Research Paper
International Journal of Aerospace System Engineering
Vol.2, No.2, pp.19-23 (2015)
ISSN 2383-4986

Buckling and Post buckling Analysis of Composite Plates with Internal Flaws

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Abstract : This work deals with the study of buckling and post buckling characteristics of laminated composite plates with and without localized regions of damage. The need of a detailed study on Finite Element Analysis of buckling and post buckling of laminated composite structures considering various aspects enhances the interest among researchers. Mathematical formulation is developed for damaged composite plates using a finite element technique based on Inverse Hyperbolic Shear Deformation Theory. This theory satisfies zero transverse shear stresses conditions at the top and bottom surfaces of the plate and provides a non-linear transverse shear stress distribution. Damage modeling is done using an anisotropic damage formulation, which is based on the concept of stiffness change. The structural elements are subjected to in-plane loading. The computer program is developed in MATLAB environment. The numerical results are presented after through validation of developed finite element code. The effect of damage on buckling and post buckling has been carried out for various parameters such as amount of percentage of damaged area, damage intensity, etc. The results show that the presence of internal flaws will significantly affect the buckling characteristics of laminated composite plates. The outcomes and remarks from this work will assist to address some key issues concerning composite structures.

Key Words : Laminated plates, Shear deformation theory, Anisotropic damage, Buckling load, FEM, Cross-ply, Angle-ply

1. Introduction

The composite plates play an essential role in current and future aerospace, civil and mechanical structural system due to its improved performance. Even though such structures are highly reliable, damage or flaws are almost unavoidable at some stage of its operative life span. Various investigators have done buckling analysis of composite structures. Most recently, Grover et al. [1] proposed new nonpolynomial shear deformation theories and implemented for structural responses of laminated composite and sandwich plates. The overall percentage error is less with new nonpolyomial shear deformation theories compared to other existing shear deformation theories. Valliappan et al. [2] developed the elastic constitutive relationship for anisotropic damage mechanics and explained the implementation of these constitutive relations in finite element analyses. However, most studies [3, 4] on free vibration and static stability behaviour with localized zones of damage were restricted to simple beams and plates. Singha et al. [5] investigated the thermal post buckling behaviour of laminated composite plates by finite element method. Keeping the above aspects in mind, an attempt has been made in this investigation to study the bending and buckling behaviour of laminated composite plates with and without localized regions of damage using a

Received: June 11, 2015 Revised: Aug. 21, 2015
Accepted: Sep. 08, 2015
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finite element technique where the governing equations are developed based on Inverse Hyperbolic Shear Deformation Theory (IHSDT).

2. Mathematical modeling

The following displacement model is considered for modeling the laminated composite plate, the present displacement field is taken from Ref [1].

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_0}{\partial x}+\left[\sinh^{-1}(rz/h)-z\left(\frac{2r}{h\sqrt{(r^2+4)}}\right)\right]\theta_x(x,y)$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_0}{\partial y}+\left[\sinh^{-1}(rz/h)-z\left(\frac{2r}{h\sqrt{(r^2+4)}}\right)\right]\theta_y(x,y)$$

$$w(x,y,z)=w_0(x,y)$$

(1)

Where u, v, w are the displacement components in x, y and z directions respectively; u_0, v_0, w_0 are the displacements of a point on the mid plane (x, y, 0). Θ_x, Θ_y are the shear deformations at the mid plane. The transverse shear stress parameter, r, is set as 3 as per the inverse method in post processing step given in Ref [1]. The prebuckling equilibrium equation is shown below.

$[K]\{\Delta\}=\{F\}$,

(2)

where [K] is linear stiffness matrix and {F} is thermal load vector.

The above equation is solved under the specified boundary condition and membrane forces are obtained. In the next step geometric stiffness matrix $[K_G]$ associated with these membrane forces is calculated by following the procedure described by Zienkiewicz (1971). The critical buckling temperature is found by solving the linear eigenvalue problem,

$([K]+\lambda_{cr}[K_G])\{\Delta\}=\{0\}$

(3)

The lowest eigenvalue is the critical buckling temperature. After obtaining the buckling condition, the nonlinear stiffness matrix $[K_{nl}]$ is incorporated as,

$([K]+[K_{nl}]+\lambda_{cr}[K_G])\{\Delta\}=\{0\}$

(4)

The nonlinear equilibrium equation for composite laminated plate as given above is solved as an eigenvalue problem, considering small displacement increment relative to previous equilibrium state to trace the path of post buckling response (for details, see Singha et al.).

Damage modeling is done using an anisotropic damage formulation, which is based on the concept of stiffness change. The effects of a region of damage are introduced by the use of an idealized model having a reduction in the elastic property in the zone of damage. By considering a parameter Γ_i, (for details, see Valliappan et al.) the anisotropic damage is parametrically incorporated into the buckling formulation.

3. Results and Discussion

3.1. Problem definition

To solve the present problem, a C^0-continuous isoparametric biquadratic-quadrilateral serendipity element with 56 degrees of freedom has been used. The structures studied here are generally thin, square plates, length (a) = breadth (b), having the thickness ratio (a/h) = 100, unless specified. The damaged area is a patch having a size 4 % of the total plate area and located at the center of the plate.

3.2. Validation study

After incorporating the effects of damage, some results for simple plate were compared with the results from Ref [3]. A plate with centre damage is considered and the plate is having a damage area of 4% of the total plate area. The buckling coefficients for a plate for three different aspect ratios are found and compared with those of reference as shown in Table 1, and validated the methodology. In Fig. 1a, comparison is done for the equilibrium thermal post buckling path for present study and that of Ref [5] using four layered (45/-45/45/-45) square plate, simply supported boundary condition, and a/h = 100. Here the w denotes maximum transverse deflection. In Fig. 1b thermal post buckling equilibrium path are traced for two different a/h conditions, a/h=50 and a/h=100.

Table 1 Buckling coefficients for various aspect ratio (a/b)

AR	Present	Ref	Variation
0.8	3.48	3.59	3.06%
1	4.32	4.45	2.92%
1.6	1.71	1.77	3.38%

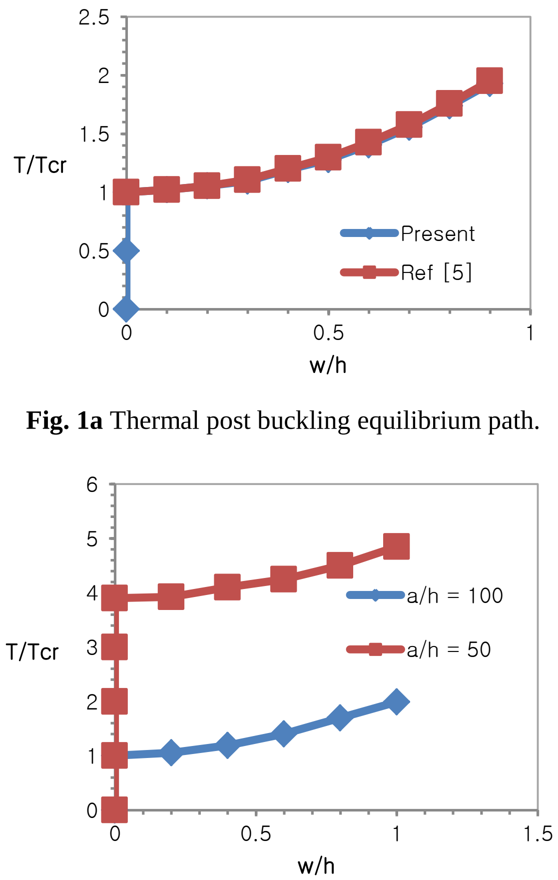

Fig. 1a Thermal post buckling equilibrium path.

Fig. 1b Effect of a/h in thermal postbuckling path, T_{cr}= $(Tcr)_{a/h}$= 100.

3.3. Effects of damage

A laminated composite plate of 4 layered $(0/90)_s$ with damage in two layers with Γ_1/Γ_2 =9 is analysed and central transverse deflection and buckling loads are found. The results obtained are shown in Fig. 2a. For plates having higher E_1/E_2, the deflection is found less. As the percentage of damaged area increases in an element, the central deflection gets increased. As damage area increases the buckling load gets decreases and is plotted in Fig. 2b. Analyses have been done for both uniaxial and biaxial loads. The variation in buckling load due to the change in size of damage is found out. The damage ratio Γ_1/Γ_2 takes values between 0.0 and 9.0. A mild damage may be represented with a damage ratio $0.0 \leq \Gamma_1/\Gamma_2 \leq 3.0$ while a heavy damage may be denoted by the range of values, $7.0 \leq \Gamma_1/\Gamma_2 \leq 9.0$. The variation of nondimensional buckling load for centrally located damage patches having damage intensities of Γ_1/Γ_2 =3 and Γ_1/Γ_2 =7 is presented in Table 2. It is found from the present analyses that the internal flaws will significantly affect the buckling characteristics of laminated composite plates.

Damage area at center layer (%)	Γ_1/Γ_2 =3	Γ_1/Γ_2 =7
35	21.4969	16.9440
45	19.9309	15.1381

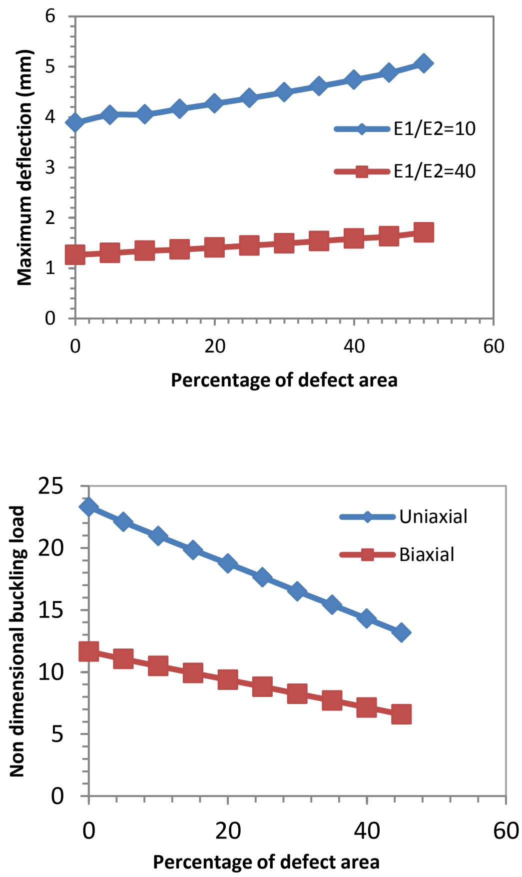

Fig. 2a Effect of damage area on the transverse central deflection. **b** Effect of damage area on the non dimensional buckling load.

Table 2 Variation of nondimensional buckling load of damaged composite plate as damage intensity varies under a uniaxial loading condition.

Buckling analysis for laminated composite plates with central damage subjected to uniform in-plane thermally induced loadings are performed using IHSDT. Critical buckling temperature of damaged plate has been obtained and compared with those of undamaged case. Buckling temperature is found for an isotropic square plate with α= $2x10^{-6}$ $^{o}C^{-1}$ and a/h = 100, for both 'all edges simply supported' and 'all edges clamped' boundary conditions. The values for two damage intensities, Γ_1/Γ_2 =3 and Γ_1/Γ_2 =7, are given in Table. 3.

Table 3 Critical buckling temperatures (^{o}C) for undamaged and damaged plates

Source	Simply Supported		
	Undamaged	Damaged	
		Γ_1/Γ_2 =3	Γ_1/Γ_2 =7
Present	63.1443	60.0667	55.8697
Ref [5]	63.2660	--	
Source	Clamped		
	Undamaged	Damaged	
		Γ_1/Γ_2 =3	Γ_1/Γ_2 =7
Present	167.2378	165.5620	161.5379
Ref [5]	167.8560	--	

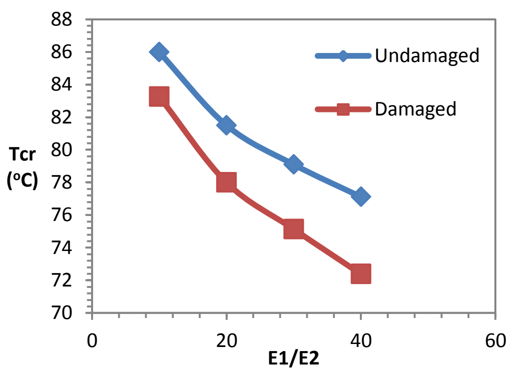

Fig. 3 Critical buckling temperature for 4 layered symmetric composite plate

Critical buckling temperature is found and plotted in Fig. 3 for a square symmetric cross-ply (0/90/90/0) laminated composite plate with and without damage. The plate considered is under simply supported boundary conditions and uniform temperature. As E_1/E_2 increases from 10 to 40, the critical buckling temperature decreases. The decrement is higher for damaged plates, as observed from the steeper curve of damaged case. Effect of damage in post buckling path is illustrated in Fig. 4.

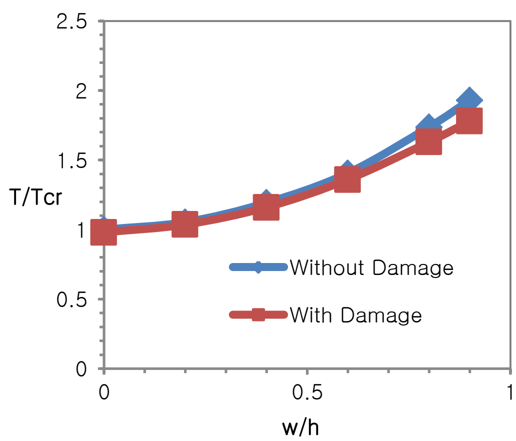

Fig. 4 Effect of damage in post buckling path

As load increases, the difference in displacements with and without internal flaw increases.

4. Conclusions

Buckling analysis of a laminated composite plate with and without damage has been studied. FEM formulation and programming in MATLAB environment using an Inverse Hyperbolic Shear Deformation Theory have been done for a damaged composite plate. The codes are providing satisfactory results when compared with references. The convergence is obtained in plate bending and buckling analysis. The results of critical buckling loads at various conditions are calculated and compared with several published results and analytical solutions available in the literature. A very good agreement of the results obtained by the present method with reference solutions, shows that the method is robust, effective and highly accurate. Thermal post buckling equilibrium path is traced for composite plates with an internal flaw and compared the results with that of undamaged case. It is found from the present analyses that the internal flaws will significantly affect the buckling and post buckling behaviour of laminated composite plates.

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