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MATHEMATICAL MODELLING AND ITS SIMULATION OF A QUASI-STATIC THERMOELASTIC PROBLEM IN A SEMI-INFINITE HOLLOW CIRCULAR DISK DUE TO INTERNAL HEAT GENERATION

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ABSTRACT. The present paper deals with the determination of temperature, displacement and thermal stresses in a semi-infinite hollow circular disk due to internal heat generation within it. Initially the disk is kept at arbitrary temperature $F(r, z)$. For times $t > 0$ heat is generated within the circular disk at a rate of $g(r, z, t)$ Btu/hr.ft^3. The heat flux is applied on the inner circular boundary $(r = a)$ and the outer circular boundary $(r = b)$. Also, the lower surface $(z = 0)$ is kept at temperature $Q_3(r, t)$ and the upper surface $(z = \infty)$ is kept at zero temperature. Hollow circular disk extends in the z-direction from $z = 0$ to infinity. The governing heat conduction equation has been solved by using finite Hankel transform and the generalized finite Fourier transform. As a special case mathematical model is constructed for different metallic disk have been considered. The results are obtained in series form in terms of Bessel's functions. These have been computed numerically and illustrated graphically.

1. INTRODUCTION

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly important. This is mainly due to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations [1].

Nowacki [2] has determined the steady-state thermal stresses in a circular plate subjected to an axi-symmetric temperature distribution on the upper surface with zero temperature on the lower surface and with the circular edge thermally insulated. Obata and Noda [3] studied the steady thermal stresses in a hollow circular cylinder and a hollow sphere made of a functionally gradient material. Ootao et al. [4] have studied the theoretical analysis of a three

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dimensional transient thermal stress problem for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction from the inner and outer surfaces. Ishihara at al. [5] discussed the theoretical analysis of thermoelastoplastic deformation of a circular plate due to a partially distributed heat supply. Eduardo at al. [6] discussed the generalized boundary integral equation for heat conduction in non-homogeneous media. Bao-Lin Wang et al. [7] have establishes a solution method for the one-dimensional transient temperature and thermal stress fields in non-homogeneous materials. Cheng-Hung Huang et al. [8] have determined an inverse hyperbolic heat conduction problem in estimating surface heat flux by the conjugate gradient method. Zhengzhu Dong et al. [9] have studied the thermal bending of circular plates for non-axisymmetrical problems. Ghadle et al. [10] have solved nonhomogeneous heat conduction problem and its thermal deflection due to internal heat generation in a thin hollow circular disk. Recently, Gaikwad [11] analysed thermoelastic deformation of a thin hollow circular disk due to partially distributed heat supply.

In this article, we analyzed the quasi-static thermal stresses in a semi-infinite hollow circular disk due to internal heat generation under unsteady-state temperature distribution and determined the expressions for temperature, displacement and thermal stresses. Initially the disk is kept at arbitrary temperature $F(r,z)$. For times $t > 0$ heat is generated within the circular disk at a rate of $g(r,z,t)$ Btu/hr.ft^3. The heat flux is applied on the inner circular boundary $(r = a)$ and the outer circular boundary $(r = b)$. Also, the lower surface $(z = 0)$ is kept at temperature $Q_3(r,t)$ and the upper surface $(z = \infty)$ is kept at zero temperature. Hollow circular disk extends in the z-direction from $z = 0$ to infinity. The governing heat conduction equation has been solved by using finite Hankel transform and the generalized finite Fourier transform. The results are obtained in series form in terms of Bessel's functions. As a special case mathematical model is constructed for different metallic disk have been considered. The results are obtained in series form in terms of Bessels functions and these have been computed numerically and illustrated graphically.

It is believed that, this particular problem has not been considered by any one. This is new and novel contribution to the field of thermoelasticity. The results presented here will be more useful in engineering problem particularly, in the determination of the state of strain in hollow circular disk constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

2. Mathematical Formulation of the problem

Consider a semi-infinite hollow circular disk occupying the space D: $a \le r \le b, 0 \le z < \infty$ under an unsteady temperature field due to internal heat generation within it. Initially, the circular disk is at arbitrary temperature $F(r,z)$. For times $t > 0$ heat is generated within the circular disk at a rate of $g(r,z,t)$ Btu/hr.ft^3. The heat flux is applied on the inner circular boundary $(r = a)$ and the outer circular boundary $(r = b)$. Also, the lower surface $(z = 0)$ is kept at temperature $Q_3(r,t)$ and the upper surface $(z = \infty)$ is kept at zero temperature. Hollow circular disk extends in the z-direction from $z = 0$ to infinity.

Under these realistic prescribed conditions, temperature, displacement and thermal stresses in a semi-infinite hollow circular disk due to internal heat generation are required to be determined.

The temperature of the disk satisfies the heat conduction equation as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{in } a \le r \le b,\ 0 \le z < \infty, \text{ for } t > 0 \tag{1}$$

with boundary conditions,

$$K\frac{\partial T}{\partial r} = Q_1(z,t) \qquad \text{at } r = a, \text{ for } t > 0 \tag{2}$$

$$K\frac{\partial T}{\partial r} = Q_2(z,t) \qquad \text{at } r = b, \text{ for } t > 0 \tag{3}$$

$$T = Q_3(r,t) \qquad \text{at } z = 0, \text{ for } t > 0 \tag{4}$$

$$T = 0 \qquad \text{at } z = \infty, \text{ for } t > 0 \tag{5}$$

and the initial condition

$$T = F(r,z) \qquad \text{in } a \le r \le b,\ 0 \le z < \infty \text{ for } t = 0, \tag{6}$$

where $T = T(r,z,t)$, K, α are the thermal conductivity and thermal diffusivity of the material of the circular disk.

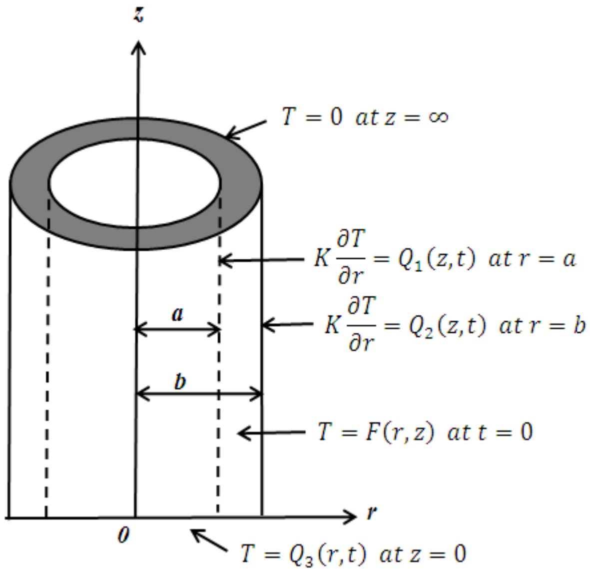

FIGURE 1. Geometry of the heat conduction problem.

The displacements equations of thermoelasticity have the form

$$U_{i,k,k} + \left(\frac{1+\nu}{1-\nu}\right) e_{,i} = 2\left(\frac{1+\nu}{1-\nu}\right) a_t T_{,i}$$

$$e = U_{k,k}; \quad k, i = 1, 2$$

where U_i is the displacements component, e is the dilatation, T is the temperature and ν and a_t are respectively, the Poisson ratio and linear coefficients of thermal expansions of the circular disk material.

Introducing $U_i = U_{,i} \quad i = 1, 2,$
we have

$$\nabla_1^2 U = (1+\nu) a_t T,$$

$$\nabla_1^2 = \frac{\partial^2}{\partial k_1^2} + \frac{\partial^2}{\partial k_2^2}$$

$$\sigma_{ij} = 2\mu(U_{,ij} - \delta_{ij} U_{,kk}) \qquad i, j, k = 1, 2$$

where μ is Lames constant and δ_{ij} is the well-known Kronecker symbol.

In the axisymmetric case,

$$U = U(r, z, t) \qquad T = (r, z, t)$$

and the differential equation governing the displacements function $U = U(r, z, t)$ is given by

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu) a_t T \tag{7}$$

$$\text{with} \quad U = 0 \qquad \text{at } r = a, r = b \quad \text{for all time } t \tag{8}$$

Initially

$$T = U = \sigma_{rr} = \sigma_{\theta\theta} = F(r, z) \qquad \text{at } t = 0. \tag{9}$$

The stress components σ_{rr} and $\sigma_{\theta\theta}$ of the circular disk are given by,

$$\sigma_{rr} = -\frac{2\mu}{r}\frac{\partial U}{\partial r} \tag{10}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \tag{11}$$

while each case of the stress functions σ_{rz} , σ_{zz} and $\sigma_{\theta z}$ are zero within the circular disk in the plane stress of the stress.

Equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

3. Solution of the heat conduction problem

To obtain the expression for temperature function $T(r,z,t)$; firstly we define the finite Fourier transform and its inverse transform over the variable z in the range $0 \le z < \infty$ defined in [12] as,

$$\overline{T}(r,\eta,t) = \int_{z'=0}^{\infty} K(\eta,z').T(r,z',t).dz' \tag{12}$$

$$T(r,z,t) = \int_{\eta=0}^{\infty} K(\eta,z).\overline{T}(r,\eta,t).d\eta \tag{13}$$

where

$$K(\eta,z) = \sqrt{\frac{2}{\pi}}\sin(\eta z).$$

and $\eta_1, \eta_2, \dots$ are the positive roots of the transcendental equation

$$\sin(\eta_p h) = 0, \quad p = 1,2,\dots$$

i.e.

$$\eta_p = \frac{p\pi}{h}, \quad p = 1,2,3,\dots$$

Applying the finite Fourier transform defined in Eq. (12) to Eq. (1) and using the conditions (2)-(6), one obtains

$$\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - \eta_p^2 + \frac{\overline{g}(r,\eta_p,t)}{K} = \frac{1}{\alpha}\frac{\partial \overline{T}}{\partial t} \tag{14}$$

with

$$K\frac{\partial \overline{T}}{\partial r} = \overline{Q}_1(\eta_p,t) \qquad \text{at } r=a, \text{ for } t>0 \tag{15}$$

$$K\frac{\partial \overline{T}}{\partial r} = \overline{Q}_2(\eta_p,t) \qquad \text{at } r=b, \text{ for } t>0 \tag{16}$$

$$\overline{T} = \overline{F}(r,\eta_p) \qquad \text{in } a \le r \le b, \text{ for } t=0, \tag{17}$$

where $\overline{T} = T(r,\eta_p,t)$.

Secondly, we define finite Hankel transform and its inverse transform over the variable r in the range $a \le r \le b$ as defined in [12] respectively as,

$$\overline{\overline{T}}(\beta_m,\eta_p,t) = \int_{r'=a}^{b} r'.K_0(\beta_m,r').T(r',\eta,t).dr' \tag{18}$$

$$\overline{T}(r,\eta,t) = \sum_{m=1}^{\infty} K_0(\beta_m,r).\overline{\overline{T}}(\beta_m,\eta,t) \tag{19}$$

where

$$K_0(\beta_m,r) = \frac{\pi}{\sqrt{2}}\frac{\beta_m J_0'(\beta_m b).Y_0'(\beta_m b)}{\left[1 - \dfrac{J_0'^2(\beta_m b)}{J_0'^2(\beta_m a)}\right]}\left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)}\right]$$

and $\beta_1, \beta_2, \beta_3, \ldots$ are the positive root of transcendental equation

$$\frac{J_0'(\beta a)}{J_0'(\beta b)} - \frac{Y_0'(\beta a)}{Y_0'(\beta b)} = 0$$

This transform satisfies the relation

$$H\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] = -\beta_m^2 \overline{T}(\beta_m, t)$$

Applying the finite Hankel transform defined in Eq. (18) to Eq. (14) and using the conditions (15)-(17, one obtains

$$\frac{\partial \overline{\overline{T}}(\beta_m, \eta_p, t)}{\partial t} + \alpha(\beta_m^2 + \eta_p^2)\overline{\overline{T}}(\beta_m, \eta_p, t) = A(\beta_m, \eta_p, t) \tag{20}$$

$$\overline{\overline{T}}(\beta_m, \eta_p, t) = \overline{\overline{F}}(\beta_m, \eta_p), \qquad \text{for } t = 0, \tag{21}$$

where

$$\begin{aligned} A(\beta_m, \eta_p, t) = &\frac{\alpha}{K}\overline{\overline{g}}(\beta_m, \eta_p, t) + \frac{\alpha}{K}\left\{aK_0(\beta_m, a)\overline{Q}_1(\eta_p, t) - bK_0(\beta_m, b)\overline{Q}_2(\eta_p, t)\right. \\ &\left. + \frac{dK_0(\eta_p, z)}{dz}\Big|_{z=0}\overline{Q}_3(\beta_m, t)\right\} \end{aligned} \tag{22}$$

Solution of the Eq. (20) is obtained as

$$\overline{\overline{T}}(\beta_m, \eta_p, t) = e^{-\alpha(\beta_m^2 + \eta_p^2)t}\left[\overline{\overline{F}}(\beta_m, \eta_p) + \int_{t'=0}^{t} e^{\alpha(\beta_m^2 + \eta_p^2)t'} A(\beta_m, \eta_p, t').dt'\right] \tag{23}$$

Finally taking inverse finite Hankel transform defined in Eq. (19) and inverse finite Fourier transform defined in Eq. (13), one obtains the expressions of the temperature $T(r, z, t)$ as

$$\begin{aligned} T(r, z, t) = &\sum_{m=1}^{\infty} K_0(\beta_m, r)\int_{\eta=0}^{\infty} K(\eta, z).e^{-\alpha(\beta_m^2 + \eta^2)t} \\ &\left\{\int_{r'=a}^{b}\int_{z'=0}^{\infty} r'.K_0(\beta_m, r').K(\eta, z').F(r', z').dr'.dz'\right. \\ &+ \int_{t'=0}^{t} e^{-\alpha(\beta_m^2 + \eta^2)t'}.\left[\frac{\alpha}{k}\int_{r'=a}^{b}\int_{z'=0}^{\infty} r'.K_0(\beta_m, r').K(\eta, z').g(r', z', t').dr'.dz'\right. \\ &+ \frac{\alpha}{K}(a.K_0(\beta_m, a) - b.K_0(\beta_m, b))\int_{z'=0}^{\infty} K(\eta, z').Q_{1,2}(z', t').dz' \\ &\left.\left. + \sqrt{\frac{2}{\pi}}\frac{\alpha}{K}.\eta.\int_{r'=a}^{b} r'.K_0(\beta_m, r').Q_3(r', t').dr'\right] dt'\right\} d\eta \end{aligned} \tag{24}$$

4. Thermoelastic Displacement Potential U

To obtain the displacement function U, using Eq. (24) in Eq. (7) one obtains,

$$\begin{aligned}\frac{\partial^2 U}{\partial r^2}+\frac{1}{r}\frac{\partial U}{\partial r} &= (1+\nu)a_t\sum_{m=1}^{\infty}K_0(\beta_m,r)\int_{\eta=0}^{\infty}K(\eta,z).e^{-\alpha(\beta_m^2+\eta^2)t}\\ &\left\{\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').F(r',z').dr'.dz'\right.\\ &+\int_{t'=0}^{t}e^{-\alpha(\beta_m^2+\eta^2)t'}.\left[\frac{\alpha}{k}\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').g(r',z',t').dr'.dz'\right.\\ &+\frac{\alpha}{K}(a.K_0(\beta_m,a)-b.K_0(\beta_m,b))\int_{z'=0}^{\infty}K(\eta,z').Q_{1,2}(z',t').dz'\\ &\left.\left.+\sqrt{\frac{2}{\pi}}\frac{\alpha}{K}.\eta.\int_{r'=a}^{b}r'.K_0(\beta_m,r').Q_3(r',t').dr'\right]dt'\right\}d\eta\end{aligned} \tag{25}$$

Solving Eq. (25), one obtains

$$\begin{aligned}U &= -(1+\nu)a_t\sum_{m=1}^{\infty}\frac{1}{\beta_m^2}K_0(\beta_m,r)\int_{\eta=0}^{\infty}K(\eta,z).e^{-\alpha(\beta_m^2+\eta^2)t}\\ &\left\{\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').F(r',z').dr'.dz'\right.\\ &+\int_{t'=0}^{t}e^{-\alpha(\beta_m^2+\eta^2)t'}.\left[\frac{\alpha}{k}\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').g(r',z',t').dr'.dz'\right.\\ &+\frac{\alpha}{K}(a.K_0(\beta_m,a)-b.K_0(\beta_m,b))\int_{z'=0}^{\infty}K(\eta,z').Q_{1,2}(z',t').dz'\\ &\left.\left.+\sqrt{\frac{2}{\pi}}\frac{\alpha}{K}.\eta.\int_{r'=a}^{b}r'.K_0(\beta_m,r').Q_3(r',t').dr'\right]dt'\right\}d\eta\end{aligned} \tag{26}$$

5. Quasi-Static Thermal Stresses

Using Eq. (26) in Eqs. (10) and (11), one obtains the expression for thermal stresses as,

$$\begin{aligned}\sigma_{rr} &= -2(1+\nu)a_t\mu\sum_{m=1}^{\infty}\frac{1}{r\beta_m}.K_1(\beta_m,r)\int_{\eta=0}^{\infty}K(\eta,z).e^{-\alpha(\beta_m^2+\eta^2)t}\\ &\left\{\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').F(r',z').dr'.dz'\right.\\ &+\int_{t'=0}^{t}e^{-\alpha(\beta_m^2+\eta^2)t'}.\left[\frac{\alpha}{k}\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').g(r',z',t').dr'.dz'\right.\end{aligned}$$

$$+\frac{\alpha}{K}(a.K_0(\beta_m,a)-b.K_0(\beta_m,b))\int_{z'=0}^{\infty}K(\eta,z').Q_{1,2}(z',t').dz'$$
$$+\sqrt{\frac{2}{\pi}}\frac{\alpha}{K}.\eta.\int_{r'=a}^{b}r'.K_0(\beta_m,r').Q_3(r',t').dr'\Bigg]dt'\Bigg\}d\eta \tag{27}$$

$$\sigma_{\theta\theta}=-2(1+\nu)a_t\mu\sum_{m=1}^{\infty}\frac{1}{\beta_m}\left(\beta_m K_0(\beta_m,r)-\frac{K_1(\beta_m,r)}{r}\right)\int_{\eta=0}^{\infty}K(\eta,z).e^{-\alpha(\beta_m^2+\eta^2)t}$$
$$\Bigg\{\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').F(r',z').dr'.dz'$$
$$+\int_{t'=0}^{t}e^{-\alpha(\beta_m^2+\eta^2)t'}.\Bigg[\frac{\alpha}{k}\int_{r'=a}^{b}\int_{z'=0}^{\infty}r'.K_0(\beta_m,r').K(\eta,z').g(r',z',t').dr'.dz'$$
$$+\frac{\alpha}{K}(a.K_0(\beta_m,a)-b.K_0(\beta_m,b))\int_{z'=0}^{\infty}K(\eta,z').Q_{1,2}(z',t').dz'$$
$$+\sqrt{\frac{2}{\pi}}\frac{\alpha}{K}.\eta.\int_{r'=a}^{b}r'.K_0(\beta_m,r').Q_3(r',t').dr'\Bigg]dt'\Bigg\}d\eta \tag{28}$$

where $K_1(\beta_m,r)=\frac{\partial}{\partial r}[K_0(\beta_m,r)]$

6. Special Cases and Numerical Calculations

Setting
$Q_1(z,t)=Q_2(z,t)=e^{-z}.e^{-At}$,
$Q_3(r,t)=(r^2-a^2)^2(r^2-b^2)^2.e^{-At}$,
$F(r,z)=(r^2-a^2)^2(r^2-b^2)^2.e^{-z}$,
$g(r,z,t)=g_i\delta(r-r_1)\delta(z-z_1)\delta(t-\tau)$Btu/hr.ft^3,
where δ is the Derac-delta function and $A>0$.

We noticed that

$$\int_{r'=a}^{b}r'.K_0(\beta_m,r').Q_3(r',t').dr'=$$
$$\frac{8\left\{\begin{array}{c} b(40a^2b^2\beta_m^4-32b^4\beta_m^4-8a^4\beta_m^4+2304b^2\beta_m^2-576a^2\beta_m^2-18432)J_1(\beta_m a)\\ -b(40a^2b^2\beta_m^4-32b^4\beta_m^4-8a^4\beta_m^4+2304b^2\beta_m^2-576a^2\beta_m^2-18432)J_1(\beta_m a)\end{array}\right\}}{\pi\sqrt{N}\beta_m^{11}J_1(\beta_m a)J_1(\beta_m b)Y_1(\beta_m b)} \tag{29}$$

$$\int_{r'=a}^{b}\int_{z'=0}^{\infty} r'.K_0(\beta_m, r').K(\eta_p, z').g(r', z', t').dr'.dz' =$$
$$\sqrt{\pi}.r_1 \frac{\beta_m J_0(\beta_m b).Y_0(\beta_m b)}{\left[1-\dfrac{J_0^2(\beta_m b)}{J_0^2(\beta_m a)}\right]^{1/2}} \left[\frac{J_0(\beta_m r_1)}{J_0(\beta_m b)} - \frac{Y_0(\beta_m r_1)}{Y_0(\beta_m b)}\right] \times \sin(\eta_p z_1) \tag{30}$$

The heat source $g(r, z, t)$ is an instantaneous line heat source of strength g_i=50 Btu/hr.ft^3, situated at center of the circular disk along radial direction and releases its instantaneously at the time $t = \tau = 2$ hr.

Dimension

The constants associated with the numerical calculation are taken as
Inner radius of a circular disk a = 1 ft,
outer radius of a circular disk b = 2 ft,
Central circular path of circular disk r_1= 1.5 ft,
Height of a circular disk z = 10 ft,
Central height of a circular disk z_1 = 5 ft,

Material properties

The numerical calculation has been carried out for a semi-infinite hollow circular disk with the material properties as,

TABLE 1. Thermal properties of materials.

Material	K, Btu/hr. ft. ^0F	c_p, Btu/lb ^{0}F	ρ, lb/ft^3	α, ft^2/hr	λ, $1/F$	E, GP$_a$	ν
Aluminum(Al)	117	0.208	169	3.33	12.84 $\times 10^{-6}$	70	0.35
Copper(Cu)	224	0.091	558	4.42	9.3 $\times 10^{-6}$	117	0.36
Iron(Fe)	36	0.104	491	0.70	6.7 $\times 10^{-6}$	193	0.21
Silver(Ag)	242	0.056	655	6.60	10.7 $\times 10^{-6}$	83	0.37

Roots of the transcendental equation

The first five positive root of the transcendental equation $\dfrac{J_0'(\beta a)}{J_0'(\beta b)} - \dfrac{Y_0'(\beta a)}{Y_0'(\beta b)} = 0$ as defined in [12] are $\beta_1 = 3.1965$, $\beta_2 = 6.3123$, $\beta_3 = 9.4445$, $\beta_4 = 12.5812$, $\beta_5 = 15.7199$.

For convenience, we set

$$A = \frac{1}{10^3}, \quad B = \frac{-2(1+\nu)\, a_t}{10^3}, \quad C = \frac{-2(1+\nu)\mu a_t}{10^3}.$$

The numerical calculation has been carried out with help of computational mathematical software Mathcad-2007, and the graphs are plotted with the help of Excel (MS Office-2007).

7. Discussion of the Results

In this study, we analyzed quasi static thermal stresses in a semi-infinite hollow circular disk due to internal heat generation under unsteady-state temperature distribution. As an illustration, we carried out numerical calculations for a hollow circular disk made up of different metals viz. Aluminium, Copper, Iron (pure), Silver and examined the thermoelastic behavior in the state for the temperature, displacement and thermal stresses in the radial direction.

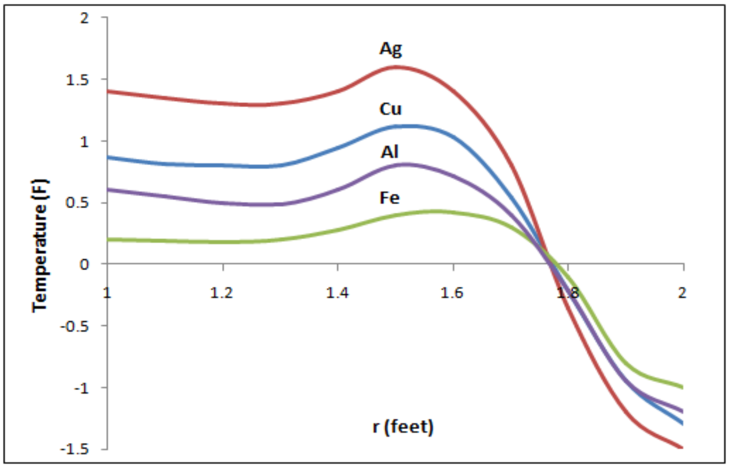

FIGURE 2. Temperature distribution T/A in radial direction.

Figure 1, shows the variation of temperature T versus radius r, it is clear that temperature is maximum at the inner boundary surface (r = 1) and decreases from outer boundary surface with the increase of radius r. It becomes zero at the $(r = 1.75)$ of the circular disk.

Figure 2, shows the variation of displacement U versus radius r, it is clear that displacement is zero at $(r = 1.9)$ and maximum is occur at $(r = 1.4)$ of the circular disk.

Figure 3, shows the variation of radial stresses versus radius, it is seen that σ_{rr} is zero at the inner boundary surface $(r = 1)$ and increases from outer boundary surface $(r = 2)$ with the increase of radius r. It is clear that maximum stress is occur at the middle surface of the circular disk.

Figure 4, shows the variation of axial stresses versus radius, it is seen that $\sigma_{\theta\theta}$ is maximum at $(r = 1.4)$ and develops the compressive stresses in radial direction.

We can summarize that, the temperature, displacement and thermal stresses occurs near heat source, due to internal heat generation in a hollow circular disk within it. The numerical values of the temperature, displacement and stresses for the disk of metals Steel, Iron, Aluminum and Copper are in the proportion and follows relation Iron $\leq$ Aluminum $\leq$ Copper $\leq$ Silver. It is clear that, these values are directly proportional to the thermal conductivity.

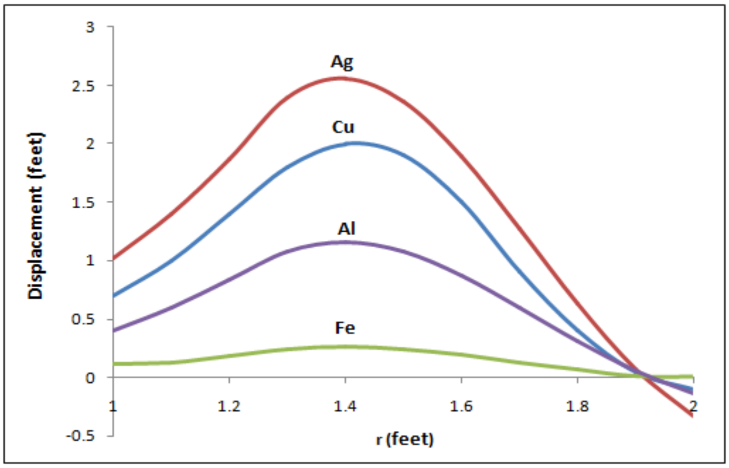

FIGURE 3. Displacement function U/B in radial direction.

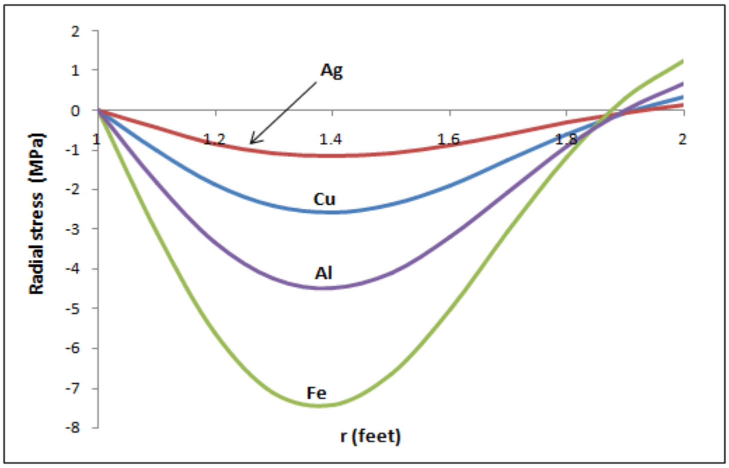

FIGURE 4. Radial Stress function σ_{rr}/C in radial direction.

8. CONCLUDING REMARKS

In this article, we analyzed the quasi-static thermoelastic problem in a semi-infinite hollow circular cylinder under unsteady-state temperature field due to internal heat generation within

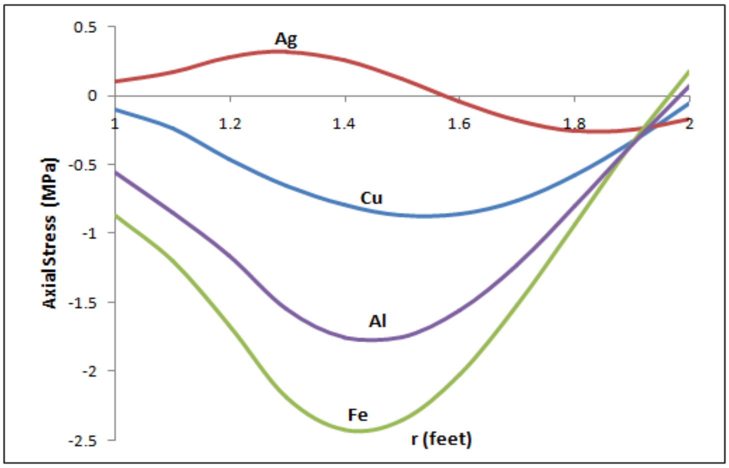

FIGURE 5. Axial Stress function $\sigma_{\theta\theta}/C$ in radial direction.

it. The present method is based on the direct method, using the finite Hankel transform and the generalized finite Fourier transform. As a special case mathematical model is constructed for different metallic disk have been considered. The numerical results are compared with different metal disks. We conclude that, the displacement and stresses are proportional to the thermal conductivity of the metal of the disk. From the figure of displacement and stresses, it can be observed that direction of heat flow and direction of body displacement are opposite. Due to heat generation within the hollow circular disk, the displacement function develops the tensile stresses, whereas the radial and axial stresses develops the compressive stresses in radial direction.

The results presented here will be useful in engineering problems, particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. Also, any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (24)–(28).

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