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Robustness Improvement and Assessment of EARSM k-ω Model for Complex Turbulent Flows

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Abstract : The main concern of this study is to integrate the EARSM into an industrial RANS solver in conjunction with the k-ω model, as proposed by Hellsten (EARSMKO2005). In order to improve the robustness, particular limiters are introduced to turbulent conservative variables, and a suitable full-approximation storage (FAS) multi-grid (MG) strategy is designed to incorporate turbulence model equations. The present limiters and MG strategy improve both robustness and efficiency significantly but without degenerating accuracy. Two discretization approachs for velocity gradient on cell interfaces are implemented and compared with each other. Numerical results of a three-dimensional supersonic square duct flow show that the proper discretization of velocity gradient improves the accuracy essentially. To assess the capability of the resulting EARSM k-ω model to predict complex engineering flow, the case of Common Research Model (CRM, Wing-Body) is performed. All the numerical results demonstrate that the resulting model performs well and is comparable to the standard two-equation models such as SST k-ω model in terms of computational effort, thus it is suitable for industrial applications.

Key Words : RANS, LEVM, Reynolds stress model, EARSM, Robustness, Boussinesq eddy viscosity hypothesis

1. Introduction

Currently, industrial and aeronautical turbulence models for use with Reynolds-Averaged Navier-Stokes (RANS) equations are mainly based on the Boussinesq eddy viscosity hypothesis, which assumes a linear dependency of the turbulent stress on the mean strain-rate tensor. Consequently, these models are known as linear eddy viscosity turbulence models (LEVMs). However, the linear relationship may be a too restricting assumption for complex problems, because a multitude of flow phenomena may be present in a single problem. Therefore, turbulence models with a wider range of applicability than LEVMs are expected. Differential Reynolds stress models (DRSMs), in which a modeled transport equation is solved for each stress component, are in principle a more general class of models possessing wider applicability. DRSMs are rarely used even although they are available in many general-purpose codes [1]. One important reason is the highly increased complexity.

Alternatively, explicit algebraic Reynolds Stress Models (EARSMs) provide an attractive framework balancing applicability and complexity. These models can be considered as a subset of nonlinear models, in which a part of higher-order description of physical process on the DRSMs level is transferred into the two-equation model level. As a result, they are of much less complexity than DRSM, and meanwhile, are capable of reproducing some important turbulent features that are beyond the capabilities of LEVMs, e.g., anisotropy in the normal stresses. Unfortunately,

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EARSMs still suffer robustness issues especially for the computation of flows around complex three-dimensional geometries, which limits their use. Therefore, the main interests of this study are to integrate the EARSM [2] into an industrial RANS solver equipped with the k-ω model in a robust way, and to perform an assessment of the resulting EARSM k-ω model.

2. Governing Equations and Numerical Method

In a compact conservation law form, the governing equations may be expressed as follows:

$$\frac{\partial \boldsymbol{W}}{\partial t}+\frac{\partial(\boldsymbol{F}_c-\boldsymbol{F}_d)}{\partial x_i}=\boldsymbol{S} \tag{1}$$

$\boldsymbol{W}=(\boldsymbol{Q},\boldsymbol{q})$ consists of conservative variables for mean-flow equations and turbulence model equations, denoted by $\boldsymbol{Q}$ and $\boldsymbol{q}$ respectively. $\boldsymbol{F}_c$ and $\boldsymbol{F}_d$ indicate the convective and diffusive flux, respectively.

$\boldsymbol{S}$ is the source term arising only from turbulence model equations. In EARSM model, the Reynolds stress depends on strain-rate tensor S_{ij}, rotation-rate tensor W_{ij} and turbulence time scale by definition and reads [3]:

$$\tau_{ij}=2\mu_t\left(S_{ij}-\frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)-\frac{2}{3}\rho k\delta_{ij}-a_{ij}^{(ex)}\rho k \tag{2}$$

The strain- and rotation-rate tensor absorbed turbulence time scale are defined S_{ij}^* and W_{ij}^*, then:

$$\begin{aligned}a_{ij}^{(ex)}=&\beta_3\left(W_{ik}^*W_{kj}^*-\frac{1}{3}II_\Omega\delta_{ij}\right)+\\&\beta_4\left(S_{ik}^*W_{kj}^*-W_{ik}^*S_{kj}^*\right)+\\&\beta_6\left(S_{ik}^*W_{kl}^*W_{lj}^*+W_{ik}^*W_{kl}^*S_{lj}^*-II_\Omega S_{ij}^*-\frac{2}{3}IV\delta_{ij}\right)+\\&\beta_9\left(W_{ik}^*S_{kl}^*W_{lm}^*W_{mj}^*-W_{ik}^*W_{kl}^*S_{lm}^*W_{mj}^*\right)\end{aligned} \tag{3}$$

The detailed definition of missing terms and turbulence model constants can be found in reference [2, 4].

The governing equations are discretized using a nominally second-order cell-based finite volume method (FVM) on a structured multi-block grid. The mean-flow convective fluxes are computed using Roe's scheme with MUSCL-type reconstruction, and turbulence advective terms are approximated using first-order upwind differences, while the central-differencing scheme is used in the calculation of all the diffusive fluxes. The discretized mean-flow and turbulence model equations are driven to the steady state in a loosely coupled manner by the first-order implicit backward Euler method. For all the computations in this work, the resulting linear systems are solved using the diagonally dominant alternating direction implicit (DDADI) method.

3. Limiters and Multi-grid Method for EARSM model

The general limiters for two-equation turbulence model such as production limiter, the limiters guaranteeing k, ω and $\tau_{ii}>0$ (i =1, 2, 3) non-negative are implemented. However, it is still not sufficient to make turbulence model equations converge especially when computing complex wall-bounded flows. To cope with this problem, $\boldsymbol{q}$ is limited to be always positive through

$$\boldsymbol{q}^{(n+1)}=\begin{cases}\boldsymbol{q}^{(n)}+\Delta\boldsymbol{q}^{(n)}, & \boldsymbol{q}^{(n)}+\Delta\boldsymbol{q}^{(n)}>\varepsilon\\ \boldsymbol{q}^{(n)} & otherwise\end{cases},$$

$$\varepsilon=1\times10^{-20}\sim1\times10^{-12} \tag{4}$$

Eq.(4) means that if the newly updated turbulence variables were negative, they are not updated. The positivity-preserving limiter can be used to either both turbulence variables or one of them. In this work we use limiter only in ω-equation to avoid too large unphysical turbulent time scale entering the viscous flux calculations, which may cause divergence in the initial stage of computation.

In order to incorporate the turbulence model equation into multi-grid method in a robust way, we consider the mean-flow multi-grid strategy (as sketched by Fig. 1). That is, mean-flow equations are solved on all grid levels, while the turbulence model equations are integrated only on the finest grid level. The turbulent stress on coarse grid levels can be computed either exactly via eq.(2) or by only the linear part of eq. (2), with corresponding quantities , except the gradient of mean-flow velocity, injected from the finest grid level. We compared these two methods in a previous validation, and observed nearly no deviation. Thus, we use the latter one in all computations.

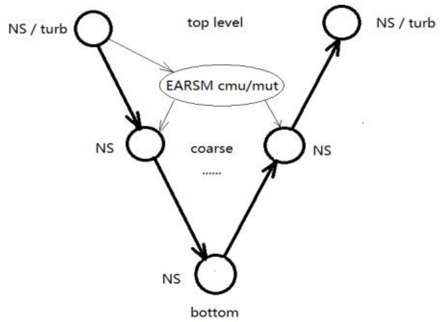
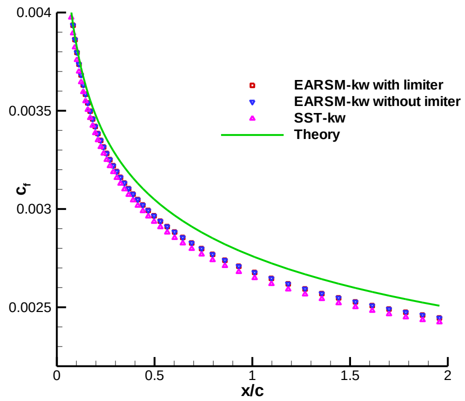
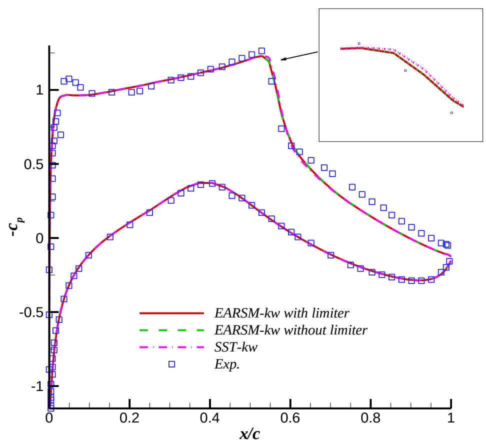


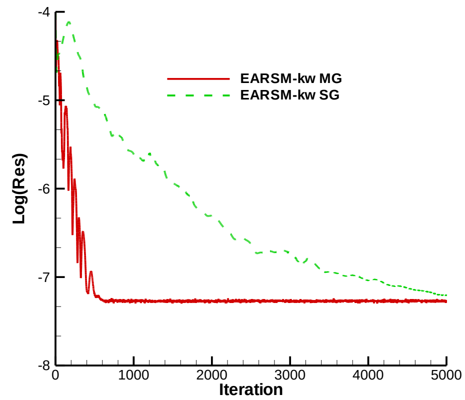
Fig. 1 Multi-grid cycling strategy



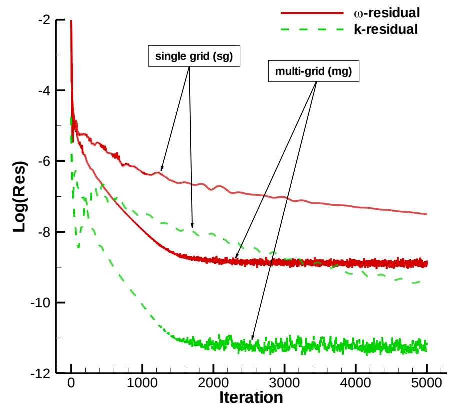
(a) Friction coefficients of flat plate



(b) RAE2822 airfoil pressure distribution
Fig. 2 Comparison of limiter effects



(a) Mean flow



(b) Turbulence model
Fig. 3 RAE2822 airfoil CASE-10 residual convergence

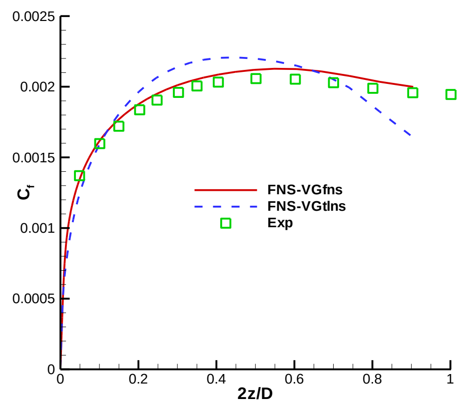


Fig. 4 3D Supersonic square duct comparison of friction coefficients at x/c=40 along z-direction

A two-dimensional boundary layer over a flat plate was computed to verify that the EARSM has been correctly implemented, which is proved by Fig. 2(a). The RAE2822 airfoil CASE-10 and the flat plate boundary layer test case were also executed to demonstrate the effectiveness of the introduction of limiters and MG strategy. The comparisons displayed in Fig. 2 confirm that the introduction of limiters promises the accuracy. Moreover, the introduction of limiters improves the robustness significantly. As we observed in the RAE2822 test case, the computation converges only when the limiters are active or when the computation starts from a good initial solution obtained by k-ω model. Fig. 3 (b) presents the convergence history of RAE2822 airfoil CASE-10 in the context of single grid (SG) and multi-grid (MG). Apparently, the efficiency in terms of number of iterations is improved by about one order of magnitude. And the turbulence model equations are better solved when MG method is used.

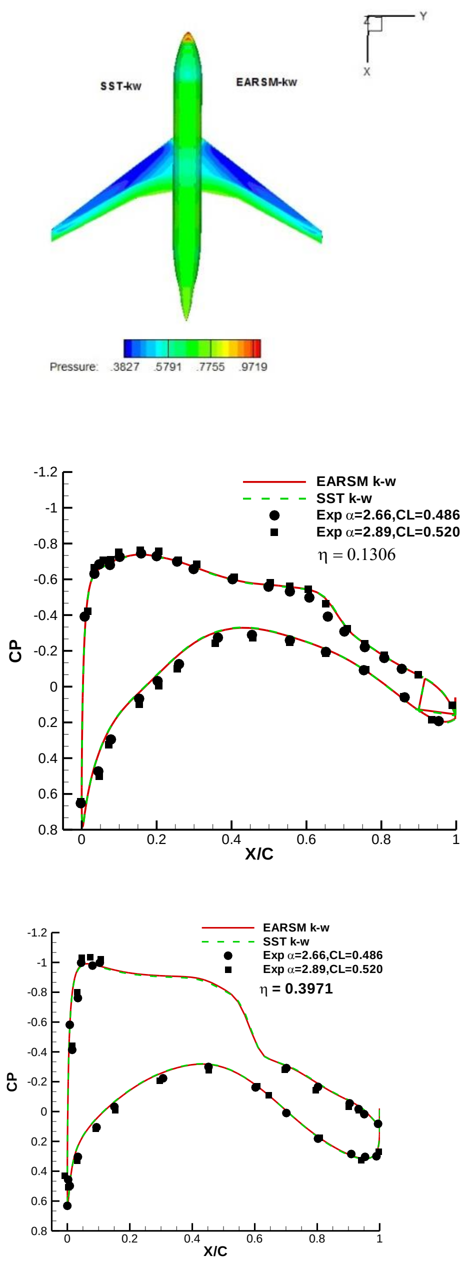
4. Discretization method for velocity gradient

One discretization of velocity gradient at cell interface is based on the thin shear-layer (TSL) assumption. The approach, denoted by VGtlns, is quite convenient, however, is not capable especially for the computation of stress anisotropy dominant flows. Thus, the discretization based on Green-Gauss theorem and arithmetic average is also implemented, indicated by VGfns.

To show the improvement, the supersonic flow at Mach number 3.9 through a square duct [5] is investigated. It is a typical case used to assess the turbulence models to capture the secondary flow features. The flow in the cross-cutting plane to the stream-wise direction should exhibit flow from the channel center region toward the corners, with two counter-rotating vortical features within each quadrant. As observed in Fig. 4, comparing the computed and measured friction coefficients, the approach of VGfns improves the accuracy essentially.

5. CRM simulations

The case 1 of Common Research Model (CRM, Wing-Body) provided by the Fifth AIAA CFD Drag Prediction Workshop (DPW-V) [6] was performed using EARSM k-ω and SST k-ω model. The comparisons are shown in Fig. 5. The numerical results show that the EARSM k-ω model behaves well and similar to the SST k-ω model, and is comparable to SST k-ω model in computational effort, thus it is suitable for industrial applications.



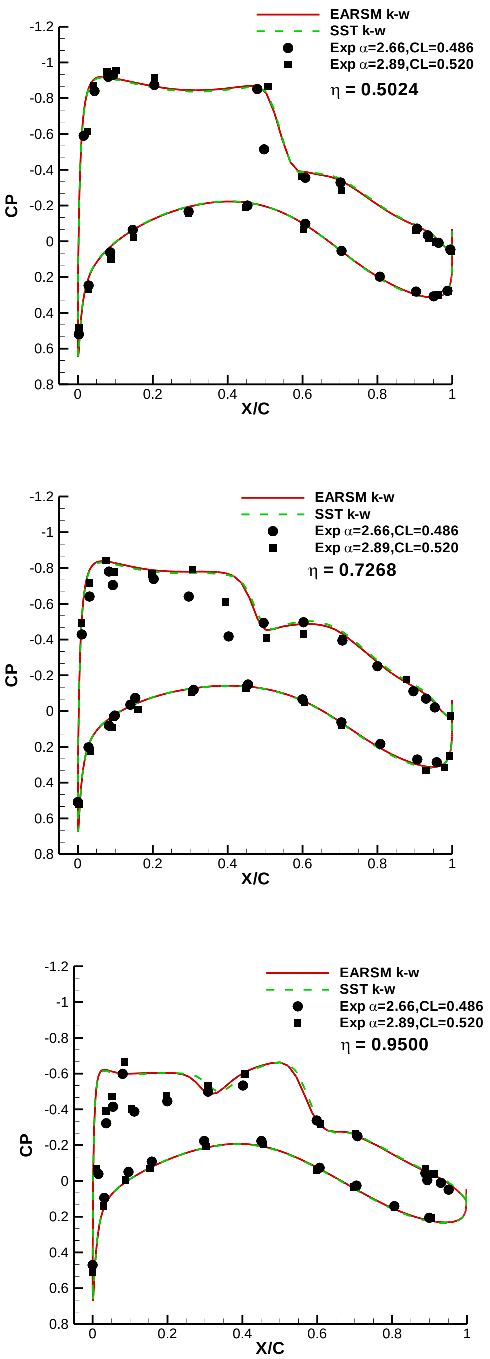


Fig. 5 CRM pressure contour and pressure coefficient calculated by SST k-w and EARSM k-w model

6. Conclusions

Hellsten's EARSM (EARSMKO2005) has been integrated into an industrial RANS solver. In order to improve the robustness, a particularly simple positivity-preserving limiter for ω-equation is introduced, and a multi-grid (MG) strategy suitable for EARSM is developed based on the FAS multi-grid method. The introduction of limiters improves the robustness significantly while guarantees the accuracy. When MG method is used, the efficiency in terms of number of iterations is improved by about one order of magnitude, and the turbulence model equations are better solved.

Since the constitutive relation of EARSM depends highly on the strain- and rotation-rate tensors, the discretization of velocity gradient plays an important role especially for the computation of flows in which the stress anisotropy is dominant, the approach based on Green-Gauss theorem improves the accuracy essentially.

All the numerical results show that the resulting EARSM k-ω model performs well and is comparable to standard two-equation models e.g. SST k-ω model in computational effort, thus it is suitable for industrial applications.

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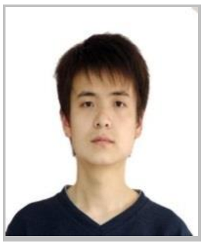

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