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# Carbon Reduction Investments under Direct Shipment Strategy

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**ABSTRACT**

Recently much research efforts have focused on how to manage carbon emissions in logistics operations. This paper formulates a model to determine an optimal shipment size with aims to minimize the total cost consisting not only of inventory and transportation costs but also cost for carbon emissions. Unlike the literature assuming carbon emission factors as a given condition, we consider the emission factors as decision variables. It is allowed to make an investment in improving carbon emission factors. The optimal investment decision is shown to be of a threshold type with respect to unit investment costs. Moreover, the findings in this work provide insights on the various elements of the investment decision and their impacts.

Keywords: Carbon Reduction Investment, Direct Shipment Strategy, Shipment Size

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## 1. INTRODUCTION

Managing carbon emissions has become a major consideration in logistics because of its recent growing importance. The emission trading system is in effect as of 2015 in Korea, and the carbon emission has a direct influence on the company's profit. Thus, it is required to design an effective distribution strategy considering carbon costs.

Generally, the trade-off between transportation cost (or ordering cost) and inventory cost has been a basis of developing a distribution decision. An interesting research question raised here is whether an optimal decision (e.g., shipment size) developed based on the conventional approach is still effective when considering carbon costs. Recently, the literature has begun to address this research question.

Benjaafar *et al*. (2010) and Chen *et al*. (2011) modify the standard EOQ (Economic Oder Quantity) model and include carbon constraints in determining an optimal order quantity. Hua *et al*. (2011) also extends the EOQ model to include carbon trading costs in constraints and objective function to find an optimal order quantity.

Instead of simply considering the carbon costs as input data, some research works have begun to consider the amount of carbon emissions as a decision variable. Such research proposes a model for determining an optimal level of carbon emissions when processing a single unit of item (i.e., carbon emission factor). Toptal *et al*. (2014) and Hua *et al*. (2011) modify the EOQ model to consider how much to invest in reducing carbon emissions. Swami and Shah (2013) addressing a supply chain coordination problem with carbon trading costs proposes an optimal sustainability level in a closed form.

Despite of the consideration of carbon reduction investment, there are some limitations to the literature. The order quantity (or shipment size) is generally considered as being independent of the value of carbon emission factors, but they are, in fact, technically highly correlated with each other. Moreover, most of the literature consider a problem of coordinating supply chain players and normally ignore the details of distribution operations. The most similar study is Jiang and Klabjan (2012) in which carbon reduction investment is addressed, but their model is based on the Newsvendor model and fails to consider transportation and inventory in detail.

The purpose of this paper is not simply to determine an optimal shipment size, but also to provide insights on how much to invest in reducing carbon emissions by determining an optimal value of emission factors. We consider a distribution system that operates under the direct shipment strategy as described in Burns *et al.* (1985). In Section 2, we extend the model proposed by Burns *et al.* (1985) to include a decision on carbon reduction investment as well as carbon costs. Section 3 provides some analytical results and managerial implications, and the conclusion is followed in Section 4.

## 2. The Model

This paper considers distributing items from a distribution center to several customers located in a delivery region based on the direct shipment distribution strategy (See Figure 1). Transportation and inventory costs are incurred in delivering items. Inventory is located in the distribution center, transit to customers.

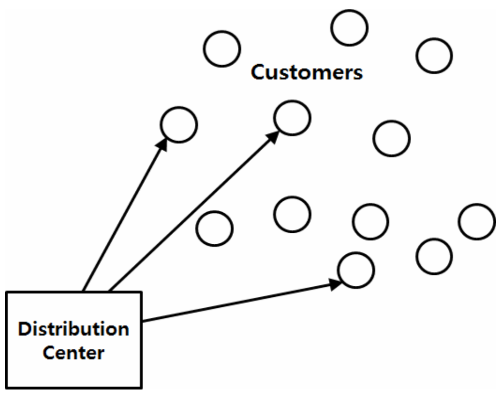


Figure 1. Illustration of Direct Shipment Strategy

We develop an expression for the total direct shipment cost that includes transportation and inventory cost adjusted by considering carbon costs and the investment in reducing carbon emissions. The optimal shipment size and carbon emission factors are specified based on the trade-offs between these three cost elements. The proposed model considers a single customer because the direct shipment strategy involves one customer-visit per load when customers are assumed to be identical.

$q$ and $D$ denote shipment size (items/load) and customer demand (items/week). If customer demand arrives at a constant rate, the average time required for consumption at each customer is equally $q/D$. Thus, each item waits, on average, $q/2D$ before being used. Similarly, the average time for production at the distribution center is also $q/D$, and each item in the same load waits on average $q/2D$ before being shipped. Thus, an item is, on average, tied up in inventory for $\tau_d = q/D + T$ period, where $T$ (weeks) is transit time from the distribution to a customer. Unit inventory cost becomes $R(q/D+T)$ when $R$ denotes unit inventory holding cost ($/item-week).

Transportation cost per load consists of three cost elements; fixed cost of initiating a truck dispatch $F_t$ ($/load), transportation cost $V_t L$ ($/km) and fixed cost for a customer visit $F_c$ ($/visit). Here, $V_t$ and $L$ represent transportation cost per unit distance and round trip distance, respectively. The transportation cost per item is given as $(F_t + F_c + V_t L)/q$.

The amount of carbon emissions depends on emission factors. Let $c$ (kgCO2/item-km) and $h$ (kgCO2/item-week) denote carbon emission factors in transportation and warehousing, respectively. For given values of $c$ and $h$, the total amount of carbon emissions under the direct shipment strategy is $cL/q+hq/D$.

Responsible firms have invested in the adoption of cleaner technologies to reduce carbon emissions (Drake and Spinler, 2013). For reducing carbon emissions in retail industry, Chen *et al.* (2011) proposed several strategies and measures including adopting green vehicles (e.g., electronic vehicles, or vehicles with better fuel efficiency), green packaging, storage facility with renewable energy, etc. To accommodate these efforts on reducing carbon emissions in the model, this paper considers these emission factors as decision variables, and it is allowed to improve emission factors with additional costs.

In general, a strategy for reducing carbon emission is designed to improve emission factors in transportation or in warehousing. Thus, we consider the costs for improving $c$ and $h$, separately. The investment to achieve carbon emission factor $c$ is $\alpha(C_o - C)$, where $c_o$ is the initial value when there is no investment and $\alpha$ is an unit cost ($/kgCO2/item-km) that is required to improve the carbon emission factor by one unit. Similarly, the investment to get emission factor $h$ becomes $\beta(h_o - h)$. As it is shown here, carbon reduction investment is assumed to be a linear cost function with respect to $c$ and $h$. We assume that there are technologically possible lower bounds on emission factors, $c$ and $h$.

We finally introduce the proposed model for distributing items under the direct shipment strategy with additional carbon costs; Eq. (1)~Eq. (5).

$$\min \quad \pi(q, x, c, h) = (F_t + F_c + V_t L)\frac{1}{q} + R(\frac{q}{D} + T) - Px + \alpha(c_o - c) + \beta(h_o - h) \qquad (1)$$

$$\frac{cL}{q} + \frac{hq}{D} + x = K \qquad (2)$$

$$0 \le q \le U \qquad (3)$$

$$\underline{c} \le c \le c_o \qquad (4)$$

$$\underline{h} \le h \le h_o \qquad (5)$$

The model considers cap-and-trade system, and the variable $x$ represents the amount of tradable emissions when $K$ (kgCO2/item) emission permit is initially allocated (2). Thus, $Px$ is either cost or revenue when the unit carbon price is given as $P$ ($/kgCO2).

By substituting (2) in (1), the total cost function is reformulated as follows.

$$\pi(q,c,h) = (F_t + F_c + (V_t + cP)L)\frac{1}{q} + \frac{(R+hP)q}{D} + RT - PK + \alpha(c_o - c) + \beta(h_0 - h) \quad (6)$$

## 3. AN OPTIMAL CARBON INVESTMENT DECISION

In this section, we first investigate several properties of the total cost presented in (6) and then find an optimal solution in a closed form. To facilitate the presentation, let $H = R+hP$ and $F = F_t + F_c + (V_t + cP)L$. Here, $F$ and $H$ now imply transportation and inventory cost considering additional cost incurred by carbon emissions.

For a given any values of $c$ and $h$, an optimal shipment size $q_o$ is obtained by taking the first derivative with respect to $q$.

$$\frac{\partial \pi(q, c, h)}{\partial q} = 0 \rightarrow q_o = \sqrt{\frac{DF}{H}}$$

The optimal shipment size is consistent with that of an EOQ (Economic Order Quantity) model.

***Proposition 1:*** An optimal shipment size $q_o$ is increasing in $c$ and decreasing in $h$.
***Proof:*** The proof is trivial. The first-order derivatives with respective to $c$ and $h$ result in $\frac{\partial q_o}{\partial c} = -\frac{q_o pL}{2F} > 0$ and $\frac{\partial q_o}{\partial h} = -\frac{q_o p}{2H} < 0.$ □

The higher $c$ is, the more the transportation emits carbon. Therefore, it is highly likely to increase the shipment size so as to facilitate the economies of scale and reduce the unit transportation cost. The higher $h$ leads to the increase in inventory holding cost $H$, and it should deliver more items per load like the EOQ model,

The total cost becomes a function of $c$ and $h$ by replacing $q$ with $q_o = \sqrt{\frac{DF}{H}}$.

$$\pi(c, h) = 2\sqrt{\frac{HF}{D}} + RT - PK + \alpha(c_o - c) + \beta(h_o - h)$$

***Proposition 2:*** The total cost function $\pi(c, h)$ is jointly concave in $c$ and $h$.
***Proof.*** The Hessian matrix of $\pi(c, h)$ is obtained.

$$H = \begin{bmatrix} \frac{\partial^2 \pi(c,h)}{\partial c^2} & \frac{\partial^2 \pi(c,h)}{\partial c \partial h} \\ \frac{\partial^2 \pi(c,h)}{\partial h \partial c} & \frac{\partial^2 \pi(c,h)}{\partial h^2} \end{bmatrix} = \begin{bmatrix} -\frac{L^2P^2}{2q_oF} & \frac{LP^2}{2q_oH} \\ \frac{LP^2}{2q_oH} & -\frac{FP^2}{2q_oH^2} \end{bmatrix}$$

Because the first leading principle minor $\Delta_1 < 0$ and determinant of Hessian matrix $|H| = 0$, $H$ is negative semidefinite. Thus, we conclude that $\pi(c, h)$ is jointly concave in $c$ and $h$. □

Because of the concavity, the first-order condition (i.e., $\frac{\partial \pi(c,h)}{\partial c} = 0$ and $\frac{\partial \pi(c,h)}{\partial h} = 0$) determines a local maximizer. However, it needs to find a point minimizing $\pi(c, h)$ rather than the local maximizer in this paper. The following Theorem 1 provides an optimal $c^*$ and $h^*$ minimizing the total cost.

***Theorem 1:*** The optimal carbon emission factors $(c^*, h^*)$ are as follows,

$$(c^*, h^*) = \begin{cases} (c_o, h_o) \text{ if } \alpha > A_o \text{ and } \beta > B_o \\ (c_o, \underline{h}) \text{ if } \alpha \le A_o \text{ and } \beta \ge \underline{B} \\ (\underline{c}, h_o) \text{ if } \alpha \ge \underline{A} \text{ and } \beta \le B_o \\ (\underline{c}, \underline{h}) \text{ if } \alpha < \underline{A} \text{ and } \beta < \underline{B} \end{cases}$$

where $A_o = 2\sqrt{\frac{H_o}{D}}\frac{\sqrt{F_o} - \sqrt{F}}{c_o - c}$, $B_o = 2\sqrt{\frac{F_o}{D}}\frac{\sqrt{H_o} - \sqrt{H}}{h_o - h}$, $\underline{A} = 2\sqrt{\frac{\underline{H}}{D}}\frac{\sqrt{F_o} - \sqrt{F}}{c_o - c}$, and $B_o = 2\sqrt{\frac{\underline{F}}{D}}\frac{\sqrt{H_o} - \sqrt{H}}{h_o - h}$.

Moreover, $F_o = F_t + F_c(V_t + c_oP)L$, $H_o = R + h_oP$, $\underline{F} = F_t + F_c + (V_t + \underline{c}P)L$, and $\underline{H} = R + \underline{h}P$.

***Proof:*** From the "Extreme Value Theorem" and Proposition 2, there exists a point that minimizes $\pi(c, h)$ when $c$ and $h$ are bounded. Because of the concavity of $\pi(c, h)$, we can find the minimizer on the boundary of $c$ and $h$. We consider four cases.

Case 1: when $h = \underline{h}$:

$$\pi(c, \underline{h}) = 2\sqrt{\frac{HF}{D}} + RT - PK + \alpha(c_o - c) + \beta(h_0 - \underline{h}).$$

$\pi(c, h)$ is concave a concave function with respect to $c$ and thus, $c^*$ is determined at $c_o$ and/or $\underline{c}$.

$$\pi(c_o, \underline{h}) - \pi(\underline{c}, \underline{h}) = 2\sqrt{\frac{HF_o}{D}} - 2\sqrt{\frac{HF}{D}} - \alpha(c_o - \underline{c})$$
$$= \sqrt{\frac{\underline{H}}{D}}\left(\sqrt{F_o} - \sqrt{\underline{F}}\right) - \alpha(c_o - \underline{c})$$

$$\pi(c_o, \underline{h}) - \pi(\underline{c}, \underline{h}) > 0 \Leftrightarrow \alpha < 2\sqrt{\frac{\underline{H}}{D}}\frac{\sqrt{F_o} - \sqrt{\underline{F}}}{c_o - \underline{c}} = \underline{A}$$

$\pi(c, h)$ is minimized at $c^* = \underline{c}$ if $\alpha < \underline{A}$.

Thus, it concludes $c^* = \begin{cases} \underline{c} \text{ if } \alpha < \underline{A} \\ c_o \text{ otherwise} \end{cases}$ (7)

Case 2: when $h = h_o$: similar derivation as in case 1 yields

$$\pi(c_o, h_o) - \pi(\underline{c}, h_o) = 2\sqrt{\frac{H_o F_o}{D}} - 2\sqrt{\frac{H_o \underline{F}}{D}} - \alpha(c_o - \underline{c})$$

$$= 2\sqrt{\frac{H_o}{D}}\left(\sqrt{F_o} - \sqrt{\underline{F}}\right) - \alpha(c_o - \underline{c})$$

$$c^* = \begin{cases} \underline{c} \ \ if \ \ \alpha < A_o \\ c_o \ \ otherwise \end{cases} \tag{8}$$

Case 3: when $c = \underline{c}$:

$$\pi(\underline{c}, h_o) - \pi(\underline{c}, \underline{h}) = 2\sqrt{\frac{\underline{F}}{D}}\left(\sqrt{H_o} - \sqrt{\underline{H}}\right) - \beta(h_o - \underline{h})$$

$$h^* = \begin{cases} \underline{h} \ \ if \ \ \beta < \underline{B} \\ h_o \ \ otherwise \end{cases} \tag{9}$$

Case 4: when $c = c_o$:

$$\pi(c_o, h_o) - \pi(c_o, \underline{h}) = 2\sqrt{\frac{F_o}{D}}\left(\sqrt{H_o} - \sqrt{\underline{H}}\right) - \beta(h_o - \underline{h})$$

$$h^* = \begin{cases} \underline{h} \ \ if \ \ \beta < B_o \\ h_o \ \ otherwise \end{cases} \tag{10}$$

According to (7)-(10), we prove the theorem. □

According to Theorem 1, we find that the optimal values of emission factors are situational. There exist conditions on the unit costs for carbon reduction (i.e., α and β) in determining whether to invest in improving the carbon emission factors from $c_o$ (or $h_o$) to $\underline{c}$ (or $\underline{h}$).

In Theorem 1, $A_o$ (and $\underline{A}$) represents additional gain (i.e., the amount of cost reduction) achieved in transportation by reducing the carbon emission factor *c* from $c_o$ to $\underline{c}$ when $h = h_o$ (and $h = \underline{h}$). In particular, $\sqrt{F_o} - \sqrt{\underline{F}}$ denotes the amount of cost reduction in transportation, and thus $\frac{\sqrt{F_o} - \sqrt{\underline{F}}}{c_o - \underline{c}}$ represents the amount of transportation cost reduced by improving the carbon emission factor by one unit.

Similarly, the amount of cost reduction associated with warehousing items by reducing the carbon emission factor *h* from $h_o$ to $\underline{h}$ are $B_o$ and $\underline{B}$.

From (7)-(10), we understand that the decision on investment in carbon reduction relies on the comparison between the cost for reducing the carbon emission factor by one unit and the amount of total cost reduction. If the total cost reduction is larger than investment, it suffices to make an investment in improving carbon emission factors. Here, an interesting finding is that cost reduction is of square root type instead of linear with respect to *F* and *H* (e.g., $\sqrt{F_o}$, $\sqrt{\underline{F}}$, $\sqrt{H_o}$ and $\sqrt{\underline{H}}$). The cost reduction is less sensitive than the improvement in carbon emission factors. This finding states that the cost reduction requires the more improvement in carbon emission factors (*c* or *h*), but at a decreasing rate.

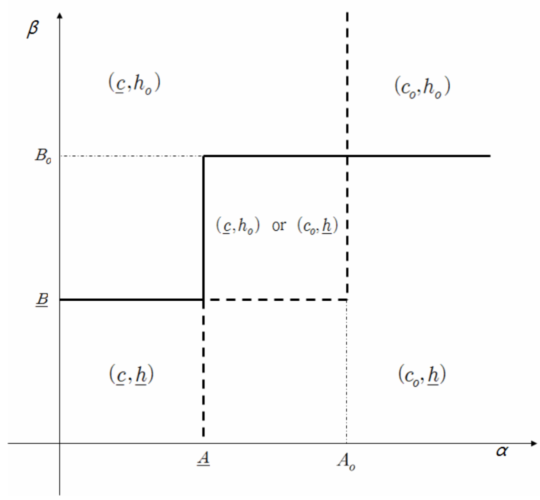


Figure 2. Optimal Value of *c* and *h*

Based on Theorem 1, an optimal decision for the carbon reduction investment is illustrated in Figure 2, and it indicates the policy is of threshold-type. For example, if *β* is beyond a threshold shown as the solid line in Figure 2, then it should not to invest in improving the carbon emission factor *h*. Instead, it should reduce the emission factor in warehousing items to accommodate the increase in inventory cost due to carbon emissions when *β* is below the threshold.

Unlike the literature that generally considers a single emission factor, the model in this paper includes *c* and *h*, separately. Figure 2 shows that the decision on *h* and the decision on *c* are associated with each other. When *α* is less than $\underline{A}$, the comparison between *β* and $\underline{B}$ decides whether to invest in reducing *h*. However, when *α* is beyond $\underline{A}$, $B_o$ instead of $\underline{B}$ should be involved in the decision.

If *α* (or *β*) is too large and exceeds $A_o$ (or $B_o$), it is better not to make an investment. On the contrary, both *α* and *β* are small enough, then we should invest to reduce the emission factors. If $\underline{A} \le \alpha \le A_o$ and $\underline{B} \le \beta \le B_o$, there is no difference between $(\underline{c}, h_o)$ and $(c_o, \underline{h})$ in reducing the total cost.

## 4. A NUMERICAL EXAMPLE

We provide an example to illustrate an overall range of parameters ($A_o$, $\underline{A}$, $B_o$ and $\underline{B}$) in Theorem 1 with aims to deliver a practical implication. An example scenario is developed based on the data used in Burns *et al*. (1985) with additional assumptions on carbon costs. Table 1 summarizes the input data for the example.

Table 1. Input Data for a Numerical Example

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $D$ | 50 | $\underline{c}$ | 5 |
| $F_t$ | 100 | $c_o$ | 10 |
| $F_c$ | 5 | $\underline{h}$ | 3 |
| $V_t$ | 10 | $h_o$ | 8 |
| $P$ | 0.02 | $K$ | 3 |
| α | 0.5 | $\beta$ | 0.1 |

According to Theorem 1, we have the following results; $A_o$ = 0.009998, $\underline{A}$ = 0.0099, $B_o$ = 0.040254 and $\underline{B}$ = 0.040157. Because both α and β are above $A_o$ and $B_o$, ($c^*$ and $h^*$) = ($c_o$, $h_o$). Moreover, the optimal shipment size becomes 20.029 by $q_o = \sqrt{\frac{DF}{H}}$.

## 5. CONCLUSIONS

We propose and investigate a model for determining an optimal shipment size and carbon reduction investment under the direct shipment distribution strategy. The proposed model particularly considers cost associated with carbon emissions in addition to the cost incurred when delivering and warehousing items. An optimal shipment size and optimal values of carbon emission factors are presented in a closed form, and their structural properties are investigated.

There are several interesting findings. First, an optimal carbon emission level is determined based on the trade-off between cost for investing carbon reduction and the total cost reduction. Moreover, the investment decision on improving carbon emission factors is of threshold-type. Second, unlike the literature, there exists an interconnection between the decision on $c$ and that on $h$. Finally, the total cost reduction requires the more improvement in carbon emission factors ($c$ or $h$) at a decreasing rate.

There are several topics for further research. We formulate the cost for reducing carbon emission factors as a linear function with respect to the emission level, but Swami and Shah (2013) and Dong *et al*. (2014) assume a quadratic function. We should extend the current proposed model to include a quadratic or general convex function in formulating the cost for reducing carbon emissions. Second, we have not considered the effects of an optimal decision on the amount of carbon emissions and the total logistics cost. Min (2015) analytically shows the possibility of reducing carbon emissions while reducing inventory and transportation costs and provides some necessary conditions. Another area of interests includes developing a model for other distribution strategies such as the peddling and consolidated distribution in addition to the direct shipment.

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