

Dynamic Stability of Liquid in a Spherical Tank Covered with Membrane under Vertical Harmonic Excitation

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Abstract : Experimental studies were conducted on the liquid sloshing characteristics in a spherical tank covered with a flexible membrane. A spherical acrylic tank with 145.2 mm in radius was used as a test tank, and it was half-filled with water. Silicon membranes with 0.2 mm thickness were used as a test membrane with plane or hemispherical types. The test tank was harmonically excited in a vertical direction by an electro-dynamic exciter. In this case, a parametric instability vibration comes up when the excitation frequency is twice the natural frequency. Parametric instability regions of natural modes were measured for three cases, i.e. liquid surface is free, covered with plane membrane and hemi-spherical membrane.

Key Words : Spherical tank, Liquid sloshing, Flexible membrane, Parametric instability, Dynamic stability, Vertical excitation

1. Introduction

A spherical tank has been adopted as a fuel tank in satellites, in which liquid fuel is covered with a membrane and is pressed out to combustion chamber by gas pressure[1]. In the space, liquid vibration, i.e., sloshing[2], may occur during a transition of its orbit or by an excitation from a compressor for cooling infrared telescopes. Since the liquid sloshing continues for a long time in a microgravity condition, it reduces the accuracies of attitude control of satellite or of astronomical observation[3]. In addition, in the ground tests, liquid sloshing also occurs, and may cause destructions of mounting portion of test tanks and peripheral devices.

There has been some studies on the liquid sloshing in a spherical tank with membrane: Stofan and Summer[4] conducted an experiment on a spherical tank with hemi-spherical membrane made of isobutylene - isoprene rubber, Dodge and Kana[5] theoretically analyzed a mechanical model of liquid sloshing in an oval spherical tank with high flexural rigidity membrane, and compared with their experimental results. Both studies examined the case when the tank is excited horizontally.

The aim of the present study is to make clear the liquid sloshing characteristics in a spherical tank covered with membrane under vertical harmonic excitation on the ground. In general, parametric instability vibration easily occurs instead of usual resonance on liquid in a vertically excited tank, when the excitation frequency is twice the natural frequency[6]. The filling ratio of liquid η (=liquid volume V_l / tank volume V_t) $\times 100$) was fixed to 50%. Three cases for the liquid surface condition were considered, i.e. liquid surface is free, covered with plane membrane and hemi-spherical membrane.

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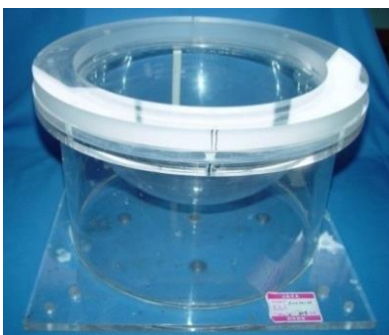
2. Test Tank and Membrane

Test tank consists of three parts, i.e. upper spherical tank (or acrylic ring), lower spherical tank and cylindrical stand (Fig. 1). Specification of the test tank is shown in Table 1. When free surface is covered with plane or hemi-spherical membrane an acrylic ring was used, instead of upper spherical tank, for convenience of measurement. A water leakage preventing tube and clamps were used to fix the acrylic ring on the test tank.

Test membranes made of silicon with 0.2 mm in thickness in plane or hemispherical shapes are used (Fig. 2) and are mounted on test tank as shown in Fig. 3.



(a) With upper spherical tank



(b) With ring

Fig. 1 Spherical test tank



(a) Plane membrane (500 mm×500 mm, 6.4 gr.)

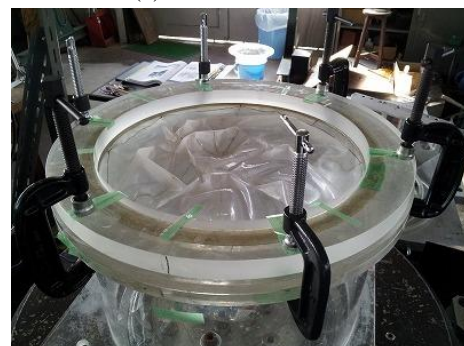


(b) Hemi-spherical membrane ($r = 142.5$ mm)

Fig. 2 Test membranes : 0.2 mm in thickness



(a) Plane membrane



(b) Hemi-spherical membrane ($r = 142.5$ mm)

Fig. 3 Test tank with membrane

Table 1 Specifications of spherical test tank

	Inner radius [mm]	Thickness [mm]	Mass [kg]	Thickness of flange [mm]	Radius of outer flange [mm]
Spherical part: upper	142.5	8.1	1.48	15.0	200.1
Spherical part: lower	142.5	8.1	1.52	10.0	200.0
Cylindrical part	168.2	7.4	4.80	13.0	200.0
Ring part	150.0	20.4	1.20	20.4	196.5

3. Experimental Equipment and Method

A block diagram of experimental set-up is shown in Fig. 4. In the test, (1) vibration controller and (2) electro-dynamic exciter CV-1000-5(IMV, max excitation force: 9807N) were used, and (3) the test tank was harmonically excited in the vertical direction. In order to excite the tank with constant acceleration, out-put of (4) an acceleration pickup (PCB-352-C22: 9.62 mV/g, 0.5 gr.) was fed back to (1) the controller. A laser Doppler vibrometer (6) was used to measure the velocity response, and out-put signal was send to (8) the vibrometer controller through (7) the laser module. Response wave was analyzed by (9) FFT Analyzer. Test tank set-up is shown in Fig. 5.

i) By exciting the test tank system with an arbitrary frequency f and acceleration ratio G (=excitation acceleration/gravitational acceleration), frequency component of velocity response is analyzed using FFT analyzer. When frequency component of the membrane response with half of the excitation frequency exceeds -20dBV (1mm/s), we define parametric instability vibration comes up(Fig. 6), and plot ● mark in a graph. While it does not come up within three minutes, we plot ○ mark. Then, changing the excitation acceleration with keeping the excitation frequency, the same operation above is repeated. A ⊙ mark is plotted to the lowest acceleration at which instability vibration occurs, as shown in Fig. 7(a).

ii) A parametric instability region can be obtained by repeating the same operation as above, as shown in Fig. 7(b).

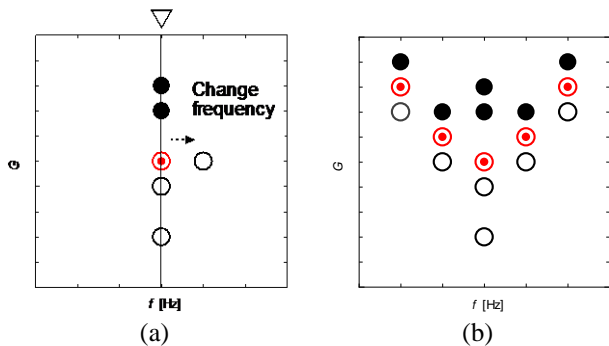


Fig. 7 Determination of a parametric instability region

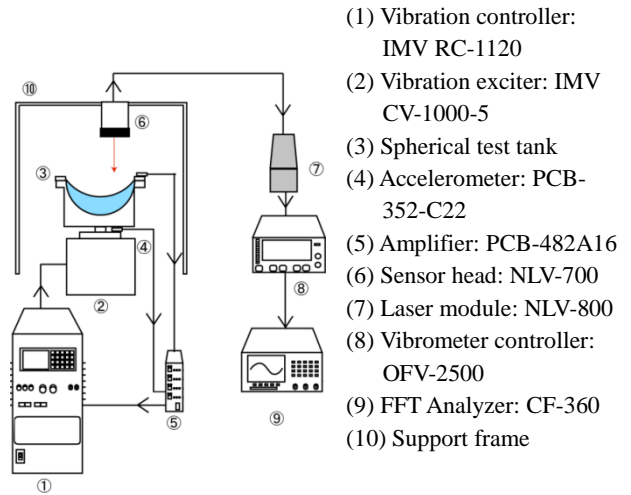


Fig. 4 Experimental set-up

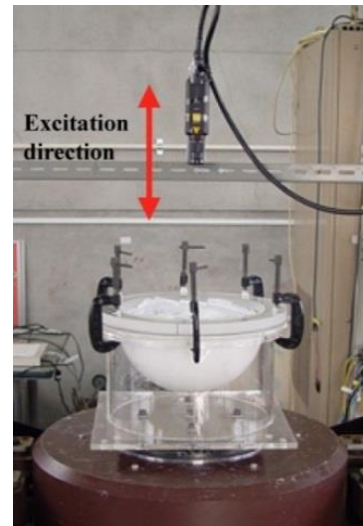


Fig. 5 Test tank

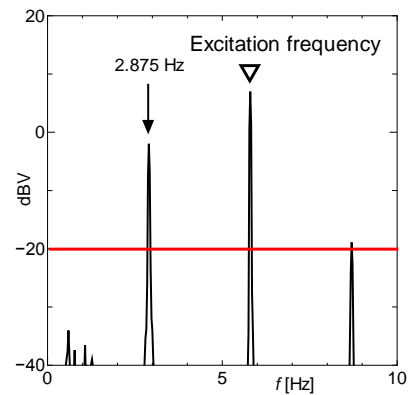


Fig. 6 Frequency components of membrane response : $f=5.75 \text{ Hz}$ (▽)

4. Experimental Results

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Vibration modes are represented by N and m (N : diametrical node, m : circular node). An example of measured parametric instability region is shown in Fig. 8: without membrane, ($N=1, m=1$) mode. In the figure, an quartic approximation curve which connects the critical accelerations points \odot is shown in a solid line. Here, we define the central frequency f_c (red line in Fig. 8) as a center frequency of the instability region and the critical acceleration G_{cr} (blue line in Fig. 8) as the lowest excitation acceleration of the instability region in which parametric vibration occurs.

Vibration modes and the central frequencies f_c observed in the test are summarized in Table 2.

Parametric instability regions are shown in Fig. 9, and variations of the critical acceleration G_{cr} with surface conditions are shown in Fig. 10.

In the case without membrane, since in the neighbor of the instability region of ($N=0, m=1$) mode and ($N=3, m=0$) mode, ($N=4, m=0$) mode and ($N=1, m=1$) mode are piled up, f_c and G_{cr} for these modes could not be obtained exactly. In addition, ($N=4, m=0$) mode is not observed with covered plane and spherical membrane case.

Comparing the values of f_c for three cases; i.e., without membrane: I, plane membrane: II and hemi-spherical membrane: III, those are f_c : I < f_c : II < f_c : III in order, as shown in Table 2, except values of f_c : I and f_c : II of ($N=1, m=1$) mode.

While, comparing the values of G_{cr} for three cases, as shown in Fig. 9 and Fig. 10, those are G_{cr} : I < G_{cr} : II < G_{cr} : III in order, and become less unstable in this order. ($N=2, m=0$) mode only takes the same value in G_{cr} : I and G_{cr} : II.

Critical acceleration of ($N=0, m=1$) and ($N=1, m=1$) modes are larger than any other modes in G_{cr} : III, however, the values of G_{cr} : I and G_{cr} : II are lower than any other mode except ($N=1, m=0$) mode. Therefore, vibration modes including axisymmetric mode, such as ($N=0, m=1$) mode and ($N=1, m=1$) mode, are hard to occur with hemispherical membrane case.

Fig. 11 shows velocity responses of ($N=0, m=1$) mode at the center of liquid surface. In the case of plane and hemi-spherical membrane, comparing the case without membrane, velocity amplitude is reduced. Thus, a liquid surface displacement is greatly reduced with membrane.

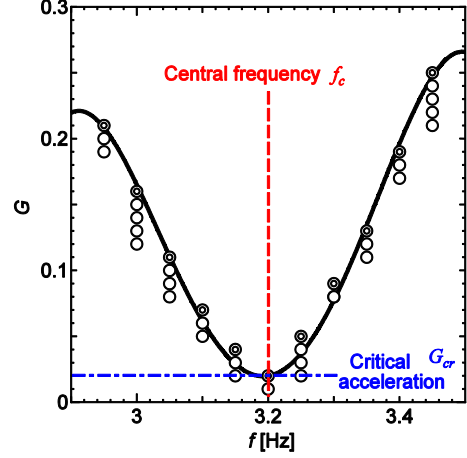


Fig. 8 Parametric instability region : $\eta=50\%$, without membrane, ($N=1, m=1$)

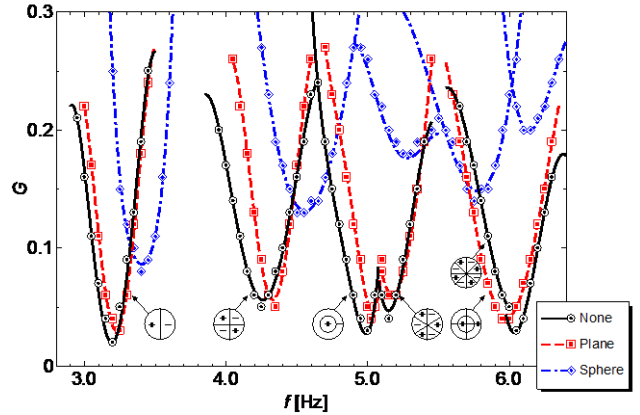


Fig. 9 Parametric instability regions for three types conditions: $\eta=50\%$

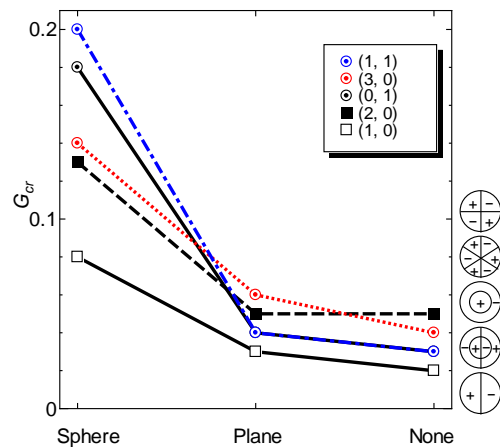
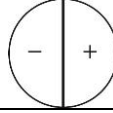
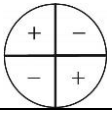
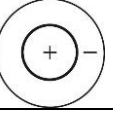
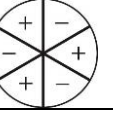
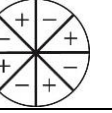
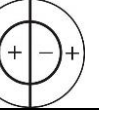
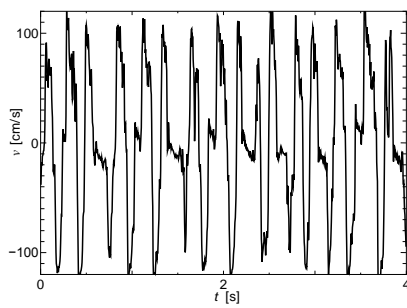


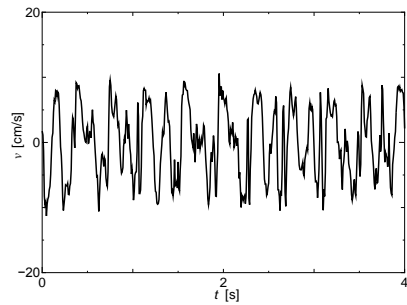
Fig. 10 Critical acceleration for three covered cases : $\eta=50\%$

Table 2 Vibration modes and central frequency [Hz]
(Spherical tank : without membrane, Plane membrane, Hemi-spherical membrane, $\eta = 50\%$)

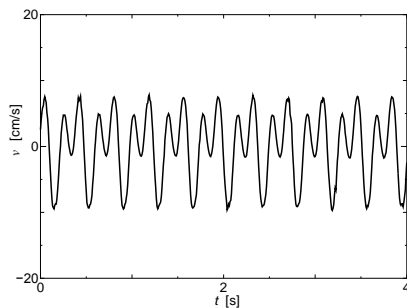
Vibration Mode							
N, m		1, 0	2, 0	0, 1	3, 0	4, 0	1, 1
f_c	None	3.18	4.25	4.98	5.15	▼6.00	6.05
	Plane	3.22	4.33	5.03	5.17	-	5.97
	Spherical	3.38	4.55	5.27	5.75	-	6.12



(a) Without membrane ($f = 4.95$ Hz)



(b) Plane membrane ($f = 5.03$ Hz)



(c) Hemi-spherical membrane ($f = 5.27$ Hz)
Liquid displacement: 2.6mm

Fig. 11 Liquid surface velocity response: $\eta = 50\%$, $G = 0.2$,
($N=0, m=1$) mode

5. Conclusions

In this study, water filled test tank was excited to make clear the liquid sloshing characteristics in a spherical tank covered with membrane under vertical harmonic excitation. Three types of free surface conditions were considered, i.e. without membrane, with plane membrane and with hemi-spherical membrane. When the system is harmonically excited with twice its natural frequency, parametric instability vibration occurs on the liquid surface. In the experiment, parametric instability region was measured in excitation acceleration G – excitation frequency f plane. Obtained results can be summarized as below.

1. Liquid surface displacement

Liquid surface displacement is largely reduced by covering with membrane.

2. Central frequency f_c of instability region

Comparing the values of central frequencies, those are f_c : without membrane < f_c : plane membrane < f_c : hemi-spherical membrane in order. ($N = 1, m = 1$) mode only became f_c : plane < f_c : without < f_c : hemi-spherical in order.

3. Critical acceleration of instability region

Comparing the values of critical accelerations, those are G_{cr} : without < G_{cr} : plane < G_{cr} : hemi-spherical, and become less unstable in this order. ($N = 2, m = 0$) mode only takes same value in G_{cr} : without and G_{cr} : plane.

Vibration modes including axisymmetric, such as ($N = 0, m = 1$) mode and ($N = 1, m = 1$) mode, are hard to occur when the free surface is covered with hemi-spherical membrane. Therefore, parametric instability vibration is hard to occur when the free surface is covered with hemi-spherical membrane than covered with plane membrane. It may due to an increased rigidity of membrane from produced wrinkles in a hemispherical membrane.

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