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Study on Safe Set and Maneuverability Envelope Protection during Arresting Landing

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Abstract: According to the characteristic of carrier-based aircraft, the method of solving safe set during arresting landing is discussed in this paper based on optimal control and invariant set theory. The safe sets of carrier aircraft are evaluated in different states on the characteristic of longitudinal augmented system by using the level set method. Then, the influence on the boundary of safe set under various factors is analyzed. At last, the maneuverability envelope protection is established based on the corresponding theory, and the validity of the system is verified through simulation. The results demonstrate preliminarily that: compared with mass and thrust, the elevator is the greatest influence factor for the boundary of safe set; the dynamic trajectory of carrier-based aircraft can be located at the interior of safe set effectively with the maneuverability envelope protection.

Key Words : Arresting landing, Flight envelope, Safe set, dynamic system, mathematical models

1. Introduction

Due to the harsh external environment, attitudes of aircraft are difficult to remain within its prescribed flight envelope during carrier landing. If the aircraft is driven outside of the zone of flight envelop by disturbances, the multi-body dynamic system should be prone to a variety of problems. The results show that the accidents caused by dynamics account for about 40% of the entire amount in the last stage during arresting landing [1]. Then, how to determine the zone of safe-set, the largest controlled invariant set within the flight envelope during carrier landing, becomes particularly important.

Currently, researches on the security of arresting landing mainly concentrate on the complexity of the landing process, as well as on the properties of multi-body dynamic system. The study on the boundary of flight envelope is hard to reflect the dynamic characteristics by ignoring variously valuable parameters [2]. So the need of the security features during arresting landing is difficult to meet. Moreover, as for safe set during arresting landing, the related research is few.

To solve these problems, the definition of safe set is explained by the theory of optimal control and invariant set in this paper. Considering constrains and physical characteristics, the system are augmented. For different environments, the flight safe sets containing the longitudinal factors augmented system are calculated based on level set method. By using the results, a maneuverability envelope protection controller is established, and verified through simulation.

2. Mathematical Preliminaries and Model

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Defining a nonlinear system as:

$$\dot{x} = f(x,u,t) \quad (1)$$

Where $x \in C \subset R^n$ is the state space, $u \in U \in R^m$ is the control space. As the control constraint, U is assumed to be bounded and f should be smooth to all variables [3].

2.1. Safe Set Formulation

If there is a subset of flight envelope, where admissible control variables that make a trajectory remains in C for each initial states always exist. The subset can be defined as S, the largest positively invariant set of flight envelope, and can be denoted as follows:

$$Viab(t,C) = \{x \in R^n \mid \exists u \in U, \forall \tau \in [t,T], \; x = \varphi(\tau,t,x,u) \in S\} \quad (2)$$

Where $x = \varphi(\tau,t,x,u)$ is the trajectory of system.

According to the optimal control theory [4], the corresponding target index can be defined as J, and takes the following form:

$$J(x,u,t) = \left\{V(x,u,t) \ge 0 \mid V : R^n \times R^m \times R \to R\right\} \quad (3)$$

Where V indicates the positive distance between the dynamic trajectory and envelope boundary based on the level set theory.

The minimum solution of target index can be taken as the optimization result. Therefore, the optimal solution can be specified as:

$$V(x,u,t) = \sup_{u \in U} \; \min V(x = \varphi(\tau,t,x,u)) \quad (4)$$

According to the description, the safe set of C at the initial time takes the following form [5]:

$$S = Viab(0,C) = \{x \in R^n \mid V(x,0) \ge 0\} \quad (5)$$

Then the problem of optimal trajectory can be converted into the dynamic problem. Combined with the above assumptions of state space, Eq. (4) can be formulated as a time dependent Hamilton-Jacobi partial differential equation as:

$$-\frac{\partial V}{\partial t} = H(x,p), \; V(x,t) = l(x) \quad (6)$$

Defining the Hamiltonian in Eq. (6) as:

$$H(x,p) = \sup_{u \in U} \; p^T \cdot f(x,u), \; p = \nabla V(x,u,t) \quad (7)$$

Due to the limit of optimal theory, the dynamic trajectory may be departure from the flight envelope during the state transition process. To solve this problem and ensure the positive definite properties of index, the Eq. (6) can be modified as:

$$-\frac{\partial V}{\partial t} = \min\{0, H(x,p)\} \quad (8)$$

For realistic model, it is hard to obtain analytic solutions. Then, the numerical level set and viscous finite element algorithms are applied to solve the safe set problem in this paper. Hence the optimal solution can be expressed as:

$$u^* = \arg\sup H(x,p,u) \quad (9)$$

2.2 The Augmented Aircraft Model

If the Hamiltonian gets the maximum numerical solution, the Eq. (7) can be described as:

$$H(x,p,u) = \max(p_1\dot{V} + p_2\dot{\gamma} + p_3 q + p_4\dot{\alpha}) \quad (10)$$

Based on the optimal theory, the corresponding result of u is the optimal solution. Therefore, the form of optimal solution of thrust can be considered by using Eq. (10):

$$T^* = \begin{cases} T_{\min}, & \dfrac{p_1\cos\alpha}{m} + \dfrac{p_2\sin\alpha}{mV} - \dfrac{p_4\sin\alpha}{mV} < 0 \\ T_{\max}, & other \end{cases} \quad (11)$$

Obviously, the output of thrust is discontinuous because of the process of switching control strategy, and harmful for the real physical system. In order to solve this problem, the first-order delay link is used to approximately simulate the characteristic of control actuator by adopting the method of high order sliding mode control. The equivalent equations take the following form:

$$\begin{cases} \dfrac{T}{T_C} = \dfrac{1}{\tau_{\delta T}s + 1} \\ \dfrac{\delta e}{\delta e_c} = \dfrac{1}{\tau_{\delta e}s + 1} \end{cases} \quad (12)$$

Where, T_C is the command signal of thrust, δe_c is the command signal of elevator.

Defining the derivative of control actuators as the input factors of states space, the system can keep the realistic physical system smooth. The augmented state space is shown as:

$$\begin{cases} \dot{V} = \dfrac{1}{m}(T\cos\alpha - D(\alpha,q,\dot{\alpha},de) - G\sin\gamma) \\ \dot{r} = \dfrac{1}{mV}(T\sin\alpha + L(\alpha,q,\dot{\alpha},de) - G\cos\gamma) \\ \dot{\alpha} = q - \dot{\gamma} \\ q = \dfrac{M(\alpha,q,\dot{\alpha},de) + D(\alpha,q,\dot{\alpha},de)\Delta x_{cg}c + M_T}{I_y} \\ \dot{T} = -\dfrac{1}{\tau_T}T + \dfrac{1}{\tau_T}T_C \\ \dot{\delta e} = -\dfrac{1}{\tau_{\delta e}}\delta e + \dfrac{1}{\tau_T}\delta e_C \end{cases} \quad (13)$$

By Eq. (13), we can obtain that the operating envelope which is defined as a 6-dimensional hypercube state space, which is described as below:

$$C = \{v, \gamma, q, \alpha, T, \delta e\}$$

And the control restraint set is:

$$u = \{T_C, \delta e_c\}$$

Then, the boundary conditions at the finial time can be described as :

$$\begin{aligned} l(x) = \min\{&V_{\max} - V, V - V_{\min}, \gamma_{\max} - \gamma, \gamma - \gamma_{\min} \\ &q_{\max} - q, q - q_{\min}, \alpha_{\max} - \alpha, \alpha - \alpha_{\min} \\ &T_{\max} - T, T - T_{\min}, \delta e_{\max} - \delta e, \delta e - \delta e_{\min}\} \end{aligned}$$

Clearly, $l(x)$ is a Lipschitz continuous convex set, and satisfies the following characteristic.

$$\begin{cases} l(x) \ge 0, & x \in C \\ l(x) < 0, & x \notin C \end{cases}$$

Therefore, $l(x)$ can be considered as the function of minimum distance between the dynamic trajectory and envelope boundary.

2.3 parameter matching characteristics

For the characteristics of arresting landing, stringent requirements on the attitudes of aircraft should be satisfied. Constrained by approaching criteria [2][6], the appropriate criteria are selected as the foundation to solve the flight envelope. By deriving the corresponding motion equations, the analytical formulas based on respective criteria are deduced.

By adopting the analytical formulas, the matching envelope of aircraft during carrier landing is calculated with the 76m high and the largest flap angle. Meanwhile, the angle of attack and gliding angle is 10.36°and -3.5°, respectively. The result can be shown in Fig. 1

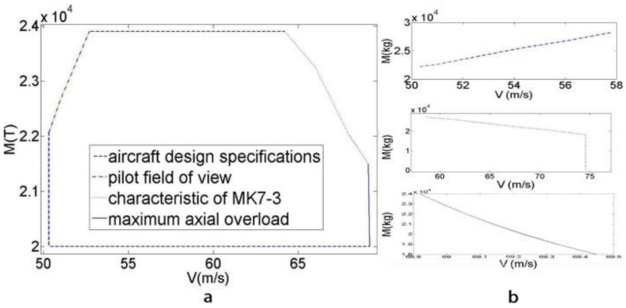

Fig 1 Matching envelope with the criteria

The closed field in Fig.1 (a) is composed by the curves calculated by constrains of approaching criteria, such as the pilot field of view, the characteristics of arresting system and the maximum axial overload. The curve of above criteria can be drawn individually in Fig.1 (b) (from upper to lower).

The results indicate that the parameter matching envelope is decided by the constraints of pilot field of view(FOV)、stalling speed、characteristics of Mk7-3 and axial overload. The minimal engaging speed is determined by FOV and design specification. Meanwhile, the maximal engaging speed is determined by the maximum axial overload and characteristics of Mk7-3.

2.4 Example Calculation and Analysis

Before calculating the safe set, an operating envelope of aircraft with the limit of approaching criteria during arresting landing is specified as:

$$\begin{aligned} C = \{(V,\gamma,q,\alpha,T,\delta_e) \mid\ & f_{\min}(m) \le V \le f_{\max}(m),\ -6° \le \gamma \le -2°, \\ & 4.8° \le \alpha \le 12.2°,\ -16°/s \le q \le 16°/s, \\ & 0Kn \le T \le 32Kn,\quad -28° \le \delta e \le 28°\} \end{aligned}$$

where $f(m)$ means the function of mass.

The control restraint set is:

$$U = \{(T_C, \delta e_C) \mid 0Kn \le T_C \le 32Kn, -28° \le \delta e_C \le 28°\}$$

First of all, the global Lax-Friedrichs schemes [7] is used to discrete the computational domain of flight envelope. Next, UpwindFirstENO2 format is adopted to make the spatial derivatives discrete, while the form of odeCFLset is selected as the time advance mechanism.

To illustrate the various situation of safe set, the velocity, attack angle and gliding angle are chosen as a 3- dimensional space to represent the safe set during carrier landing. Meanwhile, degree of the elevator and thrust are constants, four conditions with different pitch rate as a sequence, $S(V,\gamma,\alpha)$ is respectively calculated with the landing progress of generic or augmented longitudinal system. The form of discretization domain is over 21*4*7*16*18*31 grid in the computational domain, and the corresponding safe sets are discussed in Fig 2:

As shown in Fig.2, the closed convex sets are continuous and smooth. Due to the delay characteristic of augmented system, the corresponding safe sets are shrunk obviously.

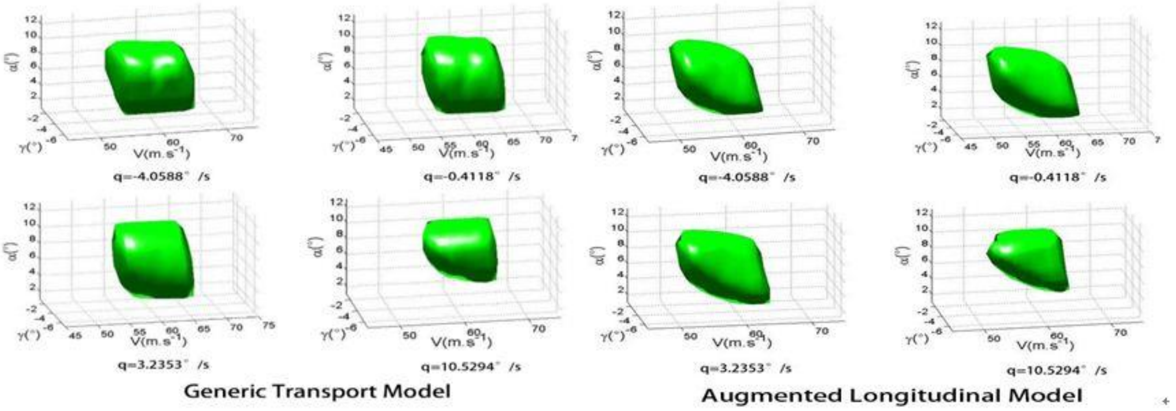

Fig.2 Variation of safe sets with typical conditions

2.4.1 The Influence of Mass

To study the influence of mass on the safe set, the landing process of carrier-based aircrafts with different mass is stimulated. The safe sets of calculation examples with the quality of 20T, 22.4T and 23.9T are compared in Fig.3:

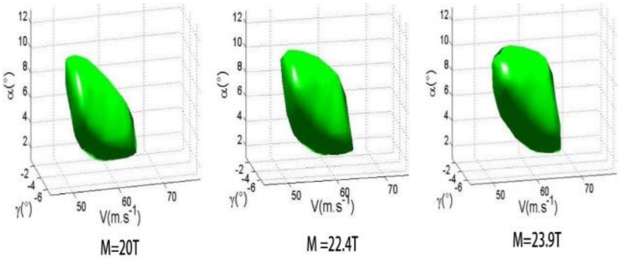

Fig.3 Mass influence

Fig.3 shows that the safe set of carrier-based aircraft expands gradually with the increasing quality from 20T to 23.9T. Moreover, we can see that the boundaries of attack angle shift upward as well. The reason for this phenomenon is that it needs larger angle of attack to obtain more lift to trim the heavier aircraft.

2.4.2 The Influence of Control Actuators

To get the influence of decreased maneuvering efficiency caused by the hydraulic hitch or thrust loss on the safe set, this paper chooses several typical conditions to analysis. For example, the engine is broken down, or the elevator is impaired caused by the hydraulic loss. The corresponding results are shown as follow:

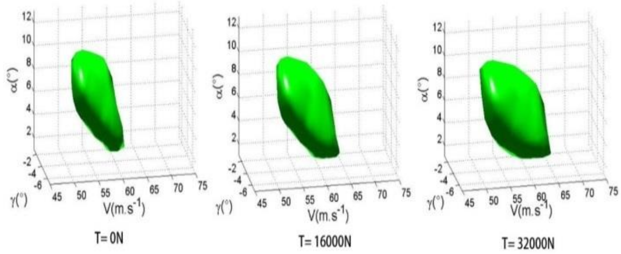

Fig.4 Thrust influence

When single engine is broken down, there is only marginally affected in the safe set. Even under the circumstance of two engines shutdown, the aircraft is still controllable in certain attitudes. But it has to be pointed out that the stability of the airplane can't be guaranteed.

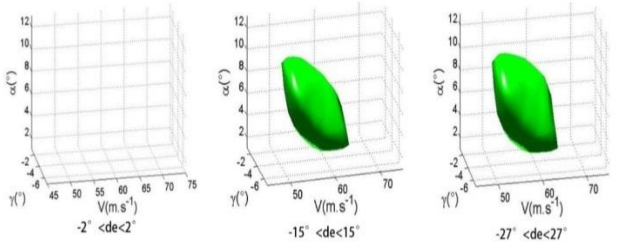

Fig.5 Elevator Influence

The safe sets shrink obviously for the damaged elevator system. When the hydraulic pressure is entirely lost, the declination range of elevator is from -2(°) to 2(°) by the effect of artificial, the zone of safe set is disappeared completely. The carrier landing should be avoided in this case.

The pictures show that when an aircraft controller is damaged, the safe sets would shrink as expected. The damaged elevator has more impact on the safe set.

3. Maneuverability envelope protection

In this section, the strategy of maneuverability envelope protection which is used to prevent the dynamic trajectory departure from the operating envelope is discussed. Meanwhile, in order to ensure the protection strategy does not influence the operability of aircraft when the dynamic trajectory is located at the interior of safe set, the pilot/automatic control strategy would be chosen. Thus, the structure diagram of maneuverability envelope protection can be drawn as:

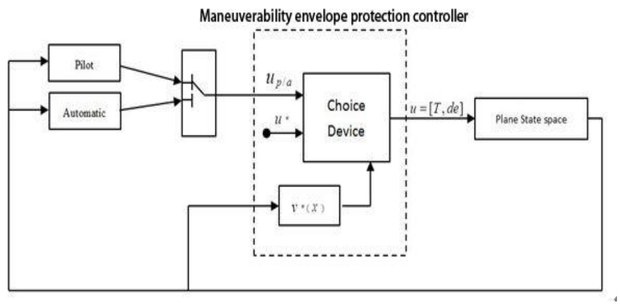

Fig.6 Maneuverability envelope protection strategy

Firstly, the boundary of safe set is defined as the zero level isocontour of $\hat{V}_i(x_i)$, which expresses a numerical approximation to the exact solution $V(x)$. Then ∂S can be shown as[3]:

$$\partial S = \{x \in C \mid \hat{V}(x) = 0\} \tag{14}$$

Then, we can discuss whether the trajectory is in the safe set or not by the value of $\hat{V}_i(x_i)$.when the value of $\hat{V}_i(x_i)$ is greater than zero. The trajectory of system belongs to the interior of safe set. In the opposite case, the state is out of the safe set. If the value of $\hat{V}_i(x_i)$ is zero, then the trajectory locates at the boundary of safe set. So the envelope protection controller can be designed as:

$$u(x) = \begin{cases} u_{p/a}, & \hat{V}_i(x_i) > 0 \\ u_{p/a}, & \hat{V}_i(x_i) = 0 \text{ and } u_{p/a} \in u_{safe} \\ u_{safe}, & otherwise \end{cases} \tag{15}$$

Obviously, when the dynamic trajectory crosses the zero level isocontour of $\hat{V}(x)$, the command signals of control actuators are discontinuous caused by switching the control strategies. However, for the augmented aircraft model described in Eq. (13), the first-order delay link can effectively eliminate the discontinuous characteristic from the command signal of control surfaces, and can ensure realistic control actuators output smooth.

Finally, combined with Eq. (9), the specific forms of the maneuverability envelope protection can be defined as below:

$$u_{safe}(x) = \begin{cases} T_C = \begin{cases} T_{\max}, & p_5 > 0 \\ T_{\min}, & p_5 < 0 \end{cases} \\ \delta e_C = \begin{cases} \delta e_{\max}, & p_6 > 0 \\ \delta e_{\min}, & p_6 < 0 \end{cases} \end{cases} \tag{16}$$

3.2 example calculation and analysis

In order to verify the effectiveness of maneuverability envelope protection, the initial state is designated, which would produce a dynamic trajectory tangent to boundary of safe set. Aiming at this situation, the simulation with and without the application of maneuverability envelope protection strategy are calculated respectively, the corresponding results are compared in Fig.7

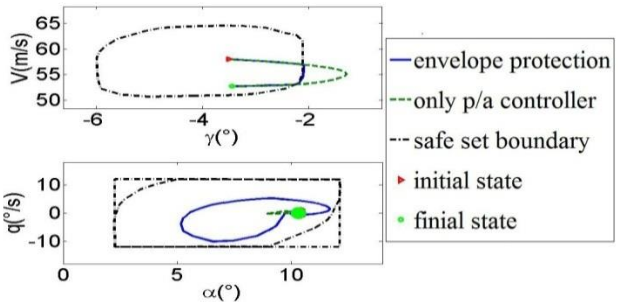

Fig 7 influence of maneuverability envelope protection

As shown in Fig.7, the trajectory only with the pilot/automatic control strategy is departed from flight envelope in the initial stage of simulation. Subsequently, this curve goes back under the effect of automatic control law, and the carrier-based aircraft reaches a desired trim condition at the finial time. By contrast, the trajectory with the maneuverability envelope protection strategy can be forced into the safe set when the trajectory intersects with the boundary of safe set. Clearly, the maneuverability envelope protection can effectively keep the flight parameters within the security lines, and doesn't interfere with the normal operation of the polite when the dynamic trajectory locates at the interior of safe set.

4. Conclusions

The relationship between safe set and the flight envelope during the arresting landing is discussed in this paper. Then the calculation method of safe set is proposed by the theory of optimal control. Based on constrains in approaching criteria and the characteristic of carrier landing, the flight envelopes of aircraft during the arresting landing are obtained. Meanwhile the generic aircraft model is augmented by the derivative of control actuators to eliminate the discontinuous characteristic, and the safe set is calculated with the numerical level set algorithm. Due to there are many advantages for the strategy of maneuverability envelope protection, such as the perfect theory basis, no effect on the normal control and comprehensive protection of attitudes. Then the relevant control system is established, and its effectiveness is verified though simulation.

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