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Observer-based Feedback Controller Design for Robust Tracking of Discrete-time Polytopic Uncertain LTI Systems

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Abstract – This paper presents an observer–based robust controller for constant reference tracking of linear time invariant systems with polytopic model uncertainties. To this end, this paper not only designs a robust integral controller gain but also suggests how to determine the robust observer gain and the observer model used in the observer. Since the observer model selection is not obvious due to the polytopic uncertainties, particular attention needs to be paid to that. This paper computes the robust controller and observer gains first. Then, the observer model is selected in a way that the whole closed-loop is stable and LMIs are used in the middle of choosing the gains and observer model. Simulation examples show that the proposed observer-based feedback control successfully achieves robust reference tracking.

Keywords: Reference tracking, Robust control, LMIs (Linear Matrix Inequalities), Non-separation principle approach, Observer model selection, Polytopic uncertainty

1. Introduction

Since it is important to take model uncertainties into account in control systems for practical engineering applications, the robustness analysis and the robust controller design for uncertain systems have been studied for a long time [1-7]. In general, output variables rather than state variables are available for use by the controller. Accordingly, the applicable scope of an output feedback control is larger than that of a state feedback control. Therefore, much research efforts have still been made for the output feedback control technique dealing with model uncertain systems [8-10].

In the previous results on observer-based feedback controls for uncertain systems, the model uncertainty is usually expressed as norm bounded uncertainties such that $(A_o + \Delta A, B_o + \Delta B)$ where (A_o, B_o) is the nominal model and $(\Delta A, \Delta B)$ is perturbation. In such a case, it is straightforward to choose the nominal model (A_o, B_o) as the observer model. However, if the model uncertainty is assumed to be polytopic type, it is not obvious how to select the nominal model of the uncertain system in the polytopic set for observer design. This is why there are rare results on this problem [8, 11, 12]. There are only a few results which use the static output feedback control [13], [14]. On the other hand, the polytopic uncertain model is popularly employed in robust control design [15-18], because many control systems are identified as polytopic uncertain systems. In addition, polytopic uncertainty type usually describes the uncertainty of nominal system more precisely than norm bounded uncertainties [19]. Since an observer-based controller can have dynamics in controller, it can handle wider class of control systems than static output feedback controller. These facts are motivation of researches on the observer-based feedback control scheme for the polytopic uncertain systems.

Recently, an observer based robust regulation scheme for polytopic uncertain type proposed in [20]. For the purpose, robust control and estimation gains are computed first and then an optimal observer model is found in a way that the closed-loop system is asymptotically stable. This paper not only extends the previous regulation scheme to the robust reference tracking but also handles more general systems. It is worth noticing that a tracking controller for polyopic uncertain system has not been reported yet. In addition, this paper presents the observer-based controller design without solving BMIs problem in contrast with previous result. To this end, an integral control law and iterative linear matrix inequality (ILMI) are adopted in this paper. The feedback gain for the integral control law and observer gain are computed such that all possible controller and observer poles of the uncertain system are located in the open unit disk of the complex plane under the assumption that the nominal model for observer design (observer model hereafter) is determined properly. Then, inspired by [20], the observer model is selected such that the tracking error dynamics is asymptotically stable under the assumption that the observer model belongs to the polytopic set. For the sake of selecting such an observer model, it is first expressed as a convex combination of known models and then the convex combination coefficients are found by solving LMIs iteratively. This means that the observer model is used as a control parameter in the proposed scheme. Owing to the stabilizing property of the observer model, robust tracking performance is achieved for polytopic uncertain systems.

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2. Preliminaries

To design the state observer for discrete–time uncertain LTI systems, we use the robust stability condition proposed in [21]. Consider the following discrete-time linear systems:

$$x(k+1) = A(\alpha)x(k) \tag{1}$$

where $A(\alpha)$ belongs to a convex polytopic set defined as:

$$\Lambda := \left\{ A(\alpha) : A(\alpha) = \sum_{i=1}^{N} \alpha_i A_i, \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0 \right\}. \tag{2}$$

Definition 1: [21], [22] System (1) is robustly stable in the uncertainty domain (2) if all eigenvalues of matrix $A(\alpha)$ have magnitude less than one for all values of α such that $A(\alpha) \in \Lambda$.

It is well-known that robust stability can be checked by using the following Lemma.

Lemma 1: [21, 22] System (1) is robustly stable in the uncertain domain (2) if, and only if, there exists a matrix $P(\alpha) = P(\alpha)^T > 0$ such that

$$A(\alpha)^T P(\alpha) A(\alpha) - P(\alpha) < 0 \tag{3}$$

for all α such that $A(\alpha) \in \Lambda$.

Lemma 2: [21, 22] Uncertain system (1) is robustly stable in uncertainty domain (2) if there exist symmetric matrices P_i and a matrix G such that

$$\begin{bmatrix} P_i & A_i^T G^T \\ G A_i & G + G^T - P_i \end{bmatrix} > 0 \tag{4}$$

for all $i = 1, \cdots, N$.

3. Problem Formulation

Consider the following discrete-time uncertain LTI system:

$$\begin{aligned} x(k+1) &= A(\alpha)x(k) + B(\beta)u(k), \\ y(k) &= C(\alpha)x(k), \end{aligned} \tag{5}$$

where $u \in R^{n_u}$, $x \in R^{n_x}$, $y \in R^{n_y}$ are the control input, unmeasurable states, and output, respectively. Matrices $A(\alpha), B(\beta)$, and $C(\alpha)$ are unknown constant matrices that belong to the polytopic uncertainty class as follows:

$$\Omega = \left\{ \begin{matrix} (A(\alpha), C(\alpha)) : (A(\alpha), C(\alpha)) = \sum_{i=1}^{N} \alpha_i (A_i, C_i), \\ \alpha_i \geq 0, \sum_{i=1}^{N} \alpha_i = 1 \end{matrix} \right\}, \tag{6}$$

$$\Phi = \left\{ B(\beta) : B(\beta) = \sum_{j=1}^{M} \beta_j B_j, \ \beta_j \geq 0, \sum_{j=1}^{M} \beta_j = 1 \right\}, \tag{7}$$

A_i, B_j, and C_i are known constant matrices but coefficients α_i for $i = 1, \cdots, N$ and β_j for $j = 1, \cdots, M$ are unknown constants. It is noted that uncertain system (5) can describe more general parametric uncertainties than a general polytopic uncertainty (i.e.$(A(\alpha), B(\alpha), C(\alpha))$).

The objective of this paper is to design an observer-based offset-free reference tracking controller for uncertain system (5) such that it results in

$$\lim_{k \to \infty} y(k) = r$$

where r denotes a constant reference.

4. Main Results

The design procedure for the proposed observer-based robust tracking control consists of three steps: the robust tracking controller gain design, the robust observer gain design, and the observer model selection.

4.1 Robust tracking controller design

If the state is measurable, the following control law can be employed for offset-free constant reference tracking

$$u(k) = K_1 x(k) + K_2 z(k) \tag{8}$$

$$z(k+1) = y(k) - r + z(k) \tag{9}$$

where K_1 and K_2 are the controller gains. With this control law, the closed-loop dynamics can be represented as:

$$\begin{bmatrix} x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} A(\alpha) + B(\beta)K_1 & B(\beta)K_2 \\ C(\alpha) & I \end{bmatrix} \begin{bmatrix} x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -r \end{bmatrix} \tag{10}$$

To achieve reference tracking, we choose the controller gains K_1 and K_2 such that the closed-loop dynamics (10) is stable, i.e. the following matrix is stable:

$$\begin{bmatrix} A(\alpha) + B(\beta)K_1 & B(\beta)K_2 \\ C(\alpha) & I \end{bmatrix}. \tag{11}$$

Stability of matrix (11) is guaranteed if the matrices

$$\begin{bmatrix} A_i + B_j K_1 & B_j K_2 \\ C_i & I \end{bmatrix}, \tag{12}$$

are all stable where $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, M$ [4,21,22]. Note that A_i, B_j, and C_i are known and the vertices of the polytopic uncertain set (6) and (7). Matrices (12) can be rewritten as follows:

$$\begin{bmatrix} A_i & 0 \\ C_i & I \end{bmatrix} + \begin{bmatrix} B_j \\ 0 \end{bmatrix} [K_1 \quad K_2] =: F_i + H_j K, \tag{13}$$

where $i = 1, \cdots, N$ and $j = 1, \cdots, M$. The matrices F_i, H_j, K are defined as:

$$F_i := \begin{bmatrix} A_i & 0 \\ C_i & I \end{bmatrix},\ H_j := \begin{bmatrix} B_j \\ 0 \end{bmatrix},\quad K := [K_1 \quad K_2].$$

Equation (13) can be interpreted as a closed-loop system of (F_i, H_j) with feedback gain K. Finding such a stabilzing feedback gain is formulated as LMIs in the following Theorem.

Theorem 1: Uncertain system (5) can be robustly stabilized in uncertainty domain (6) and (7), if there exist symmetric matrices P_{ij}, G such that

$$\begin{bmatrix} \varepsilon^2 P_{ij} & F_i G + H_j L \\ (F_i G + H_j L)^T & G + G^T - P_{ij} \end{bmatrix} > 0, \qquad (14)$$

for all $i = 1, \cdots, N$, $j = 1, \cdots, M$, and $\varepsilon \in (0,1)$ is a tuning parameter. If the (14) is feasible, we can get controller gains K_1 and K_2 as follows:

$$[K_1 \quad K_2] = K = LG^{-1}, \qquad (15)$$

which achieve reference tracking in the case of the state feedback control.

Proof: This theorem can be proved with the similar way in [Theorem 3. 21]. The difference is that this theorem computes the integral controller gain, but this does not affect the proof.

4.2 Robust observer design

Suppose that the full order state observer is of the form

$$\hat{x}(k+1) = \bar{A}\hat{x}(k) + \bar{B}u(k) + L(y(k) - \bar{C}\hat{x}(k)) \qquad (16)$$

where $\bar{A}, \bar{B}$, and $\bar{C}$ are the observer model and L is the robust observer gain. Unlike existing results in the literature in which the observer model is the same as the plant model, in this paper, the observer model is also a design parameter, which is in sharp contrast to the previous results. In the subsequent sections, the observer gain L is designed first under the assumption that $\bar{A}, \bar{B}$, and $\bar{C}$ are chosen appropriately. It is presented afterwards how to select the observer model.

The robust observer gain L is chosen such that $A(\alpha) - LC(\alpha)$ is a stable matrix. In order to make $A(\alpha) - LC(\alpha)$ stable, consider the following inequalities:

$$\left({A_i}^T - {C_i}^T L\right)^T P\left({A_i}^T - {C_i}^T L\right) - P < -(1-\delta^2)P \qquad (17)$$

where $i = 1, \cdots, N$, $\delta \in (0,1)$ is a tuning parameter. These inequalities (17) are not LMIs. However, it can be easily transformed into LMIs in the following [5]:

$$\begin{bmatrix} \delta^2 T & ({A_i}^T T - {C_i}^T S)^T \\ ({A_i}^T T - {C_i}^T S) & T \end{bmatrix} > 0,\ T > 0, \qquad (18)$$

where $i = 1,2, \cdots, N$ and $\delta \in (0,1)$ is a tuning parameter. If the LMIs (18) are feasible, then the observer gain L is obtained by

$$P = T^{-1}, \qquad (19)$$
$$L = (ST^{-1})^T. \qquad (20)$$

Based on these observer gains, the observer model is determined. To choose the observer model, it is assumed that the observer model is parameterized as follows:

$$(\bar{A}, \bar{C}) \in \{\textstyle\sum_{i=1}^N \rho_i(A_i, C_i) \,|\, \rho_i \geq 0, \sum_{i=1}^N \rho_i = 1\}, \quad (21)$$
$$\bar{B} \in \{\textstyle\sum_{j=1}^M \eta_j B_j \,|\, \eta_j \geq 0, \sum_{j=1}^M \eta_j = 1\}, \qquad (22)$$

where A_i, B_j, C_i are known constant matrices and coefficients ρ_i, η_j are design parameters. It implies that coefficients ρ_i, η_j can be chosen arbitrary for achieving the control goal. The observer model selection problem is equivalent to selection of the coefficients ρ_i (i=1,$\cdots$,N) and η_j (j=1,$\cdots$,M).

Lemma 3 proposes how to choose coefficients ρ_i (i=1,$\cdots$,N) and η_j (j=1,$\cdots$,M) defining the observer model which can stabilize the estimation error equations.

Lemma 3: Suppose that there exist symmetric matrices P_{ij} and matrices O_ρ, O_η, and G such that following conditions hold:

$$\begin{bmatrix} P_{ij} & \left(\Pi_i + \Xi_j + O_\rho \varphi + O_\eta \chi\right)^T G^T \\ G\left(\Pi_i + \Xi_j + O_\rho \varphi + O_\eta \chi\right) & G + G^T - P_{ij} \end{bmatrix} > 0 \qquad (23a)$$
$$P_{ij} > 0, \qquad (23b)$$
$$\textstyle\sum_{i=1}^N \rho_i = 1, \qquad (23c)$$
$$0 \leq \rho_i \leq 1, \qquad (23d)$$
$$\textstyle\sum_{j=1}^M \eta_j = 1, \qquad (23e)$$
$$0 \leq \eta_j \leq 1, \qquad (23f)$$

for all $i = 1,2, \cdots, N$ and $j = 1,2, \cdots, M$. The matrices O_ρ, O_η, φ, χ, and Π_i are defined as:

$$\Pi_i = \begin{bmatrix} LC_i & LC_i & 0 \\ A_i - LC_i & A_i - LC_i & 0 \\ C_i & C_i & I \end{bmatrix},$$
$$\Xi_j = \begin{bmatrix} 0 & 0 & 0 \\ B_j K_1 & 0 & B_j K_2 \\ 0 & 0 & 0 \end{bmatrix},$$
$$O_\rho := \begin{bmatrix} \rho_1 I_{n_x} & \rho_2 I_{n_x} & \cdots & \rho_N I_{n_x} \\ -\rho_1 I_{n_x} & -\rho_2 I_{n_x} & \cdots & -\rho_N I_{n_x} \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$
$$O_\eta := \begin{bmatrix} \eta_1 I_{n_x} & \eta_2 I_{n_x} & \cdots & \eta_M I_{n_x} \\ -\eta_1 I_{n_x} & -\eta_2 I_{n_x} & \cdots & -\eta_M I_{n_x} \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

$$\varphi := \begin{bmatrix} A_1 - LC_1 & 0 & 0 \\ A_2 - LC_2 & 0 & 0 \\ \vdots & \vdots & \vdots \\ A_N - LC_N & 0 & 0 \end{bmatrix}, \qquad \chi := \begin{bmatrix} B_1K_1 & 0 & B_1K_2 \\ B_2K_1 & 0 & B_2K_2 \\ \vdots & \vdots & \vdots \\ B_M & 0 & B_MK_2 \end{bmatrix}.$$

Then, matrix $\Psi(\alpha,\beta)$ defined by

$$\Psi(\alpha,\beta) = \begin{bmatrix} \bar{A} + \bar{B}K_1 + L(C(\alpha) - \bar{C}) & LC(\alpha) & \bar{B}K_2 \\ A(\alpha) + B(\beta)K_1 - \bar{A} - \bar{B}K_1 - L(C(\alpha) - \bar{C}) & A(\alpha) - LC(\alpha) & (B(\beta) - \bar{B})K_2 \\ C(\alpha) & C(\alpha) & I \end{bmatrix} \quad (24)$$

is robustly stable.

Proof: Consider the feasibility problem (23)-(a). For each i, multiply the corresponding $j = 1,\cdots,M$ inequalities by β_j and sum as mentioned below:

$$\sum_{j=1}^{M} \beta_j I_{2(2n_x+1)} \times \begin{bmatrix} P_{ij} & \left(\Pi_i + \Xi_j + O_\rho\varphi + O_\eta\chi\right)^T G^T \\ G\left(\Pi_i + \Xi_j + O_\rho\varphi + O_\eta\chi\right) & G + G^T - P_{ij} \end{bmatrix} > 0.$$

Similarly, multiply the resulting $i = 1,\cdots,N$ inequalities by α_i and sum. Then, following inequalities can be obtained:

$$\begin{bmatrix} P(\alpha,\beta) & \Psi(\alpha,\beta)^T G^T \\ G\Psi(\alpha,\beta) & G + G^T - P(\alpha,\beta) \end{bmatrix} > 0. \qquad (25)$$

where the matrix $P(\alpha,\beta) := \sum_{i=1}^{N} \alpha_i \left(\sum_{j=1}^{M} \beta_j P_{ij}\right)$. Since the inequality (25) is equivalent to following inequality:

$$\Psi(\alpha,\beta)^T P(\alpha,\beta)\Psi(\alpha,\beta) - P(\alpha,\beta) < 0, \qquad (26)$$

$\Psi(\alpha,\beta)$ is robustly stable with a parameter-dependent Lyapunov matrix $P(\alpha,\beta)$ by Lemma 1. Converting inequality (25) into inequality (26) is given in [Theorem 1. 21].

Remark 1: Since the system (5) has parametric uncertainty, the separation principle is no longer applicable. Thus, with the controller and observer gains computed in (15), (20), the closed-loop system is not guaranteed to stable. Lemma 3 is a procedure to compensate the non-separation principle by choosing the suitable observer model which stabilizes the estimation error equations.

In the feasibility problem (23), constraint (23)-(a) is form of bilinear matrix inequality (BMI) due to the structural constraints in O_ρ, O_η.

4.3 Iterative Linear Matrix Inequalities for selecting O_ρ, O_η

Unlike the linear matrix inequality (LMI), BMI formulation has some drawbacks such as difficulties for handling numerical errors and local optimization solver. Therefore, we have presented an algorithm which exploits LMI solver iteratively instead of BMI solver. This algorithm is called iterative linear matrix inequality (ILMI).

∗ ILMI Algorithm

Step 0: Select the initial observer model parameters ρ_i for all $i = 1,2,\cdots,N$ and η_j for all $j = 1,2,\cdots,M$ as the center of the vertices. (i.e.$\rho_i = \frac{1}{N}$, $\eta_j = \frac{1}{M}$).

Step 1: Solve the feasibility problem (23). If the feasibility problem (23) has the solution, these parameters can be used as observer model parameters. If feasibility problem (23) is infeasible, then go to the Step 2.

Step 2: Change the observer model parameters ρ_i (i=1,$\cdots$,N) and η_j (j=1,$\cdots$,M) into the different values. Move to the Step 1.

Since the observer model parameters ρ_i for all $i = 1,2,\cdots,N$ and η_j for all $j = 1,2,\cdots,M$ are fixed in algorithm (it implies that matrices O_ρ and O_η are constant matrices and constraints (23)-(c),(d),(e),(f) are removed), the feasibility problem (23) can be solved by using LMIs technique in Step 1. The observer model candidates can be obtained via ILMI algorithm and one may choose the suitable model for one's control goal.

Remark 2: In general, optimization packages solve an optimization problem in order to find a solution to the feasibility problem in (23). For example, a representative of such an optimization problem is given by

$$\text{minimize} \quad \gamma \qquad (27)$$

subject to (28*a*-*f*), (28-*h*)

$$\begin{bmatrix} P_{ij} & \left(G\left(\Pi_i + \Xi_j + O_\rho\varphi + O_\eta\chi\right)\right)^T \\ G\left(\Pi_i + \Xi_j + O_\rho\varphi + O_\eta\chi\right) & G + G^T - P_{ij} \end{bmatrix} > \gamma I \qquad (28a)$$

$$\gamma \geq 0, \qquad (28h)$$

where $i = 1,2,\cdots,N$ and $j = 1,2,\cdots,M$. It is obvious that a feasible solution to this optimization is also a feasible solution to (23). In addition to the optimization problem (27), in order to find a feasible solution to (23), it is also possible to consider various optimization problems whose cost functions take performance of the system into account.

4.4 Analysis of reference tracking

In this subsection, it is proved that the proposed observer-based output feedback control described in the previous sections successfully achieves offset-free piecewise constant reference tracking.

Theorem 2: Assume that the LMIs (14), (18), and feasibility problem (23) are feasible. The controller gain K_1 and K_2 are computed using the solution of (14), the observer gain L using that of (18), and the observer model $\bar{A}, \bar{B}, \bar{C}$ using that of (23). Then, the observer based feedback control

$$u(k) = K_1\hat{x}(k) + K_2 z(k) \quad (29)$$

$$z(k+1) = y(k) - r(k) + z(k) \quad (30)$$

leads to successful reference tracking, where $\hat{x}(k)$ is an estimated state which is provided from the state observer (16).

Proof: Define the estimation error as a difference between the state and its estimated value

$$e(k) = x(k) - \hat{x}(k). \quad (31)$$

Then, the estimation error dynamic is given by

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1)) \\ &= A(\alpha)x(k) + B(\beta)u(k) \\ &\quad - \left(\bar{A}\hat{x}(k) + \bar{B}u(k) + L(y(k) - \bar{C}\hat{x}(k))\right) \\ &= A(\alpha)(x(k) - \hat{x}(k) + \hat{x}(k)) - \bar{A}\hat{x}(k) + B(\beta)K_1\hat{x}(k) \\ &\quad -(\bar{B}K_1 + L(C(\alpha) - \bar{C}))\hat{x}(k) + (B(\beta) - \bar{B})K_2 z(k) \\ &\quad -LC(\alpha)e(k) \\ &= \left(A(\alpha) + B(\beta)K_1 - \bar{A} - \bar{B}K_1 - L(C(\alpha) - \bar{C})\right)\hat{x} \\ &\quad +(B(\beta) - \bar{B})K_2 z + \left(A(\alpha) - LC(\alpha)\right)e. \end{aligned} \quad (32)$$

Moreover, the estimated state $\hat{x}$ and the controller state z are written as

$$\begin{aligned} \hat{x}(k+1) &= \bar{A}\hat{x}(k) + \bar{B}u(k) + L(y(k) - \bar{C}\hat{x}(k)) \\ &= (\bar{A} + \bar{B}K_1)\hat{x}(k) + \bar{B}K_2 z(k) + L(C(\alpha)x(k) \\ &\quad -C(\alpha)\hat{x}(k) + C(\alpha)\hat{x}(k) - \bar{C}\hat{x}(k)) \\ &= (\bar{A} + \bar{B}K_1 + L(C(\alpha) - \bar{C}))\hat{x}(k) + \bar{B}K_2 z(k) + \\ &\quad LC(\alpha)e(k) \end{aligned} \quad (33)$$

$$\begin{aligned} z(k+1) &= y(k) - r(k) + z(k) \\ &= C(\alpha)x(k) - r(k) + z(k) \\ &= C(\alpha)\hat{x}(k) + C(\alpha)e(k) - r(k) + z(k). \end{aligned} \quad (34)$$

Then, the whole closed loop system becomes

$$\begin{bmatrix} \hat{x}(k+1) \\ e(k+1) \\ z(k+1) \end{bmatrix} = \Psi(\alpha,\beta) \begin{bmatrix} \hat{x}(k) \\ e(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -r(k) \end{bmatrix}. \quad (35)$$

where $\Psi(\alpha,\beta)$ is defined in (24). Since the observer model $\bar{A}, \bar{B}$, and $\bar{C}$ are selected in a way that the matrix $\Psi(\alpha,\beta)$ is robustly stable and reference r is constant, the estimated state $\hat{x}$, estimation error e, and the controller state z converge to the unique constant equilibrium point $\hat{x}^*, e^*, z^*$, respectively. Therefore, it follows from the controller equation that

$$z^* = y^* - r^* + z^*.$$

It is equivalent to

$$y^* = r^*.$$

This is the end of the proof.

5. Numerical Simulation Result

Consider the system with polytopic uncertainty (5). Simulation is done with vertex matrices A_i, B_j, and C_i given by

$$A_1 = \begin{bmatrix} 0.8 & -0.25 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.3 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C_1 = [1 \quad 0 \quad 0 \quad 0],$$

$$A_2 = \begin{bmatrix} 0.8 & -0.25 & 0 & 1 \\ 1 & 0 & 0.6 & -0.6 \\ 0 & 0 & 0.2 & 0.03 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C_2 = [1 \quad -0.5 \quad 0.5 \quad 0],$$

$$A_3 = \begin{bmatrix} 0.8 & -0.25 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0.33 \\ -0.4 & -0.5 & 1 & 0.5 \end{bmatrix}, C_3 = [1 \quad 0 \quad -0.2 \quad -0.3],$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.8 \\ 0 \\ 1 \\ 0.3 \end{bmatrix}, B_3 = \begin{bmatrix} -0.8 \\ 0.4 \\ 1 \\ 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 \\ 0.3 \\ 1 \\ -0.3 \end{bmatrix}.$$

For the numerical simulations, uncertain plant parameters $(\alpha_1 = 0.4639, \alpha_2 = 0.3123, \alpha_3 = 0.2238)$ and $(\beta_1 = 0.2264,\ \beta_2 = 0.4037,\ \beta_3 = 0.3317,\ \beta_4 = 0.0382)$ are selected arbitrarily. The reference signal is set to 35, -40, 0,

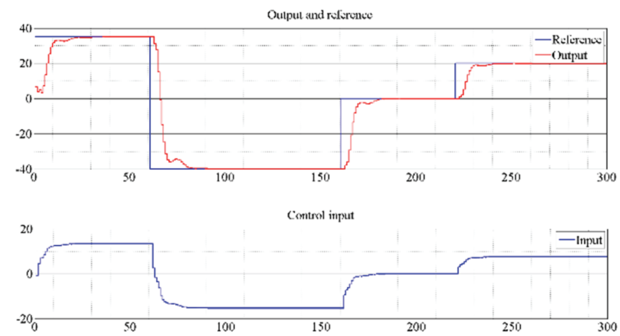

Fig. 1. Simulation result with observer model parameters $(\rho_1 = 0.8,\ \rho_2 = 0.1,\ \rho_3 = 0.1)$ and $(\eta_1 = 0.4,$: $\eta_2 = 0.3,\ \eta_3 = 0.2,\ \eta_4 = 0.1)$ The reference signal, output, and the input are plotted. Uncertain plant parameters: $\alpha_1 = 0.4639, \alpha_2 = 0.3123,\ \alpha_3 = 0.2238,\ \beta_1 = 0.2264,\ \beta_2 = 0.4037,\ \beta_3 = 0.3317,\ \beta_4 = 0.0382$.

and 20.

Consider the case where the state is unmeasurable. The controller and observer gains can be computed by solving proposed LMIs in (14), (18) as follows:

$$K_1 = [-0.0743 \quad 0.2075 \quad -0.6225 \quad -0.4873],$$

$$K_2 = [-0.1465],\ L = \begin{bmatrix} 0.8806 \\ 0.9581 \\ 0.0709 \\ 0.0179 \end{bmatrix}.$$

Besides, the observer model is computed by solving the feasibility problem (23) with ILMI algorithm. Fig. 1 shows that the resulting observer-based controller successfully leads to piecewise constant reference tracking.

To verify the robustness of the proposed controller, similar simulation is done with the different uncertainty parameters $(\alpha_1 = 0.8639, \alpha_2 = 0.1123, \alpha_3 = 0.0238,\ \beta_1 = 0.6264,$ $\beta_2 = 0.3037,\ \beta_3 = 0.0317,\ \beta_4 = 0.0382)$. As depicted in Fig. 2 the proposed controller also stabilized the different uncertain system.

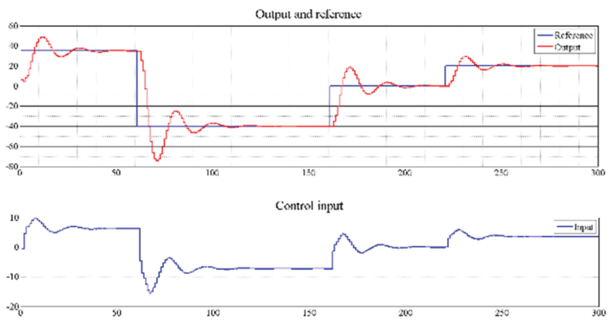

Fig. 2. Simulation result with observer model parameters $(\rho_1 = 0.8,\ \rho_2 = 0.1,\ \rho_3 = 0.1)$ and $(\eta_1 = 0.4,$: $\eta_2 = 0.3,\ \eta_3 = 0.2,\ \eta_4 = 0.1)$: The reference signal, output, and the input are plotted. Uncertain plant parameters: $\alpha_1 = 0.8639, \alpha_2 = 0.1123, \alpha_3 = 0.0238, \beta_1 = 0.6264,\ \beta_2 = 0.3037,\ \beta_3 = 0.0317,$ $\beta_4 = 0.0382$.

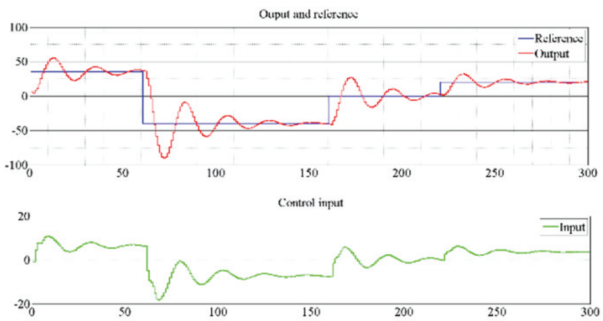

Fig. 3. Simulation result with the badly chosen observer model: The reference signal, output, and the input are plotted. Uncertain plant parameters: $\alpha_1 = 0.8639, \alpha_2 = 0.1123, \alpha_3 = 0.0238, \beta_1 = 0.6264,$ $\beta_2 = 0.3037,\ \beta_3 = 0.0317,\ \beta_4 = 0.0382$.

To fully illustrate the purpose of this scheme, we show the case that for a badly chosen observer model can cause some unwanted behavior. The plant uncertainty parameters are the same as those used in Fig. 2. It is assumed that the observer model parameters are selected as the center of the polytopic set $(\rho_1 = \frac{1}{3}, \rho_2 = \frac{1}{3}, \rho_3 = \frac{1}{3})$, $(\eta_1 = \frac{1}{4},\ \eta_2 = \frac{1}{4},\ \eta_3 = \frac{1}{4},\ \eta_4 = \frac{1}{4})$. We can find out that in Fig. 3 the control goal cannot be achieved by using the badly chosen observer model.

6. Conclusion

This paper proposes an observer-based controller which achieves reference tracking for model uncertain linear time invariant systems. To this end, robust controller and observer gains are computed. In addition, the observer model is also treated as a design parameter and it is selected such that the closed-loop system is stable. This scheme not only extends the applicable scope wider than the static output feedback controller but also deals with tracking problems for polytopic uncertain systems.

Future work includes observer-based robust tracking controller for input-constrained uncertain systems and observer-based MPC (Model Predictive Control) for the uncertain system.

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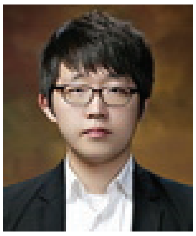

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