

# Takagi-Sugeno Fuzzy Model-Based Approach to Robust Control of Boost DC-DC Converters

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**Abstract**—This paper considers the robust controller design problem for a boost DC-DC converter. Based on the Takagi-Sugeno fuzzy model-based approach, a fuzzy controller as well as a fuzzy load conductance observer are designed. Sufficient conditions for the existence of the controller and the observer are derived using Linear Matrix Inequalities (LMIs). LMI parameterizations of the gain matrices are obtained. Additionally, LMI conditions for the existence of the fuzzy controller and the fuzzy load observer guaranteeing  $\alpha$ -stability, quadratic performance are derived. The exponential stability of the augmented fuzzy observer-controller system is shown. It is also shown that the fuzzy load observer and the fuzzy controller can be designed independently. Finally, the effectiveness of the proposed method is verified via experimental and simulation results under various conditions.

**Keywords:** Boost converter, Fuzzy system, Fuzzy controller, Fuzzy observer, Robust control, Uncertainty.

## 1. Introduction

Precise control of DC-DC converters is not easy due to the nonlinearities of converters, unavoidable uncertainties, parameter or load variations. To get around this problem various control design methods have been proposed in the literature such as [1-7]. Most of the previous control design methods require the good knowledge of the parameter values and the control performances can be severely degraded in the presence of load or parameter variations. On the other hand, the Takagi-Sugeno (T-S) fuzzy model-based control theory has been successfully applied to control of complex nonlinear or ill-defined uncertain systems [8-12].

Considering these facts, a nonlinear control design method for a boost DC-DC converter is proposed based on the T-S fuzzy approach. This paper first gives a T-S fuzzy model-based control design method. This paper introduces an error vector associated with the desired output voltage, and using this error vector a T-S fuzzy error dynamics model of a boost DC-DC converter is obtained to construct a fuzzy controller. In the fuzzy error dynamics model for controller design a boost DC-DC converter is represented as an average weighted sum of simple local linear subsystems. Local linear controllers are designed for each local subsystem and a global nonlinear controller from the local linear controllers is derived via a standard fuzzy inference method. An LMI condition for the existence of the fuzzy controller is derived and an explicit parameterization of the fuzzy controller gain is obtained

in terms of the solution matrices to the LMI condition. LMI existence conditions of the fuzzy speed controller guaranteeing  $\alpha$ -stability or quadratic performance are additionally derived. Secondly, the fuzzy controller method is applied to design a T-S fuzzy load conductance observer. It should be noted that the previous fuzzy control systems of [13-18] can suffer from lack of systematic and consistent design guidelines to determine design parameters such as fuzzy partition of the input and output spaces, membership function shapes, the number of fuzzy rules [11, 18]. However, in the proposed T-S model-based approach, one can systematically design a fuzzy controller as well as a load observer guaranteeing the asymptotic stability of the closed-loop control system. And one can handle various useful convex performance criteria such as  $\alpha$ -stability, quadratic performance. Through simulations and experiments, it is shown that the proposed method can be successfully used to control a boost DC-DC converter under load variations and it is very robust to the model parameter variations.

## 2. System Description

A boost converter shown in Fig. 1 can be represented by the following nonlinear equation [3, 4]:

$$\begin{aligned} \dot{i}_L &= -\frac{1}{L}v_C + \frac{1}{L}v_C u + \frac{E}{L} \\ \dot{v}_C &= \frac{1}{C}i_L - \frac{1}{RC}v_C - \frac{1}{C}i_L u \end{aligned} \quad (1)$$

where  $i_L$ ,  $v_C$ ,  $u$  represent the input inductor current, the output capacitor voltage, the discrete-valued control input

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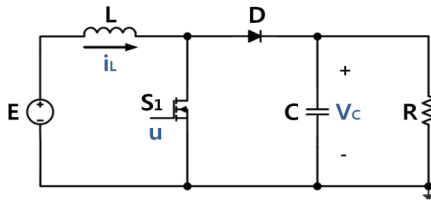


Fig. 1. Topology of boost converter

taking values in the set  $\{0, 1\}$ , and  $E, L, C, R$  are the external source voltage value, the inductance of the input circuit, the capacitance of the output filter, the output load resistance, respectively.

Following assumptions are used in this research:

- A1 :  $i_L, v_C$  are available.
- A2 :  $\dot{Y}$  can be neglected and it can be set as  $\dot{Y} = 0$  where  $Y = R^{-1}$ .
- A3 : The inductor current is never allowed to be zero, i.e. the converter is in continuous conduction mode.

By introducing the duty ratio input function  $u_c(\cdot)$  ranging on the interval  $[0, 1]$  and the following error terms  $[1, 3, 5]$ .

$$z_1 = i_L - \frac{V_r^2}{RE}, \quad z_2 = v_C - V_r, \quad z_3 = \int_0^t (v_C - V_r),$$

$$v = 1 - u_c - \frac{E}{V_r}$$

the following approximate averaged model can be derived from (1)

$$\dot{z} = \begin{bmatrix} 0 & -\frac{E}{LV_r} & 0 \\ \frac{E}{CV_r} & -\frac{1}{RC} & 0 \\ 0 & 1 & 0 \end{bmatrix} z + \begin{bmatrix} -\frac{v_C}{L} \\ \frac{i_L}{C} \\ 0 \end{bmatrix} v \quad (2)$$

where  $z = [z_1, z_2, z_3]^T$  and  $V_r$  is the desired reference output voltage such that  $V_r > E > 0$ . After all, our design

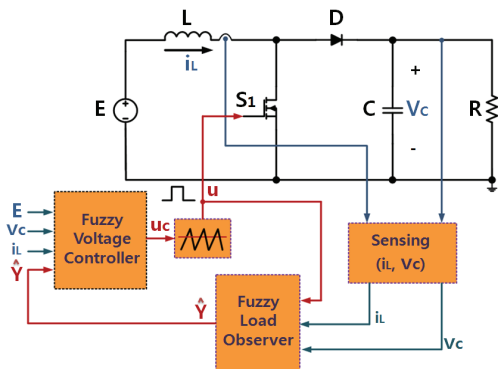


Fig. 2. Schematic diagram of proposed control system

problem can be formulated as designing a fuzzy controller for the system (2). Fig. 2 shows a schematic diagram of the control system.

### 3. Fuzzy Controller Design

Based on the T-S fuzzy modeling approach [8-12], the model (2) can be approximated by a second order  $r_c$ -rule fuzzy model to design a fuzzy controller. The  $i$ th rule of the T-S fuzzy model is of the following form:

**Plant Rule  $i$ :** IF  $x$  is  $F_{ci}$ , THEN  $\dot{z} = A_c z + B_{ci} v$

where  $F_{ci} (i = 1, \dots, r_c)$  denote the fuzzy sets,  $r_c$  is the number of fuzzy rules,  $x = [i_L, v_C]^T$ ,  $A_c$  and  $B_{ci}$  is given by

$$A_c = \begin{bmatrix} 0 & -\frac{E}{LV_r} & 0 \\ \frac{E}{CV_r} & -\frac{1}{RC} & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_{ci} = \begin{bmatrix} -\frac{X_{2i}}{L} \\ \frac{X_{1i}}{C} \\ 0 \end{bmatrix} \quad (3)$$

Each fuzzy set  $F_{ci}$  is characterized by a membership function  $m_{ci}(x)$  and the  $i$ th operating point  $x = (X_{1i}, X_{2i})$ . Via a standard fuzzy inference method, the following global nonlinear model can be obtained.

$$\dot{z} = \sum_{i=1}^{r_c} h_{ci}(x) [A_c z + B_{ci} v] \quad (4)$$

where  $h_{ci}(x) = \frac{m_{ci}(x)}{\sum_{j=1}^{r_c} m_{cj}(x)}$ ,  $m_{ci}: R \rightarrow [0, 1], i = 1, \dots, r_c$  is the membership function of the model with respect to plant rule  $i$ ,  $h_{ci}$  can be regarded as the normalized weight of each IF-THEN rule and it satisfies  $h_{ci}(x) \geq 0$  and  $\sum_{i=1}^{r_c} h_{ci}(x) = 1$ .

Let the local controller be given by the following linear controller

**Controller Rule  $i$ :** IF  $x$  is  $F_{ci}$ , THEN  $v = K_i z$

where  $K_i \in R^{1 \times 3}$  are gain matrices. Then the final fuzzy controller is given by

$$v = \sum_{i=1}^{r_c} h_{ci}(x) K_i z \quad (5)$$

and the closed-loop control system is given by

$$\dot{z} = \sum_{i=1}^{r_c} \sum_{j=1}^{r_c} h_{ci}(x) h_{cj}(x) (A_c + B_{ci} K_j) z \quad (6)$$

Assume that the following LMI condition is feasible

$$P_c > 0, M_{ij} + M_{ji} < 0, i \leq j \quad (7)$$

where  $M_{ij} = A_c P_c + P_c A_c^T + B_{ci} Y_{cj} + Y_{cj}^T B_{ci}^T$ .  $P_c \in R^{3 \times 3}$  and  $Y_{ci} \in R^{1 \times 2}$  are decision variables.

And assume that the controller gain matrices  $K_i$  are given by

$$K_i = Y_{ci} P_c^{-1} \quad (8)$$

Then there exists a matrix  $Q_c > 0$  such that

$$P_c^{-1} [M_{ij} + M_{ji}] P_c^{-1} < -2Q_c < 0, i \leq j$$

Let us define the Lyapunov function as  $V_c(z) = z^T X_c z$  where  $X_c = P_c^{-1}$ . Its time derivative along the closed-loop system dynamics (6) is given by

$$\begin{aligned} \dot{V}_c &= \frac{d}{dt} z^T X_c z = 2z^T \sum_{i=1}^{r_c} \sum_{j=1}^{r_c} h_{ci}(x) h_{cj}(x) X_c [A_c + B_{ci} K_j] z \\ &= 2z^T \sum_{i=1}^{r_c} \sum_{j=1}^{r_c} h_{ci}(x) h_{cj}(x) P_c^{-1} M_{ij} P_c^{-1} z \leq -x^T Q_c z \end{aligned} \quad (9)$$

which implies that the origin  $z=0$  is exponentially stable.

**Theorem 1** Assume that the LMI condition (7) is feasible for  $(P_c, Y_{ci})$  and the gain matrices  $K_i$  are given by (8). Then,  $x$  converges exponentially to zero.

**Remark 1** The LMI parameterization of the controller gain enables one to handle various useful convex performance criteria such as  $\alpha$ -stability, quadratic performance, and generalized  $H_2/H_\infty$  performances [12, 19]. For example, if the controller gain is set as  $K_i$  of (8) with  $P_c$  and  $Y_{ci}$  satisfying for some  $\alpha_c > 0$

$$P_c > 0, M_{ij} + M_{ji} + 4\alpha_c P_c < 0, i \leq j \quad (10)$$

then by referring to (9) the following can be obtained

$$\frac{d}{dt} x^T X_c x = -2\alpha_c x^T X_c x \leq 0$$

which implies that  $x$  converges to zero with a minimum decay rate  $\alpha_c$ . On the other hand, if the controller gain is set as  $K_i$  of (8) with  $P_c$  and  $Y_{ci}$  satisfying

$$\begin{bmatrix} \gamma & * \\ z(0) & P \end{bmatrix} > 0, M_{ij} + M_{ji} < -2Q_c < 0, i \leq j \quad (11)$$

Then the followings can be obtained

$$\begin{aligned} &\int_0^\infty z^T Q_c z dt \\ &\leq \int_0^\infty [z^T Q_c z + 2z^T \sum_{i=1}^{r_c} \sum_{j=1}^{r_c} h_{ci} h_{cj} P_c^{-1} [A_c + B_{ci} K_j] z] dt \\ &\quad + z^T(0) P_c^{-1} z(0) \\ &\leq z^T(0) P_c^{-1} z(0) < \gamma \end{aligned}$$

where the Shur complement lemma of [19] is used. After all, it can be seen that the fuzzy controller (5) guarantees the quadratic performance bound constraint  $\int_0^\infty x^T Q_c x dt \leq \gamma$ .

#### 4. Fuzzy Load Conductance Observer Design

In this section, a fuzzy observer to estimate the load conductance  $Y = R^{-1}$  will be designed. By applying the T-S fuzzy modeling methods [9, 10] to (2) or the equivalent error dynamics (2), the boost converter and the dynamics of  $Y, \dot{Y} = 0$ , can be approximated by a second order  $r_o$ -rule fuzzy model. The  $i$ th rule of the T-S fuzzy model is of the following form:

**Plant Rule  $i$ :** IF  $v_c$  is  $F_{oi}$ , THEN

$$\dot{x}_o = A_{oi} x_o + B_o u_o, y_o = C_o x_o$$

where the assumption A2 is used,  $F_{oi} (i = 1, \dots, r_o)$  denote the fuzzy sets,  $r_o$  is the number of fuzzy rules,  $x_o = [v_c, Y]^T$  is the state,  $y_o = v_c$  is the output,  $B_o = C_o^T = [1, 0]^T, u_o = i_L(1 - u)/C$ , and

$$A_{oi} = \begin{bmatrix} 0 & -\frac{V_i}{C} \\ 0 & 0 \end{bmatrix} \quad (12)$$

Each fuzzy set  $F_{oi}$  is characterized by a membership function  $m_{oi}(v_c)$  and the  $i$ th operating point  $V_i$ . Via a standard fuzzy inference method, the following global nonlinear model can be obtained :

$$\dot{x}_o = \sum_{i=1}^{r_o} h_{oi}(v_c) A_{oi} x_o + B_o u_o, y_o = C_o x_o \quad (13)$$

where  $h_{oi}(v_c) = \frac{m_{oi}(v_c)}{\sum_{j=1}^{r_o} m_{oj}(v_c)}$ ,  $m_{oi} : R \rightarrow [0, 1], i = 1, \dots, r_o$  is the membership function of the system with respect to plant rule  $i$ ,  $h_{oi}$  is the normalized weight of each IF-THEN rule and it satisfies  $h_{oi}(v_c) \geq 0$  and  $\sum_{i=1}^{r_o} h_{oi}(v_c) = 1$ .

Let the local observer given by the following linear observer

**Observer Rule  $i$ :** IF  $v_c$  is  $F_{oi}$ , THEN

$$\hat{x}_o = A_{oi}\hat{x}_o + L_i y_o - L_i C_o \hat{x}_o + B_o u_o$$

where  $L_i \in R^{2 \times 1}$  are gain matrices,  $\hat{x}_o = [\hat{v}_C, \hat{Y}]^T$ . Then the final fuzzy observer induced as the weighted average of the each local observer is given by

$$\dot{\hat{x}}_o = \sum_{i=1}^{r_o} h_{oi}(v_C) [A_{oi} - L_i C_o] \hat{x}_o + L_i y_o + B_o u_o \quad (14)$$

which gives the following error dynamics.

$$\dot{\bar{x}}_o = \sum_{i=1}^{r_o} h_{oi}(v_C) [A_{oi} - L_i C_o] \bar{x}_o$$

where  $\bar{x}_o = x_o - \hat{x}_o$ .

**Theorem 2** Assume that the following LMI condition is feasible for  $(P_o, Y_{oi})$

$$P_o > 0, P_o A_{oi} + A_{oi}^T P_o - Y_{oi} C_o - C_o^T Y_{oi}^T < 0, \forall i \quad (15)$$

where  $P_o \in R^{2 \times 2}, Y_{oi} \in R^{2 \times 1}$  are decision variables. And assume that the observer gain  $L_i$  is given by

$$L_i = P_o^{-1} Y_{oi} \quad (16)$$

Then, the estimation error converges exponentially to zero.

**Proof:** Assume that (15) is feasible. Then there exists a matrix  $Q_o > 0$  such that

$$P_o A_{oi} + A_{oi}^T P_o - Y_{oi} C_o - C_o^T Y_{oi}^T < -Q_o, \forall i$$

Let us define the Lyapunov function as  $V_o(\bar{x}_o) = \bar{x}_o^T P_o \bar{x}_o$ . Its derivative with respect to time is given by

$$\dot{V}_o = 2\bar{x}_o^T \sum_{i=1}^{r_o} h_{oi}(v_C) [P_o A_{oi} - Y_{oi} C_o] \bar{x}_o \leq -\bar{x}_o^T Q_o \bar{x}_o \quad (17)$$

which implies that  $\bar{x}_o$  is exponentially stable.

**Remark 2** The LMI parameterization of the observer gain (16) also provides some degrees of freedom which can be used to handle various useful convex performance criteria such as  $\alpha$ -stability, quadratic performance, and generalized  $H_2/H_\infty$  performances [12, 19]. For example, if the observer gain is set as  $L_i$  of (16) with  $P_o$  and  $Y_{oi}$  satisfying for some  $\alpha_o > 0$

$$P_o > 0, P_o (A_{oi} + \alpha_o I) + (A_{oi} + \alpha_o I)^T P_o - Y_{oi} C_o - C_o^T Y_{oi}^T < 0, \forall i \quad (18)$$

then  $\bar{x}_o$  converges to zero with a minimum decay rate  $\alpha_o$ . On the other hand, if the observer gain is set as  $L_i$  of (16)

with  $P_o$  and  $Y_{oi}$  satisfying for some  $Q_o \geq 0$

$$P_o > 0, P_o A_{oi} + A_{oi}^T P_o - Y_{oi} C_o - C_o^T Y_{oi}^T < -Q_o, \quad (19)$$

$$\bar{x}_o^T(0) P_o \bar{x}_o(0) \leq \gamma, \forall i$$

Then the followings can be obtained

$$\int_0^\infty \bar{x}_o^T Q_o \bar{x}_o dt$$

$$\leq \int_0^\infty [\bar{x}_o^T Q_o \bar{x}_o + \bar{x}_o^T \sum_{i=1}^{r_o} h_{oi}(v_C) [P_o A_{oi} - Y_{oi} C_o] \bar{x}_o] dt$$

$$+ \bar{x}_o(0)^T P_o \bar{x}_o(0) \leq \bar{x}_o(0)^T P_o \bar{x}_o(0) \leq \gamma$$

After all, it can be easily shown that the fuzzy observer (14) guarantees the quadratic performance bound constraint

$$\int_0^\infty \bar{x}_o^T Q_o \bar{x}_o \leq \gamma.$$

## 5. Separation Property and Design Algorithm

This section illustrates the exponential stability of the augmented control system containing the fuzzy controller and the fuzzy load observer. The following theorem implies that the separation property holds.

**Theorem 3** Assume that the LMIs (7) and (15) are feasible, and the controller (5) is replaced with the following load observer-based control law

$$v = \sum_{i=1}^{r_c} h_{ci1}(x) K_i \hat{z} \quad (20)$$

where  $\hat{z} = [i_L - \hat{Y} V_r^2 / E, v_C - V_r]^T$  and  $\hat{Y}$  is the estimated output conductance via the fuzzy observer (14). Then  $z$  and  $\bar{x}_o$  converge exponentially to zero.

**Proof:** It should be noted that because  $Y - \hat{Y} = [0, 1] \bar{x}_o$  the vector  $\hat{z}$  can be rewritten as  $\hat{z} = z + G \bar{x}_o$  where

$$G = \begin{bmatrix} 0 & -\frac{V_r^2}{E} \\ 0 & 0 \end{bmatrix}$$

Let us define the Lyapunov function as  $V(z, \bar{x}_o) = z^T P_c^{-1} z + \eta \bar{x}_o^T P_o \bar{x}_o$  where  $\eta$  is a sufficiently large scalar,  $P_c$  and  $P_o$  satisfy the LMIs (7) and (15). Its derivative with respect to time is given by

$$\dot{V} \leq -\eta \lambda_{\min}(Q_o) \|\bar{x}_o\|^2 - \eta \lambda_{\min}(Q_c) \|z\|^2 + 2\zeta \|\bar{x}_o\| \cdot \|z\|$$

where  $\zeta = \sum_{i=1}^{r_c} \sum_{j=1}^{r_c} \|P_c^{-1} B_i K_j G\|$ . If  $\eta$  is large enough to guarantee  $\eta > \zeta^2 \lambda_{\min}^{-1}(Q_c) \lambda_{\min}^{-1}(Q_o)$ , then  $\dot{V} < 0$

for all  $(z, \bar{x}_o) \neq 0$ . This proves the exponential stability of  $(z, \bar{x}_o)$ .

**Remark 3** From the standard results [19], it can be shown that if one of the pairs  $(A_c, B_{ci})$  is not stabilizable then the LMI condition (7) is not feasible. And it can be easily shown that  $(A_c, B_{ci})$  are stabilizable as long as  $X_{1i} \neq 0$  or  $X_{2i} \neq 0$ . It can be also shown that if one of the pairs  $(A_{oi}, C_o)$  is not detectable then the LMI condition (7) is not feasible. And it can easily shown that  $(A_{oi}, C_o)$  are detectable as long as  $V_i \neq 0$ . These facts imply that the LMI condition (7) and (15) is always feasible for an appropriately chosen set of the operating points.

**Remark 4** Theorems 1-2 imply that our design problem is a simple LMI problem which can be solved very easily via various powerful LMI optimization algorithms. Theorem 3 implies that the controller gains and the load conductance observer gains can be independently designed. And Remark3 implies that our design problem is always feasible for an appropriately chosen set of the operating points. Our results can be summarized as the following LMI-based design algorithm.

- [Step1] Choose an appropriate set of  $\{V_1, \dots, V_{r_o}\}$  and obtain the fuzzy model (13).
- [Step2] Solve the LMIs (15), obtain the gain matrices  $L_i$ , and construct the fuzzy observer (14).
- [Step3] Choose an appropriate set of and obtain the fuzzy model  $\{(X_{11}, X_{21}), \dots, (X_{1r_c}, X_{2r_c})\}$  and obtain the fuzzy model (4).
- [Step4] Solve the LMIs (7), obtain the gain matrices  $K_i$ , and construct the load observer-based fuzzy control law (20)

**Remark 5** Via extensive numerical simulations and experimental studies, it has been found that fuzzy models with  $r_o = 2$  and  $r_c = 2$  are enough to obtain load observer-based fuzzy control laws with satisfactory performances. As can be seen in the next section a two-rule fuzzy model (13) with the following normal membership functions is enough to design a fuzzy load observer with satisfactory performances

$$V_1 = \epsilon_{o1}V_r, V_2 = \epsilon_{o2}V_r, m_{o1} = e^{-\epsilon_{o3}(v_c - V_1)^2}, m_{o2} = e^{-\epsilon_{o4}(v_c - V_2)^2}$$

where  $\epsilon_{oi} > 0$ . A fuzzy controller with satisfactory performances can also be obtained by using a fuzzy model (4) with  $r_c = 2$  and

$$X_{11} = X_{12} = \frac{V_r^2}{RE}, X_{21} = \epsilon_{c1}V_r, X_{22} = \epsilon_{c2}V_r, m_{c1} = e^{-\epsilon_{c3}(v_c - X_{21})^2}, m_{c2} = e^{-\epsilon_{c4}(v_c - X_{22})^2}$$

where  $\epsilon_{ci} > 0$ , and the nominal values of  $E, L, C, R$  are

used.

**Remark 6** To design a fuzzy load observer under the several performance specifications, one has only to gather LMI conditions corresponding to each design performance specification, and form a system of LMIs as a subset of (15), (18), (19), and solve the system of LMIs under the assumption that the Lyapunov matrices ‘ $P_o$ ’ are common. Similarly, one can design a fuzzy controller under the several performance specifications.

### 6. Simulation and Experiment

Consider a boost converter (1) with  $L=200[\mu\text{H}]$ ,  $C = 220[\mu\text{F}]$ ,  $v_c = 20[\text{V}]$ , the PWM switching frequency  $60[\text{kHz}]$ . Assume that the nominal load resistance is  $R_o = 100[\Omega]$  and desired reference output voltage  $V_c$  is  $V_c = 20[\text{V}]$ . The parameters values used for the simulations and experiments are summarized in Table 1. Let us first design a fuzzy observer guaranteeing the minimum decay rate  $\alpha_o = 10$ . Here, the following two-rule fuzzy model to design a fuzzy observer is used.

**Plant Rule 1:** IF  $v_c$  is about  $V_r$ , THEN

$$\dot{x}_o = A_{o1}x_o + B_o u_o, y_o = C_o x_o$$

**Plant Rule 2:** IF  $v_c$  is about  $0.5V_r$ , THEN

$$\dot{x}_o = A_{o2}x_o + B_o u_o, y_o = C_o x_o$$

where

$$A_{o1} = \begin{bmatrix} 0 & -50000 \\ 0 & 0 \end{bmatrix}, A_{o2} = \begin{bmatrix} 0 & -15000 \\ 0 & 0 \end{bmatrix}, B_o = C_o^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And  $h_{o1} = m_{o1}/(m_{o1} + m_{o2}), h_{o2} = m_{o2}/(m_{o1} + m_{o2}), m_{o1} = e^{-(v_c - V_r)^2}, m_{o2} = e^{-(v_c - 0.5V_r)^2}$ . By solving (18) with  $\alpha_o = 100$  the following fuzzy observer (14) with the following gain

**Table 1.** Utilized components and parameters

Components	Parameters
$V_i$ (Input Voltage)	5~15 $V_{DC}$
$V_r$ (Reference Voltage)	20~13 $V_{DC}$
$V_o$ (Output Voltage)	20 $V_{DC}$
$P_{out}$ (Maximum output power)	20W
$f_s$ (Switching Frequency)	60kHz
$N$ (Turns ration)	52Turns (CS20060)
$C_s$ (Current Sensor)	LTS 6-NP
$L_m$ (magnetizing inductor)	200 $\mu\text{H}$
$C_o$ (Output capacitor)	220 $\mu\text{F}$
R (Load Resistance)	20~100 $\Omega$
$S_1$ (Main switches)	IRFP26N60
$D$ (Output diode)	CSD10060
DSP (Digital Signal Processor)	TMS320F28335

$$L_1 = [1642 \quad -13.85]^T, \quad L_2 = [1904 \quad -43.01]^T \quad (21)$$

Now, let us design a fuzzy controller guaranteeing the minimum decay rate  $\alpha_o = 20$ . In order to design a fuzzy controller, the following two-rule fuzzy model is used.

**Plant Rule 1:** IF  $(i_L, v_C)$  is about  $(0.4, V_r)$ , THEN

$$\dot{z} = A_{c1}z + B_{c1}v$$

**Plant Rule 2:** IF  $(i_L, v_C)$  is about  $(0.4, 0.25V_r)$ , THEN

$$\dot{z} = A_{c2}z + B_{c2}v$$

where

$$A_c = \begin{bmatrix} 0 & -2500 & 0 \\ 1250 & -125 & 0 \\ 0 & 1 & 0 \end{bmatrix}, B_{c1} = \begin{bmatrix} -10000 \\ 1000 \\ 0 \end{bmatrix}, B_{c2} = \begin{bmatrix} -30000 \\ 1000 \\ 0 \end{bmatrix}$$

and  $h_{c1} = m_{c1}/(m_{c1} + m_{c2})$ ,  $h_{c2} = m_{c2}/(m_{c1} + m_{c2})$ ,  $m_{c1} = e^{-(v_C - V_r)^2}$ ,  $m_{c2} = e^{-(v_C - 0.25V_r)^2}$ . By solving (10) with  $\alpha_c = 100$ , the following controller gain can be obtained :

$$K_1 = [-0.0795 \quad -0.0362 \quad -16.07] \quad (22)$$

$$K_2 = [-0.2618 \quad -0.0341 \quad -39.06]$$

After all, the following observer-based fuzzy controller can be obtained

$$u_c = 1 - h_{c1}K_1\hat{z} - h_{c2}K_2\hat{z} - \frac{E}{V_r} \quad (23)$$

where

$$\hat{z} = \left[ i_L - \hat{Y}V_r^2 / E, v_C - V_r, \int_0^t (v_C - V_r) ds \right]^T, \hat{Y} = [0 \quad 1] \hat{x}_o$$

$$\dot{\hat{x}}_o = \sum_{i=1}^2 h_{oi}(v_C)([A_{oi} - L_i C_o] \hat{x}_o + L_i y_o) + B_o u_o \quad (24)$$

Figs. 3 and 4 show the proposed fuzzy load observer and fuzzy controller, respectively.

A conventional cascade PI controller shown in Fig. 5 is also considered for performance comparisons. The gains of the above PI controller are designed based on the method [21]. The P and I gains of the voltage PI controller are  $K_{pV}=0.1$  and  $K_{iV}=4$ . The P and I gains of the current PI controller are  $K_{pI}=0.8$  and  $K_{iI}=1$ . Fig. 6

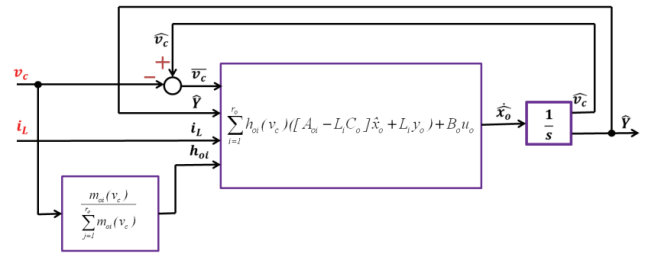


Fig. 3. Block diagram of the proposed fuzzy load observer.

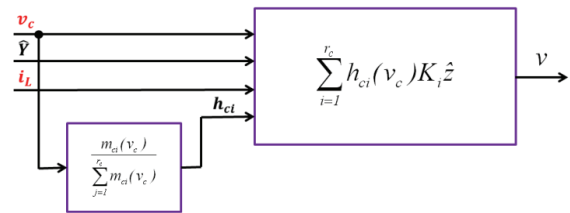


Fig. 4. Block diagram of the proposed fuzzy controller.

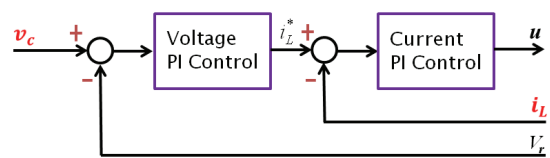


Fig. 5. PI control block diagram

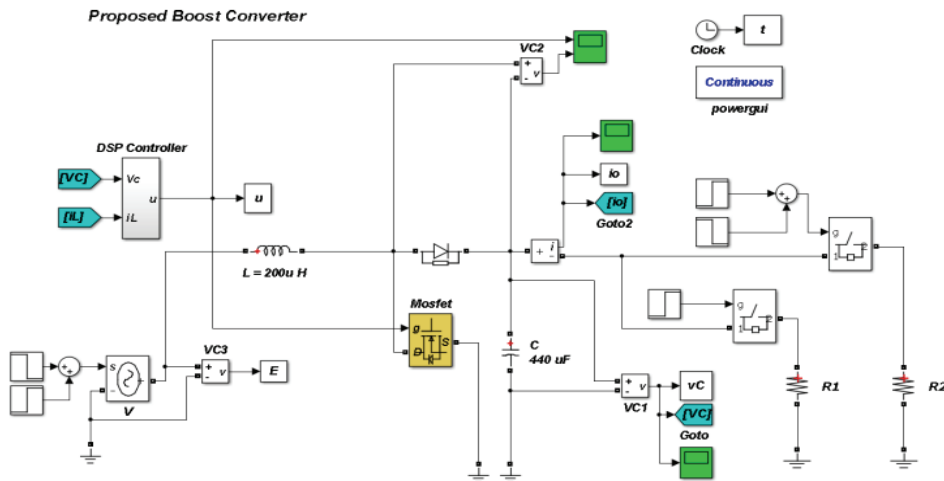


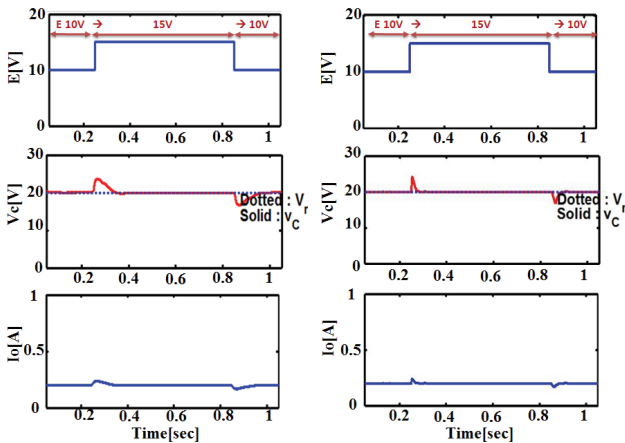
Fig. 6. Simulation model of the proposed control system implemented with Simulink.

illustrates the Matlab/Simulink simulation model of the proposed controller system.

In order to verify the effectiveness of the proposed method, the following four cases are considered :

- C1) The input voltage  $E$  changes from 10[V]→15[V]→10[V] while the load resistor  $R$  is constant at the nominal value  $R=100 [\Omega]$ .
- C2) The input voltage  $E$  changes from 10[V]→5[V]→10[V] while the load resistor  $R$  is constant at the nominal value  $R=100 [\Omega]$ .
- C3) The reference voltage  $V_r$  changes from 20[V]→13[V]→20[V] while  $E$  and  $R$  are kept constant at  $E=10 [V]$  and  $R=100 [\Omega]$ .
- C4) The load resistor  $R$  changes from 100[ $\Omega$ ]→20[ $\Omega$ ]→100[ $\Omega$ ] while  $V_r$  and  $E$  are kept constant at  $V_r=20 [V]$  and  $E=10 [V]$ .

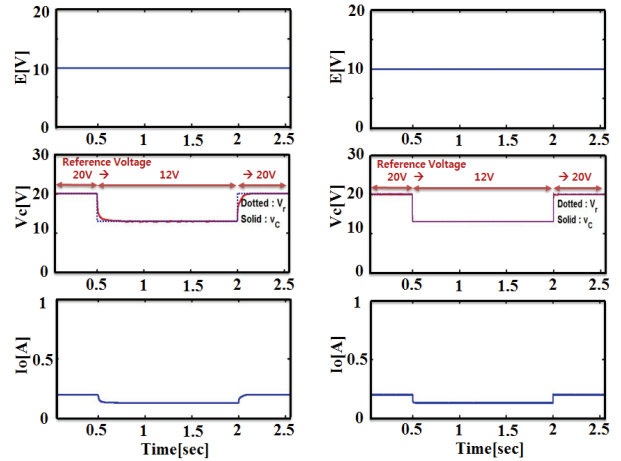
Fig. 7 shows the time responses under the case C1. Fig. 7(a) shows the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the conventional PI control method. The PI gain values are computed based on the methods given in [1-2].



(a) PI controller (b) Proposed fuzzy controller

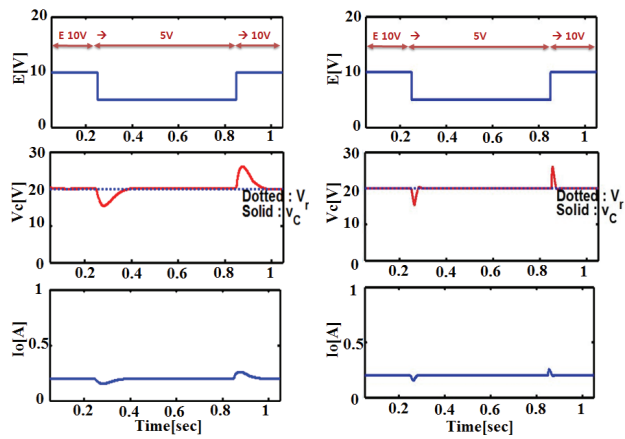
Fig. 7. Simulation results under C1.

Fig. 7(b) depicts the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the proposed load observer-based fuzzy controller (23). Fig. 8 shows the time responses under the case C2. Fig. 8(a) shows the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the conventional PI control method. The PI gain values are computed based on the methods given in [1-2]. Fig. 8(b) depicts the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the proposed load observer-based fuzzy controller (23). Fig. 9 shows the time responses under the case C3. Fig. 9(a) shows the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the conventional PI control method. The PI gain values are computed based on the methods given in [1-2]. Fig. 9(b) depicts the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the proposed load observer-based fuzzy controller (23). Fig. 10 shows the time responses under the case C4. Fig. 10(a) shows the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the conventional PI control method. Fig. 10(b) depicts the time histories of  $E$ ,  $v_c$ , and output current  $i_o$  by the following load observer-based fuzzy controller. Figs. 7, 8, 9 and 10 imply that our method gives a faster recovery time as well



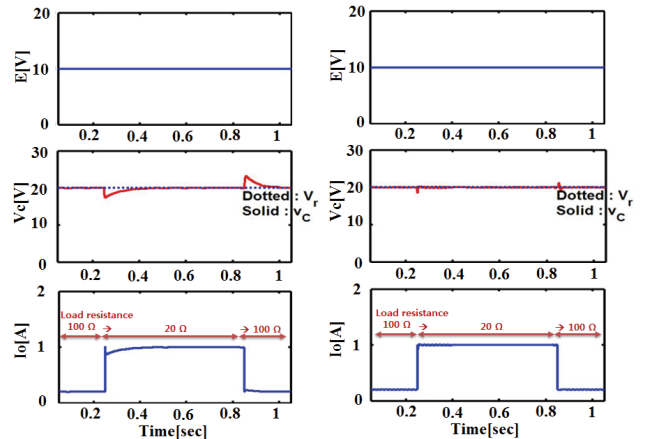
(a) PI controller (b) Proposed fuzzy controller

Fig. 9. Simulation results under C3.



(a) PI controller (b) Proposed fuzzy controller

Fig. 8. Simulation results under C2.



(a) PI controller (b) Proposed fuzzy controller

Fig. 10. Simulation results under C4.

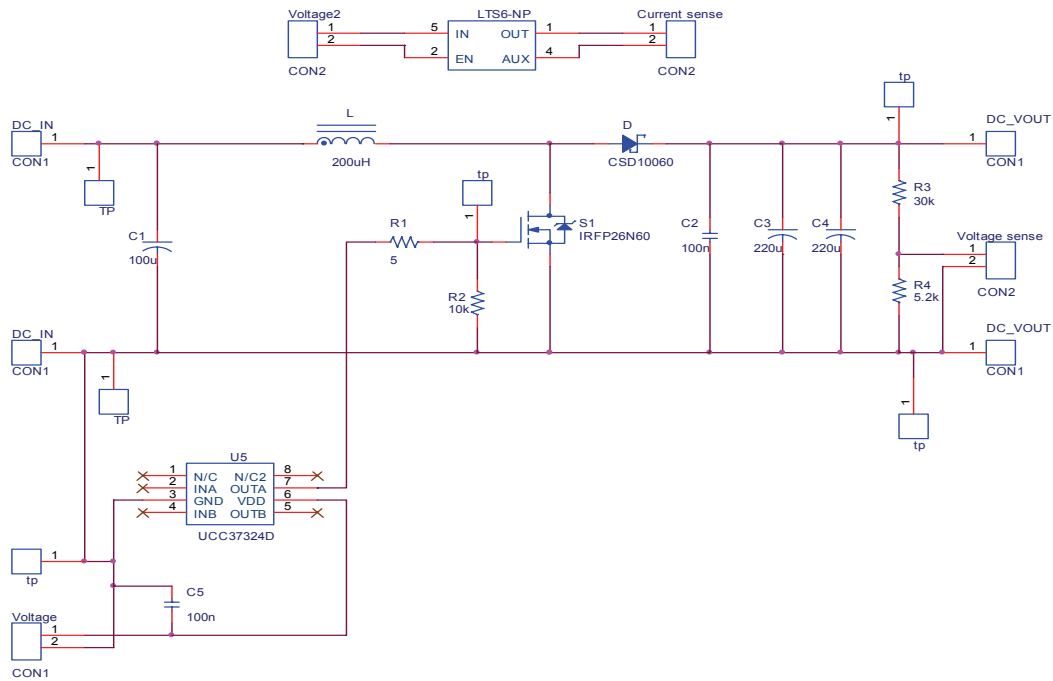


Fig. 11. Power stage circuit scheme of the boost converter

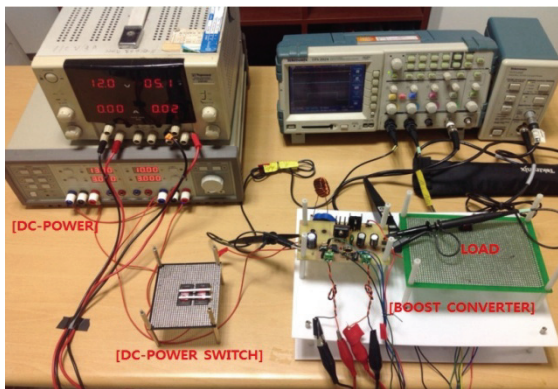


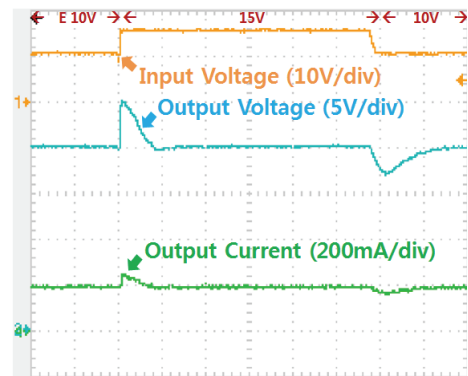
Fig. 12. Experimental setup.

as a less overshoot.

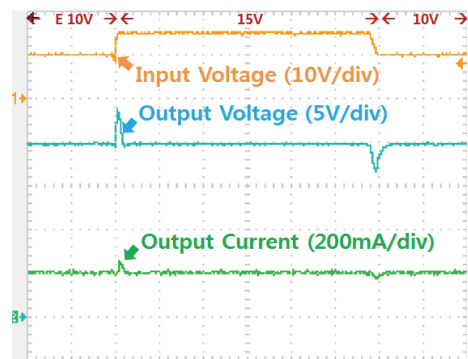
The conventional PI control algorithm as well as our method is implemented on a Texas Instruments TMS 320F28335 floating-point DSP. A Tektronix TDS5140B digital oscilloscope is used to measure and plot the signals  $v_C$ ,  $i_L$ ,  $E$ . Fig. 11 shows the circuit scheme of power stage. Fig. 12 illustrates the experimental setup. Figs. 13, 14, 15 and 16 show the experimental results. It can be seen that our method outperforms the conventional PI method.

### 7. Conclusion

A simple fuzzy load observer-controller design method was proposed for a boost converter under an unknown load resistance. Explicit parameterizations of the fuzzy controller gain and the fuzzy load conductance observer gain were given in terms of LMIs. LMI existence conditions



(a) PI controller

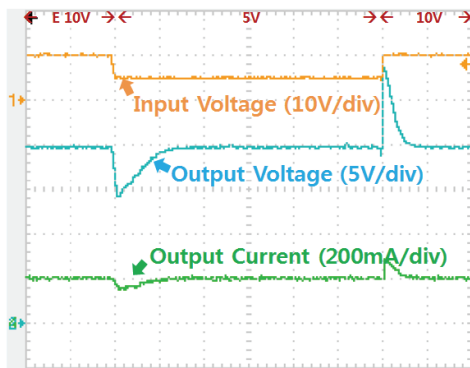


(b) Proposed fuzzy controller

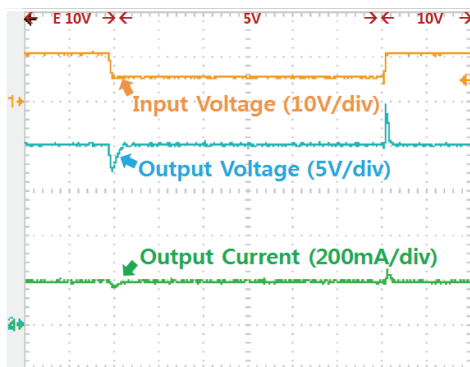
Fig. 13. Experimental results under C1.

of the fuzzy controller and the fuzzy observer guaranteeing  $\alpha$ -stability or quadratic performance were also derived. Finally, the robust performance of the proposed method was verified via numerical simulations and experiments.



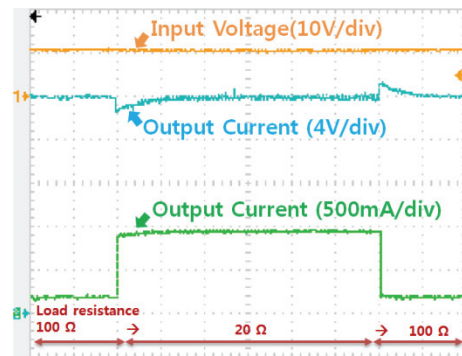


(a) PI Controller

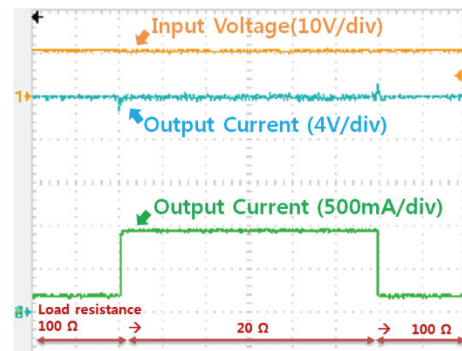


(b) Proposed Fuzzy Controller

Fig. 14. Experimental results under C2.

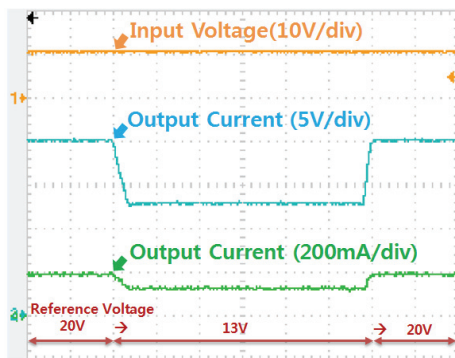


(a) PI Controller

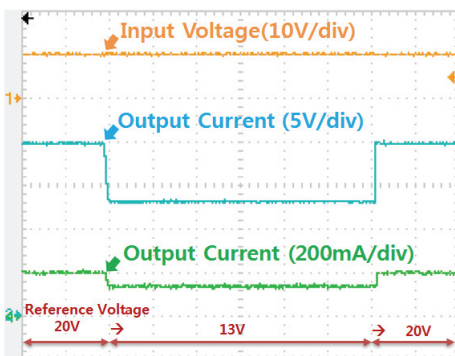


(b) Proposed Fuzzy Controller

Fig. 16. Experimental results under C4.



(a) PI Controller



(b) Proposed Fuzzy Controller

Fig. 15. Experimental results under C3.

### Acknowledgment

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