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THE n-TH TWISTED CHANGHEE POLYNOMIALS AND NUMBERS

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Abstract. The Changhee polynomials and numbers are introduced in [6]. Some interesting identities and properties of those polynomials are derived from umbral calculus (see [6]). In this paper, we consider Witt-type formula for the n-th twisted Changhee numbers and polynomials and derive some new interesting identities and properties of those polynomials and numbers from the Witt-type formula which are related to special polynomials.

1. Introduction

Let p be an odd prime number. $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic integers, the field of p-adic numbers and the completion of algebraic closure of $\mathbb{Q}_p$. The p-adic norm $|\cdot|_p$ is normalized by $|p|_p = \frac{1}{p}$. Let $\mathbb{C}(\mathbb{Z}_p)$ be the space of continuous functions on $\mathbb{Z}_p$. For $f \in \mathbb{C}(\mathbb{Z}_p)$, the fermionic p-adic integral on $\mathbb{Z}_p$ is defined by Kim to be

$$(1.1) \qquad I_{-1}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-1}(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x, \text{ (see [8,15]).}$$

Let $f_1(x) = f(x+1)$. Then, by (1.1), we get

$$(1.2) \qquad I_{-1}(f_1) + I_{-1}(f) = 2f(0), \text{ (see [3,5,7–11]).}$$

It is well known that the Euler polynomials are defined by the generating function to be

$$(1.3) \qquad \frac{2}{\varepsilon e^t + 1} e^{xt} = \sum_{n=0}^{\infty} E_{n,\varepsilon}(x) \frac{t^n}{n!},$$

(see [1–14, 16]).

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When $x=0$, $E_{n,\varepsilon}=E_{n,\varepsilon}(0)$ are called the n-th twisted Euler numbers. The Changhee polynomials are defined by the generating function to be

$$\frac{2}{t+2}(1+t)^x=\sum_{n=0}^{\infty}Ch_n(x)\frac{t^n}{n!},\ \text{(see [6])}. \tag{1.4}$$

When $x=0$, $Ch_n=Ch_n(0)$ are called the Changhee numbers, (see [6]).

The Stirling number of the first kind is defined by

$$(x)_n=x(x-1)\cdots(x-n+1)=\sum_{l=0}^{n}S_1(n,l)x^l,\ \text{(see [6])}. \tag{1.5}$$

The Changhee numbers and polynomials are introduced in [9]. Many interesting identities of those numbers and polynomials arise from umbral calculus (see [9]). We consider Witt-type formula for the n-th twisted Changhee numbers and polynomials and derive some new interesting identities and properties of those polynomials and numbers from the Witt-type formula which are related to special polynomials.

2. The n-th twisted Changhee numbers and polynomials

For $n\in\mathbb{N}$, let T_p be the p-adic locally constant space defined by

$$T_p=\bigcup_{n\geq 1}\mathbb{C}_{p^n}=\lim_{n\to\infty}\mathbb{C}_{p^n},$$

where $\mathbb{C}_{p^n}=\left\{\omega\,|\,\omega^{p^n}=1\right\}$ is the cyclic group of order p^n.

For $\varepsilon\in T_p$, the n-th twisted changhee polynomial are defined by

$$\begin{aligned}\sum_{n=0}^{\infty}Ch_{n,\varepsilon}(x)\frac{t^n}{n!}&=\int_{\mathbb{Z}_p}(1+\varepsilon t)^{x+y}d\mu_{-1}(y)\\&=(1+\varepsilon t)^x\int_{\mathbb{Z}_p}(1+\varepsilon t)^y d\mu_{-1}(y)\\&=\frac{2}{2+\varepsilon t}(1+\varepsilon t)^x,\end{aligned} \tag{2.1}$$

and

$$\begin{aligned}\int_{\mathbb{Z}_p}(1+\varepsilon t)^{x+y}d\mu_{-1}(y)&=\sum_{n=0}^{\infty}\varepsilon^n\int_{\mathbb{Z}_p}\binom{x+y}{n}d\mu_{-1}(y)t^n\\&=\sum_{n=0}^{\infty}\varepsilon^n\int_{\mathbb{Z}_p}(x+y)_n d\mu_{-1}(y)\frac{t^n}{n!}.\end{aligned} \tag{2.2}$$

Therefore, by (2.1) and (2.2) we obtained the following theorem.

Theorem 2.1. *For $n\geq 0$, we have*

$$\varepsilon^n\int_{\mathbb{Z}_p}(x)_n d\mu_{-1}(x)=Ch_{n,\varepsilon},$$

where $Ch_{n,\varepsilon}$ are called the n-th twisted Changhee numbers.

Therefore, by (2.1) and (2.2), we obtain the following theorem.

Theorem 2.2. *For $n \geq 0$, we have*

$$\varepsilon^n \int_{\mathbb{Z}_p} (x+y)_n d\mu_{-1}(y) = Ch_{n,\varepsilon}(x).$$

From (2.1) and (2.2), we note that

$$\sum_{n=0}^{\infty} \varepsilon^n \int_{\mathbb{Z}_p} \binom{x}{n} d\mu_{-1}(x) t^n = \frac{2}{2+\varepsilon t} = \sum_{n=0}^{\infty} (-\frac{\varepsilon}{2})^n t^n. \tag{2.3}$$

Thus, by comparing the coefficients on the both sides, we obtain the following theorem.

Theorem 2.3. *For $n \geq 0$, we have*

$$\int_{\mathbb{Z}_p} \binom{x}{n} d\mu_{-1}(x) = (-1)^n (\frac{1}{2})^n, \text{ (see [6]).}$$

Replacing t by $e^t - \frac{1}{\varepsilon}$ in (1.3), we get

$$\varepsilon^x \sum_{n=0}^{\infty} E_{n,\varepsilon}(x) \frac{t^n}{n!} = \frac{2}{\varepsilon e^t + 1} (\varepsilon e^t)^x = \sum_{n=0}^{\infty} Ch_{n,\varepsilon}(x) \frac{1}{n!} \Big(e^x - \frac{1}{\varepsilon} \Big)^n, \tag{2.4}$$

where $E_{n,\varepsilon}(x)$ is the n-th twisted Euler polynomials and

$$\begin{aligned} \sum_{n=0}^{\infty} Ch_{n,\varepsilon}(x) \frac{1}{n!} (e^x - \frac{1}{\varepsilon})^n &= \sum_{n=0}^{\infty} Ch_{n,\varepsilon}(x) \frac{1}{n!} \varepsilon^{-n} n! \Big(\sum_{m=n}^{\infty} S_2(m,n) \frac{t^m}{m!} \Big) \\ &= \sum_{m=0}^{\infty} \sum_{n=0}^{m} Ch_{n,\varepsilon}(x) S_2(m,n) \varepsilon^{-n} \frac{t^n}{m!}. \end{aligned} \tag{2.5}$$

By comparing the coefficients on the both sides of (2.4) and (2.5), we obtain the following theorem.

Theorem 2.4. *For $m \geq 0$, we have*

$$E_{m,\varepsilon} = \sum_{n=0}^{m} \varepsilon^{-n-x} Ch_{m,\varepsilon} S_2(m,n),$$

where $S_2(m,n)$ is the Stirling number of the second kind.

By Theorem 2.2, we easily get

$$\begin{aligned} Ch_{n,\varepsilon}(x) &= \varepsilon^n \int_{\mathbb{Z}_p} (x+y)_n d\mu_{-1}(y) \\ &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l) \int_{\mathbb{Z}_p} (x+y)^l d\mu_{-1}(y) \\ &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l) E_l(x). \end{aligned} \tag{2.6}$$

Therefore, by (2.6), we obtain the following theorem.

Theorem 2.5. *For $n \geq 0$, we have*

$$Ch_{n,\varepsilon}(x) = \varepsilon^n \sum_{l=0}^{n} S_1(n,l) E_l(x),$$

where $S_1(n,l)$ is the Stirling number of the first kind.

The n-th twisted Changhee polynomials of order k are defined by the generating function to be

$$\begin{aligned} \sum_{n=0}^{\infty} Ch_{n,\varepsilon}^{(k)}(x) \frac{t^n}{n!} &= \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{x_1+x_2+\cdots+x_k+x} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \\ &= (1+\varepsilon t)^x \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{x_1+x_2+\cdots+x_k} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \\ &= (1+\varepsilon t)^x \Big(\frac{2}{1+\varepsilon t} \Big)^k. \end{aligned} \tag{2.7}$$

We observe that

$$\begin{aligned} &\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{x_1+x_2+\cdots+x_k+x} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \\ &= \sum_{n=0}^{\infty} \varepsilon^n \int_{\mathbb{Z}_p} (x_1+x_2+\cdots+x_k+x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \frac{t^n}{n!}. \end{aligned} \tag{2.8}$$

Therefore, by (2.7), we obtain the generating function of $Ch_{n,\varepsilon}^{(k)}$ as follows.

Theorem 2.6. *The generating function of $Ch_{n,\varepsilon}^{(k)}$ is given by*

$$\sum_{n=0}^{\infty} Ch_{n,\varepsilon}^{(k)} \frac{t^n}{n!} = \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{x_1+x_2+\cdots+x_k} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k).$$

From (2.8), we have

$$\varepsilon^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} \binom{x_1+x_2+\cdots+x_k}{n} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) = \frac{Ch_{n,\varepsilon}^{(k)}}{n!}. \tag{2.9}$$

By (2.6), we get

$$\begin{aligned} Ch_{n,\varepsilon}^{(k)} &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l) \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} x_1^l \cdots x_k^l d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \\ &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l) (E_l)^k, \end{aligned} \tag{2.10}$$

where $(E_l)^k = \underbrace{E_l \times E_l \times \cdots \times E_l}_{k\text{-times}}$.

Therefore, by (2.10), we obtain the following theorem.

Theorem 2.7. *For $n \geq 0$, $k \geq 1$, we have*

$$Ch_{n,\varepsilon}^{(k)} = \varepsilon^n \sum_{l=0}^{n} S_1(n,l)(E_l)^k,$$

where $S_1(m,n)$ is the Stirling number of the first kind.

Now we consider the n-th twisted Changhee polynomials of the second kind as follows:

$$\begin{aligned} \sum_{n=0}^{\infty} \widehat{Ch}_{n,\varepsilon}(x)\frac{t^n}{n!} &= \int_{\mathbb{Z}_p} (1+\varepsilon t)^{-y+x} d\mu_{-1}(y) \\ &= (1+\varepsilon t)^x \int_{\mathbb{Z}_p} (1+\varepsilon t)^{-y} d\mu_{-1}(y) \\ &= \frac{2(1+\varepsilon t)}{2+\varepsilon t}(1+\varepsilon t)^x. \end{aligned} \tag{2.11}$$

Hence,

$$\begin{aligned} \widehat{Ch}_{n,\varepsilon}(x) &= \varepsilon^n \int_{\mathbb{Z}_p} (-y+x)_n d\mu_{-1}(y) \\ &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l)(-1)^l \int_{\mathbb{Z}_p} (y-x)^l d\mu_{-1}(y) \\ &= \varepsilon^n \sum_{l=0}^{n} S_1(n,l)(-1)^l E_l(-x). \end{aligned} \tag{2.12}$$

Therefore, by (2.12), we obtain the following theorem.

Theorem 2.8. *For $n \geq 0$, we have*

$$\widehat{Ch}_{n,\varepsilon}(x) = \varepsilon^n \sum_{l=0}^{n} S_1(n,l)(-1)^l E_l(-x).$$

By replacing t by $e^t - \frac{1}{\varepsilon}$ in (2.11)

$$\begin{aligned} &\sum_{m=0}^{\infty} \sum_{n=0}^{m} \widehat{Ch}_{n,\varepsilon}(x)\varepsilon^{-n} S_2(m,n)\frac{t^m}{m!} \\ &= \sum_{n=0}^{\infty} \widehat{Ch}_{n,\varepsilon}(x)\frac{1}{n!} n!\varepsilon^{-n} \sum_{m=n}^{\infty} S_2(m,n)\frac{t^m}{m!} \\ &= \sum_{n=0}^{\infty} \widehat{Ch}_{n,\varepsilon}(x)\frac{1}{n!}\Big(e^t - \frac{1}{\varepsilon}\Big)^n \end{aligned} \tag{2.13}$$

$$
\begin{aligned}
&= \frac{2}{\varepsilon e^t+1}\left(\varepsilon e^t\right)^{x+1} \\
&= \varepsilon^{x+1}\frac{2}{\varepsilon e^t+1}e^{t(x+1)} \\
&= \varepsilon^{x+1}\sum_{m=0}^{\infty} E_{m,\varepsilon}(x+1)\frac{t^m}{m!}.
\end{aligned}
$$

Therefore, by (2.13), we obtain the following theorem.

Theorem 2.9. *For $m \geq 0$, we have*

$$
E_m(x+1) = \sum_{n=0}^{m} \widehat{Ch}_{n,\varepsilon}(x)\varepsilon^{-n-x-1}S_2(m,n). \tag{2.14}
$$

Now, we consider the n-th twisted Changhee polynomials of the first kind relate to the n-th twisted Changhee polynomials of the second kind.

$$
\begin{aligned}
\frac{(-1)^n Ch_{n,\varepsilon}(x)}{n!} &= (-1)^n\varepsilon^n \int_{\mathbb{Z}_p}\binom{x+y}{n}d\mu_{-1}(y) \\
&= \varepsilon^n \int_{\mathbb{Z}_p}\binom{-x-y+n-1}{n}d\mu_{-1}(y) \\
&= \varepsilon^n \sum_{m=0}^{\infty}\binom{n-1}{n-m}\int_{\mathbb{Z}_p}\binom{-x-y}{m}d\mu_{-1}(y) \\
&= \varepsilon^n \sum_{m=1}^{n}\binom{n-1}{m-1}\varepsilon^{-m}m!\varepsilon^m\int_{\mathbb{Z}_p}\binom{-x-y}{m}d\mu_{-1}(y) \\
&= \sum_{m=1}^{n}\binom{n-1}{m-1}\varepsilon^{n-m}\frac{\widehat{Ch}_{m,\varepsilon}(-x)}{m!}.
\end{aligned} \tag{2.15}
$$

Therefore, for $n \geq 1$, we have

$$
\frac{(-1)Ch_{n,\varepsilon}(x)}{n!} = \sum_{m=1}^{n}\binom{n-1}{m-1}\varepsilon^{n-m}\frac{\widehat{Ch}_{m,\varepsilon}(-x)}{m!}. \tag{2.16}
$$

For $k \in \mathbb{N}$, let us consider the n-th twisted Changhee numbers of order k as follows:

$$
\varepsilon^n \int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}(-x_1-x_2\cdots-x_k)_n d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_k) = \widehat{Ch}_{n,\varepsilon}, \tag{2.17}
$$

where $n \in \mathbb{Z}_{\geq 0}$.

Then the generating function of $\widehat{Ch}_{n,\varepsilon}^{(k)}$ is given by

$$
\begin{aligned}
&\sum_{n=0}^{\infty}\widehat{Ch}_{n,\varepsilon}^{(k)}\frac{t^n}{n!} \\
&= \int_{\mathbb{Z}_p}\cdots\int_{\mathbb{Z}_p}(1+\varepsilon t)^{-x_1-\cdots-x_k}d\mu_{-1}(x_1)\cdots d\mu_{-1}(x_k)
\end{aligned} \tag{2.18}
$$

$$= \sum_{n=0}^{\infty} \varepsilon^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (-x_1 - \cdots - x_k)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \frac{t^n}{n!}.$$

Now, we observe that

(2.19)

$$\begin{aligned}
&\int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{-x_1-\cdots-x_k} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \\
&= \sum_{m=0}^{\infty} \varepsilon^m \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (-1)^m (x_1 + \cdots + x_k)^m d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_n) \tfrac{(\log(1+\varepsilon t))^m}{m!} \\
&= \sum_{m=0}^{\infty} \varepsilon^m (-1)^m (E_m)^k \sum_{n=m}^{\infty} S_1(n,m) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} \varepsilon^m (-1)^m (E_m)^k S_1(n,m) \right) \frac{t^n}{n!}.
\end{aligned}$$

From (2.18) and (2.19), we have

$$\widehat{Ch}_{n,\varepsilon}^{(k)} = \varepsilon^n \sum_{m=0}^{n} (-1)^m (E_m)^k S_1(n,m), \tag{2.20}$$

where $(E_m)^k = \underbrace{E_m \times \cdots \times E_m}_{k\text{-times}}$.

Now, we define the n-th twisted Changhee polynomials of the second kind of order k as follows:

$$\widehat{Ch}_{n,\varepsilon}^{(k)}(x) = \varepsilon^n \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (-x_1 \cdots x_k - x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k). \tag{2.21}$$

Then the generating function of $\widehat{Ch}_{n,\varepsilon}^{(k)}(x)$ are given by

$$\begin{aligned}
&\sum_{n=0}^{\infty} \widehat{Ch_n}^{(k)}(x) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} \varepsilon^n (-x_1 - \cdots - x_k - x)_n d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k) \frac{t^n}{n!} \\
&= \int_{\mathbb{Z}_p} \cdots \int_{\mathbb{Z}_p} (1+\varepsilon t)^{-x_1 \cdots x_k - x} d\mu_{-1}(x_1) \cdots d\mu_{-1}(x_k).
\end{aligned} \tag{2.22}$$

Proceeding similarly to (2.12), we have

$$\widehat{Ch}_{n,\varepsilon}^{(k)}(x) = \varepsilon^n \sum_{m=0}^{n} (-1)^m S_1(n,m) \sum_{l=0}^{m} \binom{m}{l} (E_{m-l})^k x^l, \tag{2.23}$$

where $(E_{m-l})^k = \underbrace{E_{m-l} \times \cdots \times E_{m-l}}_{k\text{-times}}$.

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