Kalman Randomized Joint UKF Algorithm for Dual Estimation of States and Parameters in a Nonlinear System

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Abstract – This article presents a new nonlinear joint (state and parameter) estimation algorithm based on fusion of Kalman filter and randomized unscented Kalman filter (UKF), called Kalman randomized joint UKF (KR-JUKF). It is assumed that the measurement equation is linear. The KR-JUKF is suitable for time varying and severe nonlinear dynamics and does not have any systematic error. Finally, joint-EKF, dual-EKF, joint-UKF and KR-JUKF are applied to a CSTR with cooling jacket, in which production of propylene glycol happens and performance of KR-JUKF is evaluated.

Keywords: Joint estimation, Kalman randomized joint UKF, Parameter estimation, CSTR

1. Introduction

Recently, control of the operation of chemical reactors has been studied enormously [1-3]. There are many process control strategies, in which information about the state of the process is essential for calculating the control input. Since getting an accurate model for a nonlinear chemical process is difficult or probably impossible, it is necessary to obtain an approximate model. Therefore, a technique which provides the best estimation and compensates for the lack of knowledge of uncertainties in the process is required.

The most popular nonlinear estimator is the extended Kalman filter (EKF). The problems associated with EKF are attributed to the approximation introduced by the linearization [4].This is the idea of the extended Kalman filter (EKF), which was originally proposed by Stanley Schmidt [4], so that the Kalman filter can be applied to nonlinear problems [6]. Since EKF estimates mean and covariance using first-order approximation of the system dynamics, it can be hard to tune and implement when dealing with significant nonlinearities and then exhibits divergence in extreme cases.

Unscented Kalman filter (UKF) overcomes this theoretical deficiency. The core of UKF is unscented transformation (UT). It uses a minimal set of determinate sample points (Sigma points) to completely assess the true mean and covariance of the states via UT. Studies show that UT is more accurate than linearization for propagating mean and covariance [7-12].

Although UKF is more accurate than EKF, it suffers from systematic error. To remove this systematic error, randomized unscented Kalman filter (RUKF) was suggested [13]. The idea of RUKF is based on Bayesian recursive relation (BRR). BRR is a general solution for estimation problems and is used to compute probability density functions (pdfs) of states conditioned by the measurements. To propagate mean and covariance matrix, multi-dimensional Gaussian integral should be computed. An improved UT was proposed to randomly generate points for calculating this Gaussian integral. Studies have demonstrated that utilizing the improved UT enables RUKF to provide estimates of higher quality than UKF and CKF [13].

In addition, most processes have time-varying dynamics or unknown parameters, and since both the EKF and UKF are model-dependent approaches, their estimations may leave the optimal region or even diverge in the case of large amplitude variations. Thus, a dual estimation algorithm which estimates both the states and parameters simultaneously is very practical for handling time varying systems.

One of the first usages of dual estimation is combining both the state vectors and unknown parameters in a joint-bilinear state-space representation [14], which is mentioned for linear dual Kalman. Another approach suggests two independent KFs [15]. These works have considered problem of state estimation in linear systems. Joint-EKF was suggested to model unknown parameters in a model reference adaptive control framework [16]. Dual-EKF [17] was a nonlinear extension of the linear dual Kalman approach. Although these filtering techniques have shown to perform better than the EKF [18], they do not perform as well as joint UKF in severe nonlinearity systems. Joint UKF provides a more accurate approximation, but suffers from containing a systematic error.

Nonlinear, time varying behaviour is common in chemical processes. CSTR (continuous stirred-tank reactor) is one of the significantly investigated benchmarks for nonlinear estimation [19]. Nonlinear functions describing a CSTR are generally of exponential form due to the Arrhenius dependence of reaction rate on temperature [20].

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In this paper, a new algorithm called KR-JUKF is presented. The KR-JUKF is derived from the combination of KF and RUKF and the concept of joint estimation. KR-JUKF propagates mean and covariance of a nonlinear dynamic system using linear measurement equations. The proposed approach is suitable for time varying systems due to utilizing the concept of joint estimation. Also the combination of the KF and RUKF makes the proposed approach proper for online estimation. The advantage of KR-JUKF is to reduce the estimation error compared to existing algorithms. To show the merits of the proposed approach, dual estimation algorithms are utilized to estimate the state and unknown parameters of a highly nonlinear dynamic system. Joint EKF, dual EKF, joint UKF and KR-JUKF are applied to a jacketed CSTR and their performances are compared.

This paper is organized as follows: In section 2, concepts of parameter estimation and Joint-EKF and Joint-UKF are presented. The next section represents the algorithm of dual-EKF. In section 4, KR-JUKF is presented. The dynamic of CSTR is studied in section 5 and simulation results are presented in section 6. Finally, section 7 concludes the paper.

2. Parameter Estimation

The idea of state estimation can be extended to unknown parameter estimation, which is due to uncertainties in system dynamics [4]. Parameter estimation was presented by Kopp and Orford [21]. In this approach, unknown parameters are transformed into a new state by combining the states with parameters. Suppose that we have continuous-time nonlinear dynamic and nonlinear measurement equation with additive noise:

$$\dot{X}(t) = f\left[X(t), P, U(t)\right] + W \qquad (1)$$

$$y(t) = g\left[X(t), P\right] + V \qquad (2)$$

Where $X(t)$ is the state vector, P is an unknown parameter vector, $U(t)$ is the input and V and W are the measurement and system noises, respectively. V and W are assumed to be independent, white and Gaussian with zero mean and finite covariance matrix. If we artificially combine the unknown parameters and states into a new state vector as follows:

$$Z(t) = \left[X(t)^T \quad P^T\right]^T \qquad (3)$$

a new state space is resulted with the augmented state vector:

$$\begin{cases} \dot{Z}(t) = f\left[Z(t), U(t)\right] + W \\ \quad y(t) = g\left[Z(t)\right] + V \end{cases} \qquad (4)$$

If the unknown parameters are assumed to be constant, then $\dot{P} = 0$. Therefore, state space can be modified as:

$$\dot{Z}(t) = \begin{pmatrix} \dot{X}(t) \\ \dot{P} \end{pmatrix} = \begin{pmatrix} f\left[X(t), P, U(t)\right] \\ 0 \end{pmatrix} + \begin{pmatrix} W \\ 0 \end{pmatrix} \qquad (5)$$

Nonlinear filters such as EKF and UKF can be used to estimate the new state vector. This approach is called joint estimation. Joint-EKF, joint-UKF and KR-JUKF are obtained by using the joint estimation concept.

3. Dual EKF Algorithm

In this approach, two separate EKFs are used. One EKF estimates the states while the other estimates the unknown parameters, although two EKFs operate simultaneously. Fig. 1 shows how this algorithm works.

Possibility of convergence of dual-EKF is less than joint-EKF. In addition, lowering rank of used matrices can decrease burden of matrix's computational time, especially computation of inverse matrix. Suppose we have a discrete nonlinear dynamic system as follows:

$$X_k = f_{k-1}\left[X_{k-1}, P_{k-1}, U_{k-1}, W_{k-1}\right] \qquad (6)$$

$$W_k \sim \left(0, Q_c\right) \qquad (7)$$

$$y_k = h_k\left[X_k, V_k\right] \qquad (8)$$

$$V_k \sim \left(0, R_c\right) \qquad (9)$$

Where X_k is the state vector, U_k U_k is the input, P_k is unknown parameter vector and V_k and W_k are the measurement and system independent white Gaussian noises, respectively. The initial condition for both filters will be set as follows:

$$\begin{cases} \hat{X}_0^+ = E\{X_0\} \\ P_{X_0}^+ = E\left\{\left(X_0 - \hat{X}_0^+\right)\left(X_0 - \hat{X}_0^+\right)^T\right\} \end{cases} \qquad (10)$$

$$\begin{cases} \hat{P}_0^+ = E\{P_0\} \\ P_{P_0}^+ = E\left\{\left(P_0 - \hat{P}_0^+\right)\left(P_0 - \hat{P}_0^+\right)^T\right\} \end{cases} \qquad (11)$$
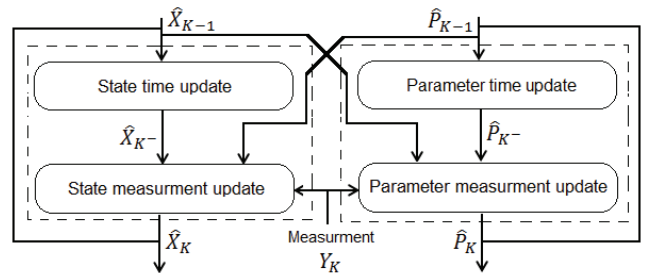

Fig. 1. Schematic of Dual-EKF.

For *k = 1,2,...,* the following are performed:

• Parameter time update

$$P_k^- = \hat{P}_{k-1}^+ \tag{12}$$

$$P_{P_k}^- = P_{P_{k-1}}^+ \tag{13}$$

• State time update

$$\hat{X}_k^- = \hat{X}_{k-1}^+ + f\left(\hat{X}_{k-1}^+, U_k\right) \tag{14}$$

$$F_{k-1} = \left.\frac{\partial f_{k-1}}{\partial X}\right|_{\hat{X}_{k-1}^+} \tag{15}$$

$$L_{k-1} = \left.\frac{\partial f_{k-1}}{\partial W}\right|_{\hat{X}_{k-1}^+} \tag{16}$$

$$P_{x_k}^- = F_{k-1} P_{x_{k-1}}^+ F_{k-1}^T + L_{k-1} Q_{k-1} L_{k-1}^T \tag{17}$$

• Parameter measurement update

$$H_{k-1} = \left.\frac{\partial h_k}{\partial X}\right|_{\hat{X}_k^-} \tag{18}$$

$$M_k = \left.\frac{\partial h_k}{\partial W}\right|_{\hat{X}_k^-} \tag{19}$$

$$K_k^p = P_{w_k}^- C_k^{p^T} \left(C_k^P P_{x_k}^- C_k^{p^T} + R_c\right)^{-1} \tag{20}$$

$$P_k^+ = \hat{P}_k^- + K_k^p \left(y_k - H_{k-1}\hat{X}_k^-\right) \tag{21}$$

$$P_{P_k}^+ = \left(I - K_k^p C_k^p\right) P_{P_k}^- \tag{22}$$

• State measurement update

$$K_k^x = P_{x_k}^- H_{k-1}^T \left(H_{k-1} P_{x_k}^- H_{k-1}^T + R_c\right)^{-1} \tag{23}$$

$$\hat{X}_k^+ = \hat{X}_k^- + K_k^x \left(y_k - H_{k-1}\hat{X}_k^-\right) \tag{24}$$

Computation of C_k^P needs recursive derivation calculations [22]:

$$C_k^P = H_k \left.\frac{\partial \hat{X}_k^-}{\partial P}\right|_{P=P_k^+} \tag{25}$$

$$\frac{\partial \hat{X}_{k+1}^-}{\partial \hat{P}} = \frac{\partial f}{\partial \hat{X}_k} \times \frac{\partial \hat{X}_k}{\partial \hat{P}} + \frac{\partial f}{\partial \hat{P}_k} \tag{26}$$

$$\frac{\partial \hat{X}_k}{\partial \hat{P}} = \left(I - K_k^x H_k\right)\frac{\partial \hat{X}_k^-}{\partial \hat{P}} + \frac{\partial K_k^x}{\partial \hat{P}}\left(y_k - H_k\hat{X}_k^-\right) \tag{27}$$

If we suppose that the term K_k^x is constant with respect to unknown parameters (*P*), then $\partial K_k^x / \partial \hat{P} = 0$. This assumption is not really true, but is very accurate. So, we have a simple Dual-EKF [23]. If

there is a sever nonlinearity, then EKF techniques which are based on linearization may lead to inaccurate estimation. Joint-UKF algorithm estimates more accurate estimations than others.

4. KR-JUKF Algorithm

UKF is still only an approximate nonlinear estimator, in which mean and covariance matrix propagate with third-order accuracy. So, it contains systematic error due to higher orders [13]. To remove this error, BRR is used. At the heart of BRR, it is necessary to compute multi-dimensional integrals which are in the form of Equation (28). The stochastic integration rule (SIR) is suitable for calculation of integrals with the following form [24]:

$$\mu = E\left(g_{(X)}\right) = \int g_{(X)} \left(2\pi\right)^{-n/2} e^{-\frac{1}{2}X^T X} \tag{28}$$

where $g_{(\cdot)}$ is an arbitrary nonlinear function and $X \in \mathbb{R}^n$ is a random vector variable with Gaussian pdf, zero mean and finite covariance matrix $X \sim N(\hat{X}, P_X)$. In the following two forms of SIR are presented. After that, based on each of the SIR formulations, the corresponding KR-JUKF algorithm is proposed.

4.1 First form of SIR

In the first form, the number of iterations (N_{max}) is a pre-defined value:

1. Set initial value of the integral $\hat{\mu}_{(0)} = 0$.
2. For *N=1,2,..., N_{max}* perform the following:

• Generate a uniformly random orthogonal matrix $\Psi_{(N)} \in \mathbb{R}^{n \times n}$. The random orthogonal matrix can be produced by using a product of appropriately chosen random reflections [24].
• Generate a random number $\rho_{(N)}$ from the chi distribution $\rho_{(N)} = chi(n+2)$ (The matrix $\Psi_{(N)}$ and scalar $\rho_{(N)}$ are regenerated in each iteration [24].)
• Compute $2n$ points $\chi_{i(N)}$ and their weights:

$$\chi_{i(N)} = \hat{X} - \left(\rho_{(N)} S_X \Psi_{(N)}\right)_i \quad for \quad i = 1,...,n \tag{29}$$

$$\chi_{n+i(N)} = \hat{X} + \left(\rho_{(N)} S_X \Psi_{(N)}\right)_i \quad for \quad i = 1,...,n \tag{30}$$

$$\omega_{0(N)} = 1 - \frac{n}{\rho_{(N)}^2} \tag{31}$$

$$\omega_{i(N)} = \omega_{n+i(N)} = \frac{1}{2\rho_{(N)}^2} \quad for \quad i = 1,...,n \tag{32}$$

where S_X is square-root of the covariance matrix, $P_X = S_X S_X^T$ and $\left(\rho_{(N)} S_X \Psi_{(N)}\right)_i$ are the *i*th row of

the matrix $\rho_{(N)}S_X\Psi_{(N)}$.

- Compute $J_{(N)}$ and $\hat{\mu}_{(N)}$

$$J_{(N)} = g_{(\hat{x})}\omega_{0(N)} + \sum_{i=1}^{n}\left(g_{(\chi_{i(N)})} + g_{(\chi_{n+i(N)})}\right)\omega_{i(N)} \qquad (33)$$

$$\hat{\mu}_{(N)} = \hat{\mu}_{(N-1)} + \left(J_{(N)} - \hat{\mu}_{(N-1)}\right)/N \qquad (34)$$

3. The value of integral μ is $\hat{\mu}_{(N)}$

Computation of μ using the above algorithm can be shown as $\mu = SIRalg\left(\hat{X}, P_X, g_{(X)}, N_{\max}\right)$. Assume that we have a discrete nonlinear dynamic with equations (6) and (7) and linear measurement equation with measurement noise of equation (9):

$$y_k = H_k X_k + V_k \qquad (35)$$

4.2 KR-JUKF (Algorithm 1)

KR-JUKF (Algorithm 1) which is based on the first form of SIR can be summarized as follows:

1. Primarily assumption will be as follows:

$$\begin{cases} \hat{X}_0^+ = E\{X_0\} \\ P_0^+ = E\left\{\left(X_0 - \hat{X}_0^+\right)\left(X_0 - \hat{X}_0^+\right)^T\right\} \end{cases} \qquad (36)$$

2. For $k = 1, 2, \ldots$, perform the following:

- Time update

$$\hat{X}_k^- = SIRalg\left[\hat{X}_{k-1}^+, P_{k-1}^+, f_{k-1}(X), N_{\max}\right] \qquad (37)$$

$$P_k^- = SIRalg[\hat{X}_{k-1}^+, P_{k-1}^+, \left(f_{k-1}(X) - \hat{X}_k^-\right) \\ \left(f_{k-1}(X) - \hat{X}_k^-\right)^T, \ N_{\max}] + Q_{k-1} \qquad (38)$$

- Measurement update

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + M_k R_k M_k^T\right)^{-1} \qquad (39)$$

$$\hat{X}_k^+ = \hat{X}_k^- + K_k\left[y_k - H_k\hat{X}_k^-\right] \qquad (40)$$

$$P_k^+ = \left(I - K_k H_k\right)P_k^- \qquad (41)$$

In Algorithm 1, the number N_{max} is assumed large enough without any optimization. So, the value of Gaussian integrals can be calculated with maximum accuracy. But, it is possible for the value of Gaussian integral which is calculated by SIR to diverge from its real value and too large value assumption for N_{max} increases computational time. The notations used in the first form of SIR and Algorithm 1 are defined in Table 1.

Table 1. SIR algorithm and KR-JUKF notation.

| Index	Definition
n	The length of vector $X \in \mathbb{R}^n$
N	The number of iterations in SIR algorithm
i	The ith row of matrix
k	The number of iterations in KR-JUKF

4.3 Second form of SIR

In this algorithm, instead of using N_{max}, convergence error vector $(J_{(N)} - \hat{\mu}_{(N-1)})/N$ specifies the number of iterations. Norm-2 of convergence error vector is compared to a threshold convergence error (ε). The iterations continue until norm-2 of convergence error vector becomes greater than the threshold convergence error. Using this approach leads to consuming less computational time, but decreases the accuracy of estimation. The second form of SIR which is regarded as time optimized SIR is as follows:

1. Set initial value of the integral $\hat{\mu}_0 = 0$
2. Perform the following:
- Generate a uniformly random orthogonal matrix $\Psi_{(N)}$
- Generate a random number $\rho_{(N)}$ from the chi distribution $\rho_{(N)} = chi(n+2)$
- Compute a set of points, their weights, $J_{(N)}$ and $\hat{\mu}_{(N)}$ While $\left(\left\|\left(J_{(N)} - \hat{\mu}_{(N-1)}\right)/N\right\|_2 < \varepsilon\right)$

3. The value of integral μ is $\hat{\mu}_{(N)}$
where ε is the threshold convergence error. A flowchart for the SIR algorithm is given in Fig. 2. Computation of μ using the above algorithm can be shown as:

$$\mu = SIRalg\left(\hat{X}, P_X, g_{(X)}, \varepsilon\right)$$
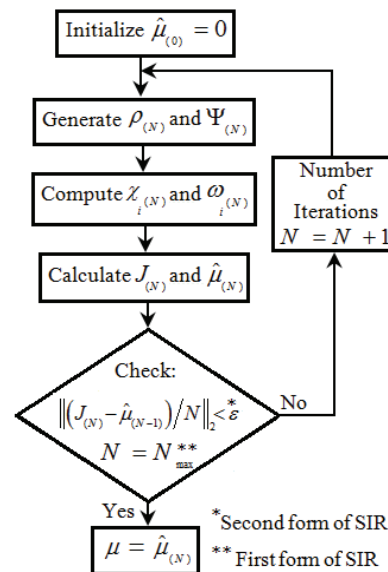
Fig. 2. Flowchart of first and second form of SIR.

4.4 KR-JUKF (Algorithm 2)

KR-JUKF (Algorithm 2) can be represented based on the time optimized SIR:

1. Use Equation (36) for primary assumption.
2. For $k = 1,2,\ldots$, perform the following:
- Time update

$$\hat{X}_k^- = SIRalg\left(\hat{X}_{k-1}^+, P_{k-1}^+, f_{k-1}(X), \varepsilon\right) \tag{42}$$

$$P_k^- = SIRalg\Big[\hat{X}_{k-1}^+, P_{k-1}^+, \left(f_{k-1}(X) - \hat{X}_k^-\right) \\ \left(f_{k-1}(X) - \hat{X}_k^-\right)^T, \varepsilon\Big] + Q_{k-1} \tag{43}$$

- For measurement updating, use Eqs. (39-41)

In several literatures, results demonstrate that the dual estimators can more effectively compute the system states in the case of unknown system parameters, compared to conventional ones [25-29]. Also, it is proved that the original UKF has a systematic error [13, 30]. In order to relax this defect, the RUKF is presented. Therefore, one can conclude that the RUKF estimates the system states more accurately compared to original UKF. However, RUKF needs more computational burden due to its iterative algorithms. Therefore, it can be concluded that our proposed KR-JUKF can estimate more accurately and more slowly than the original UKF.

5. System Dynamics

The chemical process employed in this paper is production process of propylene glycol in CSTR (Continuous Stirred Tank Reactor) with cooling jacket. In this CSTR, propylene oxide and water react in atmosphere pressure. The product of this reaction is propylene glycol. Since the reaction is exothermic, a coolant fluid circulates within the reactor jacket to maintain its temperature. Fig. 3 shows a simple flow-sheet diagram.
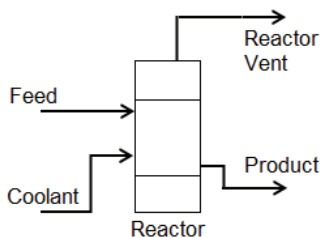

Fig. 3. Flow sheet diagram of CSTR.

Mathematical model of the CSTR is given by four nonlinear differential equations as follows [20]:

$$\dot{V}_r = \frac{\rho_0 F_0}{\rho} - F \tag{44}$$

$$\dot{C}_a = \frac{F_0}{V_r}(C_{a0} - C_a) - C_a K_0 e^{-E/RT_r} \tag{45}$$

$$\dot{T}_r = \frac{F_0}{V_r}(T_0 - T_r) - \frac{\lambda C_a K_0 e^{-\frac{E}{RT_r}}}{\rho C_p} - \frac{UA_j(T_r - T_j)}{V_r \rho C_p} \tag{46}$$

$$\dot{T}_j = \frac{F_j}{V_j}(T_{cin} - T_j) + \frac{UA_j(T_r - T_j)}{V_j \rho_j C_j} \tag{47}$$

$$y(t) = \begin{bmatrix} C_a & T_r & T_j & V_r \end{bmatrix}^T \tag{48}$$

The control input consists of the cooling water flow-rate, feed flow and product flow. Unknown vector and state vector can be defined as $P = [x_5\ x_6]^T = [UA_j\ E]^T$ and $X = [x_1\ x_2\ x_3\ x_4]^T = [C_a\ T_r\ T_j\ V_r]^T$. Therefore, state space equations can be written as:

$$\dot{x}_1 = \frac{U_{f0}}{x_4}(C_{a0} - x_1) - x_1 K_0 e^{-\frac{x_6}{R x_2}} + W_1 \tag{49}$$

$$\dot{x}_2 = \frac{U_{f0}}{x_4}(T_0 - x_2) - \frac{\lambda x_1 K_0 e^{-\frac{x_6}{R x_2}}}{\rho C_p} - \frac{x_5(x_2 - x_3)}{x_4 \rho C_p} + W_2 \tag{50}$$

$$\dot{x}_3 = \frac{U_{fj}}{V_j}(T_{cin} - x_3) - \frac{x_5(x_2 - x_3)}{V_j \rho_j C_j} + W_3 \tag{51}$$

Table 2. CSTR Parameters.

Param.	Steady-state value	Definition of parameter
C_a	0.3684 Kmol/m^3	Production concentration
T_r	333 K	Reactor temperature
F_j	9.937e-3 m^3/s	Coolant flow rate
F	2.72e-3 m^3/s	Product flow rate
F_o	2.65e-3 m^3/s	Feed flow rate
C_{ao}	7.128 Kmol	Feed concentration
T_o	296.89 K	Feed temperature
T_{cin}	288 K	Inlet coolant temperature
T_j	319.76 K	Outlet coolant temperature
V_r	6.739 m^3	Reactor volume
V_j	0.4467 m^3	Jacket volume
U_{Aj}	1e5 W/K	Heat transfer term
K_o	1.696e13 1/s	Reaction rate constant
R	8.31446 J/mol K	Universal gas constant
E	7.53664e4 J/mol	Activation energy
Landa	-9e7 J/Kmol	Heat of reaction
Row$_o$	936.7 Kg/m^3	Feed density
Row	912.9 Kg/m^3	Product density
Row$_j$	1008 Kg/m^3	Coolant density
C_p	3368 J/Kg K	Heat capacity of product
C_j	4203 J/Kg K	Heat capacity of coolant

Table 3. Variance of measurement noises.

Measured state	Noise variance
Ca	5.42e-5
Tr	0.36
Tj	0.2186
Vr	0.0182

$$\dot{x}_4 = \frac{\rho_0 U_{f0}}{\rho} - U_f + W_4 \tag{52}$$

$$\dot{x}_5 = 0 \tag{53}$$

$$\dot{x}_6 = 0 \tag{54}$$

$$y(t) = \begin{bmatrix} C_a \\ T_r \\ T_j \\ V_r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} Z(t) + R \tag{55}$$

where W_1, W_2, W_3, W_4 and R are system and measurement noises. Definition of parameters is presented in Table 2. Values of covariance of measurement noises are also presented in Table 3.

6. Simulation Results

In this section, two simulations are presented to show the superiority of the proposed algorithms. The first experiment deals with a comparison of the proposed approach with other existing algorithms. The second example investigates and compares results of the two type of KR-JUKF.

6.1 Experiment 1

In Experiment 1, unknown parameters and states are estimated using joint-EKF, dual-EKF, joint-UKF and KR-JUKF (Algorithm 1). Measurement noise characteristics are given in Table 3. The number of repetitions was chosen as $N_{max}=40$. Values of MSE (Mean Square Error) of each state and also unknown parameters are presented in Table 4.

Table 4 illustrates that KR-JUKF can estimate the unknown states and parameters more accurately than the others. Fig. 4 presents the time that each algorithm consumes per one stage for different initial values. Fig. 4 shows that joint-EKF is the fastest algorithm. The computational time of Dual-EKF is relatively the same as joint-EKF. But KR-JUKF needs more computational time than the other algorithms. The KR-JUKF algorithm estimates are significantly more accurate compared to the other ones. Although it needs more computational time, but the time of the states and parameters estimation is of the order of 10^{-3} seconds which makes the proposed algorithm suitable for online estimation.

Table 4. MSE of states.

| Algorithm	Joint UKF	Joint EKF	Dual EKF	KR-JUKF
X_1	0.0025	0.0025	0.0025	3.562e-04
X_2	4.4688e-05	6.4292e-05	6.4292e-05	6.3741e-06
X_3	3.7087e-05	6.3306e-05	5.8990e-05	5.2205e-06
X_4	1.4827e-04	1.9348e-04	1.9380e-04	1.2422e-05
P_1	4.0008e+03	4.0006e+03	4.9200e+03	3.6572e+03
P_2	8.395e+02	3.8395e+03	2.1336e+03	7.455e+02
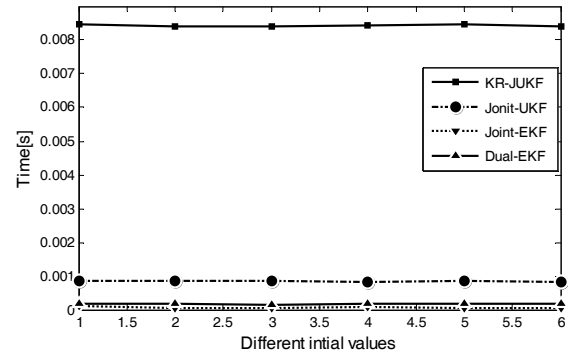
Fig. 4. Time of each algorithm.
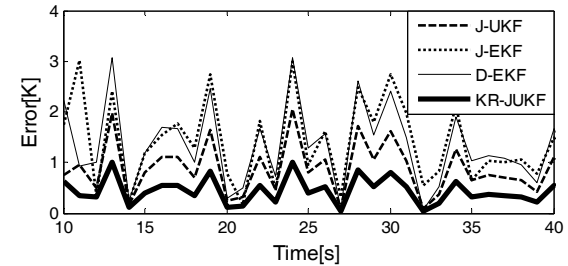
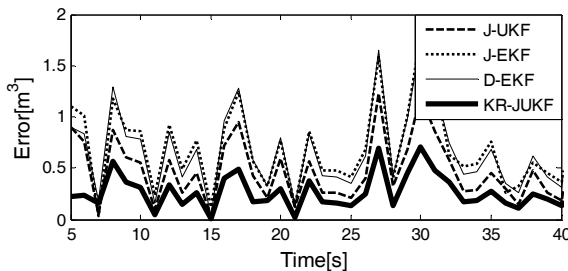Fig. 5-8 present the estimation error of each algorithm for each of states.
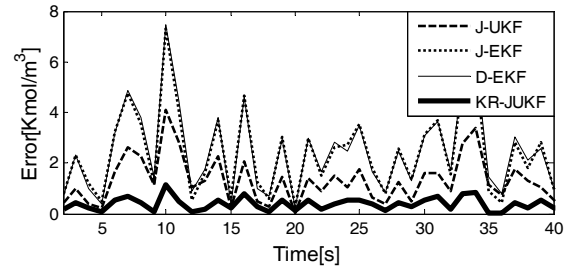
Fig. 5. Error of state *Ca*.
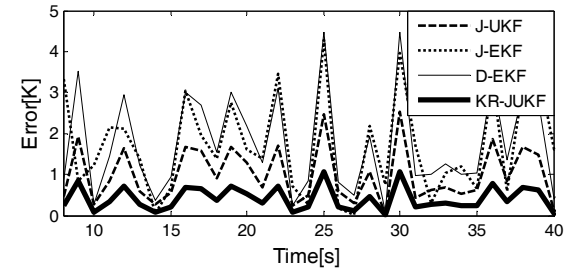
Fig. 6. Error of state *Tr*.

Fig. 7. Error of state *Tj*.

Fig. 8. Error of state *Vr*.

Figs. 5-8 show that the proposed approach greatly decreases the estimation error of the states compared to other algorithms. This is due to the usage of SIR to calculate the multi-dimensional Gaussian integrals appeared in the BRR algorithm. Figs. 9-12 show the real and the approximate value of each state for KR-JUKF (Algorithm 1).
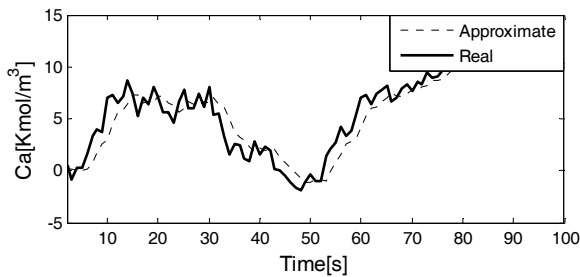

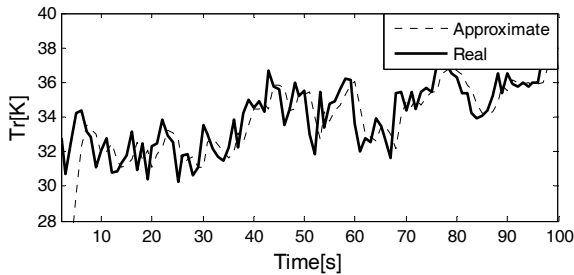**Fig. 9.** Approximation of state *Ca* (KR-JUKF Algorithm 1).

Fig. 10. Approximation of state Tr (KR-JUKF Algorithm 1).
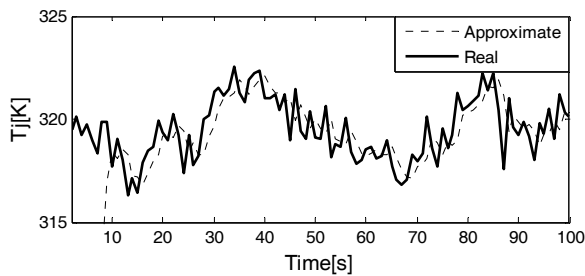

Fig. 11. Approximation of state Tj (KR-JUKF Algorithm 1).
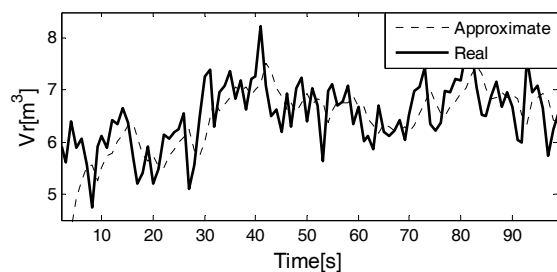

Fig. 12. Approximation of state Vr (KR-JUKF Algorithm 1).

Figs. 9-12 indicate that the proposed approach can effectively handle the state estimation issue of CSTR system with unknown system parameters.

6.2 Experiment 2

In this experiment, the variance of measurement noise is given in Table 5. Unknown parameters and states were estimated by using two algorithms of KR-JUKF. The number of repetitions is chosen as N_{max}=40 and let the convergence error vector $\varepsilon = 0.05\hat{\mu}_{(N)}$. Values of MSE of each state and unknown parameter and average time of each algorithm are given in Table 6.

Table 6 shows that the estimation accuracy of KR-JUKF Algorithms 1 and 2 are relatively similar. Algorithm 2 needs less computational time than Algorithm 1. This is due to the usage of the threshold convergence error instead of using pre-known number of repetitions. Figs. 13-16 show the errors of Algorithms 1 and 2 for each of states. These figures indicate that the Algorithm 1 provides more accurate estimations compared to the Algorithm 2. This is

Table 5. Variance of measurement noise.

Measured state	Noise variance
C_a	5.42
T_r	3.6e2
T_j	2.2e2
V_r	2.0e3

Table 6. MSE of states and average time per stage.

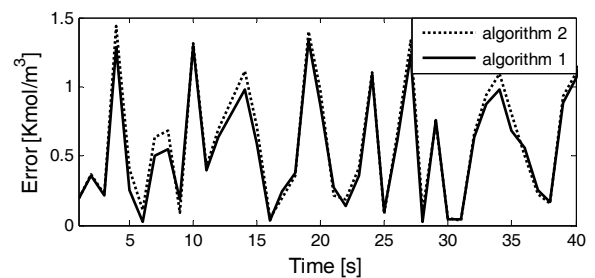| Algorithm	Algorithm 1	Algorithm 2
X_1	3.573e-04	3.822e-04
X_2	6.3743e-06	6.9483e-06
X_3	5.2211e-06	5.6274e-06
X_4	1.2418e-05	1.7906e-05
P_1	3.6572e+03	3.6583e+03
P_2	7.455e+02	7.502e+02
Time	8.8436e-2	5.3778e-3

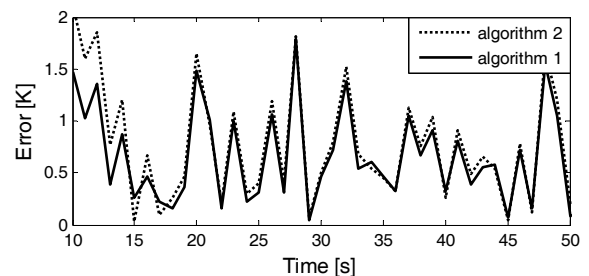**Fig. 13.** Error of state Ca (KR-JUKF Algorithm 1 and 2).

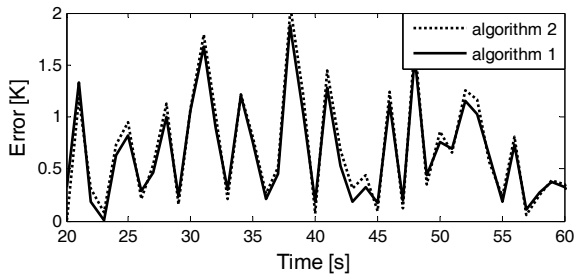**Fig. 14.** Error of state Tr (KR-JUKF Algorithm 1 and 2).

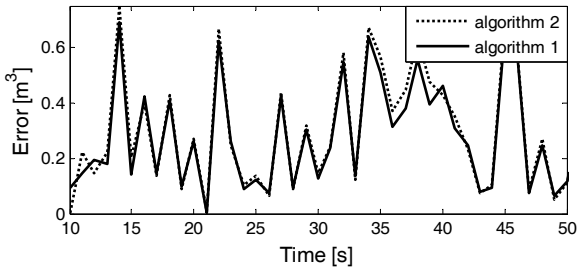**Fig. 15.** Error of state Tj (KR-JUKF Algorithm 1 and 2).

Fig. 16. Error of state Vr (KR-JUKF Algorithm 1 and 2).

due to the fact that the Algorithm 1 involves higher number of iterations in measurement and time update.

7. Conclusion

In this paper, a novel nonlinear dual-estimation algorithm based on the fusion of the Kalman filter, randomized Unscented Kalman filter and dual estimation is proposed. The proposed algorithm is proper for time varying and severe nonlinear dynamics. Moreover, the computational time decreases in this approach compared to RUKF, which makes it suitable for online estimation. To show the estimation accuracy of the proposed approach, its performance is compared to the other existing dual estimators by applying the algorithms to a severe nonlinear exothermic CSTR with cooling jacket. The applied estimators are Joint-EKF, Dual-EKF, Joint-UKF and -KR-JUKF (Algorithms 1 and 2). Simulations show that KR-JUKF (Algorithm 1) estimates more accurately than the others. This is due to the propagation of the mean and covariance through a nonlinear function using SIR algorithm; however, it needs more computational time. KR-JUKF (Algorithm 2) is an optimized method of KR-JUKF (Algorithm 1). The accuracy of Algorithm 2 is relatively equal to Algorithm 1 but needs less computational time.

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