

# Phenomenological Modeling of Newly Discovered Eclipsing Binary 2MASS J18024395 + 4003309 = VSX J180243.9+400331

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We present a by-product of our long term photometric monitoring of cataclysmic variables. 2MASS J18024395 +4003309 = VSX J180243.9 +400331 was discovered in the field of the intermediate polar V1323 Her observed using the Korean 1-m telescope located at Mt. Lemmon, USA. An analysis of the two-color VR CCD observations of this variable covers all the phase intervals for the first time. The light curves show this object can be classified as an Algol-type variable with tidally distorted components, and an asymmetry of the maxima (the O'Connell effect). The periodogram analysis confirms the cycle numbering of Andronov et al. (2012) and for the initial approximation, the ephemeris is used as follows: Min I. BJD = 2456074.4904+0.3348837E. For phenomenological modeling, we used the trigonometric polynomial approximation of statistically optimal degree, and a recent method "NAV" ("New Algol Variable") using local specific shapes for the eclipse. Methodological aspects and estimates of the physical parameters based on analysis of phenomenological parameters are presented. As results of our phenomenological model, we obtained for the inclination  $i=90^\circ$ ,  $M_1=0.745M_\odot$ ,  $M_2=0.854M_\odot$ ,  $M=M_1+M_2=1.599M_\odot$ , the orbital separation  $a=1.65\cdot 10^9\text{ m}=2.37R_\odot$  and relative radii  $r_1=R_1/a=0.314$  and  $r_2=R_2/a=0.360$ . These estimates may be used as preliminary starting values for further modeling using extended physical models based on the Wilson & Devinney (1971) code and its extensions

**Keywords:** 2MASS J18024395 + 4003309 = VSX J180243.9 + 400331, eclipsing binary, CCD photometry

## 1. INTRODUCTION

Chungbuk National University Observatory (CBNUO) is monitoring many cataclysmic variables as a part of an "Inter-Longitude Astronomy (ILA)" campaign (Andronov et al. 2010) in order to study how the physical properties of cataclysmic variables depend on time and luminosity state. CBNUO is also involved in developing an automatic observation system and an analysis program for monitoring the cataclysmic variables (Yoon et al. 2012, 2013). We have published some results from our monitoring data (Andronov

et al. 2011; Kim et al. 2004, 2005). 1RXS J180340.0+401214 is an intermediate polar, subclass of magnetic cataclysmic variables, which has a magnetic white dwarf accreting from its secondary. This object recently got an official name V1323 Her in the "General Catalogue of Variable Stars" (GCVS) (Samus et al. 2014). One eclipsing variable, 2MASS J18024395+4003309, was discovered in the vicinity of this intermediate polar, 1RXS J1803, by Breus (2012). This object was registered in the "Variable Stars index" (VSX, <http://aavso.org/vsx>) and received the name VSX J180243.9+400331 (hereafter called VSX1802 for brevity)

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and an object identifier 282837. Unfortunately, no GCVS name has yet been given to this object. The study of the main target V1323 Her was presented by Andronov et al. (2011).

Due to a relatively large angular distance (14') from V1323 Her, VSX1802 is seen in the field of V1323 Her only in CCD images with a focal reducer. This was the case for one night reported by Andronov et al. (2012), who noted a sharp profile of the minimum. Additional observations from the Catalina survey (Drake et al. 2009) allowed them to determine photometric elements  $T_0=2456074.4904$ ,  $P=0.3348837 \pm 0.0000002^d$ . This profile is typical characteristic of systems with nearly equal radii and an inclination of  $i \approx 90^\circ$  (i.e., both eclipses are nearly total). Due to incomplete coverage of the phases in the discovery paper by Andronov et al. (2012), more detailed study was needed for VSX1802. Such work can serve as one of the good by-products of the ILA campaign.

The purpose of this study is to analyse the photometry data with a complete coverage of the phase as well as to carry out phenomenological modeling. The phenomenological model is based on the theory of close binary systems e.g. presented in the classical monographs by Kopal (1959) and Tsessevich (1971). Because no detailed photometric or spectroscopic studies have been reported yet, our rough phenomenological model will help to study this object with more detail in the future.

## 2. OBSERVATIONS

The Korean 1-m telescope at Mt. Lemmon in Arizona, USA (LOAO), is equipped with a focal reducer for 2x2 K CCD. The field-of-view is 22.2 square arcminutes, which is very effective for studies not only of the main targets (typically intermediate polars), but also of other variable stars in the field. It should also be noted that another eclipsing binary GSC 04370-00206 (now called V442 Cam, Samus et al. 2014) has been discovered in the field of MU Cam = 1RXS J062518.2+733433 with this telescope (Kim et al. 2005).

In total, we obtained 196 observations in V (range 16.51<sup>m</sup>–17.51<sup>m</sup>) and 242 observations in R (range 15.88<sup>m</sup>–16.77<sup>m</sup>) between 2012 and 2014. The total duration of observations was 45.5 hours during 11 nights in R and 8 nights in the alternatively changing filters VR. The time interval of the observations was JD 2455998–2456722. To improve calibration accuracy, we have used the method of “artificial comparison star” (Andronov & Baklanov 2004; Kim et al. 2004). As the object is in the field of V1323 Her, we

have used the star C1 of Andronov et al. (2011), for which Henden (2005) published the magnitudes of V and R filters:  $V=14.807^m$ ,  $R_c=14.436^m$ . The original observations (HJD, magnitude) are available upon request. Fig. 1 is the finding chart of V1323 Her with VSX1802.

## 3. PERIODOGRAM ANALYSIS

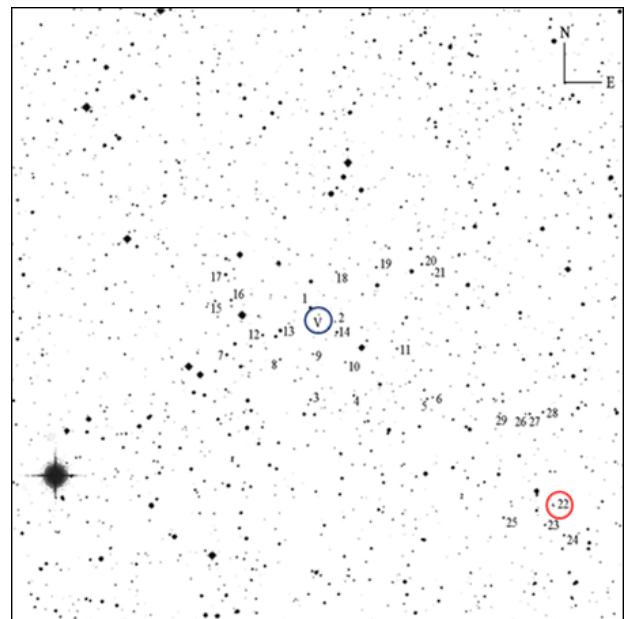
For the periodogram analysis, we have used the trigonometric polynomial fit of a degree  $s$ :

$$x_{c(t)} = C_1 + \sum_{j=1}^s (C_{2j} \cdot \cos 2\pi f t + C_{2j+1} \cdot \sin 2\pi f t) \quad (1)$$

where coefficients  $C_\alpha, \alpha=1..m, m=1+2s$  are computed using the least squares method (cf. Anderson 2003; Andronov 1994, 2003), and  $f$  is frequency (in cycles/day). For fitting, we used the statistical test function  $S(f)$  as

$$S(f) = \frac{\sigma_c^2}{\sigma_o^2} = 1 - \frac{\sigma_o^2 - C}{\sigma_o^2} \quad (2)$$

where  $\sigma_o$  is the r.m.s. deviation of the observations (O) from the sample mean, C corresponds to calculated values and O-C to the deviation of the observed values from the calculated ones (see Andronov 1994, 2003 for more details). The periodograms are shown in Fig. 2 for the filter R and  $s=1$  and  $s=2$ . One can see that the numerous peaks for  $s=1$  are much lower than those for  $s=2$ , indicating that the



**Fig. 1.** Finding chart of V1323 Her (marked with v) and reference stars (marked by numbers). Reference star 22 is VSX 1802. The size of the field is 22'x22'.

main signal corresponds not to the true frequency, but to its harmonic, which is characteristic for eclipsing binaries with nearly equal minima (as was shown by Andronov et al. (2012) from the photometry from the Catalina survey). The most prominent peak at the periodogram corresponds to  $f=2.986107$  cycles/day. A close value of the best frequency,  $f=2.986114$  cycles/day is also seen for the observations in the filter V. The frequencies in the filter V and R are close within error estimates to the value  $f=2.986111 \pm 0.000002$  cycles/day (Andronov et al. 2012), but differ by a value of 0.5/year from the estimate based on 3 minima by Parimucha et al. (2012). Since our periodogram analysis confirms the cycle numbering of Andronov et al. (2012), for the initial approximation, we use their ephemeris:

$$\text{Min I. BJD} = 2456074.4904 + 0.3348837.E \quad (3)$$

#### 4. PHENOMENOLOGICAL MODELING. MULTI-HARMONIC APPROXIMATION

The degree of the trigonometric polynomial  $s$  is often determined by visual comparison of the light curve with its approximation (cf. Parenago & Kukarkin 1936). To determine the statistically optimal degree  $s$  of approximation (1), one may use different criteria (see Andronov 1994, 2003 for detailed discussion). The classical approach is to use Fischer's criterion. In Fig. 2, the value  $L_s = -\lg \text{FAP}$  is plotted vs  $s$ . It is clearly seen that  $L$  is large for even values of  $s=2k$ , and  $L \approx 0$  for  $s=2k+1$ ;  $k=1, 2, \dots$ . This can be explained by the good symmetry of the light curve. Adopting the critical value  $L_{crit}=3$  (i.e. the false alarm probability  $\text{FAP}=10^{-3}$ ), one may suggest  $s=14$  for the filter V and  $s=8$  for R.

Another criterion proposed by Andronov (1994) is based on the r.m.s. estimate of the accuracy of the smoothing curve  $\sigma[x_c]$  at the moments of the observations. The corresponding dependence is shown in Fig. 3. For both filters, the minimum of  $\sigma[x_c]$  corresponds to  $s=6$ . In this approximation, the frequency  $f$  is a free parameter determined using differential corrections. The best fit estimate of periods is  $P=0.3348842^d \pm 0.0000005^d$  (V),  $0.3348845^d \pm 0.0000004^d$  (R), the difference from the value in (1) is not statistically significant. The corresponding moments of the primary minima are  $2456238.9186 \pm 0.0005^d$  (V),  $24\ 56320.9663 \pm 0.0007^d$  (R). The difference between these values is due to different cycle numbers close to the mean time of observations in V and R (as required for best accuracy estimates, see Andronov 1994 for details).

The phase curves in the V and R filters for statistically optimal degrees of trigonometric polynomial fit  $s=6$  (best accuracy of the smoothing function) and  $s=14$

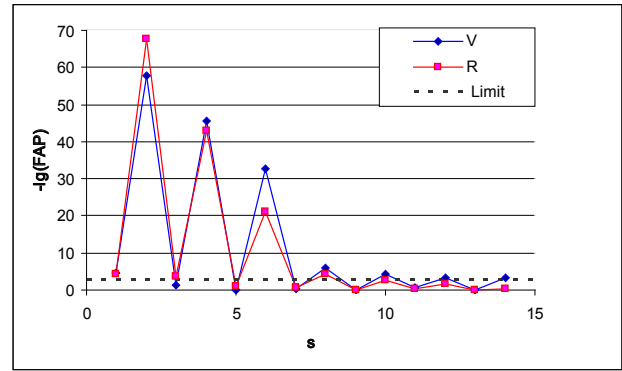


Fig. 2. Dependence of the parameter  $L_s = -\lg \text{FAP}$  on the degree of the trigonometric polynomial  $s$ . The horizontal dashed line corresponds to a critical value  $L=3$ .

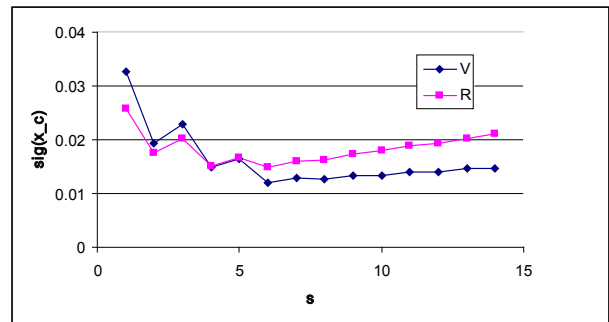


Fig. 3. Dependence of the r.m.s. value of the accuracy of the smoothing function  $\sigma[x_c]$  on the degree of the trigonometric polynomial  $s$ .

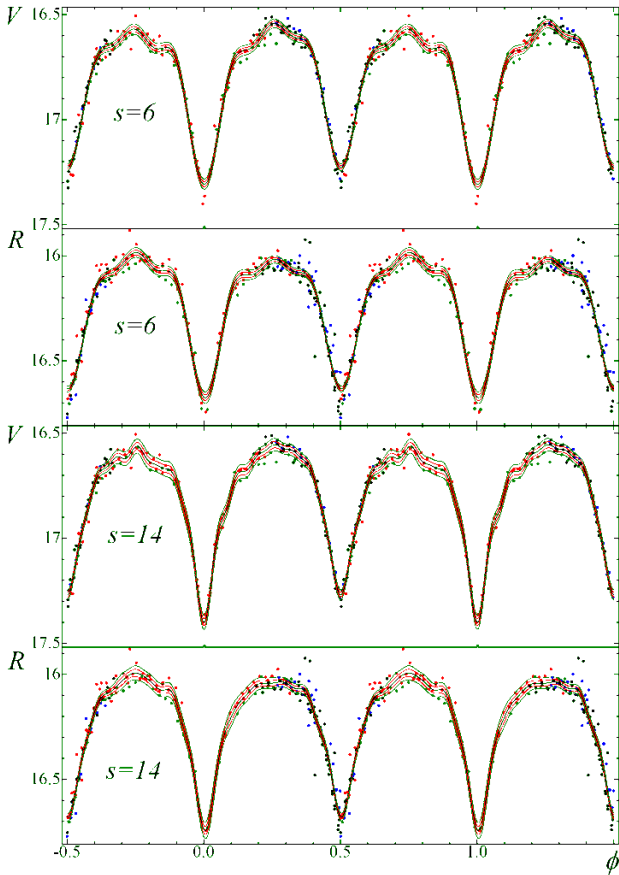
(Fischer's criterion) are shown in Fig. 4. Comparison of the trigonometric polynomial fits with different degrees  $s$  shows that, for smaller  $s$ , the depth of the eclipses is underestimated and there are formal waves at the "out-of-eclipse" part. These waves are apparently present for larger  $s$ , where the accuracy of the smoothing function is larger, but the central part of the eclipses is approximated better.

#### 5. PHENOMENOLOGICAL MODELING. THE NAV ALGORITHM

Phenomenological modeling of the light curves of eclipsing binary stars with relatively narrow eclipses was described in detail by Andronov (2012). The method was called "NAV" ("New Algol Variable") and applied to a few Algol-type variables (e.g. Kim et al. 2010b).

In short, the method may be described as follows. The smoothing function is defined as usual in the linear least squares method

$$x_c(t) = \sum_{\alpha=1}^m C_{\alpha} \cdot f_{\alpha}(t) \quad (4)$$



**Fig. 4.** Phase curves in the V and R filters for statistically optimal degrees of the trigonometric polynomial fit  $s=6$  (best accuracy of the smoothing function) and  $s=14$  (Fischer’s criterion). Filled circles are original observations, the smoothing functions  $x_s(\phi)$  are shown with  $1\sigma$  and  $2\sigma$  error corridors.

where  $C_\alpha$  are called “coefficients” and  $f_\alpha(t)$  - “basic functions”. Contrary to trigonometric polynomials with sines and cosines only, the “New Algol Variable” (NAV) algorithm combines a low - order trigonometric polynomial with a special shape for the eclipses. Andronov (2012) and Tkachenko & Andronov (2014) compared a few approaches and chose a local shape dependent on a single parameter called  $\beta$ :

$$H(z, \beta) = \begin{cases} (1 - |z|^\beta)^{3/2}, & \text{if } |z| < 1 \\ 0 & \text{else} \end{cases} \quad (5)$$

For  $\beta=0$ , the shape is narrow and is physically unrealistic; for  $\beta=1$ , the shape at the center of eclipse is triangular; for  $\beta=2$ , it is parabolic and when  $\beta$  approaches  $\infty$ , the shape tends to be a rectangle. The dimensionless variable  $z$  is related to phase  $\phi$  as

$$z = \frac{(\phi - \phi_0) - \text{int}(\phi - \phi_0 + 0.5)}{\Delta\phi} \quad (6)$$

where phase is defined typically for a given initial epoch  $T_0$  and period  $P$ :

$$\phi = \frac{t - T_0}{P} - \text{int}\left(\frac{t - T_0}{P}\right) \quad (7)$$

A discussion of the computation of phase for the case of variable period may be found in Andronov & Chinarova (2013).

Another free parameter is the eclipse half-width  $\Delta\phi = D/2$ , where  $D$  - the full width of the eclipse, in the GCVS (Samus et al. 2014) is expressed in percent of the period.

Other shapes used for determining of the parameters are based on a Gaussian function and its modifications (Mikulasek et al. 2012), which are formally of infinite width. An opposite approach is a splitting of the phase interval and approximation of the “out-of-eclipse” parts by a constant and of the eclipses by a parabola (Papageorgiou et al. 2014). The disadvantages of this model are: a) the discontinuity of the smoothing function, b) the underestimation of the depth of the minima because its width is set to a large constant and c) bad fitting of the ascending and descending branches. However, the number of parameters is only 5 (as the width of the eclipse and the phase shifts are set to constants).

Our previous approach was to use 5 parameters for the “constant+parabola” fit (brightness out of eclipse, at the primary and secondary minimum; half-width  $\Delta\phi$  and shift  $\phi_0$ ), in order to avoid discontinuity of the smoothing function. Other types of functions were discussed by Andronov (2005) and Andronov & Marsakova (2006).

In the NAV algorithm, the basic functions are:

$$f_1(\phi) = 1$$

$$f_2(\phi) = \cos(\phi), \quad \varphi = 2\pi\phi$$

$$f_3(\phi) = \cos(2\phi)$$

$$f_4(\phi) = \sin(\phi)$$

$$f_5(\phi) = \sin(2\phi)$$

$$f_6(\phi) = H((\phi - \phi_0)/C_8, C_9)$$

$$f_7(\phi) = H((\phi - \phi_0 - 0.5)/C_8, C_{10})$$

The coefficient  $C_1$  corresponds to mean stellar magnitude after reducing the observations for the eclipse, effects of reflection, ellipticity and asymmetry. Although the effects of reflection, ellipticity and asymmetry are not strictly sinusoidal, their amplitudes are typically much smaller than

those of the eclipses, thus, we use first-order approximation;  $C_2$  corresponds to semi-amplitude of the reflection effect;  $C_3$  corresponds to the semi-amplitude of the ellipticity effect; the asymmetry (O'Connell effect, usually interpreted by spots) is approximated by terms with  $C_4$  and  $C_5$ . The depths of the primary and secondary minimum are described by the coefficients  $C_6$  and  $C_7$ , respectively.

To continue numeration of the coefficients, we introduce  $C_8 = \Delta\phi$ ,  $C_9 = \beta_1$  (describing the profile of the primary minimum) and  $C_{10} = \beta_2$  for the secondary minimum. The phase shift  $C_{11} = \phi_0$  may also be added to the set of unknown variables, but we have used a fixed value  $\phi_0 = 0$  for the analysis of the present star. Typically the period and the initial epoch determined from long-term observations from different authors are more accurate than those from the smaller set of observations used for the analysis.

For each set of trial values of  $C_8, C_9, C_{10}$ , the first 7 coefficients were computed using the method of least squares (Anderson 2003; Press et al 2007). For the rest of the coefficients, one may choose among various methods (Cherepashchuk 1993; Andronov & Tkachenko 2013a; Marquardt 1963). For our analysis, we used the method of minimizing the test function at a grid.

Usually, the test function for one filter may be written in this way:

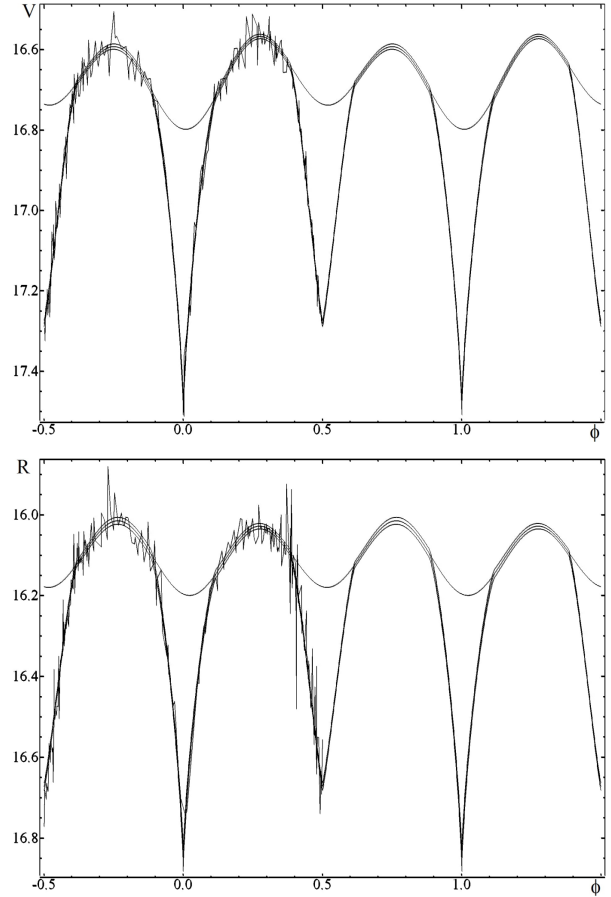
$$\Phi = \sum_{k=1}^n w_k \cdot (x_k - x_c(t_k))^2 \quad (8)$$

where  $x_k, t_k$  are signal values and times, respectively, and  $w_k$  are possible weight coefficients. The complete theory of statistical properties of the smoothing function in a case of arbitrary functions and (wavelet-like) time- and scale-dependent weight functions was presented by Andronov (1997).

We have computed a dependence  $\Phi_V(C_8) = \min_{C_8, C_9} \Phi_V(C_8, C_9, C_{10})$  for the filter V and, similarly, R. The minimum of  $\Phi(C_8)$  are slightly different due to statistical errors. To get a single value, as theoretically expected, we made a scaled sum

**Table 1.** Coefficients of the phenomenological model for our observations in the filters V, R and that for Catalina (the latter published by Andronov et al. 2012).

$\alpha$	filter V			filter R			Catalina
	$C_\alpha$	$\sigma_\alpha$	$C_\alpha/\sigma_\alpha$	$C_\alpha$	$\sigma_\alpha$	$C_\alpha/\sigma_\alpha$	$C_\alpha$
1	16.6742	0.0036	4592.10	16.1052	0.0047	3411.09	16.236
2	0.0309	0.0052	5.92	0.0090	0.0068	1.33	-
3	0.0921	0.0060	15.41	0.0815	0.0077	10.61	0.078
4	-0.0100	0.0037	-2.73	0.0078	0.0047	1.68	-
5	0.0158	0.0034	4.60	0.0204	0.0043	4.72	-
6	0.6954	0.0224	31.04	0.6633	0.0327	20.29	0.568
7	0.5457	0.0143	38.22	0.4947	0.0170	29.11	0.486



**Fig. 5.** The “NAV” fit for the phase curve and a corresponding “1 $\sigma$ ” corridor, for the filters V (up) and R (bottom). At the phases of eclipses, an additional curve corresponding to a continuation of the “out of eclipse” parts is also shown.

$$\Phi(C_8) = \frac{\Phi_V(C_8)}{\Phi_{V,min}} + \frac{\Phi_R(C_8)}{\Phi_{R,min}} \quad (9)$$

The adopted value of the filter half-width  $C_8 = 0.1177$ , corresponds to the minimum of  $\Phi(C_8)$ . Other parameters determined by the phenomenological modeling are listed in Table 1.

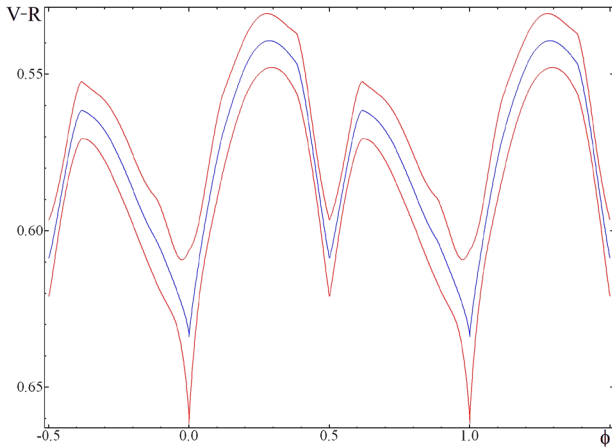
The corresponding light curves for the filters V and R are shown in Fig. 5.

Using the smoothing functions for the two filters, the smoothing function of the color index V-R was computed. It is shown in Fig. 6.

## 6. DISCUSSION

From these coefficients of the “NAV” approximation, the classical phenomenological parameters listed in the GCVS (Samus et al. 2014) were determined: Max I =  $16.567^m \pm 0.006^m$ , Max II =  $16.592^m \pm 0.006^m$ , Min I =  $17.493^m \pm 0.014^m$ ,





**Fig. 6.** The smoothing curve for the color index using the "NAV" fits for the phase curve in the filters V and R and a corresponding "1σ" corridor.

Min II = 17.281<sup>m</sup> ± 0.008<sup>m</sup>, Min I - Max I = 0.926<sup>m</sup>, Min II - Max I = 0.714<sup>m</sup> (filter V). The corresponding point at the "depth - depth" diagram (Fig.1 in Malkov et al. 2007) lies close to the line "R" of equal radii, but slightly outside the allowed region. This is caused by the "simplified" model of spherical components without effects of ellipticity, reflection and limb darkening, which Malkov et al. (2007) have used.

The parameters  $C_6$  and  $C_7$  correspond to the "reduced depth" of the minimum, i.e., not to the maximum (as in the "General Catalogue of the Variable Stars"), with respect to the continuation of the "out-of-eclipse" curve, as shown in Fig. 4. If one uses them as "depths" of the minima, the corresponding point at the diagram by Malkov et al. (2007) is located in the physically reliable region, relatively close to the line of the equal radii.

The mean color index,  $(V-R) = 16.674^m - 16.105^m = 0.569^m \pm 0.069^m$  was computed from the values of  $C_1$  for two filters. In the phase curve of the color index (Fig. 5), variations from  $0.54^m \pm 0.01^m$  (at Max I at phase 0.25) to  $0.63^m \pm 0.03^m$  (at Min I at phase 0.00) are present. At Min II at phase 0.50,  $V-R = 0.61^m \pm 0.01^m$ . The asymmetry of the maxima indicates the presence of the O'Connell effect.

These values of the color index are in the instrumental system, so, before taking into account the color transformation coefficients and determining  $(V-R)$  in the standard system, we can't estimate more precisely the "color temperature" of the "star1+star2 system" (i.e. weighted mean temperature of two components) and corresponding "spectral class" (also intermediate between that of the components).

Following Andronov (2012), we introduce residual relative intensities

$$I_1 = 10^{-0.4C_6}(\text{primary minimum}) \quad (10)$$

$$I_2 = 10^{-0.4C_7}(\text{secondary minimum}) \quad (11)$$

The intensities are in units of sum of intensities of two stars (out of eclipse, but "reduced" for the three effects mentioned above) and the "intensity depth" of minimum (how much light is eclipsed):  $d_1 = 1 - I_1$ ,  $d_2 = 1 - I_2$ .

Andronov (2012) introduced a phenomenological parameter defined as  $Y = d_1 + d_2$ . The case  $Y = 0$  corresponds to "no eclipse", 1 to "both eclipses are total", and, for the majority of cases,  $0 \leq Y \leq 1$ . For this star, we get values of

$$Y = 0.868 \pm 0.014 \text{ (filter V)}, Y = 0.823 \pm 0.019 \text{ (R)} \quad (12)$$

In the model of "spherical stars with no limb darkening" (Shulberg 1971; Malkov et al. 2005) recently discussed by Andronov & Tkachenko (2013b) as a "zero-order" approximation for determination of physical parameters), the eclipsed part of the light is proportional to the surface of projection at the eclipse  $S$ ; the surface of projection of stars  $S_1 = \pi r_1^2$ ,  $S_2 = \pi r_2^2$ . Here, we use dimensionless parameters,  $r_1 = R_1/a$ ,  $r_2 = R_2/a$ , where  $a$  is the orbital separation.

Let's introduce  $L_{1V} = 1 - L_{2V}$  as the relative contribution of the emission from the first star in V, similar to other filters. Let's define that the first star eclipses the second star at the primary (more deep) minimum. Under this definition, the second star has larger surface brightness (temperature).

So one may write a system of 4 equations

$$d_{1V} = (1 - L_{1V}) \cdot S/S_2 = L_{2V} \cdot S/S_2 = F_{2V} \cdot S \quad (13)$$

$$d_{2V} = L_{1V} \cdot S/S_1 = F_{1V} \cdot S \quad (14)$$

$$d_{1R} = (1 - L_{1R}) \cdot S/S_2 = L_{2R} \cdot S/S_2 = F_{2R} \cdot S \quad (15)$$

$$d_{2R} = L_{1R} \cdot S/S_1 = F_{1R} \cdot S \quad (16)$$

with 5 unknowns,  $L_{1V}$ ,  $L_{1R}$ ,  $S$ ,  $S_1$ ,  $S_2$ . Four parameters - the relative surface brightness  $F_{2R} = L_{2R}/S_2$  and similarly for the star 1 and filter V - are related to that mentioned above. So it is not possible to determine all 5 parameters without additional information (e.g. color index- luminosity (or absolute magnitude)-radius relation, assuming Main Sequence) - thus we need an "extra" equation.

However, we can estimate

$$L_{1V}/L_{1R} = d_{2V}/d_{2R} = 1.080 \pm 0.037 \quad (17)$$

$$L_{2V}/L_{2R} = d_{1V}/d_{1R} = 1.035 \pm 0.046 \quad (18)$$

From this, we can derive another combination

$$(d_1/d_2)_V/(d_1/d_2)_R = (L_{2V}/L_{2R})/(L_{1V}/L_{1R}) \quad (19)$$

$$= 0.958 \pm 0.053$$

i.e., equal to unity within error estimates. A small difference (even if smaller error), could be related to limb darkening.

Other combinations are for the surface brightness

$$d_{1V}/d_{2V} = F_{2V}/F_{1V} = 1.197 \pm 0.037 \quad (20)$$

$$d_{1R}/d_{2R} = F_{2R}/F_{1R} = 1.249 \pm 0.057 \quad (21)$$

These phenomenological values may be used to estimate temperature ratios, e.g. using "Main Sequence" (MS) relations". From these MS assumptions, one may make a sequence of suggestions, which will be discussed below.

The second star (eclipsed at a primary minimum) has a larger surface brightness, consequently a larger radius and luminosity than the first star.

Taking into account  $Y \approx 1$ , one may suggest that the secondary eclipse is total (or close to total). Thus,  $S = S_1$ , and the system of equations may be solved completely:

$$L_{1V} = d_{2V} = 0.395 \pm 0.008 \quad (22)$$

$$L_{1R} = d_{2R} = 0.366 \pm 0.010 \quad (23)$$

$$(S_1/S_2)_V = d_{1V}/(1 - d_{2V}) = 0.782 \pm 0.021 \quad (24)$$

$$(S_1/S_2)_R = d_{1R}/(1 - d_{2R}) = 0.721 \pm 0.029 \quad (25)$$

These values slightly differ by a value of  $0.061 \pm 0.035 = 1.7\sigma$ , and the difference is not statistically significant. The mean weighted value of the ratio of cross-sections of the two stars is  $(S_1/S_2) = 0.761 \pm 0.017$ , and the ratio of radii is  $(R_1/R_2) = 0.872 \pm 0.010$  (radius is in units of the orbital separation). For the low-mass part of the Main Sequence,  $R$ - $M$  (e.g. Faulkner 1971) so this value may be an estimate of the mass ratio,  $q = M_1/M_2 = 0.872 \pm 0.010$ . From the table of Allen (1973) for the Main Sequence one may obtain a statistical relation  $R \approx M^{0.72}$  for a wide range of spectral classes: from A0 to M0. In this case, we get a slightly different estimate  $q = M_1/M_2 \approx (R_1/R_2)^{1/0.72} = 0.826 \pm 0.013$ . From more recent tables of Cox (2000),  $R \approx 1.01M^{0.80}$ , thus

$$q = M_1/M_2 \approx \left(\frac{R_1}{R_2}\right)^{\frac{1}{0.80}} = 0.842 \pm 0.012 \quad (26)$$

However, the (unknown) deviations from the mass-radius relation may exceed this maximal 5 percent difference in estimates using different coefficients of the statistical dependence.

From the duration of eclipse, one may suggest an inequality

$$r_1 + r_2 \geq \sin(2\pi C_8) = 0.6739 \quad (27)$$

For such large values, the stars are distorted (and we see this also from large values of  $C_3$ ) and thus close to their Roche lobes.

Although a possible W UMa - type classification may not be completely rejected, these stars are not yet in thermal contact (Lucy 1976) based on the difference of mean brightness and we suggest that they are not in contact. Thus, we prefer an EA - type classification with elliptic component(s).

Using the oversimplified form of the MS mass-radius relation

$$R = R_\odot \cdot M/M_\odot \quad (28)$$

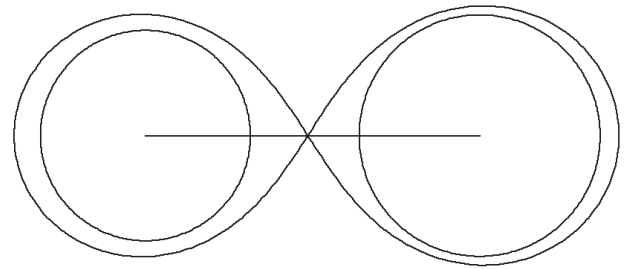
and, combining with Kepler's third law, we obtain the inclination,  $i = 90^\circ$ ,  $M_1 = 0.745 M_\odot$ ,  $M_2 = 0.854 M_\odot$ ,  $M = M_1 + M_2 = 1.599 M_\odot$ , the orbital separation  $a = 1.65 \cdot 10^8 \text{m} = 2.37 R_\odot$  and relative radii  $r_1 = R_1/a = 0.314$  and  $r_2 = R_2/a = 0.360$ .

Generally, these are minimal estimates of the radiuses and masses, since for an inclination,  $i$ , differing from  $90^\circ$ , we'll get larger estimated values. With the assumption of one total eclipse, one may write  $\cos(i) < |r_2 - r_1| = 0.046$ , thus  $87.4^\circ \leq i \leq 90^\circ$ ,  $0.999 \leq \sin(i) \leq 1$ , and for VSX 1802, the effects of inclination on estimates of radii and masses are negligible.

Even though the spectral classes of this system have not been reported yet, we can suggest the corresponding spectral classes of these stars to be G8 and K2 according to Cox (2000) and the possible Roche model of these VSX 1802 system can be estimated as in Fig. 7.

## 7. CONCLUSIONS

In this study, we estimated the parameters only from



**Fig. 7.** The model of the system VSX 1802: The Roche lobes, the line of centers and circles corresponding to estimated radii of the stars in a spherically-symmetric approximation.

phenomenological modeling of the phase light curve. The main unjustified suggestions were "no limb darkening" and "similar to MS dependencies" (no real quantitative relations, just "larger stars are brighter"). There are no detailed photometric or spectroscopic studies at present, therefore the true physical parameters of this system cannot be estimated for now.

Our phenomenological modeling could allow rough estimates to make the physical parameters. Although the errors of the phenomenological parameters are not very large (up to few percent), there may be systematic errors up to a dozen percent due to the simplicity of the model. These estimates may be used as preliminary values for further modeling using extended physical models based on the Wilson & Devinney (1971) code and its extensions (Wilson 1979, 1994, 2012, 2014; Zola et al. 1997, 2010; Bradstreet 2005; Kallrath et al, 2009; Linnel 2012; Reed 2012; Rucinski 2010; Prsa et al 2012). Such study of eclipsing binaries with the CBNU observations are reported by Jeong & Kim (2013); Kim & Jeong(2012); Jeong & Kim (2011) and Kim et al. (2010a). It should be noted that the lack of spectral data doesn't allow to study various combination of stars in different evolutionary stages. Some discussions with crude assumptions about the age and metallicity of the system are therefore out of the scope of this study. More detailed photometrical and spectroscopic information will enable more thorough studies.

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