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# Prediction of Change in Equivalent Circuit Parameters of Transformer Winding Due to Axial Deformation using Sweep Frequency Response Analysis

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**Abstract** – Power transformer is one of the major and key apparatus in electric power system. Monitoring and diagnosis of transformer fault is necessary for improving the life period of transformer. The failures caused by short circuits are one of the causes of transformer outages. The short circuit currents induce excessive forces in the transformer windings which result in winding deformation affecting the mechanical and electrical characteristics of the winding. In the present work, a transformer producing only the radial flux under short circuit is considered. The corresponding axial displacement profile of the windings is computed using Finite Element Method based transient structural analysis and thus obtained displacements are compared with the experimental result. The change in inter disc capacitance and mutual inductance of the deformed windings due to different short circuit currents are computed using Finite Element Method based field analyses and the corresponding Sweep Frequency Responses are computed using the modified electrical equivalent circuit. From the change in the first resonant frequency, the winding movement can be quantified which will be useful for estimating the mechanical withstand capability of the winding for different short circuit currents in the design stage itself.

**Key words:** Displacement, Finite element method, Short circuit forces, Transformer, Transient structural analysis, Sweep frequency response analysis

## 1. Introduction

High magnitudes of currents in the transformer windings due to short circuit events induce excessive forces in transformer and the magnitude of the fault current depends upon the short circuit reactance of the transformer [1]. In general, the winding currents produce leakage flux which can be resolved into radial and axial components. These radial and axial fluxes interact with winding current to produce axial and radial forces respectively. Depending on the distribution of forces and mechanical strength of the windings, the winding undergoes deformation. If these forces are not properly restrained, a major failure is likely to occur. For example, 1% difference in the heights of HV and LV windings leads to an axial force of nearly 50 kN/m in windings of a 5 MVA three phase transformer during short circuit [2].

As the details about the deformation in terms of displacements is necessary, the literature on computation/ measurement of the distribution of forces, displacements and analysis using SFRA are surveyed. G.B. Kumbhar and S.V. Kulkarni [3] computed the force distribution due to short-circuit current of the split-winding transformers using Finite Element Analysis (FEA). J. Faiz, B. M. Ebrahimi, and T. Noori [4] analysed the radial and axial electro-mechanical forces developed by the inrush current and short circuit current using 2D and 3D Finite Element Models (FEM). Hyun-Mo Ahn and, Yeon-Ho-Oh [5] compared the simulated and measured forces in trans-former windings. Gopalakrishna. S [2] presented the force developed in top disc of the winding and the distribution of forces in each disc using Finite Element Analysis. A.P. Purnomadi and D. Fransisco [6] analysed axial and radial winding deformation using sweep frequency response analysis for the manually displaced windings. In [7], the mechanical damage to a transformer winding is analysed with the help of sweep frequency response analysis using cross correlation method. K. Ludwikowski, K. Siodla and W. Ziomek [8] determined the frequency ranges for detection of the winding deformation for buckling phenomenon in a high voltage power transformer.

In the present work, force distribution and the winding displacements are computed using FEA and compared with the measured values. The change in winding capacitance and inductance due to deformation are calculated using FEA and the corresponding shift in resonance frequencies are obtained from sweep frequency responses. From the change in resonant frequency due to short circuit current, it is easy to quantify the winding displacement.

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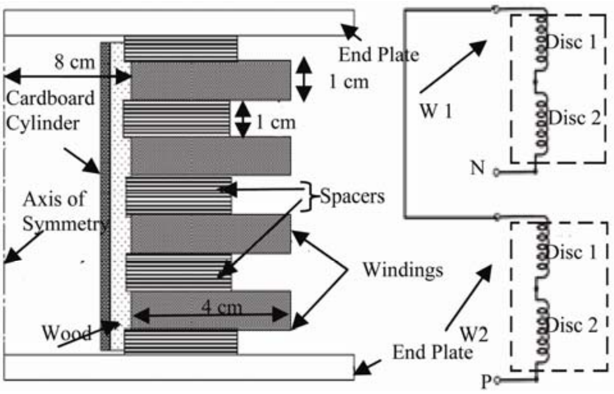
## 2. Transformer under Analysis

A 722VA, 10V aluminium wound transformer used by Gopalakrishna [2] is considered for the analysis. The transformer has two identical windings (W1 and W2) with 2 discs per winding as shown in Fig. 1(a) and 1(b). Each disc has 10 turns. The windings are connected in such a way to simulate a transformer with the winding currents in the opposite directions resulting in only axial displacement due to repulsive force.



(a) (b)

**Fig. 1.** (a) Winding model (b) Winding connection

## 3. Transient Structural Analysis using Finite Element Analysis (FEA)

As the short-circuit events result in high mechanical forces in the transformer windings, the short circuit withstand capability of a transformer is of a mechanical nature. As the short circuit current varies with time, the induced force also varies with time resulting in continuous movement of discs. Hence, it is essential to estimate the displacements using transient structural analysis using winding equivalent structural model [9]. To do the transient structural analysis, each disc is represented as lumped mass (M), spacer as spring (K) and the insulation between the discs as dashpot (C). The winding equivalent structural model is shown in Fig. 2.

The governing equation for transient structural analysis is given as,

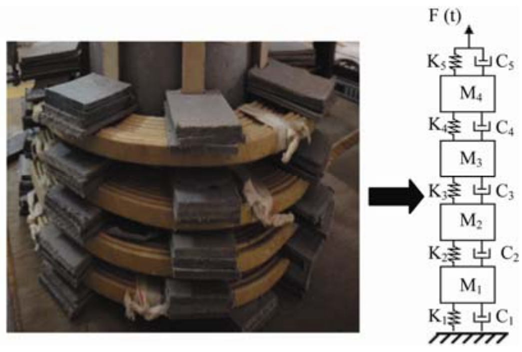


**Fig. 2.** Winding and equivalent structural model

$$[M]\frac{\partial^2}{\partial t}\{u\}+[C]\frac{\partial}{\partial t}\{u\}+[K]\{u\}=\{F^a(t)\} \qquad (1)$$

where:

$[M]$ = structural mass matrix
$[C]$ = structural damping matrix
$[K]$ = structural stiffness matrix
$\{u\}$ = nodal displacement vector
$\{F^a(t)\}$ = applied load vector

To carry out the analysis, the following parameters are required, Short circuit force vector $F^a(t)$

Mechanical parameters of the winding like mass ($M$), stiffness ($K$) and damping coefficient ($C$) of the winding

Parameters from (3.1) and (3.2) are given as input to Transient Structural Analysis to simulate the transient displacement of the winding

### 3.1 Short circuit forces $f^a(t)$ using fem (simulation)

The electromagnetic force distribution in the windings is computed using FEM based Magneto-structural analysis. Both the windings are energised with same current density in the opposite directions. The magnetic vector potential is calculated from magnetic field analysis (2).

$$\nabla^2 A=-\mu J \qquad (2)$$

where $\mu$ is the magnetic permeability, $A$ represents the magnetic vector potential and $J$ (A/m$^2$) is the current density. Electromagnetic forces are produced due to winding currents and the leakage flux in the winding regions and the force on the windings is given by the Lorentz force as,

$$d\vec{F}=i\,\vec{dl}\times\vec{B} \qquad (3)$$

where $F$ represents the force (N).

The magnetic field analysis on the windings is carried out for short circuit current density of 12 A/mm$^2$. Fig. 3 shows the magnetic flux plot of the winding.

From the magnetic flux plot, it is observed that the axial flux is negligible (as the axial flux produced by W1

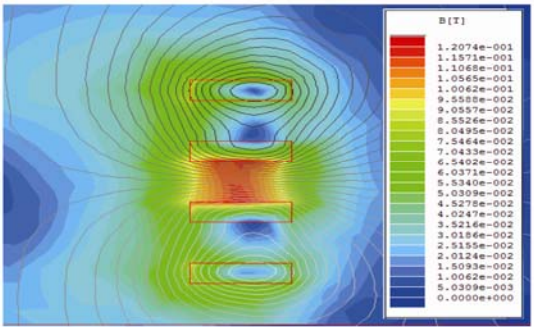
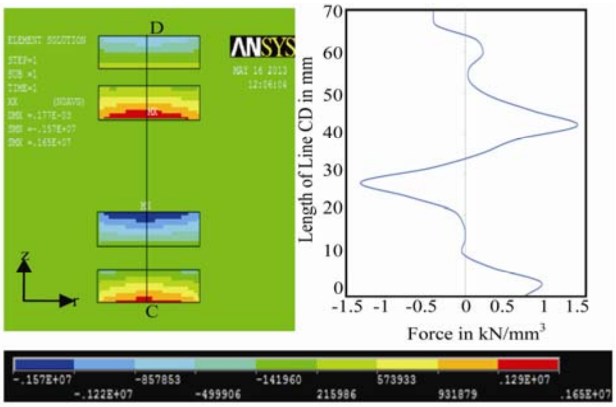


**Fig. 3.** Magnetic Flux plot of the winding



(a) (b)

**Fig. 4.** Distribution of axial force; (b) Axial force along line CD

and W2 gets cancelled) compared to the radial flux. As a result, the forces on the windings are only in axial directions and repulsive. Fig. 4(a) and 4(b) show the axial force distribution of the transformer.

In general, for the energising current

$$I(t) = I_{max} \ \ sin(\omega t) \tag{4}$$

the corresponding force is given by,

$$F^a(t) = F_{max}\left[\frac{1-cos(2\omega t)}{2}\right] \tag{5}$$

where $F_{max}$ is obtained from Magneto-structural analysis and $F^a(t)$ is given as an input to the transient structural analysis.

### 3.2 Measurement of structural parameters (experimental)

Modal testing is a form of vibration test on an object by which natural frequencies, stiffness, masses and damping ratios can be determined. This test is carried out in the Council of Scientific & Industrial Research (CSIR), Government of India, Chennai and the response is recorded. The typical instrumentation setup is shown in Fig. 5.

The natural frequency is the rate at which an object vibrates when it is not disturbed by an outside force. Each degree of freedom of an object has its own natural frequency, expressed as $\omega_n$ (radians per second). The mass ($M$) of the winding is 15 kg. The measured natural frequency of the test object is 21.52 Hz (Fig. 6).

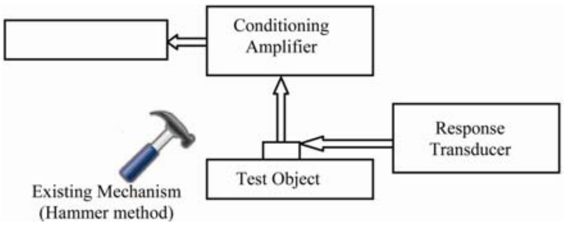


**Fig. 5.** Block diagram for modal analysis using Hammer Method (measured)

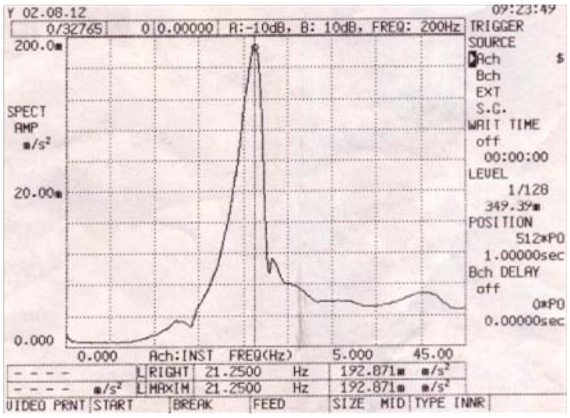


**Fig. 6.** Natural frequency of the test object (measured)

By using (6), the stiffness ($K$) of the total system is calculated as 282.47×10$^3$ N/m.

$$\omega_n^2 = \frac{K}{M} \tag{6}$$

A convenient way to measure the amount of damping present in a system is to measure the rate of decay of free oscillations. The larger the damping, the greater is the rate of decay. The rate of decay of free oscillations of the test object is recorded (Fig. 7).

The damping ratio is calculated as 811.5 using (7).

$$\zeta = \frac{C}{C_c} \tag{7}$$

where:

$\zeta$ = damping ratio
$C$ = damping
$C_c$ = damping coefficient

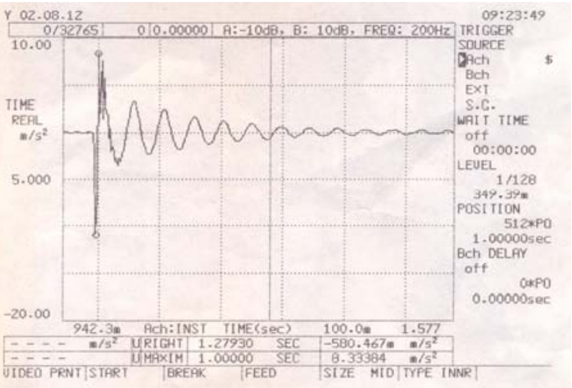


**Fig. 7**. Output of vibration analysis (measured)

Thus measured structural parameters are used in structural model (Fig. 2) to estimate the winding displacement.

### 3.3 Displacement due to short circuit force

#### *3.3.1 Displacement using transient structural analysis (simulation):*

In the present work, displacement in windings are computed using load transfer (from electromagnetic to structural analysis) method using FEM based transient structural analysis. Transient analysis is a technique used to determine the dynamic response of a structure (here windings) under the action of any general time-dependent loads (force due to short circuit current). This type of analysis is used to determine the time-varying displacements and is solved by transient structural equation is given in (1). Here, the force on each disc ($F^{a}(t)$) calculated from Magneto-structural analysis (section 3.1) and measured structural parameters (section 3.2) act as a load for each and every node in transient structural analysis as given in Fig. 2. The displacements of all the discs are computed for an excitation period of 0.3s and the Fig. 8 shows the dynamic displacement of the top disc of W1.

#### *3.3.2 Measurement of winding displacement (measured):*

To validate the simulated results, short circuit test as per IEC Standard 60076-5 [10] is conducted on the test object at On Load Gears, Ambattur, Chennai (Fig. (9)) and the outputs are recorded. The linear displacement transducer with 1.5 mm linear stroke length placed on top of the winding (on Disc 1 of W1) is used to measure the displacement.

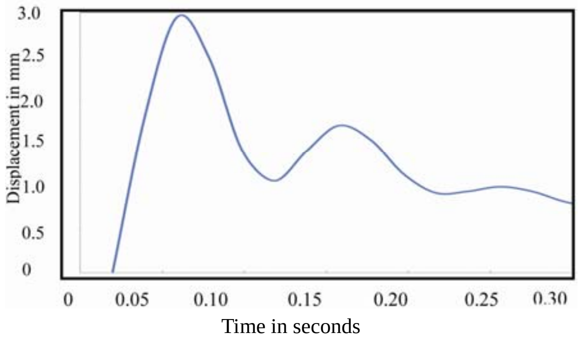


**Fig. 8**. Displacement of top disc of W1 (simulation)

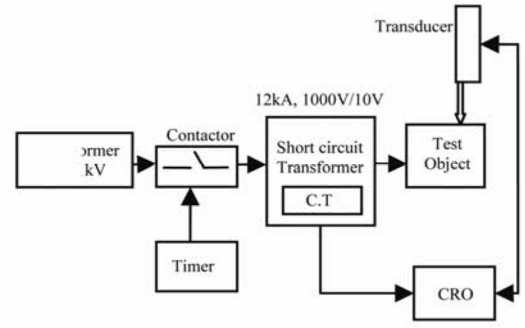
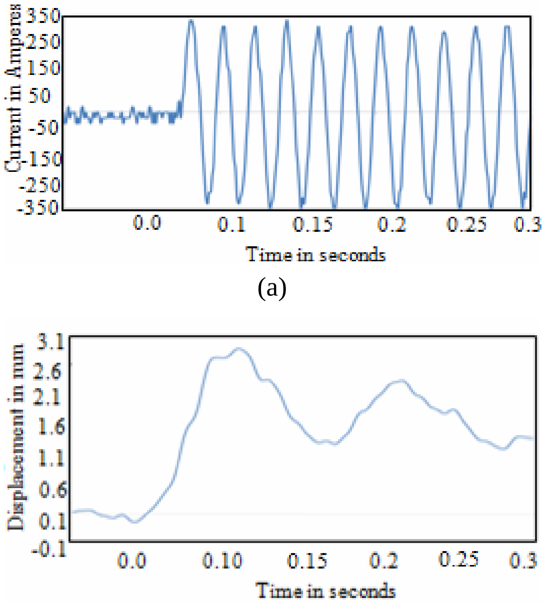


**Fig. 9.** Block diagram of experimental setup

The test is conducted for a period of 0.3s and the movement of the top disc is recorded and shown in Fig. 10(a) and 10(b). The maximum displacement is found to be 3.02mm at 0.11s.



(a)

(b)

**Fig. 10.** (a) Excitation current (measured); (b) Displacement of top disc of W1 (measured)

The dynamic movement of the winding Fig. 10(b) is compared with the simulated response Fig. 8 and observed that the disc movement pattern is similar and the percentage error in the maximum displacement is 4% for the same current density. As the simulation methodology is validated using experimental results, further analyses are carried out for different currents using simulation methodology. Fig. 11 shows the displacement profile for different current densities.

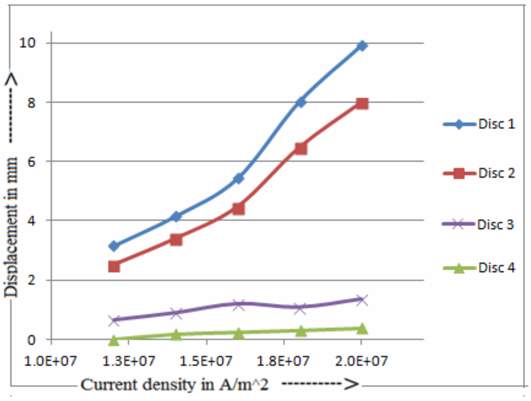


**Fig. 11.** Current density Vs displacement

In the next section, the change in the equivalent circuit parameters of the winding due to axial displacement for different current densities is incorporated by modifying the existing equivalent circuit.

## 4. Change in Winding Parameters and Resonance Frequencies due to Winding Deformation

### 4.1 Electrical equivalent circuit and change in resonant frequencies (simulation)

Electrical equivalent circuit of transformer winding is represented by a combination of winding resistance, self-inductance, mutual inductance, inter-turn capacitance, inter-disc capacitance and stray capacitance [11-12]. The basic electrical equivalent circuit of a transformer winding is shown in Fig. 12.

For axial deformation of transformer winding, both the inter turn capacitance and the self inductance do not vary with different short circuit currents. In the basic equivalent circuit, the inter-turn and inter-disc capacitances are represented as single capacitance ($C_s$) as given in [13]. Due to axial deformation, there will be a change in inter disc capacitance which necessitates the separation of the series winding capacitances ($C_s$) into inter turn ($C_t$) and inter-disc capacitance ($C_d$) and $C_d$ is placed across the two consecutive discs. The change in the mutual inductance can be incorporated through coupling factor (k). Fig. 13 shows the proposed 'modified electrical equivalent circuit' with separated $C_t$ and $C_d$.

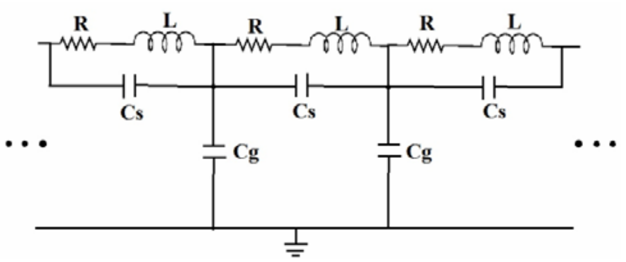


**Fig. 12.** Electrical equivalent circuit of transformer inding

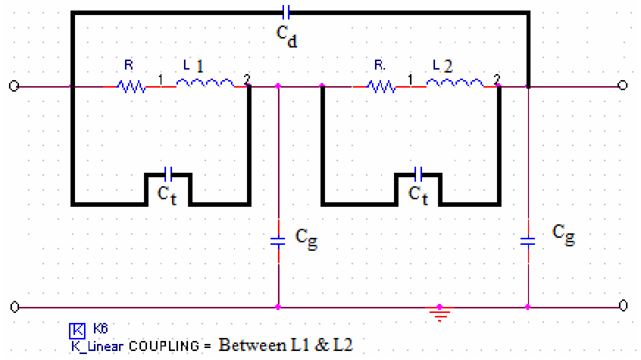


**Fig. 13.** Modified electrical equivalent circuit for 2-Disc winding

In the subsequent section, the change in equivalent circuit parameters of the winding due to displacement is calculated using Finite Element Method (FEM).

### 4.2 Determination of winding parameters due to different displacement

Axisymmetric 2D modeling of the transformer winding is done using FEM and the transformer winding model is shown in Fig. 14. Due to axial displacement, the inter disc capacitance and the mutual inductances change.

As both the inter turn capacitance and the self inductance do not vary with short circuit currents and the same are computed as 10.58pF and 32.27μH using FEM using electrostatic and magnetostatic solver respectively. Changes in inter-disc capacitance and magnetic coupling coefficient (for mutual inductance) for different distances between the discs are computed and given in Table 1. Stray / Shunt capacitance ($C_g$) is also calculated using Electrostatic solver.

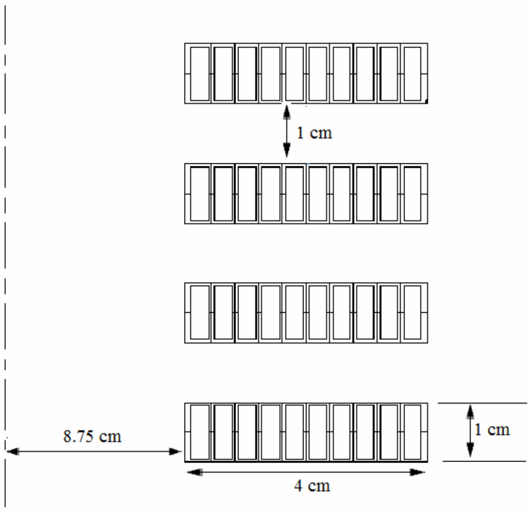


**Fig. 14**. Modeling of transformer winding using FEM

**Table 1.** Inter-disc capacitance and coupling coefficient (k) of W1 and W2 for different current densities

| Current density (A/m$^2$) | W1 | | W2 | |
|---|---|---|---|---|
| | Inter-disc Capacitance | Magnetic Coupling Coefficient | Inter-disc Capacitance | Magnetic Coupling Coefficient |
| | $C_d$ (pF) | K | $C_d$ (pF) | K |
| Unexcited (Healthy) | 14 | 0.627 | 14 | 0.627 |
| 12e6 | 12.82 | 0.612 | 16.56 | 0.653 |
| 14e6 | 12.49 | 0.606 | 17.52 | 0.661 |
| 16e6 | 12.23 | 0.602 | 17.7 | 0.663 |
| 18e6 | 11.77 | 0.595 | 18.48 | 0.669 |
| 20e6 | 11.32 | 0.587 | 19.47 | 0.676 |

### 4.3 Resonance frequencies due to winding deformation

The change in resonant frequencies due to displacements for different short circuit currents is computed using Circuit Simulation Package (OrCAD PSpice) and shown in Fig. 15(a) and 15(b). It is observed that the first resonant frequency ($f_{r1}$) increases with the increase in current for W1 and decreases with the increase in current for W2.

The simulated results are checked with measurements done using sweep frequency response analyser ( FRAX 101 of Megger make) and shown in Fig. 16(a) and 16(b). The maximum difference in first resonance frequency between the measured and the simulated is less than 4%.

The percentage shift in $f_{r1}$ (with reference to unexcited case) for both the windings is calculated (Fig. 17) from which the displacement profile can be predicted for any short circuit current from Fig. 11.

From the change in first resonant frequency for different currents, the axial displacement and the corresponding change in equivalent circuit parameters of each and every disc can be predicted. The above analysis can also be used to predict the withstand capability of transformer by calculating the winding impedance of the deformed winding due to short circuit at the design stage itself as per IEC Standard 60076-5 [10].

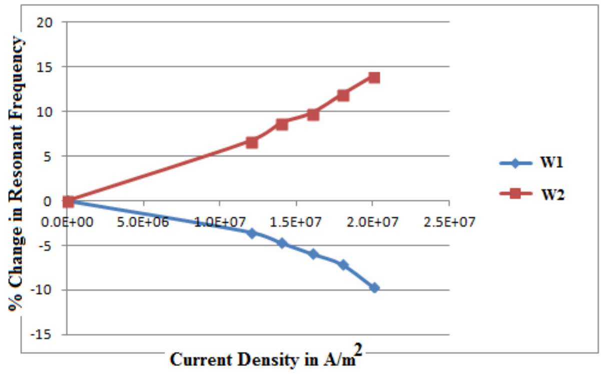
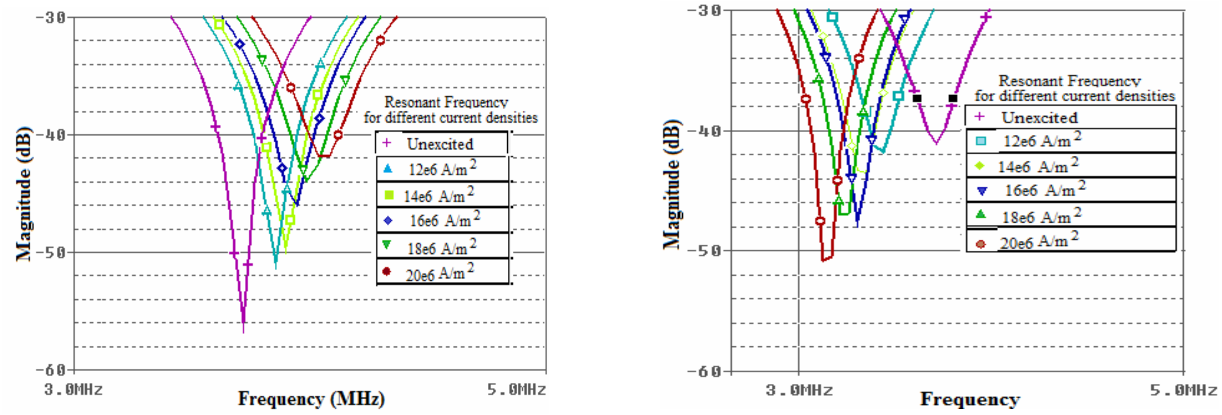


**Fig. 17.** Percentage change in resonant frequency for W1 and W2 (simulation)

## 5. Conclusions

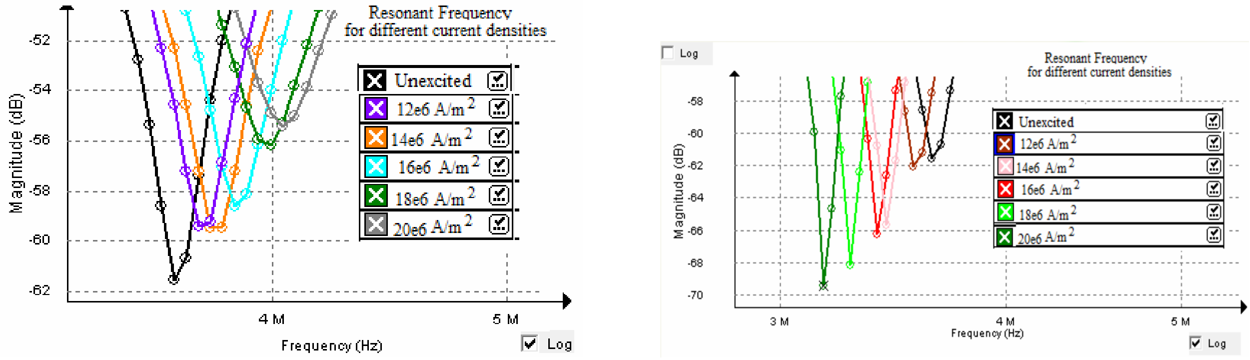
The Finite Element Method is used to compute the electromagnetic force and displacement profiles acting on the transformer winding. In the present work, the transient deformation profile of the winding due to short circuit current is simulated and compared with the measured value.



(a)

(b)

**Fig. 15.** (a) Change in resonant frequency for W1 (simulated); (b) Change in resonant frequency for W2 (simulated)



(a)

(b)

**Fig. 16.** (a) Change in resonant frequency for W1 (measured); (b) Change in resonant frequency for W2 (measured)

Changes in equivalent circuit parameters (inter disc capacitance and mutual inductance) due to displacements are calculated. The transformer equivalent circuit is modified to incorporate the axial displacement of each disc and the resonant frequencies for different short circuit currents are computed. Thus computed results are checked with measured resonant frequencies using sweep frequency response analyser. From the change in the first resonant frequency due to short circuit current, it is easy to quantify the winding movements. Though the methodology is applicable for transformers with both radial and axial displacements of continuous disc windings, the same need to be checked for other types of winding.

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