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The Relationship between Pre-service Teachers' Geometric Reasoning and their van Hiele Levels in a Geometer's Sketchpad Environment[1]

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In this study, I investigated how pre-service teachers (PSTs) proved three geometric problems by using *Geometer's SketchPad* (GSP) software. Based on observations in class and results from a test of geometric reasoning, eight PSTs were sorted into four of the five van Hiele levels of geometric reasoning, which were then used to predict the PSTs' levels of reasoning on three tasks involving proofs using GSP. Findings suggested that the ways the PSTs justified their geometric reasoning across the three questions demonstrated their different uses of GSP depending on their van Hiele levels. These findings also led to the insight that the notion of "proof" had somewhat different meanings for students at different van Hiele levels of thought. Implications for the effective integration of technology into pre-service teacher education programs are discussed.

Keywords: geometric reasoning, pre-service teacher education, van Hiele levels, geometric proofs.
MESC Classification: E50, G40
MSC2010Classification: 97E50, 97G60, 97U70

INTRODUCTION

The van Hiele Level model of geometric thought, which identifies five levels of sophistication in the way learners think about two-dimensional geometry, was first published in 1957 as a doctoral dissertation co-authored by Dina van Hiele-Geldof and Pierre van Hiele (Crowley, 1987) and remains influential to the present day. According to the theory, advancement through the five levels of geometric thought is not age-related but depends on geometric experience. The levels are sequential, which means the learner

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cannot achieve one level of thinking without having passed through the previous levels.

Research has shown that the van Hiele Level model appropriately describes the geometric thinking of students in 2-dimensional geometry and has a relationship with students' mathematical learning in various topics of geometry (Burger & Shaughnessy, 1986; Senk, 1989; Usiskin, 1982). For example, Senk (1989) found that high school students' achievement on standard geometry content is positively related to van Hiele levels of geometric thought. In addition, Gutiérrez (1992) suggested some hypotheses about the use of the van Hiele Level model for understanding 3-dimensional geometry thinking.

Table 1. Van Hiele Levels

Level	Description
Level 1 (Visualization)	Students can recognize and classify shapes based on visual characteristics of the shape.
Level 2 (Analysis)	Students can identify some properties of shapes and use appropriate vocabularies.
Level 3 (Informal Deduction)	Students know the relationship among properties of geometric objects and are able to do informal logical reasoning.
Level 4 (Deduction)	Students know the deductive systems of properties and can create formal proof.
Level 5 (Rigor)	Students can do analysis of deductive systems and compare different axiom systems.

In particular, there has been some research on investigating the relationship between students' ability to write geometry proofs and their van Hiele Levels (Battista & Clements, 1995; Senk, 1989). According to the van Hiele Level model, students below level 3 are not able to prove but rely on memorizing. Students at level 3 might be able to understand a short proof based on properties acquired from concrete experience. But only students who have reached levels 4 or 5 would be able to write formal proofs.

Consistent with this prediction, Senk (1989) showed that students at levels 3 or higher noticeably outperformed students at levels 1 or 2 in writing proofs. That is, she found that less than 22% of students below level 3, 57% at level 3, 85% at level 4, and 100% at level 5 had mastered geometric proofs. However, her finding that some students below level 3 were able to do some mathematical proofs is not consistent with van Hiele predictions.

With respect to the integration of technology into geometry education, this inconsistency in van Hiele predictions was also found in other studies. Along with the development of technology, some researchers have investigated the possibility of improving students' van Hiele levels through use of Dynamic Geometry Software such as *Geometer's SketchPad* (GSP) (Battista & Clements, 1995; de Villiers, 2004; Gawlick, 2005; Patsiomitou, Barkatsas & Emvalotis, 2010). Patsiomitou, Barkatsas, and Emvalotis, *ibid.*, found that GSP impacted students' conjecturing and proving processes so that students at lower van Hiele levels could demonstrate some features of mathematical proofs. De Villiers (2004) also found that students below van Hiele level 3 provided arguments to prove geometric problems using GSP software.

Also, Gawlick (2005) maintained that GSP software can raise levels of geometric thinking, although, to be able to prove in some way, students at different van Hiele levels may require different uses of the three main GSP instruments. More specifically, in his study of loci of ortho-centers and in-centers, he found that the drag mode is an important tool to progress from level 2 to level 3, and Macros and loci modes support advancement from level 3 to level 4. Finally, families of loci can be used to advance from level 4 to level 5.

However, although there is some evidence that students at lower van Hiele Levels can prove their geometric reasoning using GSP software, how *pre-service teachers* (PSTs) at different van Hiele levels justify geometric problems using GSP software has rarely been investigated. Accordingly, to examine PSTs' geometric reasoning in a GSP-integrated problem solving situation, in this study a task-based test comprising three problems was conducted with eight PSTs enrolled in a geometry content knowledge course. The participants were selected based on their van Hiele levels, which were assessed using both classroom observations and a selection test.

The research questions pursued in the study were as follows:

1. How do pre-service teachers employ GSP to justify three geometric problems designed for explanatory purposes, constructive purposes, and combined explanatory/constructive purposes respectively?
2. How does GSP impact the geometric reasoning of pre-service teachers at different levels of the van Hiele model?

THEORETICAL BACKGROUND

The use of Geometers' Sketch Pad software in geometry education

The Geometer's Sketch Pad (GSP) is an interactive and dynamic geometry software program that allows students to explore and analyze mathematical concepts in the fields of algebra, geometry, trigonometry, calculus and other areas by creating mathematical objects (artifacts) (Jackiw, 2001). GSP constructions can be dragged, squeezed, stretched, or otherwise changed while keeping all mathematical properties intact. To operate GSP, one first selects one or more objects in the toolbox situated on the left side of screen and then chooses an action in the menu bar situated on the upper bar of the screen, which offers five major functions: Display, Construct, Transform, Measure, and Graph. The Display menu commands allow one to control the appearance of objects in a sketch and the tools to work with them. The Construct menu provides commands for accomplishing many important geometric constructions. The Transform menu commands apply geometric transformations to figures in sketches, allowing one to create translations, rotations, dilations, reflections, tessellations, scale models, kaleidoscopes, fractals, and much more. The Measure menu commands allow one to measure numeric properties of selected objects. The Graph menu allows one to create and manipulate coordinate systems, to create parameters and functions, to find the derivative of a function, to plot points and functions on the coordinate axes, and to tabulate measured values.

In GSP, students can change the original objects by moving components (such as a point, segment, and/or circle) to different locations on the screen. As the original objects are transformed, the results of all constructions are immediately applied to those objects on the screen. The students also can measure the lengths, angles, and areas of objects on the screen and then observe how these are dynamically changed according to the manipulation that is applied.

Owing to this special feature, dynamic geometry technology is considered more effective than a paper and pencil environment in geometry education. At the elementary level, dynamic geometry technology helps children abstract the essence of a shape by allowing them to discern its component parts rather than understand the shape in terms of what it resembles (Goldenberg & Cuoco, 1998; Laborde & Laborde, 2008). At the secondary level, dynamic geometry technology provides students with opportunities to create their own mathematics (Goldenberg & Cuoco, 1998; Hollebrands, 2007) and also enables them to connect the geometrical representation of a changing phenomenon with an algebraic representation of the varying quantities (Olive, 2000). In higher education, dynamic geometry technology acts as a bridge between Euclidean geometry and its analysis and helps undergraduates improve their geometric reasoning (de Villiers, 2003; Goldenberg & Cuoco, 1998; Govender & de Villiers, 2003; Haja, 2005).

In this study, GSP was also used as a tool to bridge Euclidean geometry and its analysis. That is, PSTs were allowed to justify or prove their geometric reasoning in solving three geometric problems with the five major functions of GSP.

Geometric proofs

Battista and Clements state that if problems are posed, mathematicians analyze examples, make conjectures, offer counterexamples, revise the conjectures, and establish a theorem when this convergence of processes produces a valid answer to a given problem (*cf.* Battista & Clements, 1995). Consequently, they argue, “formally presenting the results of mathe-matical thought in terms of proofs is meaningful to mathematicians as a method for establishing the validity of ideas”, and therefore most mathematics instruction and textbooks let us believe that "mathematicians make use only of formal proof – logical, deductive reasoning based on axioms" (p. 48). Higher van Hiele Levels are also distinguished by whether students are able to formally prove through logical interpretation of geometric statements such as axioms, definitions and theorems.

However, often mathematicians find a theorem through intuitive and empirical methods (Eves, 1972). Also, the methods for establishing the validity of ideas can be other than formal proofs. That is, although students do not use formal proof statements, they can justify their mathematical ideas in meaningful ways by making conjectures, resolving conflicts via presenting arguments and evidence, and formulating proof hypotheses (Battista & Clements, 1995).

In line with this alternative perspective on proofs, GSP software can be used to facilitate students' making and testing conjectures, which are crucial elements of mathematical discovery in formulating proof hypotheses (Battista & Clements, 1995). The GSP allows users to explore the generality of consequences of constructions by helping them construct a shape and change it while maintaining some properties. In the process of using the GSP, users can invent definitions, make conjectures, deal with significant problems, justify their generalizations, and devise original proofs.

Similarly, according to de Villiers (2003), GSP can be a useful tool to allow learners to recognize the need for a proof when asked why they think a particular result is always true. That is, inductive verification through GSP can lead to a deductive argument that provides explanation of what they have observed in GSP. In this way, de Villiers (2004) expanded the role of proofs in a GSP environment beyond verification and suggested five roles: explanation, discovery, verification, intellectual challenge, and systematization.

Proof as a means of explanation involves providing the reason why a specific result is true. Proof as a means of discovery refers to inventing a new result in the process of explaining why a result is true. Proof as a means of verification entails determining whether a statement or an observation is always true or not. Proof as a means of intellectual challenge relates to trying to find what has puzzled other people and results in self-realization and fulfillment. Proof as a means of systematization refers to logically organizing "various known results into a deductive system of axioms, definitions, and theorems" (de Villiers, 2004, p. 9). De Villiers recommended introducing the various functions of proof in the sequence of explanation, discovery, verification, intellectual challenge, and systematization based on a spiral rather than a linear approach. That is, teachers should allow students to revisit functions of proof that were introduced earlier, depending on their needs.

In this study, I took an alternative perspective on proofs (Battista & Clements, 1995; de Villiers, 2004) and asked PSTs to justify their geometric reasoning rather than to prove formally after solving the given problems. Also, to investigate PSTs' geometric reasoning in a GSP environment, I provided three geometric problems, which I judged would require the first three functions of proof, namely, explanation, discovery, and verification, but not intellectual challenge and systematization for two reasons:

(1) This semester was first time for PSTs to learn GSP technology, and
(2) The PSTs were preparing to teach primary and middle school rather than high school mathematics and so could be expected to have less prior mathematics knowledge.

Van Hiele Levels

To explore the geometric reasoning of PSTs at different levels of geometric knowledge, participants were selected based on their van Hiele Levels. According to van Hiele Level model, students at level 1 can recognize and name shapes by appearance. However, they either do not recognize properties or do not use the properties for sorting or recognition. Thus, they may not recognize a shape in a different orientation. Students at level 2 can identify some properties of shapes and use appropriate terminology. However, they cannot explain the relationship between shapes and their properties. Students at level 3 can recognize interrelationships among shapes or classes of shapes and follow informal proofs. However they cannot create a formal proof or see which steps of a proof can be interchanged. Students at level 4, which they usually do not reach before high school, can create formal proofs in an axiomatic system and understand how postulates, axioms, and definitions are used in proofs. Finally students at level 5, which only some students attain in college, can compare different axiom systems such as a spherical geometry system beyond a Euclidean system.

In the van Hiele Level model, formally proving or justifying geometric reasoning can be expected of students at Level 4, at which, I assumed initially, most PSTs were functioning because they were college students. However, from a selection test, I found that PSTs were at a surprisingly wide range of van Hiele levels, from 1 to 4. Nevertheless, after referring to prior research (de Villiers, 1991; de Villiers, 2004; Mudaly & de Villiers, 2000), I did not change the requirements of the original tasks to justify geometric reasoning. According to de Villiers (2003), "the functions of proof such as explanation, discovery, and verification can be meaningful to students outside a systematization context, in other words, at van Hiele levels lower than van Hiele level 3, provided the arguments are of an intuitive or visual nature" (de Villiers 2003, p. 18). For this reason, I thought it would be interesting to examine how PSTs at a variety of van Hiele levels justify or prove their geometric reasoning with GSP technology. This aspect makes the present study distinctive in that many studies have been conducted using the van Hiele Level model but rarely if ever in a dynamic geometric environment.

RESEARCH METHOD

Participants

The van Hiele levels of 27 PSTs enrolled in a geometry content knowledge course were evaluated using both classroom observation and a selection test of 20 questions about quadrilaterals, which were selected from Mayberry's (1981) instrument to assess the PSTs' van Hiele levels of geometric thinking. Afterwards, depending on their van Hiele levels, two PSTs from each of levels 1 to 4 were selected as participants for this study because I could not find any PSTs who were at level 5. Seven participants were female and one participant was male and they wanted to be middle school teachers in the future. Then a task-based test was conducted with the eight participants to examine their geometric reasoning in a GSP environment.

Data Collection

The PSTs learned how to use the GSP software in the geometry content knowledge course at least once a week for two months of a semester. Then, at the end of the semester, the PSTs participated in one 90 minute computer-based test (see Table 2), which included three types of tasks:

Table 2. Three Different Types of GSP Tasks

Types of Tasks	The use of GSP	Tasks being given to PSTs
Explanatory Task	PSTs are expected to explore a pre-made GSP file in order to solve the problem.	In any trapezoid, when the diagonals are drawn, two regions (the shaded regions) are created that may look very different but always have the same area. Using GSP, justify why the two regions have the same area, explaining and showing how you used GSP in the text tool box in GSP.
Constructive Task	PSTs are expected to construct a quadrilateral using GSP.	Construct a rhombus using GSP in various ways and justify why you are sure you have created a rhombus.
Combination of explanatory and constructive Task	PSTs are expected to construct a quadrilateral and use it for a further problem.	Construct a quadrilateral ABCD and connect the midpoints (E, F, G, and H) of the sides. What does the new quadrilateral EFGH look like? Justify your speculation using GSP.

(1) Asking PSTs to explore pre-made artifacts and explain why their observations in

GSP were always true,

(2) Asking PSTs to construct an artifact by themselves using their mathematical knowledge and justify their constructions, and

(3) Asking PSTs to use GSP in both ways.

The reason for providing tasks that required different uses of GSP was to probe the functioning of PSTs at different van Hiele levels. According to de Villiers (2004), learners at lower levels might have difficulty constructing a geometric artifact using GSP; thus it would be appropriate for learners to begin with pre-made artifact in GSP, after which attempting to construct could assist their transition from a lower to a higher level.

Regarding the explanatory use of GSP, the task related to area was asked to observe how PSTs justified why two regions made by two diagonals on a trapezoid were the same. Concerning the constructive use of GSP, the quadrilateral construction task[2] was asked to observe how PSTs constructed a rhombus in various ways. I chose constructing a rhombus as very important in that one can use the properties of a rhombus to do basic constructions such as a perpendicular bisector, a parallel line, and an angle bisector. Thus, it would be meaningful to observe different ways that PSTs used to construct a rhombus. Finally, in order to investigate the combination of explanatory and constructive uses of GSP technology, the task related to a medial quadrilateral was asked to observe how PSTs constructed a new medial quadrilateral by connecting midpoints on each side of the original quadrilateral and justified what the new quadrilateral was. The area task and the medial quadrilateral task were adapted from Driscoll's (2007) *Fostering Geometric Thinking.* The three tasks were provided to PSTs in GSP files.

The test was administrated in a computer lab using GSP. The PSTs wrote and justified their answers in GSP files, using various functions of the GSP software, including a text toolbox. In the test, the PSTs were asked to show how they used GSP in solving problems and to justify their reasoning in the text toolbox. Results of the test were saved in GSP files and submitted to the author online. I collected data only at one point because the purpose of my study was to qualitatively investigate PSTs' geometric reasoning about rhombus construction and area in a GSP environment rather than to make claims about improved knowledge or reasoning as a result of incorporating GSP technology into geometry content knowledge course over time.

Data Analysis

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[2] Construction is defined as a drawing of shapes using only a compass and a straightedge in a paper and pencil environment. Thus, in construction, measurement of lengths or angles is not allowed. In a GSP environment, construction is defined as a drawing of shapes using the Construct menu including midpoint, intersection, segment, ray, line, parallel line, perpendicular line, angle bisector, circle by center and point, circle by center and radius, etc.

To analyze the data, I first engaged in repeated viewings of the GSP files and in recursive rounds of note-taking (Cobb & Gravemeijer, 2008). Then I produced detailed analytical notes, which included summaries of each PSTs' work on each problem and interpretations of his/her reasoning through memos (Corbin & Strauss, 2008). The aim of this phase was to examine the PSTs' geometric reasoning in a GSP environment. The results from this phase of analysis provided a portrait of the PSTs' geometric reasoning, which corresponds to the first research question. In the second phase of analysis, I generated a written synthesis of the results for the two PSTs at each van Hiele level from the narrative summaries and discussed it with other math educators to establish trustworthiness. Then I looked across the syntheses to articulate the differences in how PSTs at different van Hiele levels solved the problems in the test, which corresponds to the second research question. The goal of this phase was to illuminate how GSP technology impacts the geometric reasoning of PSTs at different Van Hiele levels.

DISCUSSION AND ANALYSIS

In this study, I have described how PSTs solved three geometric problems in a GSP environment. Regarding solving a task to demonstrate explanatory use of GSP, the PSTs at level 1 did not suggest any justification but described what they observed on the GSP screen. For example, one PST described, "The areas of two triangles in a trapezoid between two parallel lines would be always the same, no matter which points along the parallel lines are moved," without justification. The PSTs at van Hiele Level 2 provided their justifications by measuring the area of the given figure using the Measure function of GSP. However, they did not suggest any generalized justification beyond measures of a specific example. For instance, as shown in Figure 1, the PST justified that the area of two triangles were equal with evidence that both triangles had the same measure of 12.16 cm sq. After describing how she used GSP to measure the areas of the given triangles and to change the given points B and D for formulating different orientations of triangles, she reported that the area measure of 12.16 did not change no matter how she changed the position of a given point in GSP because other points adjusted accordingly.

The PSTs at level 3 justified their answers in terms of a specific example. That is, the PSTs at this level changed the given trapezoid into a parallelogram, which is a special case of a trapezoid in that a parallelogram has two pairs of parallel lines whereas a trapezoid has at least one pair of parallel lines, by stretching the side ED and making it parallel to the side BC. Then she explained that the two triangles were congruent from the specific case.

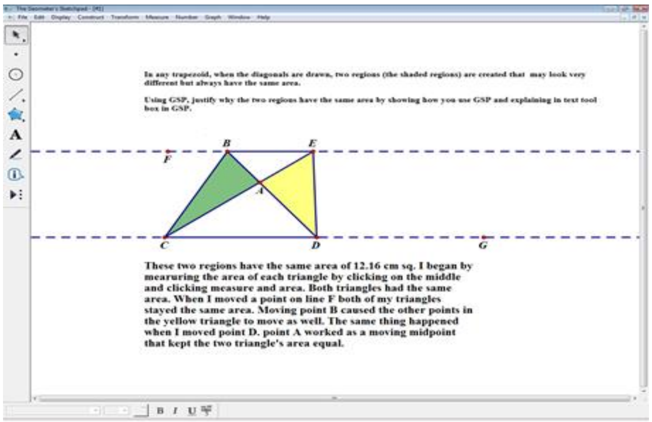

Figure 1. An example of a PST's work at van Hiele Level 2

The PSTs at level 4 justified their answers by referring to a definition, properties, or principles. They constructed mid points on a pair of parallel lines of the given trapezoid using GSP, and then justified their reasoning by referring to the invariance property of area of triangles between parallel lines when two triangles share a base. For example, as shown in Figure 2, the PST constructed two midpoints F and G and then connected the two points. Afterwards, the PST used the mathematical knowledge that the area of two shapes is the same when they have the same base and the same height by using the formula for the areas of a triangle and a trapezoid.

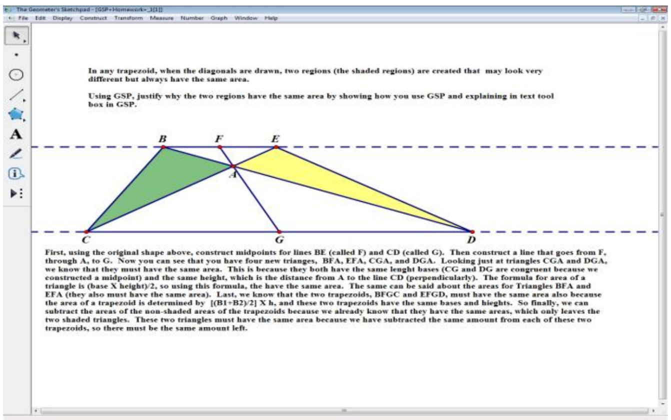

Figure 2. An example of a PST's work at van Hiele Level 4

In constructing rhombi, the PSTs used their content knowledge of side property, angle property, diagonal property, and symmetric property. However, while the PSTs at van Hiele Level 1 tried to use the properties of a rhombus, their artifacts did not make sense (see Figure 3). The PSTs at level 2 drew a rhombus based on its properties, but they did not use the Construct function in GSP. That is, they drew a rhombus by connecting four straight lines rather than using the Transformation function or a Construct function such as a circle tool. The PSTs at level 3 first constructed a parallelogram by creating two parallel lines via the Construct function and changed it into a rhombus by measuring the length of four sides using the Measure function in GSP. That is, even though two parallel lines can be one of the properties of a rhombus, the process of measuring the four sides to make them equal in length after constructing parallel lines did not guarantee that the PST constructed rather than drew a rhombus.

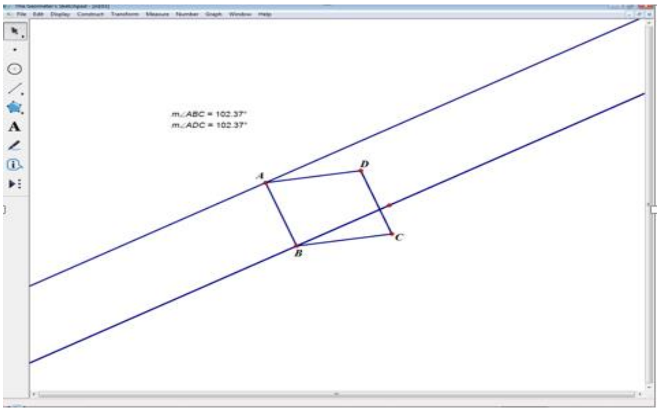

Figure 3. An example of a PST's work at van Hiele Level 1

The PSTs at level 4 constructed a rhombus based on various properties of a rhombus using the Construct function of GSP (see Figure 4). For example, in Figure 4, one PST drew three overlapping circles, all having the same radius, using the "circle by center and radius" function in the Construct menu of GSP. The PST also provided the following justification of her construction: "Rhombus has four equal sides. Sides AB, BC, CD, DA are the same in length because they are a radius of circle AB, circle BC, and circle DA, which have been constructed with a same radius"

Also, in solving a task for combination of constructive and explanatory uses of GSP, almost all PSTs were able to construct a new quadrilateral, but their justifications about the type of the quadrilateral were different. The PSTs at van Hiele Level 1 answered that the new quadrilateral was a parallelogram based on its appearance. The PSTs at van Hiele Level 2 answered that the new quadrilateral was a rhombus, which is incorrect. However they justified their answer by commenting on specific properties of a parallelogram such as that the quadrilateral seems to have two parallel lines and two equal opposite angles.

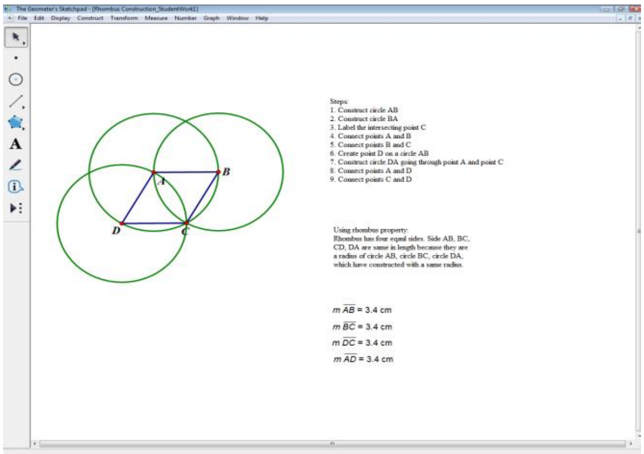

Figure 4. An example of a PST's work at van Hiele Level 4

The PSTs at van Hiele Level 3 answered the new quadrilateral was a parallelogram based on the property that it had two congruent opposite sides and angles, which they determined by measuring them using GSP (see Figure 5, left). The PSTs at van Hiele Level 4 justified that the new quadrilateral was a parallelogram by constructing diagonals and parallel lines with GSP and employing triangle similarity (see Figure 5, right).

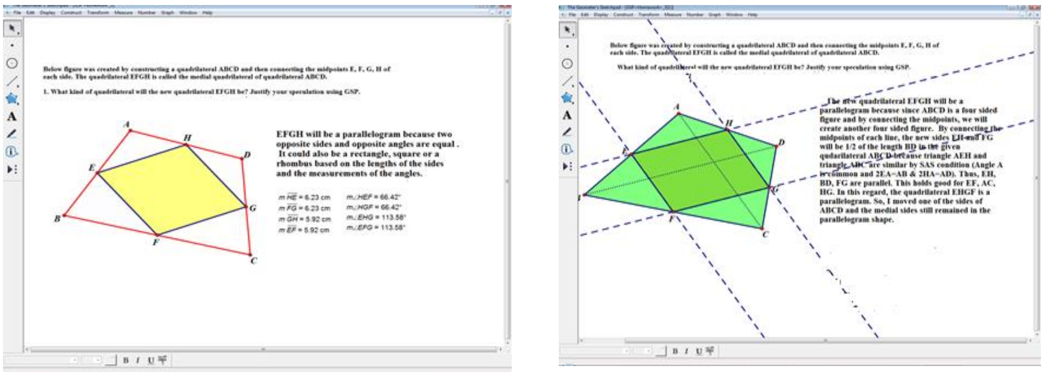

Figure 5. Example of PSTs work at van Hiele Level 3 (left) and Level 4 (right)

These findings indicate that the ways the PSTs justified their geometric reasoning across the three questions demonstrated their different uses of GSP depending on their van Hiele levels (see Figure 6). More specifically, the PSTs at van Hiele level 1 (the Visualization level) or 2 (the Analysis level) solved problems by simply measuring some properties of the given figures using the Measure function of GSP, even though the figures were provided only as examples in order to specify the questions. By contrast, the PSTs at van Hiele level 3 (the Informal deduction level) solved the problems by using both the Measure function and the Construct function of GSP in an inductive way. That is, they justified their reasoning by using the Measure function after constructing an artifact using GSP. Finally, the PSTs at van Hiele level 4 (the Deduction level) used only the Construct function of GSP in order to justify their geometric reasoning in a deductive way.

These findings also lead to the insight that functions of proof could have different meanings depending on level of geometric thought (see Figure 6). In this study, I provided three geometric problems to which three functions of proof, explanation, discovery, or verification, could be applied. However, after exploring GSP technology, the PSTs at van Hiele levels 1 or 2 tended to describe what they observed when moving one point or side, or measuring some properties of the given figures, rather than explain in terms of geometric reasoning. That is, for them, proof seemed to be a means of explanation, which focuses on providing the reason why a specific result is true by mainly using the Measure function of GSP.

The PSTs at van Hiele Level 3 seemed to regard proof as a means of explanation or inductive verification in that their verifications partially included a specific measure. They used the Construct function of GSP and created auxiliary lines or artifacts to determine whether an observation or statement was always true. However, in the process of providing the reason, they also partially depended on a specific observation by measuring angles or side lengths from the given figures using the Measure function of GSP.

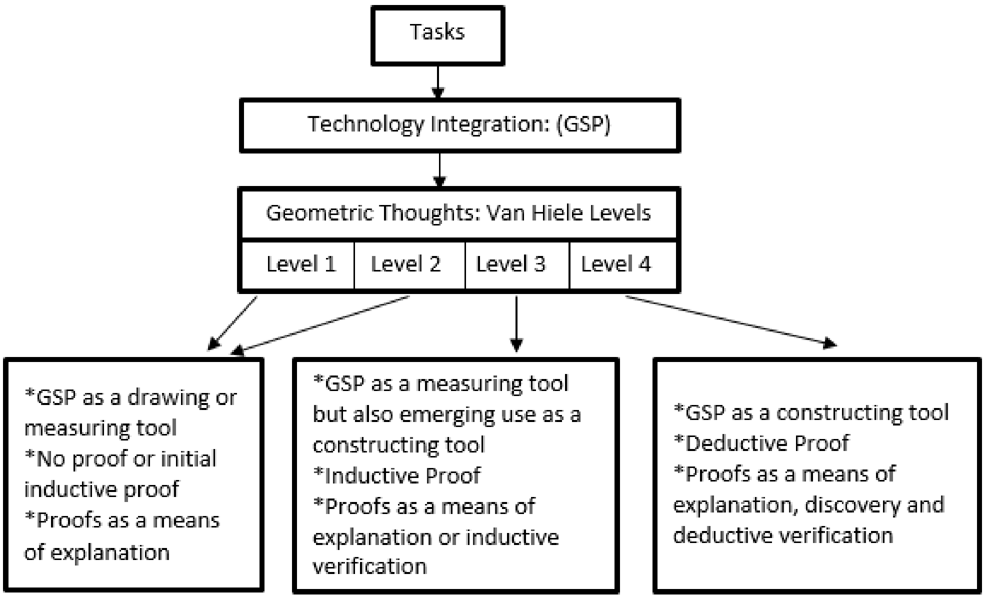

Figure 6. Impact of GSP technology on PSTs at different van Hiele Levels

The PSTs at van Hiele Level 4 appeared to regard proof as a means of explanation, discovery, and verification. That is, these PSTs mainly depended on the Construct function of GSP to verify why an observation or a statement was always true although they used the Measure function of GSP for confirmation. Also, in the process of explaining why a result was true, one PST discovered a new result and verified it (e.g. the types of the new quadrilateral created by connecting four midpoints in a quadrilateral could be different depending on what quadrilateral was given).

Some results, particularly the ways that PSTs justified the type of the quadrilateral in the first and the third tasks, are consistent with prior research (de Villiers, 2003) in that learners at van Hiele Level 1 depended on visual appearance of quadrilaterals, and learners at van Hiele Level 2 listed all properties of the given quadrilateral and would not allow the inclusion of special cases.

However, unlike prior research indicating that learners at van Hiele Level 3 were able to focus on a key feature of a quadrilateral to distinguish the quadrilateral and include special cases, the PSTs at van Hiele Level 3 in this study still demonstrated some difficulties identifying a key property to define the given quadrilateral. For this result, I offer two possible explanations. First, GSP integration might have acted as a constraint for the PSTs at level 3 but not for those at level 4, who tended to use the Construct function of GSP in more meaningful ways. Second, van Hiele Level 3 might be too broad to distinguish among different stages of geometric thought. That is, either there might be sub-levels, within van Hiele Level 3, especially when problem-solving is integrated with a technology tool, or some learners at level 3 might have difficulty engaging in problem-solving that requires them to map out the hierarchy of quadrilaterals based on the relationships among properties of quadrilaterals as would be expected for their level of geometric thought.

CONCLUSIONS

GSP technology could impose constraints until teachers and students become familiar with its use (Hollebrands, 2007). Nevertheless it can provide meaningful affordances for both teachers and students (Govender & de Villiers, 2003; Olive, 2000). From a teacher's perspective, in this study, GSP was useful for detecting learners' misconceptions about properties of a rhombus and areas of geometric figures because the ways they used the GSP reflected their geometric knowledge.

From the perspective of learners, using GSP provided opportunities for them to check their understandings by making connections among different areas of mathematical knowledge when they justified their observations. Also, GSP allowed learners to experience various functions of proofs beyond verification. In this study, learners at all four levels could be motivated to work on proofs by being convinced that a specific observation is true through GSP exploration (de Villiers, 2003), and at least they could experience proofs as a means of explanation. This result is also consistent with prior research. According to de Villiers (1991; 2003), with GSP technology, "the functions of proof such as explanation, discovery, and verification can be meaningful to students outside a systematization context, in other words, at van Hiele levels lower than van Hiele Level 3" (de Villiers, 2003, p. 18).

Implications

This study suggests some implications for ways to integrate technology into pre-service teacher education. First, when making technology part of instruction, mathematics educators need to consider PSTs' levels of mathematical thinking. In this study, PSTs demonstrated different uses of GSP technology depending on their van Hiele Levels. Thus, to effectively incorporate technology in the curriculum, mathematics educators need to go beyond technological and content knowledge, and acquire technological pedagogical content knowledge about PSTs' geometric thinking and the relationship between the levels of thinking and the use of technology (Niess, 2005).

Second, mathematics educators need to consider various instructional strategies within a specific technology framework. For example, in GSP-enhanced instructional activities, it could be helpful for PSTs to be exposed to the functions of proof such as explanation, discovery, and verification by allowing them to engage in arguments based on an intuitive or visual environment through using the Transformation function of GSP (i.e. reflect, translate, and rotate) as de Villiers (2003) recommended. From this experience, PSTs at lower van Hiele levels might raise their levels and, further, be ready to experience more advanced functions of proof such as deductive verification. Also, with regard to general instructional strategies in the integration of GSP, it seems to be beneficial for PSTs, at whatever levels of geometric thought, to be exposed gradually to different situations, moving from explanatory tasks (pre-made) to constructive tasks. According to Govender and de Villiers, GSP helps students make their own constructions to test geometric statements but it is recommended to first "provide them with ready-made scripts that they could play through step by step and observe as the figure was gradually constructed." (Govender & de Villiers, 2003, p. 57). Through this sequencing, PSTs would have opportunities to develop understanding of different functions of proofs or justifications embedded in the three types of tasks in a GSP environment.

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