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Forecast of Korea Defense Expenditures based on Time Series Models

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Abstract

This study proposes a mathematical model that can forecast national defense expenditures. The ongoing European debt crisis weighs heavily on markets; consequently, government spending in many countries will be constrained. However, a forecasting model to predict military spending is acutely needed for South Korea because security threats still exist and the estimation of military spending at a reasonable level is closely related to economic growth. This study establishes two models: an Auto-Regressive Moving Average model (ARIMA) based on past military expenditures and Transfer Function model with the Gross Domestic Product (GDP), exchange rate and consumer price index as input time series. The proposed models use defense spending data as of 2012 to create defense expenditure forecasts up to 2025.

Keywords: Defense expenditures, GDP, exchange rate, Consumer price index, ARIMA model, Transfer Function model.

1. Introduction

Military expenditures include any spending for the purpose of national security; consequently, they are a necessary expense to generate public goods of 'security' (Baek *et al.*, 2002). Spending is necessary because it protects people and their property and because national security is related to economic growth that it demonstrates a nation' willingness to protect national interests. This is particularly relevant in South Korea, a divided nation with an ongoing military confrontation between South and North. The development of a forecasting model to predict defense expenditures is salient due to internal and external security factors such as the transfer of wartime operational control. High spending on national defense is considered to weakens trade competitiveness and disperses investment with a negative influence on economic growth (Mintz and Huang, 1990; Rothschild, 1973); however, Korea has achieved a high rate of economic growth while maintaining high national defense spending (Chan and Mintz, 2002). In this case, the threat of military confrontation between the two Koreas means that a strong commitment to defending national security plays a critical role to boost the South Korean economy. The correlation between Korea's spending on national defense and its economic growth rate supports the hypothesis of Aizenman and Glick (2006), which claims that defense expenditures in the presence of threats have a positive impact on economic growth despite the direct negative impact of military expenditures and security threats on economic growth (Chan and Mintz, 2002).

Factors that influence defense expenditures are largely categorized into economic conditions such as fiscal capability and national security threats. A country's existence would be at risk if defense

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spending were too low in the presence of security threats. Excessive spending on defense disproportionate to general economic circumstances would hamper economic growth. Economic conditions have a stronger impact on Korean military expenditures than any other factor because the national policy direction focuses on the growth of the economy (Korean Defense Ministry, 2006). Other issues (such as security threats, the modernization of military facilities, and internal and external issues in response to the transfer of wartime operational control) must also be considered when estimating Korea's defense expenditures.

An appropriate forecast defense spending model is important if the ongoing European sovereign-debt crisis continues to make it impossible for the Korean government to increase spending. Current studies have focused on choosing variables that affect the estimation of defense expenditures and how to allocate them; however, economic conditions matter most when forecasting a country's military expenditures. This study suggests using an ARIMA model (Anderson, 1971; Box and Jenkins, 1976) based on past defense expenditures and a Transfer Function model (William, 1996) with GDP, exchange rate and consumer price index as input variables to generate an appropriate military spending forecasting model. Based on the estimates obtained from these models and other variables that affect the forecast of military spending, security threats and economic conditions, it would be rational for economic and military experts to revise forecasting methods to produce more reasonable forecasts.

This paper sees a sense of urgency to develop a mathematical model based on time series data of past defense expenditures. This paper determines an appropriate forecast model that is mathematically feasible and provides basic data to estimate defense expenditures. This study utilizes and analyzes military spending data from 1970 to 2012 published by the Ministry of National Defense, the Population Projection for Korea released by Statistics Korea, GDP, exchange rate and consumer price index released by the Bank of Korea.

2. The Forecasts of Korea Defense Expenditures based on the Univariate Time Series Model

The defense expenditure per person(z_t) is a sequence of observations taken over time and it is needed to investigate the stationary, trend, periodicity and autocorrelation before analyzing the defense expenditure data.

The ARIMA model developed by Box and Jenkins (1976) assumes that the time series is stationary; therefore, raw data is transformed to defense expenditure per person to obtain stationary time series that has a constant mean and a constant variance. It gradually increases over time and shows that variance stabilization is needed through data transformation (Figure 1). We used square root transformation in order to obtain data with constant variance.

Table 1 shows that since the p-values from a unit root test of $\sqrt{z_t}$ are not significant, unit root exists. If the presence of a unit root is not rejected, then one should apply the difference operator to the series. After differencing, we consider ARIMA model. Figure 2 confirms that the first differenced square root transformed data is stationary, with a constant mean and a constant variance. The three types of parameters in the ARIMA(p, d, q) model are: the autoregressive parameter "p", the number of differencing passes "d", and the moving average parameter "q". In general, the parameters p and q in the model can be identified based on the shape of the autocorrelation function (ACF) and partial auto correlation function (PACF). Since ACF and PACF show that there is drastic decreasing after time lag 1 from Figure 3, we start with an ARIMA$(1, 1, 1)$ model, then the ARIMA$(1, 1, 0)$ model, finally, the ARIMA$(0, 1, 1)$ model. Table 2 compares two models having significant estimates using Akaike Information Criterion(AIC) and Schwartz Bayesian Criterion(SBC). In Table 2, AIC and SBC

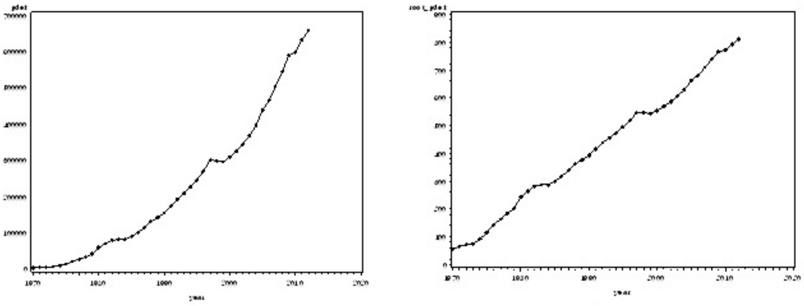

Figure 1: *Time series plot of* z_t *(left) and* $\sqrt{z_t}$ *(right)*

Table 1: Unit root test of $\sqrt{z_t}$

Model	White noise	AR(1)	AR(2)	AR(3)
p-value	0.994	0.967	0.959	0.945

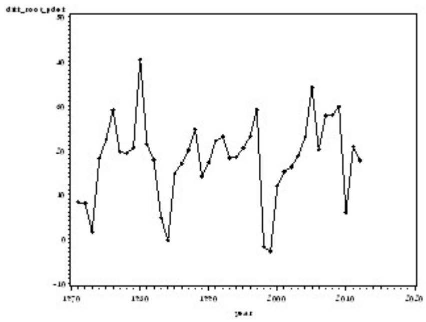

Figure 2: *Time series plot of* $\triangledown \sqrt{z_t}$

Table 2: AIC and SBC for the suggested models

Model	AIC	SBC
ARIMA(1, 1, 0)	309.26	312.78
ARIMA(0, 1, 1)	309.89	313.41

choose the ARIMA(1, 1, 0) model as the best identified model.

Table 3 shows estimates and standard errors of parameter from the identified model. The estimates are significant with p-values less than 0.05. After we verify the residuals are not autocorrelated and are white noise, the identified ARIMA(1, 1, 0) model is considered to be a proper model for forecasting.

Table 4 shows Portmanteau's chi-square test to check the assumption that the residuals are a random series such as white noise. We can conclude that the ARIMA(1, 1, 0) model is considered to be proper because the p-value is adequate based on the autocorrelation diagnosis proposed in Table 4.

Following is the final ARIMA(1, 1, 0) model for $\sqrt{z_t}$, the square root transformed defense expenditure per person.

$$\sqrt{z_t} = 11.05 + \sqrt{z_{t-1}} + 0.38\left(\sqrt{z_{t-1}} - \sqrt{z_{t-2}}\right) + \epsilon_t. \tag{2.1}$$

The correlation coefficient between the square root transformed defense expenditure per person of this

Lag	Correlation	9 8 7 6 5 4 3 2 1	1 2 3 4 5 6 7 8 9	Correlation	9 8 7 6 5 4 3 2 1	1 2 3 4 5 6 7 8 9
1	0.38250		********	0.38250		********
2	0.06649		*	-0.09350	**	
3	-0.15721	***		-0.17683	****	
4	-0.10496	**		0.03291		*
5	-0.14557	***		-0.12915	***	
6	-0.18425	****		-0.14324	***	
7	-0.34117	********		-0.28445	******	
8	-0.21966	***		-0.05141	*	
9	-0.19444	***		-0.20751	****	
10	-0.03438	*		-0.07445	*	
11	0.14181		***	0.09150		**
12	0.21618		****	-0.01221		
13	0.01035			-0.26150	*****	
14	0.10259		**	0.09747		**
15	0.18918		****	0.12738		***
16	0.18776		****	-0.08453	**	
17	0.15648		***	0.15724		***
18	-0.12013	**		-0.15854	***	
19	-0.17844	****		-0.01564		
20	-0.09200	**		0.07088		*

Figure 3: *ACF(left) and PACF(right) of* $\nabla \sqrt{z_t}$

Table 3: The parameter estimation of ARIMA(1, 1, 0) model

Parameter	Estimate	Standard Error	t-value	P-value	Lag
MU	17.82	2.10	8.49	<.0001	0
AR1,1	0.38	0.14	2.68	0.01	1

Table 4: The Portmanteau's chi-square test of residuals

To Lag	χ^2	DF	P	Autocorrelations					
6	2.02	5	0.85	0.034	−0.017	−0.187	−0.005	−0.065	−0.032
12	11.23	11	0.42	−0.276	−0.059	−0.144	−0.026	0.108	0.219
18	18.77	17	0.34	−0.135	0.054	0.133	0.096	0.181	−0.155
24	25.53	23	0.32	−0.145	−0.004	0.015	−0.131	−0.095	−0.159

year and the square root transformed defense expenditure per person of last year is 0.38.

3. The Forecasts of Korea Defense Expenditures using the Transfer Function

Defense expenditure is affected by national financial affairs. Korea's policy direction focuses on growing the economy; consequently, economic conditions have a stronger impact on its military expenditures than other factors (Aizenman and Glick, 2006). Therefore, we propose a Transfer Function model that has GDP (g_t), exchange rate(c_t) and consumer price index(x_t) as input time series variables, and defense expenditure per person(z_t) as the output time series variable. Transfer function model is a statistical model describing the relationship between an output variable and one or more input variables. If input series $X_{1,t}, \ldots, X_{k,t}$ can be represented as an ARIMA(p, d, q) model, then we obtain the transfer function model

$$Y_t = \sum_{i=1}^{k} \frac{w_{i,s}(B)}{\delta_{i,r}(B)} B^b X_{i,t} + N_t,$$

where N_t is a zero-mean stationary process, uncorrelated with $\{X_{i,t}, i = 1, \ldots, k\}$, $w_{i,s}(B) = w_{i,0} - w_{i,1}B - \cdots - w_{i,s}B^s$ and $\delta_{i,r}(B) = 1 - \delta_{i,1}B - \cdots - \delta_{i,r}B^r$.

The parameter b is time lag of input time series having first effects on output time series, s is time lag enduring affects of output time series at time t from input time series at time $t - b$ and r is the

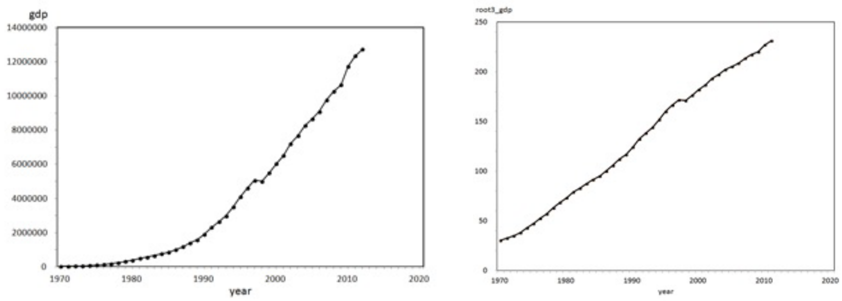

Figure 4: *Time series plot of GDP(KRW)(left) and Time series plot of the cube root transformed GDP(KRW)(right)*

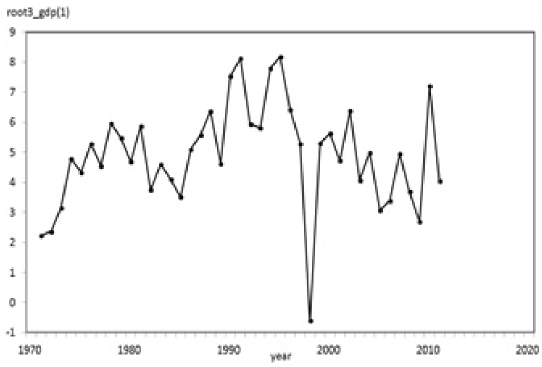

Figure 5: *Time series plot of the first differenced cube root transformed GDP* $\nabla \sqrt[3]{g_t}$

parameter representing the relation between after input time series at time $t - b - s$ and output time series at time t. It predicts future values of a time series based on past time series values and on the values of one or more time series related to the time series to be predicted. Our purpose in using the Transfer Function model is to improve the one-variable time series model by analyzing the dynamic relation between input time series data and output time series data. Before setting a transfer function model, the input-output data are first transformed to stationary data, and the ARIMA(p, d, q) model for input data and output data are found through the prewhitening procedure which changes the input (output) data into white noise beforehand. In order to apply the prewhitening procedure, it is first necessary to investigate the time series plot of GDP. The increase gradually becomes larger as time goes by in Figure 4 (left), so the variance can be stabilized by a cube root transformation of data. The cube root transformed GDP has a trend factor from Figure 4 (right), and it can be removed from first order differencing.

Figure 5 verifies that the data is transformed into an approximately stationary time series which has a constant mean. We choose ARIMA$(1, 1, 1)$ as the fitted model since the ACF and PACF for first order differencing GDP after cube root transforming show a spike at time lag 1 in Figure 6. All estimates in Table 5 are significant, and because p-value of Portmanteau's chi-square test is large enough so that the hypothesis "The autocorrelation is equal to zero" cannot be rejected in every time lag as a results of autocorrelation diagnosis of residuals from Table 6.

We can verify that the ARIMA$(1, 1, 1)$ model of the cube root transformed GDP is suitable since

Lag	Correlation	9 8 7 6 5 4 3 2 1	1 2 3 4 5 6 7 8 9	Correlation	9 8 7 6 5 4 3 2 1	1 2 3 4 5 6 7 8 9
1	0.28612		******	0.28612		******
2	0.15438		***	0.07898		**
3	0.10258		**	0.04307		*
4	-0.02628	*		-0.08315	**	
5	0.08315		**	0.10655		**
6	-0.04993	*		-0.10065	**	
7	-0.07912	**		-0.05545	*	
8	-0.02533	*		0.00828		
9	-0.04810	*		-0.00683		
10	-0.07255	*		-0.07418	*	
11	0.01732			0.07363		*
12	-0.12793	***		-0.14308	***	
13	0.05254		*	0.13356		***
14	-0.03512	*		-0.09452	**	
15	-0.11071	**		-0.06336	*	
16	0.00912			0.01720		
17	-0.18228	****		-0.15238	***	
18	-0.18295	****		-0.14482	***	
19	-0.16884	***		-0.07287	*	
20	-0.14778	***		-0.02547	*	

Figure 6: *ACF and PACF of* $\nabla \sqrt[3]{g_t}$

Table 5: The parameter estimation of ARIMA(1, 1, 1) model

Parameter	Estimate	Standard Error	t-value	P-value	Lag
MA1,1	0.672	0.130	5.18	<.0001	1
AR1,1	0.990	0.018	55.24	<.0001	1

Table 6: The Portmanteau's chi-square test of residuals

To Lag	χ^2	DF	P	Autocorrelations					
6	1.52	4	0.824	0.084	−0.056	0.068	−0.083	0.093	−0.035
12	3.04	10	0.981	−0.007	0.013	−0.004	−0.068	0.042	−0.136
18	11.24	16	0.795	0.118	0.191	0.071	0.236	−0.050	−0.074
24	13.13	22	0.930	0.047	0.021	−0.031	−0.103	0.010	−0.076

the ACF and PACF of residuals are all included in twice of standard error.

Figure 7 shows that the exchange rate (c_t) and consumer price index (x_t) have to be transformed into the stationary time series by an appropriate transformation. The variance of c_t is stabilized by root transformation and then we consider the difference in the transformed data $\sqrt{c_t}$. By inspecting the ACF and PACF of $\nabla \sqrt{c_t}$, we choose the white noise as the fitted model. Figure 7(right) shows that since the increase gradually becomes larger as time goes by, we can eliminate the trend by first order differencing; consequently, ∇x_t is transformed into an approximately stationary time series. By inspecting the ACF and PACF, we choose the ARIMA(1, 1, 1) model as the fitted model of ∇x_t.

The parameters (b, s, r) of the Transfer Function model should be decided through cross covariance and cross correlation after fitting the same model to the square root of defense expenditures per person which is the output time series data. First, we consider GDP (g_t). The cross correlation between $\nabla \sqrt{z_t}$ and $\nabla \sqrt[3]{g_t}$ can be decided to be 0 in negative time lag, which means it is reasonable to use the first order differencing GDP data after square root transforming as the input time series variable of our Transfer Function model. We determine that the parameters (b, s, r) are $(1, 0, 0)$ or $(1, 0, 1)$ since the cross correlations between $\nabla \sqrt{z_t}$ and $\nabla \sqrt[3]{g_t}$ are significant at time lag 1 and not significant after time lag 1 and display a cutoff. Second, we consider exchange rate (c_t). The cross correlations between $\nabla \sqrt{z_t}$ and $\nabla \sqrt{c_t}$ are significant at time lag 1 and 2 and cut off after lag 2. Thus, we determine that the parameters (b, s, r) are $(1, 1, 0)$ or $(1, 1, 1)$. Finally, we consider consumer price index (x_t). By inspecting the cross correlations between $\nabla \sqrt{z_t}$ and ∇x_t, since cross correlations are not significant

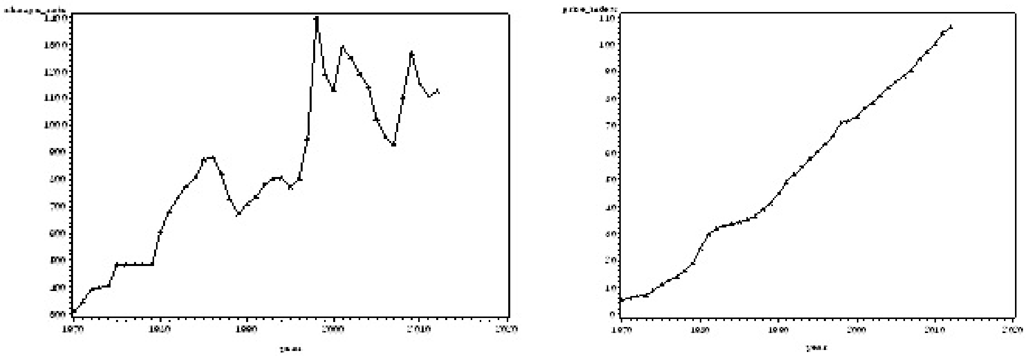

Figure 7: *Time series plot of the exchange rate c_t (left) and consumer price index x_t (right)*

Table 7: The maximum likelihood estimation for final model

Input variable	TF model	estimate	Standard Error	t-value	P-value
$\nabla \sqrt[3]{g_t}$	$(1, 0, 0)$	2.34	0.638	3.67	0.0002
∇x_t	$(0, 0, 0)$	2.54	1.209	2.10	0.0360

Table 8: The Portmanteau's chi-square test of residuals

To Lag	χ^2	DF	P	Autocorrelations					
6	6.54	6	0.366	0.229	0.267	0.075	−0.022	−0.003	−0.119
12	9.67	12	0.645	−0.149	−0.077	−0.104	−0.021	−0.124	0.051
18	14.90	18	0.669	−0.161	−0.082	−0.096	−0.096	0.044	−0.151
24	24.09	24	0.456	0.073	−0.037	−0.039	−0.162	−0.174	−0.174

and cut off after lag 0, we decide that the parameters (b, s, r) are $(0, 0, 0)$ or $(0, 0, 1)$.

We must search the most satisfactory Transfer Function model with significant independent variables and parameters using parameters of the Transfer Function model to the input variables $\nabla \sqrt[3]{g_t}$, $\nabla \sqrt{c_t}$ and ∇x_t. We can test eight models using the parameters identified above. From results of maximum likelihood estimation to eight models, we choose significant transfer function models $\nabla \sqrt[3]{g_t}(1, 0, 0)$ and $\nabla x_t(0, 0, 0)$ at level of significance 0.1. Table 7 shows the maximum likelihood estimation for the selected final model with input variables of GDP and consumer price index and an output variable of defense expenditure.

After deciding the parameters of the Transfer Function model, it is necessary to estimate the model for noise series of the Transfer Function model based on the residual series of the model. Table 8 shows estimates and standard errors of parameter from the final Transfer Function model for noise series. In Table 8, the p-value of Portmanteau's chi-square test for verification of autocorrelation of residual series is satisfactory that the null hypothesis, "There is no correlation", cannot be rejected, which shows this residual series is a random series such as white noise. Hence, the final Transfer Function model is a good fitted model.

The cross-correlations of the final residuals and prewhitened input time series in Table 9 and Table 10 suggest that both series are independent. The hypothesis "The cross-correlation is equal to 0" cannot be rejected since the p-values of both series are adequate.

From Table 7, Transfer Function model is derived as follows:

$$\nabla \sqrt{z_t} = \frac{w_{s1}(B)}{\delta_{r1}(B)} B^{b1} \nabla \sqrt[3]{g_t} + \frac{w_{s2}(B)}{\delta_{r2}(B)} B^{b2} \nabla \sqrt{x_t} + \epsilon_t, \tag{3.1}$$

Table 9: Cross-correlation Check of Residuals with input $\sqrt[3]{g_t}$

To Lag	χ^2	DF	P	Autocorrelations					
5	2.46	5	0.782	0.016	0.192	−0.037	−0.121	0.092	−0.007
11	11.83	11	0.376	−0.196	−0.301	−0.171	−0.146	−0.230	0.038
17	14.30	17	0.646	0.142	0.137	0.028	0.061	0.010	0.135
23	17.81	23	0.768	0.066	−0.173	0.007	−0.124	−0.117	−0.157

Table 10: Cross-correlation Check of Residuals with input x_t

To Lag	χ^2	DF	P	Autocorrelations					
5	5.10	5	0.40	0.013	0.220	0.028	−0.208	−0.170	−0.052
11	9.39	11	0.59	0.076	0.055	−0.035	−0.026	−0.272	0.142
17	12.29	17	0.78	−0.026	−0.047	0.046	0.082	0.216	0.111
23	16.79	23	0.82	−0.003	0.025	−0.144	−0.062	−0.164	−0.240

where $b1 = 1, s1 = 0, r1 = 0, b2 = 0, s2 = 0$ and $r2 = 0$.

Finally, the following model can be constructed using the estimates in Table 7.

$$\sqrt{z_t} = \sqrt{z_{t-1}} + 2.34\,(\sqrt[3]{g_{t-1}} - \sqrt[3]{g_{t-2}}) + 2.54\left(\sqrt{x_t} - \sqrt{x_{t-1}}\right) + \epsilon_t. \tag{3.2}$$

As a result, defense expenditure can be estimated from the above formula.

4. Forecasting

After fitting a model, we estimate a future value at time n based on the fitted model, while the actual value is unknown. We proposed the forecasts for the defense expenditure constructed by the ARIMA model and the Transfer Function model. We let the forecasted value $w_n(l)$ of $\sqrt{z_t}$ at time $t = n + l$. For the ARIMA model, we use the following equation to calculate the values of the forecast:

$$w_n(l) = \mu + \phi_1^l\left(\sqrt{z_n} - \mu\right).$$

From the equation (2.1) of the fitted ARIMA model, $\hat{\mu} = 17.82$, $\hat{\phi}_1 = 0.38$ and $\hat{\sigma}^2 = 74.08$. We can also obtain the standard error from the equation $\sqrt{\hat{\sigma}^2[1 + (\hat{\phi}_1 - \hat{\theta}_1)^2/(1 - \hat{\phi}_1^2)]}$. The forecasts can be obtained by iterating recursion for equation (3.2) of the fitted transfer function model. The final forecasts of defense expenditures are obtained by multiplying the forecasted defense expenditure by the population of Korea. Table 11 shows the forecasts and standard errors of defense expenditures up to 2025. Based on our final fitted models and Korea's projected population from the National Statistical Office, we can predict that defense expenditures in 2025 will reach 48667.334 billion KRW.

AIC, SBC and MSE are model selection criteria and the best model is the model that minimizes that criterion. Table 12 shows that the Transfer Function model is an appropriate model for the defense expenditure since AIC, SBC and MSE values achieved in the Transfer Function model are smaller than those achieved in the ARIMA model.

5. Concluding Remarks

This paper proposed forecasting time series models for Korean defense expenditures using an ARIMA model based on past defense expenditures and a Transfer Function model with GDP, exchange rate and consumer price index as input time series. We omitted some complex formulas and intermediate results for the sake of simplicity. We can see that the Transfer Function model is an appropriate model for defense expenditures, since AIC, SBC and MSE values achieved in the Transfer Function model

Table 11: The actual value and forecasts of the defense expenditure (Unit is one hundred - million KRW and standard error is given in parenthesis.)

Year	Actual value	Arima model	residual	TF model	Residual
1972	1738	2048.07 (24.82)	−310.07	1692.62 (23.76)	−45.38
1973	1843	2525.81 (25.26)	−682.81	2080.82 (24.18)	237.82
1974	2910	2510.10 (25.70)	399.90	2520.41 (24.60)	−389.59
1975	4588	4230.71 (26.14)	357.29	4144.26 (25.02)	−443.74
1976	7327	6402.23 (26.56)	924.77	5913.67 (25.42)	−1413.33
1977	9626	9919.77 (26.97)	−293.77	9154.22 (25.82)	−471.78
1978	12223	12126.29 (27.39)	96.71	11759.77 (26.22)	−463.23
1979	15366	15043.59 (27.80)	322.41	15497.76 (26.62)	131.76
1980	22465	18651.38 (28.24)	3813.62	19976.29 (27.03)	−2488.71
1981	26979	28073.30 (28.69)	−1094.30	27583.40 (27.46)	604.40
1982	31207	31513.18 (29.13)	−306.18	31508.74 (27.89)	301.74
1983	32741	35797.82 (29.57)	−3056.82	34314.00 (28.30)	1573.00
1984	33061	36173.76 (29.93)	−3112.76	36142.07 (28.65)	3081.07
1985	36892	35964.39 (30.23)	927.61	36168.02 (28.94)	−723.98
1986	41580	41488.20 (30.53)	91.80	39937.96 (29.23)	−1642.04
1987	47454	46742.86 (30.83)	711.14	45951.69 (29.52)	−1502.31
1988	55202	53367.08 (31.14)	1834.92	53643.52 (29.81)	−1558.48
1989	60148	62231.10 (31.45)	−2803.10	62228.33 (30.10)	2080.33
1990	66378	66139.10 (31.76)	238.90	67276.22 (30.40)	898.22
1991	74764	73146.07 (32.07)	1617.93	76980.87 (30.70)	2216.87
1992	84100	82782.40 (32.41)	1317.60	85548.42 (31.02)	1448.42
1993	92154	92794.77 (32.74)	−640.77	92959.59 (31.34)	805.59
1994	100753	100554.09 (33.07)	198.91	102370.98 (31.66)	1617.98
1995	110743	109656.02 (33.40)	1086.98	112668.48 (31.98)	1925.48
1996	122434	120470.78 (33.72)	1963.22	124152.68 (32.28)	1718.68
1997	137865	133216.20 (34.04)	4648.80	134356.78 (32.59)	−3508.22
1998	138000	150333.41 (34.29)	−12333.41	151807.33 (32.82)	13807.33
1999	137490	144287.13 (34.53)	−6797.13	139016.41 (33.06)	1526.41
2000	144774	143715.94 (34.82)	1058.06	147187.43 (33.34)	2413.43
2001	153884	154131.10 (35.08)	−247.10	156936.89 (33.58)	3052.89
2002	163640	163983.21 (35.28)	−343.21	163765.12 (33.77)	125.12
2003	175148	174237.35 (35.45)	910.65	176974.49 (33.94)	1826.49
2004	189412	186534.32 (35.59)	2877.68	185772.85 (34.07)	−3639.15
2005	211026	201960.83 (35.66)	9065.17	200554.36 (34.14)	−10471.64
2006	225129	227775.31 (35.83)	−2646.31	219874.85 (34.30)	−5254.15
2007	244972	238751.04 (36.00)	6220.96	235275.59 (34.46)	−9696.41
2008	266490	262017.93 (36.26)	4472.07	262489.58 (34.71)	−4000.42
2009	289803	283728.73 (36.43)	6074.27	278945.66 (34.88)	−10857.34
2010	295627	308399.94 (36.60)	−12772.94	301543.75 (35.04)	5916.75
2011	314031	308114.35 (36.88)	5916.65	319011.48 (35.30)	4980.48
2012	329576	330682.07 (37.04)	−1106.07	327455.38 (35.46)	−2120.62
2013	344970	345612.61 (37.20)	−642.61	340648.36 (40.66)	4321.64
2014	·	361148.67 (37.35)	·	354283.63 (98.20)	·
2015	·	377851.78 (109.35)	·	367790.70 (173.53)	·
2016	·	394992.40 (198.07)	·	381177.15 (270.93)	·
2017	·	412522.43 (294.06)	·	394430.61 (394.17)	·
2018	·	430406.65 (393.28)	·	407529.58 (546.61)	·
2019	·	448629.59 (494.04)	·	420463.80 (731.17)	·
2020	·	467180.06 (595.66)	·	433224.67 (950.45)	·
2021	·	486051.62 (697.85)	·	445807.56 (1206.69)	·
2022	·	505226.63 (800.42)	·	458197.93 (1501.83)	·
2023	·	524677.65 (903.24)	·	470373.46 (1837.47)	·
2024	·	544436.62 (1006.31)	·	482365.88 (2215.18)	·
2025	·	564469.42 (1109.51)	·	494148.06 (2636.07)	·

Table 12: AIC, SBC and MSE for ARIMA and Transfer Function model.

Model	AIC	SBC	MSE
ARIMA	309.26	312.78	74.78
Transfer function	293.02	296.45	70.91

are smaller than those achieved in the ARIMA model. The forecasting accuracy improved with the Transfer Function model. We verified that the proposed forecasting models are mathematically feasible to help provide basic data to estimate defense expenditures. We would like to also compare the trend of national defense expenditures on the two Koreas, but we cannot obtain proper data. GARCH models account for certain characteristics such as Volatility that are commonly associated with financial time series; therefore, it is not considered an alternative model. In the next study, we will analyze national defense expenditures compared to the GDP of Korea and neighboring nations (China, Russia and Japan) and will consider other time series models such GARCH models.

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