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Effects of Span-to-depth Ratio and Poisson's Ratio on Elastic Constants from Bending and Plate Tests[1]

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ABSTRACT

The goal of this study is to evaluate the limitation of ASTM D 198 bending and ASTM D 3044 in determination of elastic modulus and shear modulus. Different material properties and span to depth ratios were used to analyze the effects of material property and testing conditions. The ratio of true elastic modulus to apparent elastic modulus evaluated from ASTM D 198 bending sharply decreased with increment of span to depth ratio. Shear modulus evaluated from ASTM D 198 bending decreased with increment of depth, whereas shear modulus evaluated from ASTM D 3044 was hardly influenced by increment of depth. Poisson's ratio influenced shear modulus from ASTM D 198 bending but did not influence shear modulus from ASTM D 3044. Different shearing factor was obtained for different depths of beams to correct shear modulus obtained from ASTM D 198 bending equivalent to shear modulus from theory of elasticity. Equivalent shear modulus of materials could be obtained by applying different shearing factors associated with beam depth for ASTM D 198 bending and correction factor for ASTM D 3044.

Keywords : ASTM D 198 bending, ASTM D 3044, shear modulus, elastic modulus, Poisson's ratio

1. INTRODUCTION

Use of standard test methods is having a great advantage for researcher in many ways including saving time for preparation, obtaining reliable test results, and comparing mechanical properties of tested materials with other materials under the same testing conditions. However, the evaluated results could be unreliable if the limitations of the standard testing methods are not understood.

Gromala (1985) introduced an alternative testing method for measuring shear modulus of structural size lumber, which was adapted in the 1984 edition of ASTM D 198 bending and continued to be used in ASTM D 198 bending (ASTM 2005a). The new approach was taking advantage over the old method because the

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shear modulus, true elastic modulus, and apparent elastic modulus were evaluated using the deflections over multiple spans. Those three components participated in different proportions of the deflections associated with multiple spans. Although the revised ASTM D 198 bending was theoretically improved, previous studies showed that shear modulus of wood materials can be varied by evaluating procedures (Yoshihara *et al*. 1998; Biblis 2001; Yoshihara and Kubojima 2002; Murata and Kanazawa 2007; Hindman *et al*. 2006; Harrison and Hindman 2007; Jeong and Hindman 2008).

Hindman *et al*. (2006) used ASTM D 198 bending and five point bending test (FPBT) to evaluate elastic constants for machine stress rated (MSR) lumber and structural composite lumbers including laminated veneer lumber (LVL), parallel strand lumber (PSL), and laminated strand lumber (LSL). The results indicated that the edgewise elastic modulus of four different types of lumber tended to be higher than the flatwise elastic modulus. While difference between elastic constant ratios ($E_1 : G_{12}$) of MSR evaluated from ASTM D 198 bending and the elastic constant ratios of MSR evaluated from FPBT showed 2.9%, difference between the elastic constant ratio of PSL evaluated from ASTM D 198 bending and the elastic constant ratio of PSL evaluated from FPBT was 56.8%. This indicated that these two methodologies were subjective to structure of testing materials. Comparing shear modulus evaluated using ASTM D 198 and FPBT, the shear modulus of four different types of lumber evaluated in the flatwise created higher difference compared to the shear modulus evaluated in the edgewise.

Harrison and Hindman (2007) investigated ASTM D 198 bending, ASTM D 198 torsion, and FPBT in determination of in-plane shear modulus. Test specimens included nominal 2 by 8 size (3.8 cm by 19.1 cm) southern pine laminated veneer lumber (LVL) and machine stress rated (MSR) southern pine lumber. Shear modulus of the LVL evaluated from three different testing methods was significantly different. Shear modulus values of MSR evaluated from ASTM D 198 bending and FPBT were not significantly different but the shear modulus of MSR evaluated from ASTM D 198 torsion was significantly different from the shear modulus of MSR evaluated from ASTM D 198 bending and FPBT. The results indicated that the evaluated shear moduli were significantly different by testing methods and were related to properties of materials.

Jeong and Hindman (2008) compared four different testing methods including ASTM D 198 bending, FPBT, ASTM D 3044 (ASTM 2005b), and plate bending test in determination of material properties of engineered wood products and solid lumbers. The results indicated that the evaluated material properties were different by different testing methods. The elastic moduli from plate tests were compared to the results from ASTM D 198 bending and FPBT. The elastic moduli of PSL from plate tests were significantly different from the elastic moduli from the two bending tests. However, the elastic

Table 1. Different size beam and plate models used for the evaluation of elastic modulus and shear modulus using ASTM D 198 bending and ASTM D 3044

	Width (w)	Depth (h)	Span (L)	L/h	$(h/L)^2$
ASTM D 198	38 mm	89 mm	1,500 mm	16.85	0.0035
			1,200 mm	13.48	0.0055
			900 mm	10.11	0.0097
			600 mm	6.74	0.0220
		140 mm	2,700 mm	19.28	0.0026
			2,400 mm	17.14	0.0034
			2,100 mm	15.00	0.0044
			1,800 mm	12.85	0.0060
			1,500 mm	10.71	0.0087
			1,200 mm	8.57	0.0136
			900 mm	6.42	0.0241
		184 mm	3,600 mm	19.56	0.0026
			3,300 mm	17.93	0.0031
			3,000 mm	16.30	0.0037
			2,700 mm	14.67	0.0046
			2,400 mm	13.04	0.0058
			2,100 mm	11.41	0.0076
			1,800 mm	9.78	0.0104
			1,500 mm	8.15	0.0150
			1,200 mm	6.52	0.0235
	Width (w)	Depth (h)	w/h		
ASTM D 3044	150 mm	6.000 mm	25		
		5.357 mm	28		
		4.838 mm	31		
		4.411 mm	34		
		4.054 mm	37		
		3.750 mm	40		

modulus of MSR lumber from plate tests was not significantly different from the elastic modulus from the two bending tests. Shear modulus of MSR from ASTM D 3044 was significantly different from the shear modulus of MSR from the two bending tests. Shear modulus of PSL evaluated from ASTM D 198 bending was significantly different from that of PSL evaluated from ASTM D 3044. The results showed that the elastic constant ratios from plate tests were not directly comparable to the elastic constant ratios derived from the two bending tests.

To use the wood materials more efficiently, a reliable test methodology for evaluating material properties of wood is required. Previous studies showed that mechanical properties of structural size lumber and composite materials were varied by different testing methods. However, a clear explanation for different shear modulus evaluated by different test methods was not provided. The limitation of theory for evaluating shear modulus between different test methodologies should be evaluated. The goal of this study is to evaluate ASTM D 198 bending and ASTM D 3044 in determination of shear modulus. Different span to depth ratios and different Poisson's ratio were used to analyze the influence of those variables on elastic modulus and shear modulus from the two standard test methods.

2. MATERIALS and METHODS

2.1. Modeling

Different size of structural lumbers and plates were constructed for ASTM D 198 bending and ASTM D 3044 using a finite element method. Table 1 shows the different depths of beams modeled. While the width of the beam was identical as 38 mm, three different depths (89 mm, 140 mm, and 184 mm) of the beam were simulated. Four node quad elements were used to represent the beam models for ASTM D 198 bending. Plates of 150 mm (width) by 150 mm (length) with different thicknesses were constructed using eight node solid-shell elements to simulate ASTM D 3044.

To decide the optimum element size for the ASTM D 198 bending and ASTM D 3044 models, five continuous series of mesh sizes were applied to analyze the convergence of deflection and stress. When the element expansion of 1 for ASTM D 198 bending and the element expansion of 0.125 for ASTM D 3044 were applied, the differences in deflection and stress from different element size models were converged to less than 0.1%. For ASTM D 198 bending, a size of 1 mm (L) by 1 mm (h) elements was used. For ASTM D 3044, a size of 0.125 mm (L) by 0.125 mm (h) by 0.125 mm (w) elements was used. Elastic modulus of 14.7 GPa and Poisson's ratio of 0.3 were used as a baseline input material properties for ASTM D 198 bending and ASTM D 3044 models. To evaluate the effects of Poisson's ratio on shear modulus determination, different Poisson's ratios of 0.1, 0.2, 0.3, and 0.4 with the same elastic modulus of 14.7 GPa were applied for ASTM D 198 bending and ASTM D 3044 models.

2.2. ASTM D 198 bending

38 mm by 89 mm (2 by 4), 38 mm by 140 mm (2 by 6), and 38 mm by 184 mm (2 by 8) were used to analyze the effects of depth on elastic modulus and shear modulus. Different spans were used to evaluate true elastic modulus and shear modulus for different depths of beams based on the recommendation from ASTM D 198 bending (2003) indicating that $(h/L)^2$ aspect ratios of test specimens ranged from 0.035 to 0.0025. Equations below show the calculation for elastic modulus and shear modulus of structural size lumber from ASTM D 198 bending.

$$\mathrm{MOE} = \left(\frac{P}{\Delta}\right)\frac{L^3}{48I} \quad \cdots\cdots (1)$$

$$\frac{1}{MOE} = \frac{1}{E_t} + \frac{1}{KG}\left(\frac{h}{L}\right)^2 \quad \cdots\cdots (2)$$

Where,
MOE = apparent elastic modulus (MPa)
E_t = true elastic modulus (MPa)
P = applied load (N)
Δ = deflection (mm)
K = shape factor (5/6 for rectangular beams)
G = in-plane shear modulus (MPa)
h = height of the beam (mm)
I = second moment of inertia (mm^4)
L = span of the beam (mm)

2.3. ASTM D 3044 plate twisting

The following equation was used to evaluate shear modulus from ASTM D 3044. Based on the recommendation from ASTM D 3044, the FEM models had width to thickness ratios ranging from 25 to 40. Different width to thickness ratios of 25, 28, 31, 34, 37, and 40 were simulated to analyze the effects of thickness on shear modulus. To measure the deflection of plate, two paths were defined along the two diagonals of the plate. Averaged deflection values at the same distance from the center along the diagonal of the plates were used to calculate shear modulus at each point along the diagonal.

$$G = \frac{3u^2P}{2h^2\Delta} \quad \cdots\cdots (3)$$

Where,
G = shear modulus (MPa)
u = distance from the center of the plate (mm)
P = applied load (N)
h = thickness of plate (mm)
Δ = deflection (mm)

3. RESULTS and DISCUSSION

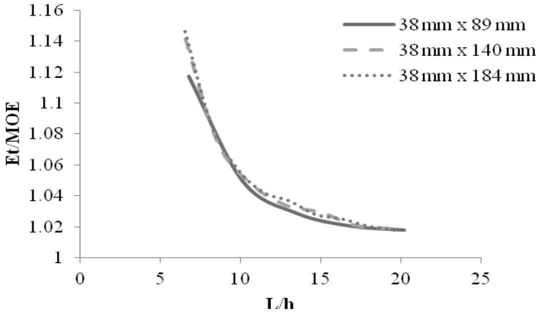
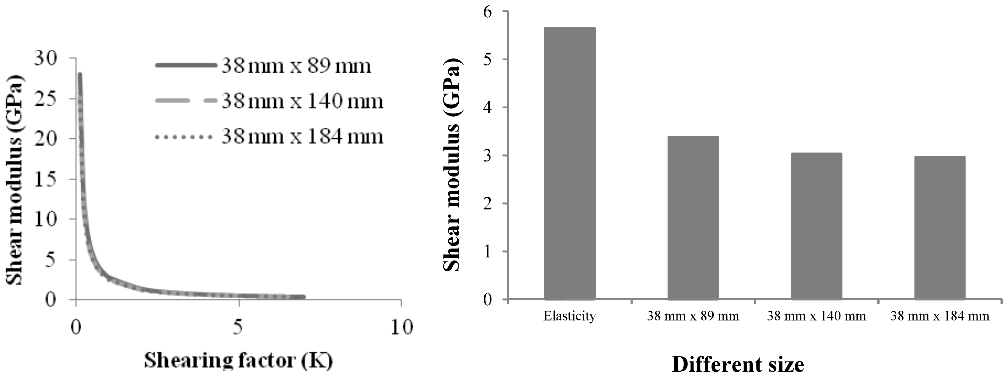


Fig. 1. Evaluated ratio of true elastic modulus (Et) to modulus of elasticity (MOE) against span to depth ratios (L/h) using ASTM D 198 bending.

Fig. 1 shows the ratio of true elastic modulus to apparent modulus of elasticity (MOE) as a function of span to depth ratio. The ratio of true elastic modulus to apparent MOE from three different depths of beams showed similar trends with increment of span to depth ratio. As the span to depth ratio increased, the ratio of true elastic modulus (E_t) to apparent MOE decreased. However, the decreasing rates associated with span to depth ratios could be divided into three ranges.

In the span to depth ratio from 6 to 10, the ratio of E_t/MOE sharply decreased. In the span to depth ratio from 10 to 15, the ratio of E_t/MOE moderately decreased. In the span to depth ratio from 15 to 20, the ratio of E_t/MOE slowly decreased. It can be interpreted that as span to depth ratio increased, the shear components involvement in elastic modulus evaluation decreased. If one span was used to evaluate elastic modulus of materials, span to depth ratio from 15 to 20 is recommended to minimize the shear deflection and produce apparent elastic modulus close to true elastic modulus.

Fig. 2 shows the shear modulus evaluated from different depth of beams as a function of shearing factor (K). As it can be seen from Fig. 2a, shear modulus was dramatically changed by shearing factor. The shape factor of 0.83 was recommended to use for shear modulus calculation in ASTM D 198 bending. Fig. 2b showed that with fixed shearing factor of 0.83, shear modulus decreased as the depth of



a) Shear modulus against shearing factor (K)

b) shear modulus of different size of lumber

Fig. 2. Shear modulus of different depth of beams from ASTM D 198 bending a) shear modulus against shearing factor (K), b) shear modulus of different size of lumber.

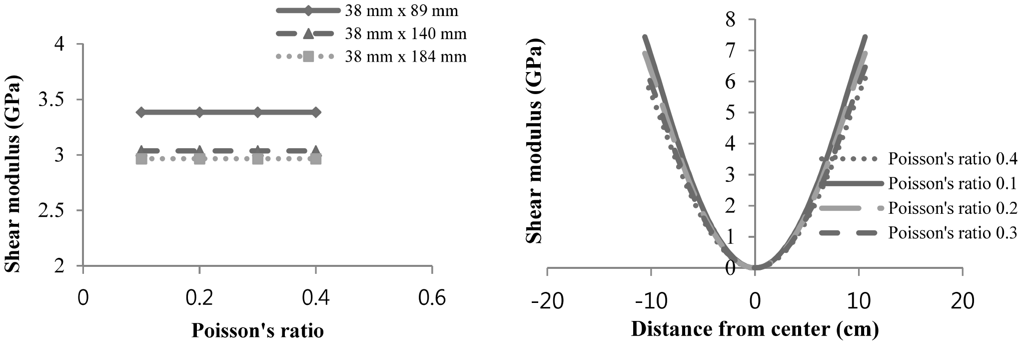
Table 2. Correction factors for shear modulus evaluated from ASTM D 198 bending and ASTM D 3044

ASTM D 198 bending		ASTM D 3044
Different size of beams	Shearing factor (K) for different depth of beams	Correction factor
38 mm by 89 mm (2 by 4)	0.440	1.652
38 mm by 140 mm (2 by 6)	0.451	
38 mm by 184 mm (2 by 8)	0.497	

beam increased. If the theory from ASTM D 198 bending is correct, the shear modulus evaluated from ASTM D 198 bending should be equivalent to the shear modulus from theory of elasticity.

The limitation of the theory before the 1984 edition of ASTM D 198 bending was that the measurement of shear modulus and true elastic modulus was counteracted by different span. Longer span increased the predictability of true elastic modulus by decreasing shear deflection, whereas shorter span increased the predictability of shear modulus by increasing the shear deformation. Although revised ASTM D 198 bending counted the different proportion of shear and bending deformation over multiple spans, shearing factor was not changed by different depths of beams. If the shape factor K was not changed by different span to depth ratio, shearing area was assumed to be identical regardless of span to depth ratio.

This is physically untrue. Biblis (2001) investigated the effect of depth on modulus of rupture (MOR) of LVL and plywood with the same span to depth ratio of 14. Larger depths of LVL and plywood indicated lower MOR values. The results indicated that larger depth of beam was more exposed to shear stress, which is weak stress for wood compared to bending and tension stresses. The dependency of shear strength of different structural composites associated with different



a) ASTM D 198 bending

b) ASTM D 3044

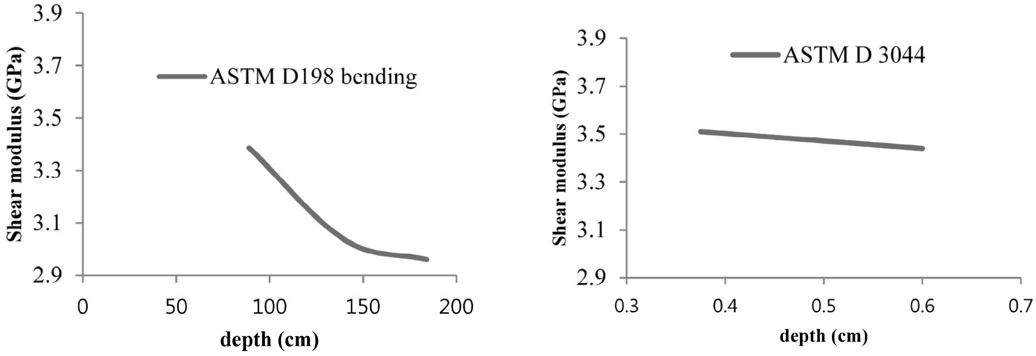
Fig. 3. Shear modulus varied by Poisson’s ratio evaluated from a) ASTM D 198 bending and b) ASTM D 3044.

aspect ratios was also emphasized from torsion test (Gupta and Siller 2005). Therefore, when ASTM D 198 bending was used to evaluate shear modulus, different shearing factors should be applied to correct shear area associated with different depth of beam. The current study found different shearing factors 0.440, 0.451, and 0.497 for 38 mm by 89 mm (2 by 4), 38 mm by 140 mm (2 by 6), and 38 mm by 184 mm (2 by 8) beams, respectively.

Fig. 3 shows the effects of Poisson’s ratio on shear modulus evaluated using ASTM D 198 bending and ASTM D 3044. Shear modulus evaluated from ASTM D 198 bending was independent upon Poisson’s ratio, whereas shear modulus evaluated from ASTM D 3044 was influenced by Poisson’s ratio. With increment of Poisson’s ratio, the deflection of plate increased. It can be indicated that theoretically ASTM D 198 bending was less sensitive to properties of materials compared to ASTM D 3044.

As it can be seen from Fig. 3b, shear modulus evaluated from ASTM D 3044 can be greatly varied by a deflection measurement point. The deflection along the diagonal showed the parabolic shape. Deflection of plate at the center was the lowest and deflection of plate at the end of plate where load applied was the highest. In ASTM D 3044, there is no guideline for the deflection measurement point. Therefore, shear modulus of materials evaluated from ASTM D 3044 was not directly comparable unless deflection measurement position indicated. Authors recommended the deflection measurement location at the four middle points from the center to the edge along the diagonal of a plate to measure shear deflection and minimize the influence of loading and boundary conditions.

Fig. 4 shows shear modulus evaluated from ASTM D 198 bending and ASTM D 3044 varied by depth. Shear modulus evaluated from ASTM D 198 bending decreased as the depth



a) ASTM D 198 bending

b) ASTM D 3044

Fig. 4. Variation of shear modulus associated with different depth determined from a) ASTM D 198 bending and b) ASTM D 3044.

of beam increased. The total decrement of shear modulus evaluated from ASTM D 198 bending associated with depth variation from 89 mm to 184 mm was 12.56%, whereas the decrement of shear modulus evaluated from ASTM D 3044 associated with depth variation from 6.00 mm to 3.75 mm was 2.00%. It could be concluded that the shear modulus evaluated from ASTM D 3044 is not much different by thickness to width or length ratio within the range suggested from ASTM D 3044.

4. CONCLUSION

The limitation of ASTM D 198 bending and ASTM D 3044 was found in determination of elastic modulus and shear modulus of wood and wood based materials. For ASTM D 198 bending, applying load at the beams with different span created different proportion of shear and bending deflections. Small span created more shear deformation, whereas small span created less pure bending deformation. With increment of span to depth ratio, apparent modulus of elasticity to true elastic modulus decreased. Span to depth ratio from 15 to 20 produced apparent elastic modulus close to true elastic modulus by minimizing shear deflection. However, shear modulus determined by using ASTM D 198 bending appeared to be influenced by different span to depth ratio. In this study, different shearing factor K of 0.440, 0.451, and 0.497 was suggested for 38 mm by 89 mm (2 by 4), 38 mm by 140 mm (2 by 6), and 38 mm by 184 mm (2 by 8) beams, respectively.

For ASTM D 3044, problem was found in deflection measurement of plate. Deflection of a plate differed along the diagonal of plate and ASTM D 3044 did not specify the deflection measurement point. Thus shear modulus was varied by the deflection measurement point. Authors recommended that the four middle points between the center of plate and the edge of plate were suitable for deflection measurement to avoid stress interference from the edge support and loading. While shear modulus evaluated from ASTM D 198 bending was not influenced by Poisson's ratio, shear modulus evaluated from ASTM D 3044 was influenced by Poisson's ratio. This indicated that ASTM D 3044 was more subjective to material properties compared to ASTM D 198 bending.

When shear modulus of material is evaluated using ASTM D 198 bending, different shearing factor should be applied to achieve equivalent shear modulus from theory of elasticity.

Equivalent shear modulus could be evaluated by applying different shearing factor associated with span to depth ratio for ASTM D 198 bending and correction factor of 1.652 for ASTM D 3044.

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