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Review of Data-Driven Multivariate and Multiscale Methods

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** Regular Paper*

** Review Paper: This paper reviews the recent progress possibly including previous works in a particular research topic, and has been accepted by the editorial board through the regular reviewing process.*

***Abstract*:** In this paper, time-frequency analysis algorithms, empirical mode decomposition and local mean decomposition, are reviewed and their applications to nonlinear and nonstationary real-world data are discussed. In addition, their generic extensions to complex domain are addressed for the analysis of multichannel data. Simulations of these algorithms on synthetic data illustrate the fundamental structure of the algorithms and how they are designed for the analysis of nonlinear and nonstationary data. Applications of the complex version of the algorithms to the synthetic data also demonstrate the benefit of the algorithms for the accurate frequency decomposition of multichannel data.

***Keywords*:** Nonlinearity, Nonstationarity, Empirical mode decomposition, Local mean decomposition, Multivariate algorithm

1. Introduction

Conventional frequency decomposition algorithms, such as Fourier analysis, mostly have assumptions on the data, stationarity and linearity, modeling input data using fixed basis functions [1]. In particular, Fourier analysis is based on linear and orthogonal sinusoids and estimates the frequency components for long period data, which is not possible to but cannot monitor time-varying frequency components. However, most real-world signals, such as like electroencephalogram (EEG) and electrocardiogram (ECG) are nonstationary and nonlinear, so that it is suboptimal to decompose their frequency components using conventional algorithms.

When it comes to the analysis of multichannel data, such as EEG and ECG, there are few algorithms to decompose it considering the information of the multichannel input simultaneously. However, the use of common oscillations across the multichannel data would improve the performance of the frequency decomposition. For example, a study of the relationship among multichannel EEG has been emphasized to understand the brain network in neuroscience [2]. Therefore, a univariate algorithm only for single channel decomposition could have a limit for an accurate multivariate data analysis.

This paper reviews time-frequency decomposition algorithms, empirical mode decomposition (EMD) [1] and local mean decomposition (LMD) [3], which can deal with the nonlinearity and nonstationarity of the real-world data. Both EMD and LMD are fully data-driven approaches to producing adaptive basis functions depending on the input data; thus, they do not need to fit the data to a set of fixed basis functions. In addition, the decomposed components using EMD are well designed to produce narrowband signals, which is a suitable form to apply the Hilbert transform for an estimation of the instantaneous frequency (IF) [1].

Recently, EMD and LMD were extended to a complex version to decompose complex data. These complex algorithms can be applied to multichannel data analysis, more specifically bivariate data. Complex data formed by putting one channel data in the real part and the other in the imaginary part can be decomposed using the complex version of EMD and LMD, through which the decomposed components can be estimated by considering the common oscillations between two channels. In that way, the complex version (or bivariate) EMD and LMD produce more accurate frequency components compared to the other univariate methods.

Section 2 addresses the background of the time-frequency analysis and instantaneous information of non-stationary data. In addition, the basic theory of the EMD algorithm is reviewed in section 2 with its extension to the complex domain, bivariate EMD. Section 3 introduces the LMD algorithm and its complex version. The final section, Section 4, concludes the review of these data-driven time-frequency decomposition algorithms.

2. Empirical Mode Decomposition

Empirical mode decomposition is a data-driven time-frequency analysis algorithm, producing amplitude modulation (AM)/frequency modulation (FM) components of an input signal [1]. In particular, there is no assumption of linearity or stationarity of the input signal in the process of EMD, which makes it possible to extract more accurate features compared to other conventional time-frequency decomposition algorithms, such as Fourier and wavelet analysis.

2.1 Background

In general, there are several obstacles to analyze real-world data in the study of natural science and engineering, such as [4]

- Nonlinear data,
- Nonstationary data and
- Short length of data.

Fourier analysis is the most popular spectral analysis for extracting the frequency components in a signal. However, the length of data should be long enough for Fourier analysis, which is impractical for the analysis of a nonstationary signal.

The frequency components in a signal keep changing with time, causing the nonstationarity of a signal, which cannot be modeled using Fourier analysis due to the assumption of the algorithm, sufficient length of data. The short-time Fourier transform (STFT), which involves segmenting input data into signals with a short length and estimating the frequency information for them, has been applied to obtain the time information as well as the frequency components. Nevertheless, it still limits the estimation of low frequency components, whose time periods are longer than the length of the segmented data. In addition, STFT still uses the basis functions of sine and cosine, which are suboptimal for the analysis of real-world signals that do not have consistent patterns like them. Wavelet analysis is also a popular time-frequency analysis algorithm, which can dynamically change the length of the basis function to deal with most of the frequency components more efficiently than STFT. Therefore, wavelet analysis could be a better option for solving the problem of the nonstationarity of real-world signals. On the other hand, it still uses wavelet basis functions, called the mother wavelet, and it is still suboptimal for the analysis of nonlinear signals.

2.2 Instantaneous Frequency using Intrinsic Mode Function

Instantaneous frequency can provide more realistic frequency information of real-world signals compared to Fourier and wavelet analysis [4]. The Hilbert transform defined below was used to estimate the instantaneous frequency.

$$\mathbf{Y}(\mathbf{t}) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{X(t')}{t - t'} dt' \tag{1}$$

where $\mathbf{X}(\mathbf{t})$ is the input data and P Cauchy principal value. The Hilbert transformed signal and original input form the analytic signal

$$\mathbf{Z}(\mathbf{t}) = \mathbf{X}(\mathbf{t}) + \mathbf{jY}(\mathbf{t}) = \mathbf{a}(\mathbf{t})\mathbf{e}^{\theta(\mathbf{t})} \tag{2}$$

where $\mathbf{a}(\mathbf{t})$ and $\boldsymbol{\theta}(\mathbf{t})$ are the instantaneous amplitude and phase derived by

$$\mathbf{a}(\mathbf{t}) = \left[\mathbf{X}^2(t) + Y^2(t)\right]^{\frac{1}{2}} \tag{3}$$

$$\boldsymbol{\theta}(\mathbf{t}) = \mathbf{arctan}\left(\frac{\mathbf{Y}(\mathbf{t})}{X(t)}\right) \tag{4}$$

In addition, the instantaneous frequency can be obtained by differentiating the instantaneous phase such as [5]

$$\boldsymbol{\omega}(\mathbf{t}) = \frac{\mathbf{d}\boldsymbol{\theta}(\mathbf{t})}{dt}. \tag{5}$$

A signal is called a monocomponent signal when it contains only one frequency component at every instant of time, and this signal can obtain reliable and accurate information of the instantaneous frequency [6]. A monocomponent signal can be defined as a narrowband signal, meaning a signal whose number of extrema is different from the number of its zero-crossing at most by one [7]. Furthermore, the signal needs to be symmetric across zero to produce an accurate instantaneous frequency, as suggested by Norden Huang [1].

2.3 Empirical Mode Decomposition

Empirical mode decomposition was developed by Norden Huang to decompose a signal into its intrinsic mode functions (IMFs), which contains only one frequency component at each time point as a monocomponent/narrowband signal [1]. The IMF needs to meet two conditions in order to be a monocomponent signal, i.e., (1) the number of extrema and the number of zero crossings differ at most by one, and (2) the mean of

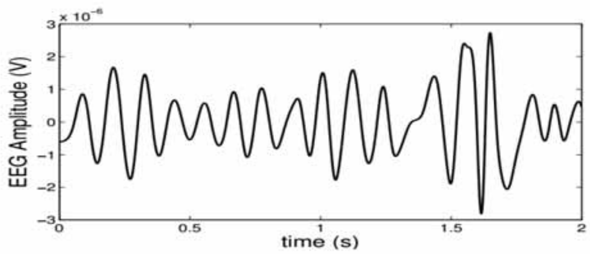


Fig. 1. Typical intrinsic mode function [4].

Table 1. Steps of the empirical mode decomposition algorithm [1].

The standard EMD algorithm
1. Let $\tilde{x}(t) = x(t)$ ($x(t)$ is an input signal)
2. Find all local maxima and minima of $\tilde{x}(t)$
3. Identify a upper and lower envelope, $e_u(t)$ and $e_l(t)$, interpolating all local maxima and minima
4. Estimate the local mean, $\overline{m} = (e_l(t) + e_u(t))/2$
5. Subtract $\overline{m}(t)$ from $\tilde{x}(t)$, $c_i(t) = \tilde{x}(t) - \overline{m}(t)$ (i is an order of IMF)
6. Let $\tilde{x}(t) = c_i(t)$ and go to step 2) and repeat the same process until $c_i(t)$ becomes an IMF

the local maxima and minima is approximately zero. Fig. 1 displays a typical intrinsic mode function.

There is no basis function used in the process of EMD algorithm and it is fully data-driven method. Thus, it is best to analyze a nonlinear signal and extract more reliable monocomponent signals to estimate accurate instantaneous frequencies compared to other conventional methods. The details of the algorithm are listed in Table 1 and Fig. 2 shows the flow chart of the algorithm. In short, the local mean value is subtracted from the input data repeatedly until the residue becomes an IMF. The same process is then conducted using the residue obtained by subtracting the IMF from the input signal until it contains no more oscillations. The input signal, *x(t)*, can be represented using the IMFs, $c_i(t)$, as follows:

$$\mathbf{x}(\mathbf{t}) = \sum_{i=1}^{M} c_i(t) + r(t). \quad (6)$$

Fig. 3 shows the IMFs of an electroencephalograph (EEG) decomposed using EMD.

2.4 Multivariate Empirical Mode Decomposition

To analyze the multichannel data, the standard EMD algorithm has a limit for two reasons. First, each channel data would have a different number of IMFs, and the same order of IMFs of two different datasets cannot guarantee they have the same frequency components. Second, with the mode-mixing problem, a mode represented in a wrong order of IMF, as shown in Fig. 5, hinders a reliable comparison of the frequency components across multiple

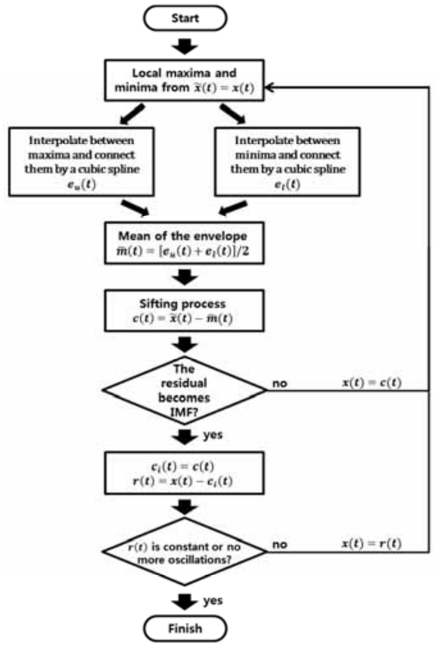


Fig. 2. Flow chart of the EMD algorithm.

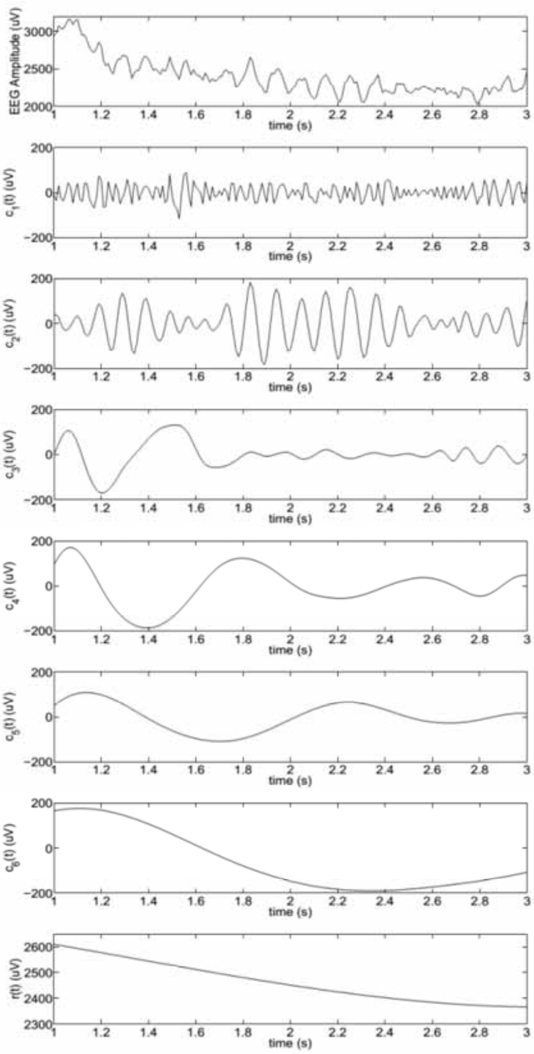


Fig. 3. IMFs of an electroencephalograph decomposed using EMD [4].

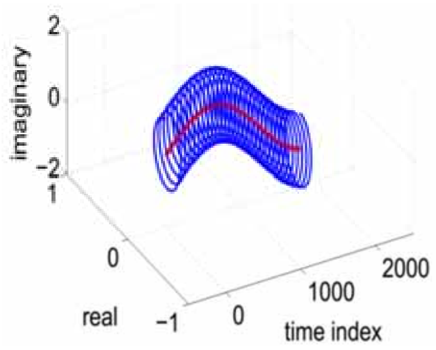


Fig. 4 Complex data represented in a 3 D domain [4].

channels. To overcome these problems, bivariate/complex [8, 9] and multivariate EMD [10, 11] has been developed. This review focuses on the complex extension of EMD, bivariate EMD for two channel data analysis.

The bivariate EMD (BEMD) was developed by Rilling et al. [8] for two channel data decomposition. They formed a complex-valued data by putting one set of data in a real part and the other in the imaginary part of a complex data and applied the complex version of EMD algorithm, (more details can be found in [8]). Fig. 4 presents a complex signal as a tube in the 3 D domain, real, imaginary and time axis. Using the standard EMD as an example, the local mean value of the tube is calculated by the mean of the estimated envelops in several directions and is subtracted iteratively from the input signal until the residue of the real and imaginary parts of the complex signal meets the condition to be an IMF. The decomposed complex-number IMFs can be separated into two real-number IMF datasets from the real and imaginary parts corresponding to each channel data. This can guarantee the same number of IMFs between two channel data and the similar frequency components between them because they are produced from the same complex-valued IMFs [12-14].

The performance of the BEMD compared with that of the original EMD was tested using two synthetic datasets.

$$\mathbf{f}_1 = 13\ \boldsymbol{Hz} \text{ and } \mathbf{f}_2 = 47\ \boldsymbol{Hz}$$

$$\text{t}=1/\mathbf{f_s},\ldots 2 \text{ s and } \mathbf{f_s} = 10\ \boldsymbol{kHz}$$

$$s_1(t) = \cos(2\pi f_1 t) + \cos\left(2\pi f_2 t + \frac{\pi}{6}\right) + v_1(t) \tag{7}$$

$$\mathbf{s}_2(t) = 1.7\cos\left(2\pi f_1 t + \frac{\pi}{4}\right) + 1.3\cos\left(2\pi f_2 t + \frac{2\pi}{3}\right) + v_2(t) \tag{8}$$

where $v_1(t)$ and $v_2(t)$ are white Gaussian noise (WGN) at 0 dB SNR. EMD and BEMD are used to decompose those two signals and their performances are compared in terms of the stability of the frequency component decomposition. In particular, complex data, $\mathbf{s}_1(t) + js_2(t)$, is formed and decomposed using BEMD. Fig. 5 shows the IMFs of two signals decomposed using EMD while Fig. 6 shows the IMFs decomposed using BEMD. The results of BEMD obtained more reliable IMFs with similar scales in

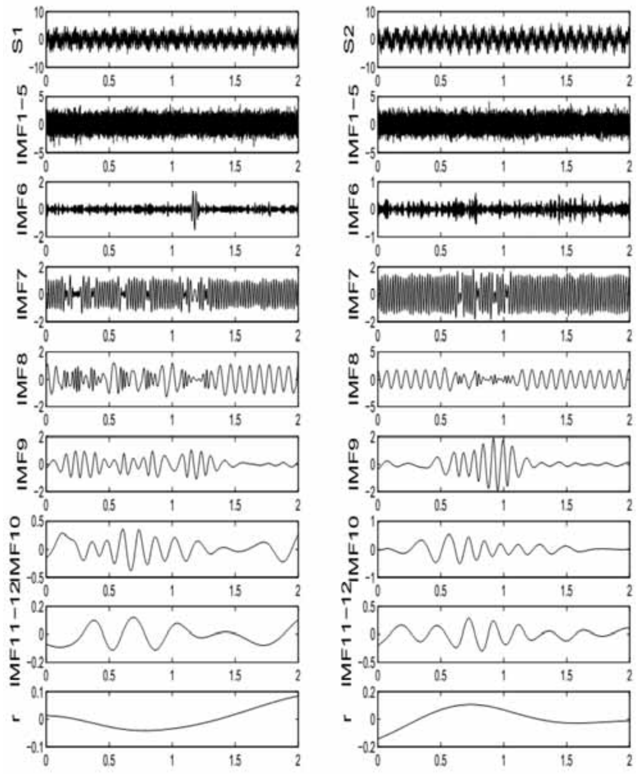


Fig. 5. IMFs of two synthetic signals (obtined using Eq (7) and (8)), decomposed using EMD [4].

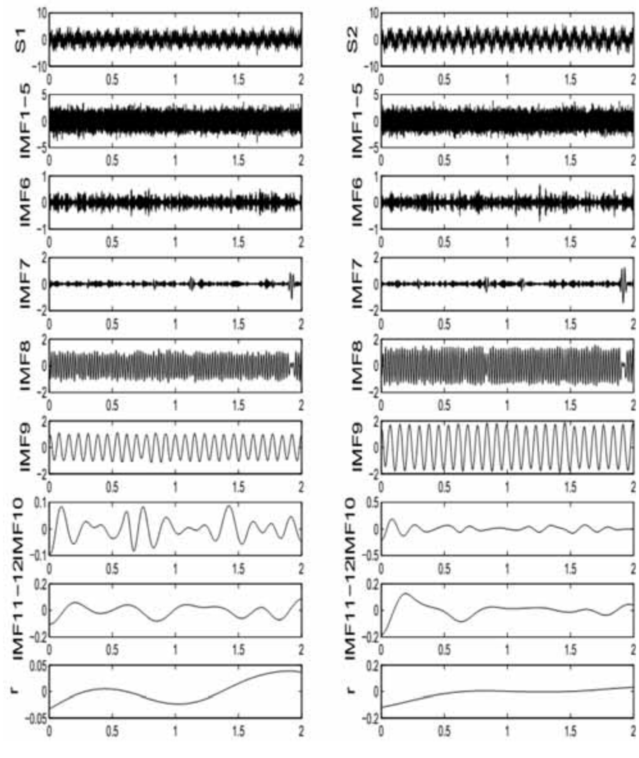


Fig. 6. IMFs of two synthetic signals, Eqs. (7) and (8), decomposed using bivariate EMD [4].

the same order of IMF and fewer mode-mixing problems compared to those of EMD. In addition, the IMFs estimating the two frequency components, 13 Hz and 47

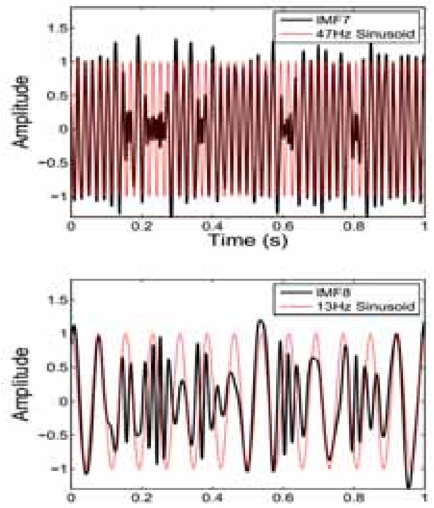


Fig. 7. EMD IMFs corresponding to the input frequency components [4].

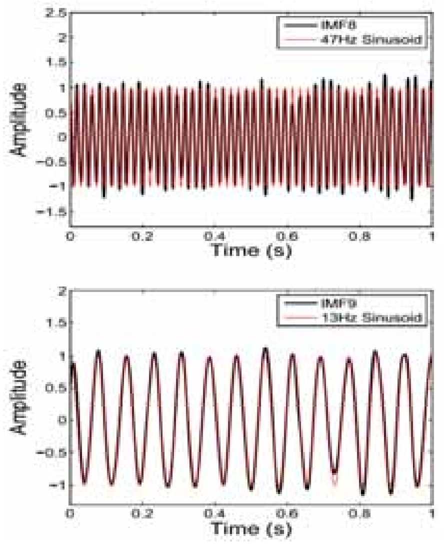


Fig. 8. MEMD IMFs corresponding to the input frequency components [4].

Hz, are shown in Figs. 7 and 8. Note that the IMFs extracted using BEMD are closer to the original sinusoidal components than those of EMD. The Hilbert transform was applied to the IMFs and their instantaneous frequency and instantaneous amplitude were calculated. Fig. 8 displays the time-frequency representations of the instantaneous information in the 3 D domain. The frequency components of two sinusoids, 13 Hz .and 47 Hz, are more salient on the right obtained using BEMD, compared to those of EMD.

3. Local Mean Decomposition

A local mean decomposition (LMD) is another data-driven time-frequency component analysis algorithm that is suitable for the analysis of nonlinear and nonstationary data [3]. While EMD uses a cubic spline to calculate the

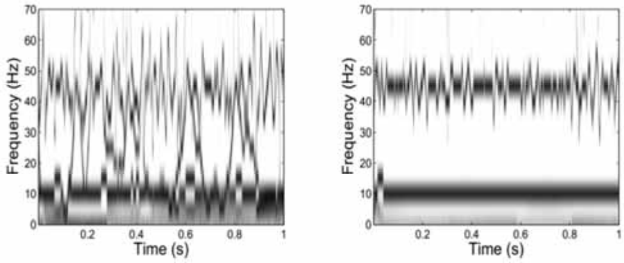


Fig. 8. Time-frequency representations obtained using EMD (left) and MEMD (right) [4].

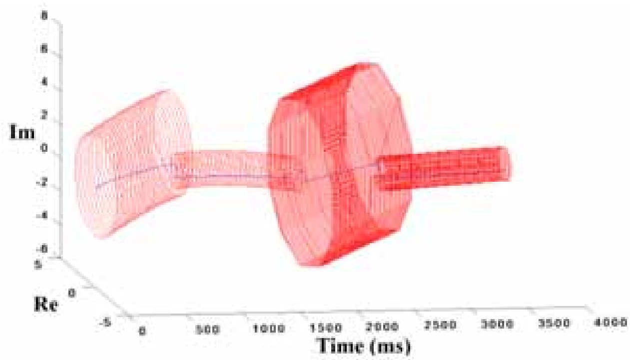


Fig. 10. Synthetic signals produced using Eq. (9)-(12). The blue line represents the local mean of the complex data. 'Im' denotes the imaginary part of the complex number and 'Re' is real part of that.

upper and lower envelope for an estimation of the local mean value, the LMD utilizes the smoothed local mean method. For an analysis of two channel data, the complex version of LMD was developed recently [15].

3.1 The standard LMD

J. S. Smith developed the local mean decomposition, which is a fully data-driven approach and does not rely on any basis function, such as the EMD. Amplitude modulated and frequency modulated components are produced using LMD, and they derive the information of time-varying instantaneous amplitude and frequency. The LMD does not use cubic splines, which are computationally expensive and time-consuming to produce. In addition, the LMD estimates the instantaneous frequency without a Hilbert transform, which sometimes induces a loss of amplitude and frequency information [15-20]. J. S. Smith reported that the smoothed local mean method can obtain an instantaneous frequency directly from the decomposed components and they are more stable than those of EMD [3]. Table 2 provides details of the algorithm. EEG was originally investigated using the LMD and Wang *et al.* applied it to the analysis of rub-impact fault diagnosis [19, 20].

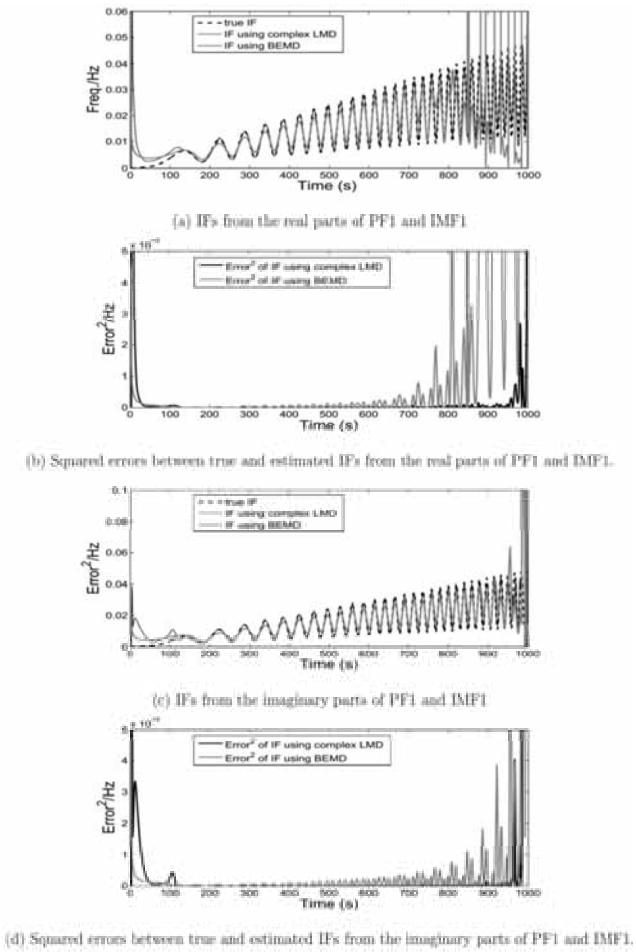
3.2 The complex LMD

To implement the complex version of LMD, the local mean value was estimated in a similar manner to that of

Table 1. Steps of the standard local mean decomposition algorithm [3].

The standard LMD algorithm
1. Let $\tilde{x}(t) = x(t)$ ($x(t)$ is an input signal)
2. Find the successive maximum and minimum of $\tilde{x}(t)$, $n_{k,c}$ and $n_{k,c+1}$, where k and c are the iteration number and the index of the extrema
3. Estimate the local mean and local magnitude $m_{i,k,c} = \frac{n_{k,c}+n_{k,c+1}}{2}$, $a_{i,k,c} = \frac{\lvert n_{k,c}-n_{k,c+1}\rvert}{2}$ where i denotes the order of product function (PF)
4. Interpolate straight lines of the local mean and local magnitude values between successive extrema, $m_{i,k}(t)$ and $a_{i,k}(t)$
5. Smooth the interpolated local mean and local magnitude using moving average method, $\tilde{m}_{i,k}(t)$ and $\tilde{a}_{i,k}(t)$
6. Subtract the smoothed mean signal from the original signal, $x(t)$ $h_{i,k}(t) = x(t) - \tilde{m}_{i,k}(t)$
7. Obtain the frequency modulated signal, $\bar{s}_{i,k}(t)$, by dividing $h_{i,k}(t)$ by $\tilde{a}_{i,k}(t)$ $\bar{s}_{i,k}(t) = \frac{h_{i,k}(t)}{\tilde{a}_{i,k}(t)}$
8. Check whether $\bar{s}_{i,k}(t)$ is a normalized frequency-modulated signal ($\tilde{a}_{i,k}(t)$ is close to 1) then go to step 11
9. If not, multiply $\tilde{a}_{i,k}(t)$ and $\tilde{a}_{i,k-1}(t)$ and go back to the first step to repeat the same procedure for $\bar{s}_{i,k}$
10. Envelope function, $\tilde{a}_i(t)$, can be derived by multiplying all $\tilde{a}_{i,k}(t)$ until $\tilde{a}_{i,k}(t)$ equals one $\tilde{a}_i(t) = \tilde{a}_{i,1}(t) \times \tilde{a}_{i,2}(t) \times ... \times \tilde{a}_{i,l}(t) = \prod_{q=1}^{l} \tilde{a}_{i,q}(t)$ where l is maximum iteration number
11. Using the envelope function, $\tilde{a}_i(t)$, and the final frequency modulated signal, $\bar{s}_{i,l}(t)$, derive PF by their multiplication $\mathrm{PF_i}(t) = \tilde{a}_i(t) \times \bar{s}_{i,l}(t)$
12. Subtract $\mathrm{PF}_i(t)$ from $x(t)$ $u_i(t) = x(t) - PF_i(t)$ then the smoothed data, $u_i(t)$, is treated as a new input, $x(t)$, and the procedure is repeated from steps 1-11, until $u_i(t)$ becomes a monotonic function
13. From the frequency modulated signal, an instantaneous phase can be calculated $\emptyset_i(t) = \arccos(\bar{s}_{i,l}(t))$
14. The phase data unwrapped and its differentiation defines the instantaneous frequency

EMD except the cubic spline. The complex-valued local mean values in several directions on a 3 D tube of the complex data, as shown in Fig. 4, were estimated using smoothed local mean method and their average becomes a common local mean value for the two channel data (more details can be found in [15]). To visualize the process of a complex LMD, four rotational signals were tested, which were made by concatenating the sinusoids as follows:



(a) IFs from the real parts of PF1 and IMF1

(b) Squared errors between true and estimated IFs from the real parts of PF1 and IMF1.

(c) IFs from the imaginary parts of PF1 and IMF1

(d) Squared errors between true and estimated IFs from the imaginary parts of PF1 and IMF1.

Fig. 11. Estimated instantaneous frequencies (IFs) using complex LMD (PF1) and BEMD (IMF1). Note the smaller error of the complex LMD results than those of BEMD [15].

$$\mathbf{f}_1 = 1kHz \;\; \mathbf{f}_2 = \;\; kHz \;\; f_3 = \;\; Hz$$

$$\mathbf{s}_1(\mathbf{t}) = 3\times\left(\cos(2\pi\mathbf{f}_1 t) + \cos(2\pi f_3 t) + jsin(2\pi f_1 t)\right) \quad (9)$$

$$\mathbf{s}_2(t) = \cos(2\pi f_1 t) + jsin(2\pi f_1 t) + \mathbf{cos}(2\pi f_3 t) \quad (10)$$

$$\mathbf{s}_3(t) = 5\times\left(\cos(2\pi f_2 t) + \cos(2\pi f_3 t) + jsin(2\pi f_2 t)\right) \quad (11)$$

$$\mathbf{s}_4(t) = \cos(2\pi f_2 t) + \cos(2\pi f_3 t) + jsin(2\pi f_2 t) \quad (12)$$

The synthetic data is displayed in the 3 D domain of time, real and imaginary parts of a complex number in Fig. 10, and the blue line is the complex-valued local mean value. The local mean value was subtracted iteratively like the standard LMD and this process guarantees the common oscillations of the frequency components between the real and imaginary parts, i.e., two channel data.

The performance of complexes LMD and BEMD, which were made for the standard LMD by [20], was compared using the Duffing wave signals. The complex signals were produced by

$$\mathbf{s}_1(t) = e^{-t/256}\cos\left[\frac{\pi}{64}\left(\frac{t^2}{512}+32\right)+0.3\sin\left(\frac{\pi}{32}\left(\frac{t^2}{512}+32\right)\right)\right] + 0.06e^{\frac{2t}{1024}}$$

$$\mathbf{s}_2(t) = e^{-t/512}\cos\left[\frac{\pi}{64}\left(\frac{t^2}{512}+32\right)+0.3\sin\left(\frac{\pi}{32}\left(\frac{t^2}{512}+32\right)\right)+\frac{\pi}{8}\right] + 0.03e^{\frac{2t}{1024}}$$

where the phase difference between $\mathbf{s}_1(t)$ and $\mathbf{s}_2(t)$ is $\frac{\pi}{8}$ and both signals have the same instantaneous frequency as follows:

$$\mathbf{f}(\mathbf{t}) = \frac{\theta'(\mathbf{t})fs}{2\pi} = \frac{t}{32768}\left(1+0.6\cos\left[\frac{\pi}{32}\left(\frac{t^2}{512}+32\right)\right]\right)$$

A complex number was constructed using those signals and decomposed using the complex LMD and BEMD. Fig. 11 shows the estimated instantaneous frequency using two algorithms [15]. Note the fewer errors of complex LMD than those of BEMD for both the real and imaginary parts of the complex-valued components.

3. Conclusion

This paper reviewed empirical mode decomposition and local mean decomposition algorithms for an analysis of nonlinear and non-stationary data and the advantage of the complex extensions of the algorithms for multichannel data analysis are also addressed. An analysis of the physiological data, e.g. EEG and ECG, would be improved using the algorithms because most of them are nonlinear and non-stationary and are recorded using multichannel sensors.

Acknowledgement

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