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# Reflection as Professional Knowledge for Mathematics Teachers[1]

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In this study, we examined the prompted reflections of four middle school mathematics teachers after their lessons. We used Cohen and Ball's instructional triangle (1999) to investigate teachers' reflections. With this framework, we addressed questions of what characteristics in reflections the participant teachers have and how the reflections differ over time. Findings indicated that the teachers showed differences in the instances of assessing and changes over time in the ways they gained more insights about students' understanding.

*Keywords:* reflection, middle school, in-service teacher, professional knowledge
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## I. INTRODUCTION

While there are different perspectives on what constitutes professional knowledge, there is considerable agreement that professional knowledge for teachers includes those actions and understandings that teachers need to support their students' learning (Krainer, 1996; Darling-Hammond, 2008). Much emphasis has been placed on the importance of teachers' development of professional knowledge, including their content and pedagogical knowledge (e.g., Borko & Whitcomb, 2008; Li & Kaiser, 2011; Loucks-Horsley,

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Love, Stiles, Mundry & Hewson, 2003; Petrou & Gouldin, 2011).

Reflection (Schön, 1983) can be seen as a form of professional knowledge that allows teachers insights into their own practices. These insights can then promote changes in practice. Researchers have shown that reflection can help teachers develop understanding of their instructional practice (e.g., Artzt & Armour-Thomas, 2002; Ticha & Hospesova, 2006). Studies about mathematics teachers' reflection on action (Schön, 1983) have demonstrated that reflection can support their learning (e.g., Chamoso, Cáceres & Azcárate, 2012; Cooney, 1996), while other studies of reflection in action (Schön, 1983), such as teacher noticing, have shown connections to the development of teacher expertise (e.g., Sherin, Jacobs & Philipp, 2011; Sherin & van Es, 2005).

To capitalize on the promise of reflection for supporting better teaching, the professional development community needs to better understand how teacher reflection develops. In an earlier study (Kwon & Orrill, 2007), we found that one case study participant changed how she perceived and discussed classroom teaching and learning as she engaged in reflective activity over time. Reviewing literature on reflection, most studies emphasized the shifts of reflection (Lyons, 1998; van Es & Sherin, 2010) or the development using reflection (Artzt & Armour-Thomas, 2002; Freese, 1999; NCTM, 2007; Scherer & Steinbring, 2006; Ticha & Hospesova, 2006) without investigating the characteristics of reflection. Therefore, in the present study, we extend our earlier study to explore characteristics of mathematics teachers' reflections across multiple teachers.

For this study, we addressed two research questions:

1) What characteristics do the participating teachers' reflections exhibit? and
2) How do the teachers' reflections shift over time?

For the purposes of this study, we define reflection as the act of a teacher interpreting her own practices and students' thinking. We assert that engaging in this kind of activity can impact how teachers think about their students, their teaching, and themselves as teachers. This study contributes to the growing understanding of how mathematics teachers make of sense their practices and their students' thinking.

## II. BACKGROUND

### 1. Teacher Reflection and Noticing

Existing research has provided insight into what teacher reflection means and how reflection could affect teachers and their practice (e.g., Artzt & Armour-Thomas, 2002; Scherer & Steinbring, 2006). Artzt and Armour-Thomas (2002) contributed a definition for reflection as thinking about teaching and asserted that reflection could help teachers

gain an intuitive understanding of their instructional practice. Scherer and Steinbring (2006) studied a *joint reflection* in which teachers carried out reflections of teaching with researchers. The collaboration helped participating teachers better understand their students' learning which led them to become more aware of the relationship between teacher and students. This finding was consistent with our earlier study (Kwon & Orrill, 2007).

In one of the most mature lines of research on teacher reflection, Sherin and her colleagues (e.g., Sherin & Han, 2004; Sherin, Jacobs, & Philipp, 2011; Sherin & van Es, 2005; van Es & Sherin, 2008, 2010) developed the concept of *teacher noticing* using video clubs. These studies showed that by participating in video clubs teachers changed the ways they talked about classroom interactions. Specifically, the teachers shifted to focus more on students than on themselves as the teachers, commented more on students' mathematical thinking, and provided more detailed information as they discussed classroom events. The researchers asserted that the video clubs played a crucial role in changing teachers' perspectives because the video clubs encouraged teachers to examine students' mathematical ideas and to reflect on their teaching practice.

Our study is relevant to the work of Sherin and her colleagues in that we engage teachers in reflecting on their own practice using videos. However, unlike Sherin's efforts, we focus teachers' reflection on students' thinking rather than focusing on instruction more broadly. Because reflection in our study was undertaken to understand how teachers make sense of their students' reasoning, there is limited dialogue between the teacher and the researcher during reflection sessions. In essence, we isolated one facet of the video club model—teacher reflection on videos—to determine whether teachers' discussion changes through engagement with the videos without substantial feedback or scaffolding.

## 2. Professional Knowledge and the Act of Reflection

Professional knowledge has been a topic of interest to researchers for decades. Shulman's (1986) seminal discussion of the knowledge teachers need for effective teaching are now nearly three decades old, yet considerable research is still being conducted on all aspects of those propositions (see for example Manizade & Mason's (2011) synthesis of research on pedagogical content knowledge; Ben-Peretz (2011) review of research on teacher knowledge; or the work on mathematical knowledge for teaching from Ball, Thames & Phelps (2008)). Shulman (1986) identified three key kinds of knowledge teachers need: pedagogical knowledge, content knowledge, and pedagogical content knowledge. We argue that reflection is particularly useful for developing pedagogical content knowledge (PCK) which Shulman defined as knowledge that "goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge *for*

*teaching*" (p. 9, emphasis in the original).

Reflection is one tool for helping a teacher make critical connections between her own understanding of content, her understanding of how her students understand content, and the materials she is using to support students' learning (Cohen & Ball, 1999). This is consistent with the framework proposed by Silverman and Thompson (2008) who proposed a framework for thinking about how to support teachers in moving from having personally meaningful mathematical understandings and being able to teach that mathematics in ways that allowed students to develop their own meaningful understandings. They suggested that a teacher needs to have understandings of mathematics that serve as "powerful springboards for understanding" (p. 502) first. Then, the teacher needs to have opportunities to understand how powerful mathematical ideas can support their students' mathematics learning and the actions that a teacher could take to support the students' development of the powerful mathematical ideas. We see the act of reflection as being one means for teachers to make this transition between having personally-meaningful powerful mathematical ideas to having ways of supporting students in building the same. Hence, reflection is one of the professional knowledge that can support pedagogical content knowledge.

Situating MKT within the Silverman and Thompson (2008) framework, we view reflection as the teacher's interpretation of her own practices, interpretation of students' learning, and the teaching moves that could be made to foster understanding. Reflection includes the teacher's ability to design, practice, and reflect on his/her teaching practices and students' learning. How teachers reflect on teaching and learning may ultimately affect their practice (NCTM, 2007). Therefore, understanding how reflection focuses teachers' attention on their practice over time is an avenue worth considering.

## 3. Theoretical Framework

Teachers' professional knowledge is the knowledge that allows teachers to teach their subject matter using pedagogical skills and principles appropriate for their learners (Ben-Peretz, 2011). It is related to content, pedagogy, and students' cognition within a given context. Thus, the investigation of reflection as professional knowledge requires reflection to be grounded in the learning environments in which teachers interact with students. Classroom learning environments are shaped by interactions between the teacher, instructional materials, and students (e.g., Cohen & Ball, 1999). Using this framework in the present study, the interactions among the elements become the critical aspect for shaping classroom instruction rather than any of the individual elements on their own. Reflection offers teachers an opportunity to consider interactions from the lessons to better understand the classroom instruction of which they were a part. Further, as we saw in our

earlier case study, the teacher's perspective about the interactions in the classroom can change, thus fundamentally altering the focus of reflection as well as the teacher's interpretation (Kwon & Orrill, 2007). We posit that this kind of reflection could be a critical element for improving instruction because it ties together the practice and the outcomes in a way that simply looking at any one element of the learning environment cannot.

## III. METHODOLOGY

This research builds on our previous work (Kwon & Orrill, 2007; Kwon, 2010), in which we found that the instances in which the teacher extended reflection to focus on herself increased dramatically, while her tendency to focus on assessing student learning (e.g., 'they are right') decreased during a single mathematics unit. More importantly, we found that her reflections began to include more complex interpretations in which she tied her teaching practices to student learning. Given the promising findings from that study, we decided to replicate our analysis with a larger sample of mathematics teachers for this study.

The present study is an exploratory study of four mathematics teachers. The data were collected as part of a larger project focused on students' and teachers' knowledge, interactions, and sense making of shared events. These data included daily observations of each teacher working with a single class of students using the *Connected Mathematics Project* (CMP) materials (Lappan, Fey, Fitzgerald, Friel & Phillips, 2002) for an entire unit of instruction (typically 8–10 weeks). From Spring 2003 to Spring 2006, project teams conducted case studies in Pierce Middle School[2] in which they videotaped teachers' classrooms and student interviews about events in those classrooms.

### 1. Participants

We considered the cases of four teachers in this study. Three of the teachers were experienced mathematics teachers (see Table III-1). The fourth was a new sixth-grade teacher who had experience teaching one seventh-grade CMP unit while serving as a long-term substitute in the year prior to her case study. We chose to consider these four teachers because the variance in their experience could give us information about their reflections and differences among them for this study.

The data analyzed for this study came from our first case study with each of the teach-

[2] All names are pseudonyms consistent with those used in other publications of the CoSTAR project.

ers (Table III.1), though Ms. Moseley[3] had participated in a two-week pilot study before these data were collected. Ms. Moseley and Ms. Reese were in their second year of implementation of the materials during this study while the seventh-grade teacher, Ms. Bishop, was in her third year of implementation at the time of data collection. Ms. Archer was using the sixth-grade materials for the first time.

**Table III.1.** Participant demographics

| Teacher | Grade Level | Years Teaching | Degree/Certification | Unit Title | Case Length |
|---|---|---|---|---|---|
| Ms. Archer | 6 | <1 | B.S./In Alternative Certification Program | *Bits & Pieces I* and *Bits & Pieces II* | 15 wks |
| Ms. Bishop | 7 | 12 | Ed.S./Middle Grades | *Variables & Patterns* | 7 wks |
| Ms. Moseley | 6 | 11 | Masters Student/Middle Grades | *Bits & Pieces II* | 10 wks |
| Ms. Reese | 6 | 16 | M.Ed./Middle Grades | *Bits & Pieces II* | 12 wks |

## 2. Data Collection

We relied on videotaped interviews for this analysis. In each interview, the teacher reflected on videotaped episodes from her classroom, selected by the research team, that were shown on a laptop computer. Each interview also provided the opportunity for the teacher to see excerpts from student interviews conducted with several pairs of students from her own class. In each interview, students were asked to reflect on video segments from the class and/or to work mathematics tasks similar to those assigned in the classroom videos so that their mathematical understanding could be more fully explored.

Each teacher was interviewed weekly. The interview focused on events that had happened since the previous interview, and typically they were of events from one to three days before the interview. In each interview, the interviewer asked the participant to discuss the mathematics and the learning in the selected scene—for instance, by recalling and explaining classroom incidents or by evaluating students' work.

For this study, we selected three interview transcripts from each teacher's case: the first interview in which each teacher was asked to discuss her students' thinking, an interview from the approximate middle of the case study, and the final interview for each case study. The videos were transcribed verbatim. The transcripts were the primary data analyzed for this study; however, the videos were used to clarify any points of confusion

[3] Ms. Moseley was the participant in our earlier work (Kwon & Orrill, 2007), but that study focused on a subsequent case study. This study focused on her first full-length case study with us.

from the transcripts.

## 3. Data Analysis

The analysis occurred in two stages. In the first stage, we engaged in a data reduction process by identifying specific transcript segments to be analyzed. The first author analyzed each of the three transcripts according to each question being asked of the teacher. Building on Cohen & Ball's (1999) triangle of interactions, the questions were classified as

(a) Questions about teaching,
(b) Questions about students, and
(c) Questions about curriculum and materials.

Questions about logistics of the research were also coded. For this analysis, we chose to focus only on those interactions that arose from questions about students. This decision built from research by Sherin and her colleagues (Sherin & Han, 2004; Sherin & van Es, 2005; van Es & Sherin, 2008) that indicated that teachers became more focused on students' thinking as they watched classroom interactions over time.

In the second stage of analysis, both authors analyzed the interviews using a set of categories and then, using *inductive analysis* (Patton, 2002) for understanding teachers' interpretation of student thinking. Our initial categories, drawn from Wallach and Even (2005) included: (1) Assess, (2) Describe, (3) Interpret, (4) Justify, (5) Extend.

Assess instances included those in which the teacher assigned a categorical review of the student's work or thinking. For example, 'this is good' or 'he didn't get it.' Describe instances were verbatim descriptions of actions that occurred without commentary. For example, Ms. Archer described one episode by saying, "I told them to think about multiples of three and multiples of two." The most prevalent category was Interpret in which the teacher provided her understanding of what the students were doing or thinking. For example, in interpreting one student's fraction addition, Ms. Reese commented, "He was trying to associate what he learned in fifth grade and he forgot that you don't add the denominator. That's what I think." Justify instances provided a clear rationale for beliefs about what was happening. For example, in providing a justification for suggesting a pedagogical strategy to understand fractional parts, Ms. Moseley provided the following justification: "…because I think they could probably do it if they had a pizza in front of them or they might make a mess and we'd all end up with slices like half an inch wide, you know." The Extend category emerged from our own analyses and was not part of the Wallach and Even (2005) coding scheme. It included interactions in which the teacher reflected on her teaching, her understanding of the materials, or her beliefs about the

materials. For example, Ms. Moseley continued her discussion of pizzas saying, "…What would have been nice to do, I think, at the end – let's draw pizzas in half and see where we go. Let's draw pizzas in thirds and see where we go. Draw it in fourths, one they've already done their discovery at their desks and let them see. But, we just didn't have time to fully do that lesson."

Both researchers identified and coded the instances in each of the selected transcripts. An instance could range from a phrase to several sentences and it was defined as a segment of transcript that was focused on a single topic and that fit within only one category. A single paragraph could have one or many instances within it, depending on what the teacher discussed. Once each researcher completed this analysis independently, we met to determine our level of agreement in both instances and categories. All disagreements were resolved in those meetings.

## IV. RESULTS

### 1. What characteristics in reflections do the participant teachers exhibit?

**Table IV.1.** Percentage of instances by teacher

| | **Ms. Moseley** | | | **Ms. Reese** | | |
|---|---|---|---|---|---|---|
| **Date** | **3/13/03** | **4/08/03** | **5/22/03** | **3/17/03** | **4/08/03** | **5/13/03** |
| **Assess** | 6.90 | 15.56 | 14.29 | 16.13 | 6.56 | 4.17 |
| **Describe** | 16.09 | 8.89 | 5.36 | 12.90 | 9.84 | 8.33 |
| **Interpret** | 42.53 | 46.67 | 41.07 | 35.48 | 49.18 | 43.75 |
| **Justify** | 9.20 | 15.56 | 16.07 | 8.06 | 13.11 | 8.33 |
| **Extend** | 25.29 | 13.33 | 23.21 | 27.42 | 21.31 | 35.42 |
| **Total** | 100 | 100 | 100 | 100 | 100 | 100 |
| | **Ms. Archer** | | | **Ms. Bishop** | | |
| **Date** | **2/10/04** | **3/01/04** | **5/25/04** | **3/17/03** | **4/08/03** | **5/13/03** |
| **Assess** | 13.04 | 10.45 | 10.13 | 3.85 | 9.52 | 10.64 |
| **Describe** | 26.09 | 5.97 | 10.13 | 11.54 | 11.90 | 8.51 |
| **Interpret** | 13.04 | 50.75 | 44.30 | 55.77 | 30.95 | 40.43 |
| **Justify** | 13.04 | 11.94 | 11.39 | 3.85 | 10.71 | 10.64 |
| **Extend** | 34.78 | 20.90 | 24.05 | 25 | 36.90 | 29.79 |
| **Total** | 100 | 100 | 100 | 100 | 100 | 100 |

Note: Due to rounding, some of the columns do not add up to 100%.

To find the characteristics in reflections, we considered differences and similarities among the four mathematics teachers' reflections in their approaches to analyzing their students' understanding. One of the key differences we found among the four teachers was in the ways and the extent to which they interpreted their students' understanding. Initially, we only considered the relative percentage of instances of each category for

each teacher (see Table IV.1). We analyzed the results of post surveys in this section. On the 8th week of the 1st semester in 2011 and on the 1st week of the 2nd semester in 2014, students participated in the surveys.

However, we quickly realized that for these participants the relatively frequency provided insufficient data for understanding their reflection-on-action (Schön, 1983). Therefore, we conducted further analysis of the content of each of the instances and found which uncovered interesting patterns in the instances of Assess and Extend.

### *A. Patterns in the instances of Assess*

The four teachers' instances of *Assess* showed different patterns as presented in Table IV.1. Ms. Moseley and Ms. Bishop increased in their frequency of Assess instances, but Ms. Reese and Ms. Archer had a decline in these instances. Further analysis showed a difference between the two pairs of teachers. During interviews, Ms. Moseley provided 17 positive comments of her 21 assess instances (80%) and Ms. Bishop offered 12 positive comments of the 15 assess instances (80%). Most of these positive Assess instances focused on students' understanding or things that the students were able to do. For example, "She got the right percent though, so that is good" (Ms. Moseley). "I think she understood the concept" (Ms. Bishop). In contrast, Ms. Reese and Ms. Archer talked about their students' misunderstanding or things that the students were not able to do when they made Assess comments. For example, "She's misunderstanding that they're different sizes" (Ms. Reese) and "Yeah… he would've found some stuff out. It wouldn't have come out right" (Ms. Archer). Of Ms. Reese's 16 Assess instances, 11 were negative (69%). Ms. Archer's Assess instances included 12 negative comments of the 16 Assess comments made (75%). It is possible that engaging in reflective examination of student understanding allows teachers to focus more on positive aspects of understanding.

Based on our analysis, the decreasing pattern of the instances of Assess of Ms. Reese and Ms. Archer aligned with their increase in discussion of their students' understanding. For developing professional knowledge, moving away from focusing on students' lack of understandings and toward focusing on how students understand or why they are struggling to make sense of a topic seems worthwhile for mathematics teachers because there is more likely to be an actionable outcome. Therefore, we see the movement away from focusing on what the students do not understand indicating that these teachers were becoming more sophisticated in their ability to reflect on students' understanding.

### *B. Patterns in the instances of Extend*

The instances of Extend highlighted aspects of participants' reflection related to their teaching (Table IV.2). In this study, Extend instances included those interactions in which

the teachers reflected on their teaching or their understanding of the materials. The Extend category was our attempt to capture those instances that included a primary focus on the teacher or the materials. We noticed that when our participants were asked questions about their students, they often responded in ways that included reflection on their practices as the teacher or in ways that linked student understanding to certain aspects of the CMP materials. Hence, we examined the instances of Extend in-depth. The instances included comments that Reported, Evaluated, Reasoned, or Reconstructed situations as they related to their students' understanding, and assessed aspects of the curriculum materials.

**Table IV.2.** Number of Instances of Extend

| **Extend instances** | **Ms. Moseley** | | | **Ms. Reese** | | |
|---|---|---|---|---|---|---|
| **Date** | **3/13/03** | **4/08/03** | **5/22/03** | **3/17/03** | **4/08/03** | **5/13/03** |
| **Report** | 4 | 0 | 0 | 8 | 7 | 2 |
| **Evaluation** | 4 | 1 | 4 | 1 | 0 | 2 |
| **Reasoning** | 3 | 2 | 4 | 4 | 5 | 5 |
| **Reconstruct** | 8 | 2 | 3 | 2 | 1 | 4 |
| **Others** | 2 | 1 | 2 | 2 | 0 | 4 |
| **Total** | 21 | 6 | 13 | 17 | 13 | 17 |
| **Extend instances** | **Ms. Archer** | | | **Ms. Bishop** | | |
| **Date** | **2/10/04** | **3/01/04** | **5/25/04** | **3/17/03** | **4/08/03** | **5/13/03** |
| **Report** | 1 | 2 | 3 | 6 | 9 | 2 |
| **Evaluation** | 2 | 0 | 2 | 0 | 5 | 2 |
| **Reasoning** | 3 | 7 | 5 | 6 | 7 | 2 |
| **Reconstruct** | 0 | 3 | 4 | 0 | 6 | 4 |
| **Others** | 2 | 2 | 5 | 1 | 4 | 4 |
| **Total** | 8 | 14 | 19 | 13 | 31 | 14 |

A) Report

Extend instances that Report focused on the teacher's instructional moves or instructional moves described in the instructional materials. These differed from Describe instances because they focused on teaching or materials rather than students. For instance, Ms. Reese explained one situation saying, "I was saying that the reason we're not drawing pictures like brownie pans or thermometers is because its gotten a lot more complicated because its mixed numbers".

B) Evaluate

The subset of Extend instances that we identified as Evaluate involved the teachers critiquing their teaching practice. For example, Ms. Moseley reflected on one video clip focused on a student's fraction model saying:

> ... I started to say I think I should've directed them more that way in the beginning, but then that would have taken away my fifth people that got there…I could have made them get tenths probably a little more easily, but I think it was very worthwhile for the people that found the fifth way.

This was Evaluate because she analyzed and critiqued her instructional decision.

C) Verify

Verify instances in the Extend category were those in which the teachers gave their rationales for instructional moves in particular situations. These instances were different from Justify instances in that they were not focused on the students' thinking, rather they were focused on the teachers' discussion of their instructional moves. These instances were crucial for providing insight into teachers' understandings of their practices. For example, Ms. Bishop discussed a student's understanding of "230 over 100" adding, "That's why I asked her what did we do with 100?". Ms. Bishop assumed that her student knew 230 over 100 means 230 divided by 100.

D) Reconstruct

In these interviews, Reconstruct instances were those in which the teachers reconstructed a situation based on assumptions about how things may have been different had one element of the interaction changed. Typically, the teacher pondered, "what if…" and then presented an alternative path that could have happened in the classroom setting. For example, Ms. Moseley reflected on a fraction problem in which students were asked to share eight pizzas among ten people. After watching a video clip and recalling what happened in class, she proposed a new scenario saying, "I think there might have been a situation where rather than drawing, if we could have had, if I could have actually had manipulatives on the table… To see if they could sort of see, okay, I've got 10 people… maybe with the hands-on they could have seen more". Ms. Moseley's reflection led her to conceive of a hypothetical situation and project what might have happened based on the hypothetical change in her instructional move.

E) Other

Comments about mathematics or other concerns related to teaching materials were coded as "Other". These instances only included comments about mathematics or curriculum without any relation to students' learning. For instance, Ms. Archer reflected on a video clip in which a student used a fraction strip in the class. To divide the paper strip into fifths, Ms. Archer's students folded the strip into four pieces from the center and made two little folds on the end. She commented on this saying, "The concept was to

have five equal parts in the strip…when you fold it, it should be five equal parts". We counted this as Other because it was a comment that explained Ms. Archer's mathematical understanding of the activity in which she had asked her students to engage.

## 2. How do the teachers' reflections shift over time?

### *A. Change in Novice/Expert Teachers*

As mentioned previously, Ms. Archer was the sole first-year teacher in this sample. We found that in her first interview, Ms. Archer used many Descriptions and only a few Interpretations. However, in later interviews, she included more Interpretations. By the later interviews, her pattern of instances was not substantially different from those of the other participants. We assert that this shift may be linked to her development of new PCK (Shulman, 1986) throughout the study. As a new teacher, she may have been limited in her ability to interpret classroom instances when we began. However, through her experiences in the classroom and in our study, she was learning to notice.

Ms. Archer's shift from Description to Interpretation suggests that providing this novice teacher with opportunities to watch and analyze her students' work with the mathematics in her classroom supported her in moving quickly to analyzing and interpreting her students' work. If this is typical, it could suggest that reflecting on student understanding can help develop professional knowledge for teaching rapidly.

### *B. Increased Incidents of "No Idea"*

Rather than becoming clearer and less hesitant in their reflections on students' understanding, these participants increased the frequency with which they declared that they did not understand a student's thinking saying, "I don't know how he/she got this" or "I have no idea" when asked about a student's thinking. We speculate on two different explanations for this. Our first hypothesis is that reflection heightened teachers' awareness of individual student thinking, thus causing them to question their preconceived ideas about student understanding. Our second hypothesis was that teachers were becoming more comfortable with the interviewers and felt safer admitting when they did not understand a student's thinking. Regardless of the cause of this increase, it was clear that the teachers did not always know how to interpret their students' thinking.

### *C. Change in the instances of Extend*

As noted above, Extend instances included a wide variety of comments including reporting, evaluating, reasoning, reconstructing teacher moves as they related to their students' understanding, and assessing aspects of the curriculum materials. The novice

teacher in this study, Ms. Archer provided the highest percentage of instances of Extend at the outset. However, the ratio of Extend comments decreased in later interviews. We found that Ms. Archer shifted over time to include fewer reasoning comments and more reconstructing comments (See Table IV.2). Careful analysis showed that while there was a proportional decrease in Ms. Archer's Extend comments, the depth of those comments increased as Ms. Archer began to spend less time reasoning about just her teaching practice and more time reconstructing her thinking about teaching as it related to her students.

This shift was noted in the other teachers' reflection as well. Generally, they shifted from reporting on or reasoning about their teaching to considering various aspects of the learning environment including evaluation of their teaching or curriculum materials. For example, in their first interviews, the teachers commented specifically on only a few aspects of teaching such as reporting what they did and providing their rationales for teaching moves. However, in later interviews, the teachers generally reflected on aspects of teaching while also talking about student thinking. This suggests that as they gained experience with analysis, these teachers were more able to coordinate their teaching with student thinking. Building from our theoretical framework, this shift suggests that these teachers attended to the interactions between the interplay of the elements of the interaction triangle (Cohen & Ball, 1999) rather than just the elements (e.g., teachers, materials, and students).

## V. CONCLUSION AND DISCUSSION

### 1. Summary

The aim of this study was to investigate in-service teachers' reflection-on-action related to student understanding. We found that there were shifts in the tone of the reflections and that, consistent with prior research (e.g., Kwon & Orrill, 2007; Kwon, 2010; Sherin & Han, 2004; Scherer & Steinbring, 2006), the participants shifted to thinking more about students and interactions among elements of the instructional triangle (Cohen & Ball, 1999).

The tone of the interview changed over the course of the case studies. At the outset, Assess statements were either positive (Ms. Moseley and Ms. Bishop) or focused on negative aspects of students' understanding (Ms. Reese and Ms. Archer). The teachers who used Assess to highlight positive aspects increased in the number of Assess instances across the case. In contrast, those who used Assess to focus on shortcomings moved away from Assess statements over time. This change suggests that the teachers moved to focus more on how students do think rather than simply categorizing that thinking as they

began to relate that understanding to their own practices.

Consistent with Sherin & Han's (2004) study, these participants shifted to focus more on student conceptions than pedagogies. Engaging these teachers in reflection promoted attention to student understanding as a product of classroom interactions. Furthermore, these participants became more sophisticated in their ability to link teaching practices, the materials, and their students' thinking. This extends the result of Sherin & Han (2004) in an important way. Our study focused on one-on-one interactions between the teacher and the researcher rather than group interactions among teachers and, in our study the researcher selected the video rather than the teacher. Then our findings similar to Sherin & Han's suggest that the reflection itself, rather than details surrounding it, might be the critical feature for invoking these sophisticated conversations.

## 2. Discussion

From this study, we suggest three ways related to encouraging teacher reflection: opportunity to discuss students' understanding, preparation of appropriate data, and systematic assistance. First, the opportunities to discuss students' understanding help teachers gain insights into their students and their teaching practices. To become a reflective practitioner, Artzt & Armour-Thomas (2002) suggest several ways for teachers to reflect on their own lessons. Based on our findings, we propose that simply having an opportunity to reflect and describe students' thinking to another person is powerful. Second, we note the importance of data preparation in supporting reflection. Reflection is successful when it is based on data such as classroom discussion or students' notes. In this study, we provided the video clips that allowed the participant teachers to revisit their classrooms or learn more about their students' thinking. Reflection based on tangible evidence from the teacher's classroom such as video or student work supports teachers in rebuilding their experience. Finally, we suggest systematic assistance from schools or school districts to support teachers in engaging in reflection activities. We believe that, as Sherin and her colleagues (Sherin & Han, 2004; van Es & Sherin, 2008) point out, facilitators are important to start the discussion and ask questions about significant moments. The systematic assistance includes many ways to provide facilitators to discuss students' thinking from school districts, to permit times for reflecting on lessons and teaching practices from schools, etc.

Researchers can examine various aspects and features of teachers' reflection from preparation to assessment in further study. The current study only examined teachers' reflection-on-action as prompted in one-on-one interviews. Our findings suggest that using reflection may be one way to increase teachers' professional knowledge. And, we note that such experiences allow teachers to move from focusing only on their students or

their teaching, to focusing on the interaction between their teaching practices and their students' learning. As stated earlier, previous studies have mostly focused on teachers' changes in their reflection. While investigating shifts in teachers' reflection is important, we assert that researchers should also consider reflection that occurs in the process of teaching. We expect further studies on reflection-in-action beyond what we have investigated in our study. This may be linked to recent teacher research such as self-study (e.g., Samaras & Freese, 2006).

To extend the research area of teacher reflection, teacher educators need to examine the features of teachers' reflection in each stage from teacher preparation to professional development. In our study, novice teachers show different patterns from expert teachers in the assessment instances. Further studies may capture this kind of variation in different stages of teachers. The studies on reflection in different stages of teachers help teacher educators design and run teacher education programs for preservice teachers and professional development programs for in-service teachers.

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