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Optimal Design of Multi-DOF Deflection Type PM Motor by Response Surface Methodology

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Abstract – This paper uses response surface methodology as the optimization method of torque of multi-DOF deflection type PM motor. Firstly, the application of Taguchi algorithm selects structural parameters affecting the motor torque largely which simplifies the optimization process greatly. Then, based on the central composite design (CCD), response surface equation numerical model is constructed by the finite element method. With the aid of experiment design and analysis software, the effects of the interaction among factors on the index are analyzed. The results show that the analytical method is efficient and reliable and the experimental results can be predicted by response surface functions.

Keywords: PM motor, Multi-degree-of-freedom, Response surface methodology, Optimization

1. Introduction

With the development of technology, engineering calculation is more complex which takes more time. Many engineering problems are nonlinear, so the optimization of engineering calculation forced scholars to find new and reliable method of mathematical programming. Response surface methodology is a progressive approximation optimization method which can efficiently solve this nonlinear optimization problem.

The response surface method combines statistics and mathematics [1]. In recent years, due to the high speed development of statistics, the response surface method is no longer confined to the biological and chemical industry, in the ecology, food science and other new areas also show its strong advantages [2].

Traditionally, multi-DOF motion is often completed by the combination of many single-DOF motion motors and mechanical transmission mechanisms. This not only results in low precision devices, but also decreases dynamic and static performance. Meanwhile, many of the applications of reduction gears increase nonlinear friction and energy losses [3]. Motors which have simple structure and can achieve multi-DOF motion become the best alternative means, therefore, research and application of multi-DOF motion of motor has attracted wide attention of scholars at home and abroad [4,5].

This paper uses response surface methodology as the optimization method of torque of multi-DOF deflection type PM motor. Based on Taguchi algorithm, appropriate optimization parameters are screened out. Based on the central composite design (CCD), the response surface equation numerical model is constructed by the finite element method. By Design-Expert 8.0.5.0 software, the optimal motor structure is obtained. Results of the improved motor torque are analyzed and compared with the initial model.

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2. Application of Taguchi Method

Traditionally, if four different parameters have to change and each parameter has three variations, 81 experiments needed to be performed [6]. The traditional methods are often time-consuming and laborious .The application of Taguchi algorithm selects structural parameters affecting the motor torque largely which simplifies the optimization process greatly.

2.1 Initial structure data of the motor

While achieving a continuous movement, the motor can complete the tilting movement within a certain angle [7]. The novel multi-DOF deflection type PM motor has a 6-pole permanent magnet rotor of the “drum” shape. The height of the rotor is 10mm and the outer diameter of the rotor is 31 mm, the inner diameter of rotor is 10 mm. Magnetic N pole and S pole are arranged alternately. The motor contains two layers of stator coils; each layer has 8 stator coils which can be controlled independently. The height of the stator is 17.5 mm and the outer diameter of the stator is 90 mm, the inner diameter of the stator is 78mm, the distance between the two stators is 5.5 mm. By giving different current to stator coils, the rotor and stator realize the multi-degree of freedom movement around a spherical bearing maintaining a constant gap (5.5 mm).The

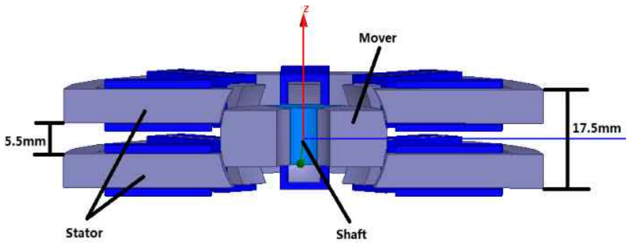

Fig. 1. Three-dimensional structure of the motor

cross-section of this motor is shown in Fig. 1. For the PM motor, the electromagnetic torque generated by the interaction between the current-carrying coils and PM poles making the motor achieve the z-rotation, x-rotation and y-rotation [8].

By controlling the energization of the stator coils, the motor can generate a continuous rotation torque in order to achieve continuous motion, which is the core of the electrifying strategy.

2.2 Determining the optimal structure parameters of the motor

In fact, the purpose of optimizing the performance of the motor is to allow the motor to be further improved, for example, motor efficiency, power factor, torque performance, a minimum amount of material, etc. Multi-DOF deflection Type PM Motor as a new special motor, the purpose of the study is to allow the motor to achieve better multi-DOF motion, so the performance index of the motor efficiency and power factor is not the primary consideration. Therefore, the optimization object of this motor is torque characteristics. The ultimate aim is to pick out the optimal motor structure size for maximum torque in a certain space occupied. According to the structure characteristics of the motor, preliminary selection of optimization parameters variable number is 4. The optimization parameters are the permanent magnet rotor diameter (*R*r), height of the rotor (*H*), air gap length between stator and rotor (σ) and the distance between the double stator (*L*). Parameter values of the variables can be shown from Table 1.

According to Taguchi algorithm, this paper selects orthogonal table $L_9(3^4)$ and establishes the experimental matrix, where *L* represents an orthogonal matrix, "9" represents the number of tests, "3" represents parameter variables number, "4" represents the number of parameters [6]. According to the experimental matrix, the maximum torque is got by finite element method, as shown in Table 2.

Table 1. Motor optimization parameters and parameter values of the variables

	Rr [mm]	H[mm]	σ[mm]	L [mm]
Variable A	11.5	6	0.2	4.5
Variable B	15.5	10	1.1	5.5
Variable C	19.5	14	2	6.5

Table 2. Orthogonal table and the results of finite element analysis

Test times	*Rr*[*mm*]	*H*[*mm*]	σ[*mm*]	*L* [*mm*]	*T* [*Nm*]
1	*A*	*A*	*A*	*A*	0.07712
2	*A*	*B*	*B*	*B*	0.10445
3	*A*	*C*	*C*	*C*	0.10491
4	*B*	*A*	*C*	*C*	0.06699
5	*B*	*B*	*B*	*A*	0.14217
6	*B*	*C*	*A*	*B*	0.23013
7	*C*	*A*	*C*	*B*	0.08866
8	*C*	*B*	*A*	*C*	0.15903
9	*C*	*C*	*B*	*A*	0.19412

Table 3. Influence rate of optimization parameters on motor torque

	YY	Influence rate(%)
*R*r [*mm*]	0.00527	17.5
H[*mm*]	0.014777	49.0
σ[*mm*]	0.008382	27.8
L [*mm*]	0.001713	5.7
Total	0.030149	100

To get the influence rate of each parameter, Taguchi method calculates the average value of each parameter, and then analyzes the variance [9]. The influence rate of each optimization parameters calculated by the following formula:

$$YY = 3\sum_{i=1}^{3}(m_{x_i}(T_i) - m(T))^2 \quad (1)$$

where *xi* represent *R*r, *H*, σ, *L* respectively, *T* represents the maximum torque of the new multi-DOF deflection type PM motor, $m_{x_i}(T_i)$ represents the average value of *T*.

Optimization of all the variables is impractical and may not be necessary for real applications [10]. Table 3 shows that the influence rates of four parameters are 17.5%, 49.0%, 27.8%, 5.7% respectively, which can provide the basis for an economic solution.

3. Experimental Design and Analysis

3.1 Response surface method

Since the motor is used for robot joints, eyes and other small space components, optimization on the motor under the condition of constant volume, has a vital significance. When the motor volume is constant, the torque of the motor is mainly related with the permanent magnet rotor diameter (*R*r), height of the rotor (*H*) and air gap length between stator and rotor (σ). This paper uses multiple linear regression method to establish the response surface model. A second-order function is used to approximate Response Surface model, which can be described as:

$$f(x_1, x_2, x_3) = \beta_0 + \sum_{i=1}^{3}\beta_i x_i + \sum_{i=1}^{3}\beta_{ii} x_i^2 + \sum_{i=1}^{2}\sum_{j>i}^{3} x_i x_j + \varepsilon \quad (2)$$

where β_0, β_i, β_{ii} are estimated by a regression from experiments, ε is the error estimate [11].

Response surface model in terms of the observation can be expressed by the following matrix:

$$y = X\beta + \varepsilon \tag{3}$$

Based on least squares estimation, the vector of estimator b can be found as follow:

$$b = (X^T X)^{-1} X^{'} Y \tag{4}$$

The fitted regression model is:

$$\hat{y} = Xb \tag{5}$$

It is convenient to transform the natural design variables to coded variables, which are usually defined to be dimensionless quantities with mean zero and the same spread or standard deviation [12]. The coding rule is described as follow:

$$\overline{x}_i = \frac{x_i - {}^{[\max(x_i)+\min(x_i)]}\!/_2}{{}^{[\max(x_i)-\min(x_i)]}\!/_2} \tag{6}$$

where x_i represent the permanent magnet rotor diameter (Rr), height of the rotor (H) and air gap length between stator and rotor (σ).

The appropriate selection of the points for experiments can reduce the variance of the coefficients, which makes the response surface more reliable. The central composite design (CCD) is applied as the experimental design method to estimate the regression. Model of CCD consists of eight runs at the corners of a cubic (±1, ±1, ±1), plus six runs at the star points, $(0,\pm\alpha,0)$, $(0,0,\pm\alpha)$, and one run at the cubic centre (0, 0, 0). CCD model can be described as follow:

In order to verify the accuracy of the obtained fitting equation, it is necessary to analyze the variance of response surface model. Table 4 is called an analysis-of-variance (ANOVA) table, where N presents total number of experiments; p presents the number of unknown parameters.

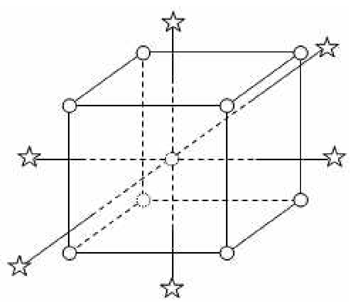

Fig. 2. The CCD model

Table 4. Analysis of variance

Source of Variation	Degree of Freedom	Sum of Squares(SS)	Mean Square (MS)
Regression	*p-1*	*SSR*	*SSR/*(*p-1*)
Residual(Error)	*N-p*	*SSE*	*SSE/*(*N-p*)
Total	*N-1*	*SST*	

The total variation in a set of data is called the total sum of squares (*SST*). In the following formula (7)-(11), Y_u represents response value, $\overline{Y}$ represents the average value of test dates, and $\hat{Y}_u$ represents Predicted value of response surface model.

$$SST = \sum_{u=1}^{N} (Y_u - \overline{Y})^2 \tag{7}$$

The sum of squares due to regression (SSR) reflects the random error and system error. The formula is

$$SSR = \sum_{u=1}^{N} (\hat{Y}_u - \overline{Y})^2 \tag{8}$$

The sum of squares (SSE) unaccounted for by the fitted model is

$$SSE = \sum_{u=1}^{N} (Y_u - \hat{Y}_u)^2 \tag{9}$$

The ratio between mean square regression and mean square residual is F- statistic, which is used to test the computed statistics. It is define as

$$F = \frac{\text{Mean Square Regression}}{\text{Mean Square Residual}} = \frac{SSR/(p-1)}{SSE/(N-p)} \tag{10}$$

The coefficient of determination R^2 is used to determine the precision of the regression equation from CCD. It is shown as

$$R^2 = \frac{SSR}{SST} \tag{11}$$

3.2 Analysis of the results

3.2.1 Response surface model

The value of α is 1.682 to guarantee the rotation and robustness of the model. The design parameters are provided in Table 5.

By the finite element software, the maximum torque for different test sets are obtained, as shown in Table 6.

Based on least squares estimation, equation of mathematical models of response surface for the motor torque can be shown in (12)

Table 5. Application of CCD designing parameter variables

Factor	Level				
	$-\alpha$	-1	0	1	α
Rr [*mm*]	11.59	15	20	25	28.41
H [*mm*]	3.27	6	10	14	16.73
Σ [*mm*]	0.26	0.7	1.35	2	2.44

Table 6. Experimental arrangement and its dates

No.	Factor			*T* [Nm]
	*X*1(*Rr*)	*X*2(*H*)	*X*3(σ)	
1	-1	-1	-1	0.11191
2	-1	-1	1	0.09887
3	-1	1	-1	0.20364
4	-1	1	1	0.14815
5	1	-1	-1	0.07062
6	1	-1	1	0.05778
7	1	1	-1	0.1936
8	1	1	1	0.1582
9	-*a*	0	0	0.14647
10	*a*	0	0	0.08527
11	0	-*a*	0	0.05301
12	0	*a*	0	0.22024
13	0	0	-*a*	0.17082
14	0	0	*a*	0.12029
15	0	0	0	0.16524
16	0	0	0	0.16982
17	0	0	0	0.16246
18	0	0	0	0.16632
19	0	0	0	0.16208

$$
\begin{aligned}
Y = {} & 0.17\text{-}0.016X_1 + 0.047X_2 - 0.012X_3 \\
& -8.126\times10^{-3}X_1X_2 + 2.536\times10^{-3}X_1X_3 \\
& +0.010X_2X_3 - 0.018X_1^2 - 0.010X_2^2 - 7.018\times10^{-3}X_3^2
\end{aligned} \qquad (12)
$$

where *X*1, *X*2, *X*3 represent the permanent magnet rotor diameter (*Rr*), height of the rotor (*H*) and air gap between stator and rotor (σ) from CCD respectively.

3.2.2 The analysis of variance

From Table 7, the model *P* value is less than 0.01 indicating a good fit of the model, and the lack of fit value *P* is equal to 0.0849 (greater than 0.05). Furthermore, the coefficient of determination R^2 is equal to 0.9939 (greater than 0.8000). The analysis of variance indicates that linear relationship between response value and variables is significant and the data regularity can be reflected by the response model accurately. *X*1, *X*2, *X*3, *X*1*X*2, *X*2*X*3, $X1^2$, $X2^2$, $X3^2$ influence the motor torque greatly and the remaining items are not significant.

Table 7. Analysis of variance

Source of Variation	Degree of Freedom	Sum of Squares(SS)	Mean Square(MS)	*F*-value	*P*-value
Model	9	0.043	$4.745*10^{-3}$	161.64	<0.0001
Lack of Fit	5	$2.24*10^{-4}$	$4.487*10^{-5}$	4.50	0.0849
Pure Error	4	$3.984*10^{-5}$	$9.960*10^{-6}$		
Total	18	0.043			

3.2.3 Analysis of each factor interaction

Fig. 3 to Fig. 5 describe the interaction between the various factors in the response surface model. From Fig.3 and Fig. 5, large changes in the magnitude of the response surface can be visually seen which indicates that the interaction between the various factors influences the motor response significantly. In Fig. 4, response surface is smooth revealing that the influence between the rotor diameter and rotor diameter on response values is smaller.

Fig. 3 shows that when air gap length is 20 mm, the interaction between the rotor diameter and the height of rotor is remarkable. Compared with the rotor diameter, magnitude of response surface changes more greatly with increasing height of the rotor, which indicates that the height of rotor has bigger influence on response surface than rotor diameter. The bigger the rotor height is, the larger the toque of motor is. However, with the rotor diameter increasing, motor torque increases first and then decreases. The reason is that when the volume of motor is constant, with the increase of rotor diameter, the stator coil and the stator core of motor corresponding decrease, resulting in reducing flux. So, when the rotor diameter is at 15-19 mm, the response value of motor torque is bigger.

Fig. 4 indicates that when the rotor height is 10 mm, the interaction between the rotor diameter and the air gap length is not significant. It can be seen from the graph that

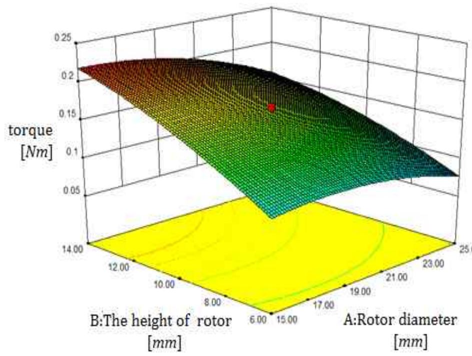

Fig. 3. Response surface for effects of the rotor diameter and the height of rotor on response value

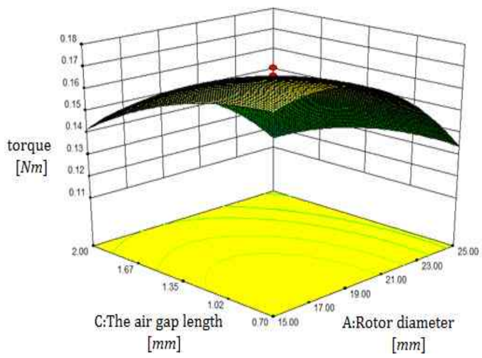

Fig. 4. Response surface for effects of the rotor diameter and the air gap length on response value

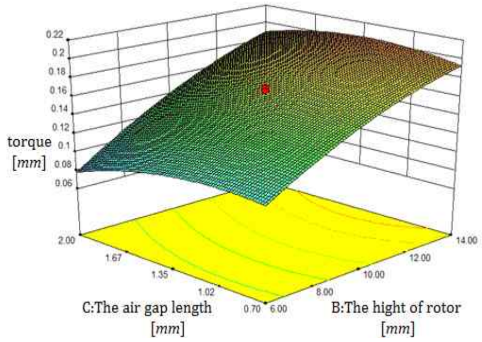

Fig. 5. Response surface for effects of the height of rotor and the air gap length on response value

the rotor diameter has bigger influence on response surface than the air gap length. With the increase of air gap length, the torque of multi-DOF deflection type PM motor is smaller. However, with the rotor diameter increasing, motor torque increases first and then decreases. When the rotor diameter is at 15-17.5 mm and air-gap length is at 0.70-1.35 mm, motor torque is bigger.

Fig. 5 shows that when the rotor diameter is at 10 mm, the interaction between the height of rotor and the air gap length is obvious. It can be seen from the graph that the rotor height has bigger influence on response surface than the air gap length. The bigger the rotor height is, the larger the toque of motor is. However, with the air gap length increasing, motor torque decreases accordingly.

By Design-Expert 8.0.5.0 software, when Rr =16.45 mm, H =14 mm, σ=1.18 mm, torque of the new multi-DOF deflection type PM motor reaches maximum value. While the motor air gap is larger, the magnetic flux leakage is greater, so the efficiency of the motor will be reduced. Taken together, the optimal motor structure is adjusted as: Rr =16.45 mm, H = 14 mm, σ=10.7 mm.

3.2.4 Validation of the numerical model

Two groups of representative parameters are selected to verify the reliability of the response surface model. Comparative results are shown in Table 8.

Table 8 shows that Relative errors between predicted values and measured values are less than 5%.The torque of improved motor is increased by 29.4%, improving the performance of the motor.

Table 8. Experiment date for test validation

Test No.	Rr [mm]	H [mm]	σ [mm]	T-Predicted values	T-measured values	Relative error/%
Initial	15.5	10	0.5	0.1725	0.1655	4.06
Proposed	16.45	14	0.7	0.2074	0.2142	3.29

4. Conclusion

This paper uses response surface methodology as the optimization method of torque of multi-DOF deflection type PM motor. Optimization parameters selected by Taguchi algorithm is the permanent magnet rotor diameter (Rr), height of the rotor (H) and air gap length between stator and rotor (σ). Based on the central composite design (CCD), response surface equation numerical model is constructed by the finite element method. The analysis of variance indicates that linear relationship between response value and variables is significant and the data regularity can be reflected by the response model accurately. With the aid of software Design-Expert 8.0.5.0, when Rr =16.45 mm, H =14 mm, σ =0.7 mm, torque of this motor reaches maximum value and is increased by 29.4% compared with the initial model. The results prove the effectiveness and correctness of the presented optimal design scheme, which provides the reference for further related optimal design of multi-DOF motors or actuators.

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