1) is much smaller than 0. This is because $0<\alpha_i(i=1, 2)<1$, $0<\beta_i(i=1, 2)<1$ and the term $2\gamma c_1$ is divided by $(s-r+k)$. However, it clear that it doesn't affect the optimal order quantity. Therefore, it can be proved that the optimal order quantity to Manufacturer 1 under ISC is larger than that under DSC. In similar way, in order to verify analytically the magnitude relation between optimal order quantity to manufacturer 2 (M_2) under DSC and that under ISC, Eq. (E-2) is compared with Eq. (E-4). It is shown that all terms in left part of Eq. (E-2) is equal to that in Eq. (E-4). Also, it can be seen that the difference between the right part of Eq. (E-2) and that of Eq. (E-4) is the magnitude relation between c_2 and w_2. Here, we can see the magnitude relation where (E-2) > (E-4) due to the condition $c_2 < w_2$. This indicates that the value of the cumulative density function of the product demand under ISC in Eq. (E-2) corresponding to Eq. (43) is larger than that under DSC in Eq. (E-4) corresponding to Eq. (49). Moreover, it can be seen that the third term in the right part of Eq. (E-2) is much smaller than 0. It is because $0<\alpha_i(i=1, 2)<1$, $0<\beta_i(i=1, 2)<1$ and the term $2\gamma c_2$ is divided by $(s-r+k)$. However, it is clear that it doesn't affect the optimal order quantity. Therefore, it can be proved that the optimal order quantity to Manufacturer 2(M_2) under ISC is larger than that under DSC.

The similar way, the magnitude relation where the optimal order quantities to two manufacturers, M_1 and M_2, under ISC are larger than those under DSC can be proved analytically in scenario 1 of the demand information.