Enhanced-Precision LHSMC of Electrical Circuit Considering Low Discrepancy

Eun-suk Park, Deok-keun Oh, and Ju-ho Kim

Abstract—**The Monte-Carlo (MC) technique is very efficient solution for statistical problem. Various MC methods can easily be applied to statistical circuit performance analysis. Recently, as the number of process parameters and their impact, has increasingly affected circuit performance, a sufficient sample size is required in order to consider high dimensionality, profound nonlinearity, and stringent accuracy requirements. Also, it is important to identify the performance of circuit as soon as possible. In this paper, Fast MC method is proposed for efficient analysis of circuit performance. The proposed method analyzes performance using enhanced-precision Latin Hypercube Sampling Monte Carlo (LHSMC). To increase the accuracy of the analysis, we calculate the effective dimension for the low discrepancy value on critical parameters. This will guarantee a robust input vector for the critical parameters. Using a 90nm process parameter and OP-AMP, we verified the accuracy and reliability of the proposed method in comparison with the standard MC, LHS and Quasi Monte Carlo (QMC).**

Index Terms—**Fast Monte Carlo, latin hypercube sampling, low discrepancy sequence, performance analysis, yield analysis**

I. INTRODUCTION

As the CMOS device becomes smaller, statistical variability of process parameters on circuit increases. There are two methods for considering this characteristic of circuits. The first method is an analytical method that can obtain statistical or deterministic results using given constraints [1]. However, it is difficult to predict circuit characteristics such as complex statistics, high dimensionality, and expensive performance evaluation. We cannot identify various performance of circuit, since it is applied to a specific target. The second method is a Monte-Carlo (MC) method, which extracts the input vector for simulation in process variation space and represents the performance of circuit as a statistical distribution. In MC simulation, when we predict the performance of circuit by process parameter, we can easily consider the characteristics of non-standard distribution and high dimensionality in process variation space [2]. In addition time-to-market (TTM) and turn-around-time (TAT) can respond quickly.

However, we have to extract sufficient samples in order to confirm the accuracy of the analysis. Also, there are some problems with the high runtime cost. The simple MC technique is known to have low efficiency. Variance reduction methods can be employed to increase its efficiency. These techniques, which predict parametric yield and timing analysis, have been studied in [3-6]. In [3], a Latin Hypercube Sampling (LHS) approach for parametric yield estimation is proposed. In [4], mixture importance sampling for statistical SRAM design and analysis is proposed. The approach in [5] uses the control variates technique in conjunction with importance sampling for timing yield estimation. However, while several approaches are reviewed, no results are presented. In [6], the authors propose to use Quasi Monte Carlo (QMC) analysis for yield estimation. It is not clear how

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this approach can be extended to systems with a large number of dimensions, which is often the case with process variation. Also, optimized QMC for accurate analysis has been studied in [7]. [7] defined a measure of 2-D uniformity and proposed a search algorithm to find a set of initial value with a high defined uniformity. The drawback to their technique is that the number of samples and dimension must be known in advance. Moreover, the technique re-produces Sobol sequence and re-evaluates their defined discrepancy measure in each iteration (after an initial value update), substantially increasing the runtime for a large number of samples and dimensions.

In this paper, the Fast MC method is proposed for an accurate analysis of circuit performance that considers high dimension problem and 2-D uniformity on critical parameters. Section II describes the process variation and variance reduction methods. We present overall flow of our proposed Fast-MC, the sample extraction method, and pairing method in Section III. Section IV presents the results in detail, while the concluding remarks are provided in Section V.

II. BACKGROUND AND RELATED WORK

In this section, we briefly describe the process variation and standard techniques for variance reduction of MC, which include QMC techniques and LHS.

1. Process Variation

Examples of variation during the manufacturing process include shifts in the values of parameter such as the effective channel length (L_{eff}), the oxide thickness (t_{ox}), and the transistor width (w). Process variations can be classified into the following categories, depending on their physical range on a die or wafer [8]:

- Die-to-die (D2D) variations correspond to changes from on die to another (Fig. 1(a)).
- Within-die (WID) variations correspond to variability within a single die (Fig. 1(b)).

D2D variations affect all the devices on the same chip in the same way; for example, they can cause the transistor gate lengths of devices on the same chip to all be larger or all be smaller than the nominal WID variations, however, these may affect different devices differently on the same chip; for example, they could cause some devices to have smaller transistor gate lengths and others to have larger transistor gate lengths than the nominal one. These D2D variations have been a longstanding design issue, and for several decades, designers have been striving to make their circuits robust under the unpredictability of such variations. This has typically been achieved by corner-based analysis that considers the worst variation. In nanometer technologies, WID variations have become significant and can no longer be ignored. Corner-based methods are adequate in cases where all variations are D2D, and no WID variations are seen. MC can be considered a high dimensionality problem with increasing WID variations during circuit performance analysis.

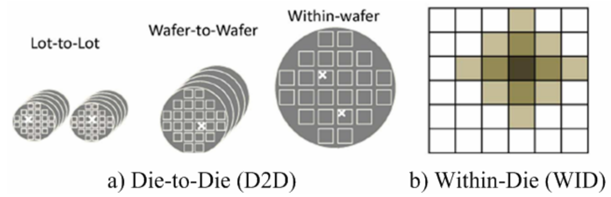


Fig. 1. Types of variations.

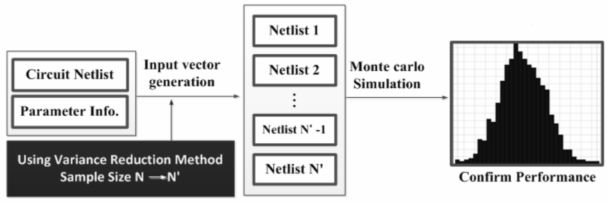


Fig. 2. Fast MC for circuit performance analysis.

2. Variance Reduction Method

Variance reduction method can maintain accuracy while reducing the N' from N sample size in MC simulation as shown in Fig. 2. We briefly describe LHS and QMC

A. Quasi Monte-Carlo

The standard MC method addresses the problem of approximating the integral of a function *f(x)* over the s-dimensional hypercube $C^S = [0,1)^S$, where x represents a point in an s-dimensional space. The MC estimate of the

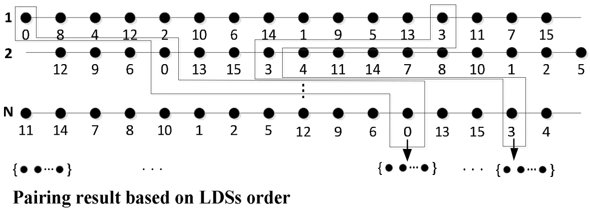
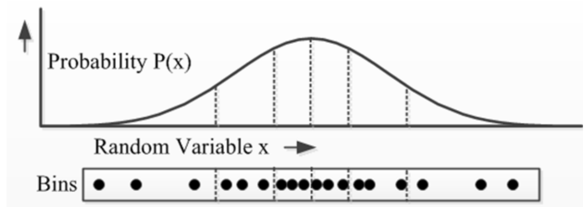


Fig. 3. Generate sample using LDS.

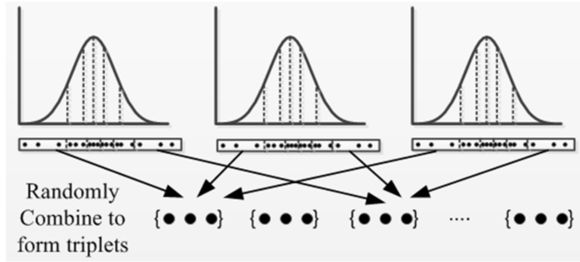
integral $\overline{f}$ is given by the arithmetic mean of f_i, which are values of the function *f(x)* evaluated at n samples distributed throughout the hypercube. The Koksma-Hlawka inequality relates the error bound of a method to numerically estimate an integral using a sequence of samples to a mathematical measure of uniformity for the distribution of the points, which is called discrepancy [9]. This inequality suggests that we should use a sequence with the smallest possible discrepancy for evaluating the function in order to achieve the smallest possible error bound. Such sequence constructed to reduce discrepancy is called LDSs (low discrepancy sequences). QMC techniques are characterized by their use of LDSs to generate samples. LDSs are deterministic sequence, in other words, there is no randomness in their generation such as Fig. 3. Intuitively these sequences are well dispersed through the domain of the function, minimizing any gap and/or clustering of points. Sobol [10] and Faure and Niederreiter [6] are LDSs that have been studied extensively. In this work, we consider Sobol sequences, which are known to be simple to construct and more resistant to the pattern dependency issue, in comparison with other sequences. However, LDSs are imperfect, and as the number of dimensions in the problem increases, the uniformity is degraded. It can lead to large errors in the integration.

B. Latin Hypercube Sampling

LHS is a technique in variance reduction which deals with multidimensional system [11]. This technique tries to sample each variable involved uniformly by dividing the variable into equal probability bins. The samples from bins in variables are combined across dimensions to obtain faster convergence than random sampling as shown in Fig. 4. This is in contrast with taking all permutations of the bins across variables to define strata, and then sampling within each stratum as in stratified sampling [12]. This means that LHS can deal with large



(a) Sampling of a variable in equal probability bins



(b) Forming triplets by randomly combining individual samples

Fig. 4. LHS sampling method.

dimensions, however with a moderate rate of convergence compared to full stratification. Each random variable is divided into equal probability bins. One sample is generated within each bin. Such samples are combined across variables to obtain LHS. This is the procedure to obtain *k* samples, where *k* is the number of bins per variable. To obtain *mk* number of samples, we repeat the LHS procedure *m* times.

III. PROPOSED FAST MONTE CARLO

The objective of the proposed Fast MC is an exact performance analysis of the circuits. It minimizes the loss of analysis accuracy by decreasing the number of samples and increasing dimensions that are applied to the simulation. In addition it produces reliable results without depending on the number of executions. In this paper, we propose a Fast MC structure, as shown in Fig. 5.

Proposed system consists of a sampling method for extracting the process parameter information, and a pairing method for generating the input vector for the simulation using the extracted samples. To improve the accuracy, pairing method calculates the number of effective dimensions using the sample size applied in the simulation and applying the critical parameter that impacts the performance of the circuit. Further description will be provided in the section of pairing method. Finally, MC simulation is performed using the

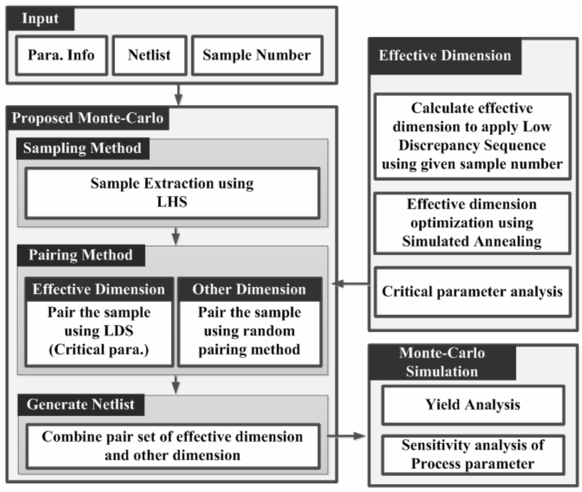
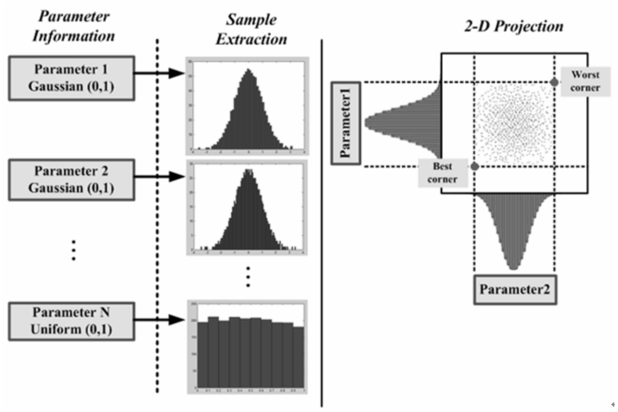


Fig. 5. Overall flow of proposed fast MC.



(b)

Fig. 6. Importance of sample extraction.

generated input vector, and the circuit's yield and performance are analyzed.

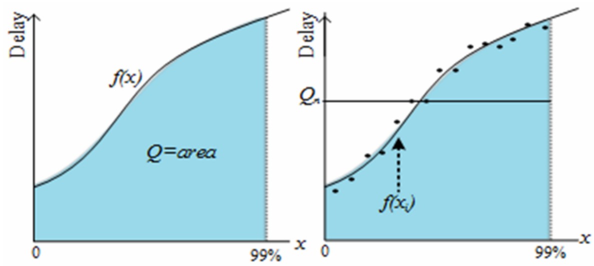
1. Sample Extraction of Process Parameters

In MC method, the number of samples needed for simulation will be extracted based on process parameter information. This is a method that considers only the accuracy of sample extracted from one parameter (considering 1-dimension). To generate an input vector for MC simulation, we have to pair extracted samples, and also evaluate pairing result of samples and uniformity of 2-dimension in the valuation basis (considering 2-dimension).

In this section, we will explain the sample extraction method for given process parameter. The sample extraction of process parameters is needed to consider both 1-D and 2-D, as shown in Fig. 6.

We have to extract samples that exactly reflect the information of parameter variation as shown Fig. 6(a) and reflect the corner value through the sample pair, as shown in Fig. 6(b). We introduce the theoretical error rate of each sampling method [13] for the above property. Using standard MC method, we typically compute some metric like yield, or 99-th percentile delay. Such a computation is essentially a numerical approximation of some integral as follows:

$$Q = \int_{C^S} f(x)dx, \quad x = (x_1, \cdots x_s), \tag{1}$$



(a) (b)

Fig. 7. Monte-Carlo integration method.

where $C^S = [0,1)^S$ is the s-dimensional unit (number of process parameters), For example, if we are estimating the mean of the worst circuit delay, f is the circuit delay as a function of the statistical parameters of all the transistors. The numerical estimate computed from an n-point MC is

$$Q_n = n^{-1} \sum\nolimits_{i=1}^{n} f(x_i), \tag{2}$$

In general, x_i are n independent and identically distributed samples drawn from the s-dimensional uniform distribution U $[0,1)^s$. Process variables with different variable ranges, arbitrary statistical distributions, arbitrary nonlinearity, etc., can always be transformed into this canonical integral form (i.e., these can always be included in our function f without any loss of generality). Fig. 7. shows integration method in MC. Fig. 7(a) represents timing yield at 99 percentile point using formula (1). Fig. 7(b). shows MC integration using expectation method as formula (2).

Suppose we use MC to estimate the standard deviation of the worst circuit delay in the presence of presence of process variations. Different n-point MC runs will give us slightly different estimates. It is well known in [14] as the convergence properties of the standard MC. If f has finite variance

$$\sigma^2(f) = \int_{[0,1)^s} [f(x) - Q]^2 dx \quad (3)$$

where $\sigma^2(f)$ is the variance of the underlying integrand f in (1). The mean square error of the MC integral approximation is given as

$$E[(Q_n - Q)^2] = \sigma^2(f) / n \quad (4)$$

Thus, the expected MC error is $O(n^{-1/2})$. The advantage of standard MC is that this error does not depend on the dimensionality.

LHS ensures high uniformity of the point set along any dimension. This, however, does not guarantee similar uniformity in two (or more) dimensional projections of the point set, because the variance reduction property of LHS is given as

$$f(x) = f_1(x) + f_{>1}(x), \quad f_1(x) = \sum_{i=1}^{s} f_{\{i\}}(x_i), \quad (5)$$

where $f_{\{i\}}(x_i)$ is that part of f which depends exclusively on the i-th variable. Thus, f_1 is the purely one-dimensional part of f, and $f_{>1}$ is the purely multi-dimensional part of f. The variance of the LHS estimate is

$$\sigma_{LHS}^2 = n^{-1}\sigma_{>1}^2 + o(n^{-1}), \quad (6)$$

where $\sigma_{>1}^2$ is the variance of $f_{>1}$, which is similar to $\sigma^2(f)$ for f. If f is only one-dimensional, we get large variance reduction and lower errors than MC, because of $O(n^{-1})$.

QMC is specifically designed to place sample points as uniformly as possible. The QMC algorithm for the evaluation of the integral has a form similar to formula (2).

$$Q_n = n^{-1}\sum_{i=1}^{N} f(q_i), \quad q_i = (q_i^1, \cdots q_i^n), \quad (7)$$

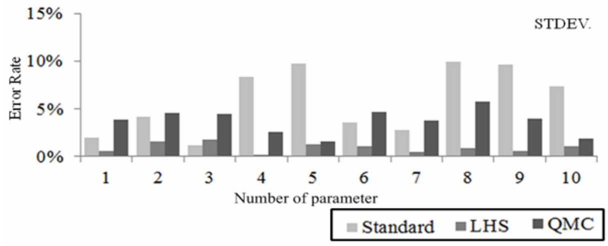


Fig. 8. Error rate of each variation on each method.

where q_i is a set of LDS points uniformly distributed in a unit hypercube. The Koksma-Hlawka inequality [7] gives an upper bound for the QMC integration error.

$$\sigma^2 \le V(f)D_N, \quad (8)$$

where $V(f)$ is the variation of $f(x)$ and D_N is the sample discrepancy. After D_N is applied, error rate of QMC is as below [11].

$$\sigma^2 = O(n^{-1}\log^s n) \quad (9)$$

The asymptotic integration error rate of QMC is $O(n^{-1})$.

We can confirm that the more effective ways in 1-D are LHS and QMC through integration error rate of sampling method. In order to verify the theoretical error rate, we carried out the following two experiments. First, we confirmed the accuracy of the sample extraction in each method. Fig. 8 shows the error rate for each variance by ten parameters sample extraction (sample size: 100) for normal distribution, in which the mean was 0 and the standard deviation was 1.

We can see that standard sampling is not valid for Fast MC, because its accuracy rate is not steady in certain parameters such as in parameter 4,5,8,9, and 10.

Second, we identified the coverage of the extracted samples. QMC generates samples using deterministic LDSs. When samples are generated deterministically, the mean and variance are accurately generated. However, it may have a limitation in term of given parameter information. Fig. 9 shows the extracted sample's max and min parameter value (dimension: 100), in which the mean was 0 and standard deviation was 1. It shows that LDSs have a deterministic min/max value on given parameter information and that there can be problem with the generation of the corner value.

As we can see from Figs. 8 and 9, LHS is the most

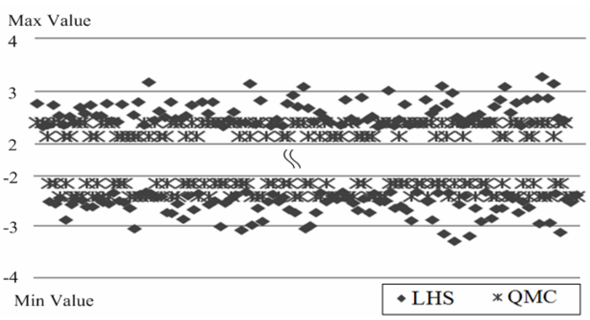


Fig. 9. Generated min/max value on LHS and QMC.

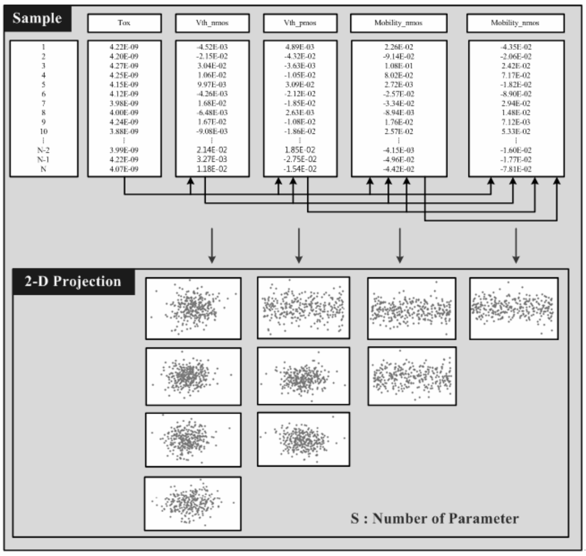


Fig. 10. Pairing method for circuit analysis.

efficient method for extracting samples in 1-D. However, in formula (6), we can observe that LHS cannot guarantee accuracy in multi-dimension. To overcome this limitation, we will describe pairing method that considers multi-dimension in the next section.

2. Pairing Method for 2-D Uniformity

In the MC simulation for circuit performance analysis, if the S process parameter and N sample size are applied, it produces a pairing result using the extracted sample, as shown in Fig. 10. When S number of parameter is applied to MC simulation, S(S-1)/2 pairing results are generated. In order to reduce the integration error of MC, the low discrepancy value of the pairing results is fairly important.

However, it is not easy to expect low discrepancy in a pairing that uses LHS. In the LDSs that be used in QMC, it can keep uniformity 2-D projection at initial several dimensions. However, pattern dependency can occur during sample pairing at higher dimensions. Fig. 11 shows the pairing results for various methods (sample size: 1000).

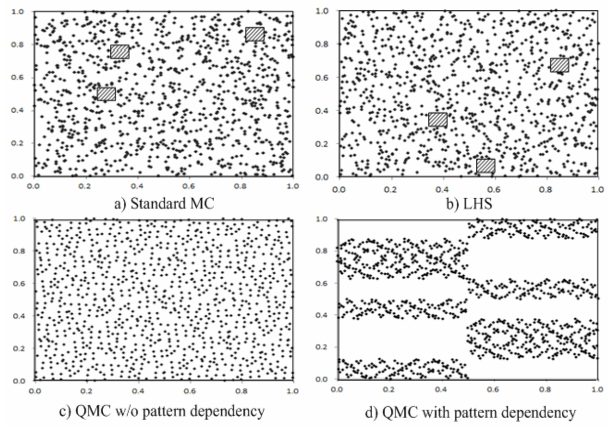


Fig. 11. Pairing result on each method.

As can be seen from a) and b) in Fig. 11, we cannot say that pairing results of LHS have greater uniformity than that of random method in 2-D projection. In c), in the case of LDSs, we can identify the highest uniformity in 2-D projection. However, as the dimensions increased, the pattern dependency in 2-D projection increased, such as d) in Fig. 11. This is the main cause of the increased integration errors in MC. For example, a set of 10K Sobol's points in 100 dimensions will have well distributed points in dimensions 1-10 and undesirable patterns in dimensions 91-100 [13].

By considering these characteristics, this paper calculates the number of dimensions that can represent the strength of LDSs according to the number of samples, and these dimensions apply to the extracted samples of the critical parameter. Therefore, we have to determine the effective dimension that can reflect the effect of LDSs. This can be calculated using the discrepancy value. Determining dimension of LDSs that may have a smaller discrepancy than LHS is limited. In other words, there are various definitions of discrepancy, and these are relative values.

We solved this problem by relating relative values with computed discrepancy, as [15] represented computed discrepancy using generated samples, as follows:

$$E(L_2(x)) = \sqrt{\frac{\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\prod_{k=1}^{2}(1-\max(x_i^{(k)},x_j^{(k)})) - \frac{1}{2N}\sum_{i=1}^{N}\prod_{k=1}^{2}(1-x_i^{(k)^2})+3^{-2})}{}/n} \tag{10}$$

where N is the sample size, k is the axis and x is the generated sample. Using formula (10), we can build the relation between the LHS and LDSs theoretical discrepancy value and define the effective dimension that can be applied to LDS as follows:

$$\frac{\sum_1^S LDS_{Discrepancy(n)}}{sP2} \le LHS_{Discrepancy(n)}$$
$$\Rightarrow LDS^S_{Average\ Discrepancy(n)} \le LHS_{Average\ Discrepancy(n)} \tag{11}$$

where S is the dimension and n is the sample size. We set the upper bound on the discrepancy of the LDSs using LHS's average discrepancy.

First, we can build the relation between the computed discrepancy and the theoretical square discrepancy of LHS. In [16], the square discrepancy formula was represented as follows:

$$E(L_2(N))_{theoretical} = (\frac{13}{12})^2 - 2(\frac{13}{12}+\frac{1}{24N^2})^2 + \frac{1}{N}(\frac{5}{4})^2 + (1-\frac{1}{N})(\frac{13}{12}-\frac{1}{6N})^2, \tag{12}$$

where N is the sample number with even values . We can express formula in the form (13) as follows:

$$E(L_2(x))_{LHS} \approx \alpha \cdot E(L_2(N))_{theoretical}, \tag{13}$$

where α can represent the coefficient value for the sample size and confirm the computed discrepancy using the average square discrepancy. Fig. 12 shows the discrepancy according to sample size, and it is the result of Eq. (13). We can confirm that the difference between two values has an error rate of about 5%.

Second, we can build the relation between the theoretical and computed discrepancy of LDS. In this work, we used the Sobol sequence generation algorithm, which is introduced in [17]. Formula (14) is defined as the theoretical discrepancy of LDS.

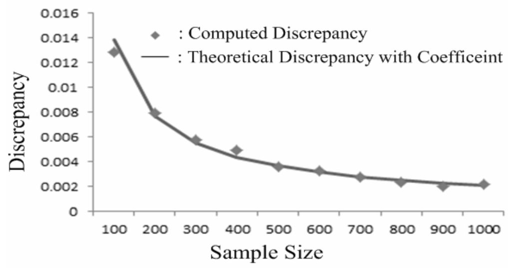


Fig. 12. Relation with computed discrepancy on LHS.

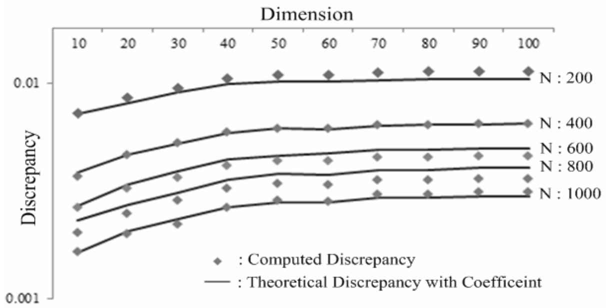


Fig. 13. Relation with computed discrepancy on QMC.

$$D_n^{(d)} \le c(s)\frac{\log^S n}{n}, \tag{14}$$

where *c(s)* is a constant depending only on dimension s, s is the dimension and n is the sample size. We can express the formula from formula (10) and (14) as follows:

$$E(L_2(x))_{sobol} \approx \beta(s) \cdot D_n \tag{15}$$

By using formula (15), coefficient can be extracted. As LDS's discrepancy is dependent on dimension, according to formula (14), there is a correlation between and dimension as well. After extracting of specific dimension, it is stored in a table. The others can be gained using interpolation. Fig. 13 shows the discrepancy according to the sample size and dimension, and it is the result of Eq. (15), and error rate is about 7%.

Therefore, we can determine the dimension to apply to LDS using formula (13) and (15) as below.

$$E(L_2(x))_{LHS} \ge E(L_2(x))_{sobol}$$
$$\Rightarrow \alpha \cdot E(L_2(N))_{theoretical} \ge \beta(s) \cdot (D_N)_{theoretical}, \tag{16}$$

where α and β are calculated as the coefficients of each method. We can obtain the effective dimension that

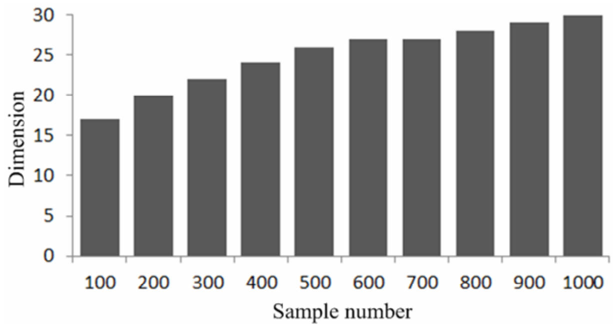


Fig. 14. Calculated dimension depend on sample size.

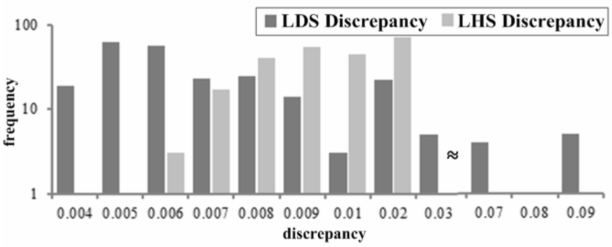


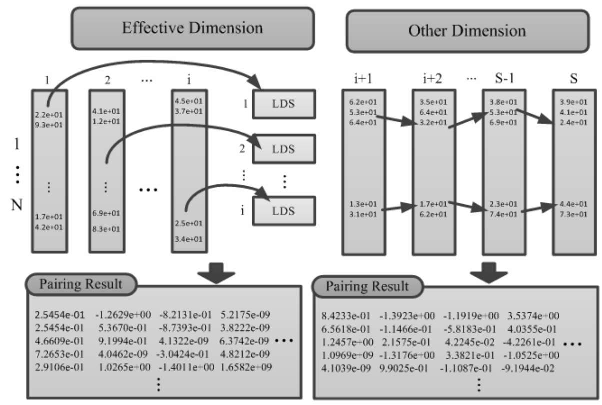
Fig. 15. The result of LDS and LHS discrepancy.

satisfies the above condition using the sample size.

Fig. 14 shows the calculated dimension using formula (16). The number of dimensions to apply to LDS increases, when the number of samples is not large enough. However, when the number of samples is large, an increase in dimensions will be small. This phenomenon is due to the fact that the discrepancy of LHS is not affected by dimension and will be sufficiently small as the number of samples grows.

Pair sets using effective dimensions from the above method can sometimes have a higher discrepancy than the average discrepancy of LHS. In Fig. 15, the discrepancy distribution of LHS is uniform, but QMC has average discrepancy by good pairing results and bad pairing results.

To improve the effect of low discrepancy in the pairing result by LDS, we have to increase the frequency in the good pairing part and decrease or remove frequency in bad pairing part. In [7, 18], by controlling the initial values of the Sobol sequence, methods are suggested for optimizing the discrepancy of LDS. In our method, using [18], we optimize the discrepancy of the dimension paired using the Sobol sequence. We have a low runtime cost, since the number of dimensions that should be optimized is not large. In the case of critical parameters, the samples are paired using LDSs and the others are paired by LHS.



(a) Effective dimension for CP (b) Other dimension

Fig. 16. Pairing result considering critical parameters.

Analysis of the critical parameters that will apply to the pairing method will be done with the following two methods. First, the designer selects the parameters that have the most effect on the relevant performance metrics and assigns these to the lower coordinates of the MC. Second, the global sensitivity of the metric to circuit parameters is used a measure of their importance, and the parameters are sorted in decreasing order of importance. This sorted list is then mapped to the corresponding pairing using LDS.

The measure of sensitivity we use is the absolute value of the Spearman's Rank Correlation Coefficient [19]. Supposing that R_i and S_i are the ranks of corresponding values of a parameter and a metric, the their rank correlation is given as

$$r_S = \frac{\sum_i (R_i - \overline{R})(S_i - \overline{S})}{\sqrt{\sum_i (R_i - \overline{R})^2}\sqrt{\sum_i (S_i - \overline{S})^2}} \qquad (17)$$

The rank correlation can be computed by first running a smaller MC run [6]. Finally, the input vectors are generated for fast MC simulation, as shown in Fig. 16. N is the sample size, i is the number of calculated effective dimensions, and S is the number of process parameter.

IV. Experimental Results

In this section, we compare the performance of our Fast-MC against the performance of MC, LHS, and QMC. First, we make some observations about various

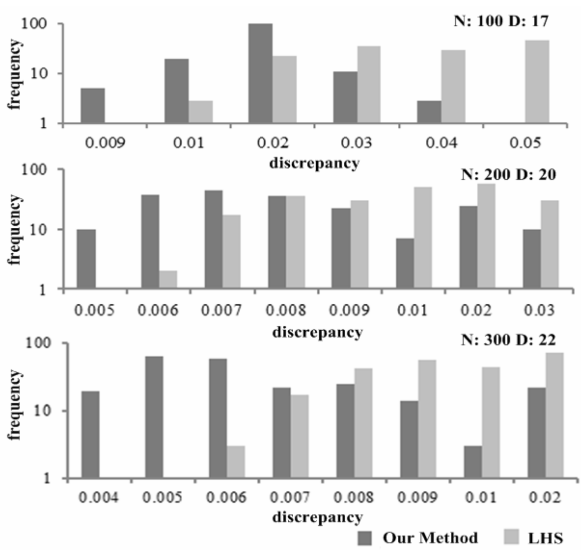


Fig. 17. Discrepancy distributions on the effective dimension.

MCs and our Fast MC implementations.

- A Linear Congruential Generator (LCG) [20] was used to generate the pseudo random sequence for MC because of its widespread popularity.
- Latin Hyper Cube method [12] was implemented for statistic
- The sobol sequence generation algorithm [17] was used to generate LDS for QMC. For optimization of effective dimensions, the initial value of each dimension was chosen by simulated annealing [18].

Now we describe the test cases and the experiments. Fig. 17 shows the discrepancy distribution on the effective dimension. N denotes the number of sample, and D refers to the effective dimension corresponding to the critical parameters.

In comparison with the discrepancy distribution of the LHS pairing results, it can be confirmed that the discrepancy distribution of the pairing results of applying the LDS's will now have a lower discrepancy. Even though a distribution that has 0.02 discrepancy exists in the condition of N:300 and D:22, LHS method provides more than 70 pair sets that have about 0.02 discrepancy value , while there are only 13 pair sets using the method that we suggested. The accuracy and reliability of the proposed Fast MC can be improved by considering the critical parameters that can have a significant impact in the performance of the circuit.

The proposed Fast MC is implemented in C++

```
.input
    Number of .sp file
    Name of .sp file
.output
    Location of output file and file name
.sample size
    Number of sample
.critical_parameter
    Automation or manual
    Parameter name
.critical_parameter_end
.constraint
    Constraint information
.end
```

Fig. 18. Configuration file for the proposed Fast MC.

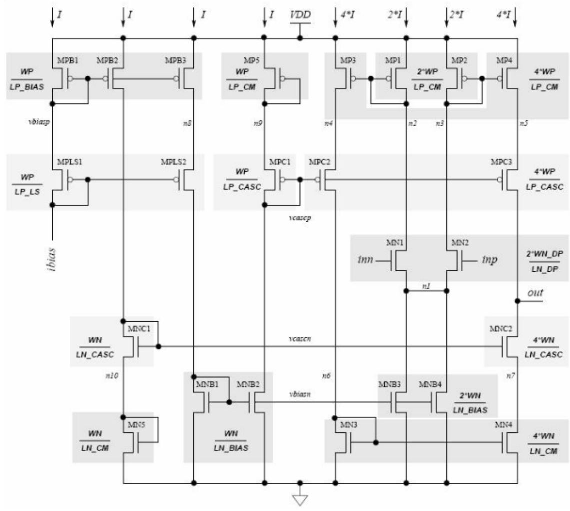


Fig. 19. OP-AMP for experimental result.

Table 1. Operating parameters

Operating Parameter	Nominal	Unit
Supply Voltage	1.1	V
Temperature	27	'C
Bias Current	10	uA

language and the input vector that was generated using the configuration file, as shown in Fig. 18, was simulated using Hspice. We measured the AC and DC for OP-AMP using 90 nm model library, and we applied 75 process parameters (dimension) as shown Fig. 19.

As shown in Fig. 19, the simulated OP-AMP consist of 13 PMOSs and 11 NMOSs, and it includes three local variations such as threshold voltage, mobility, and gate oxide thickness and global variations such as gate length, gate width, gate oxide thickness consists of 75 dimensions for simulation. Table 1 shows operating parameters for OP-AMP simulation.

Fig. 20 shows measure of gain in OP-AMP, and it also

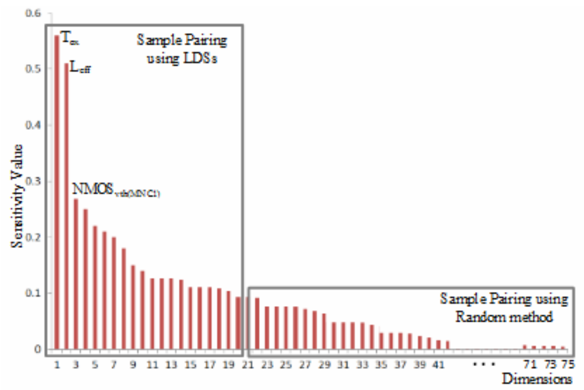


Fig. 20. Critical parameter analysis for OP-AMP gain (sample size :200).

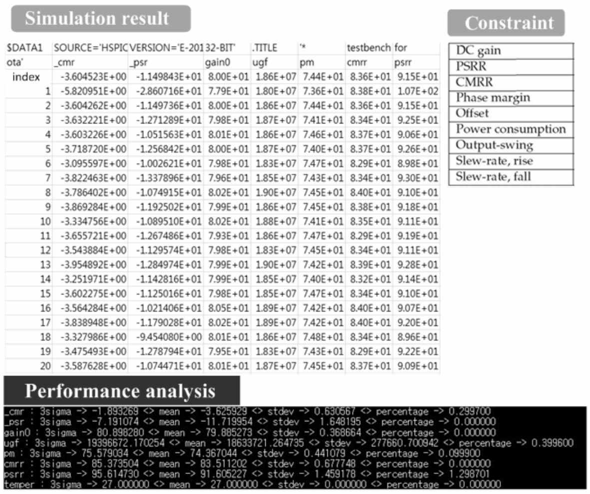


Fig. 21. Simulation result and performance analysis.

shows the absolute value of rank correlation about process parameters consisting of dimension. The variables are sorted according to decreasing importance. This is now the order they will be mapped to the effective dimension for sample pairing. When considering 200-sample size, 20 effective dimensions are generated, and sample pairing applied to LDSs are implemented based on sorted critical parameter. Fail analysis can be done using the fail index number and the sample size.

Table 2 shows the simulation results of OP-AMP. The methods presented above were used in ten simulations in same condition, and they were compared with the standard MC (sample size: 10000). Then we calculated the average error rate of variance. As shown in the table above, we can identify that the method proposed in this paper has the more low error rate result for performance

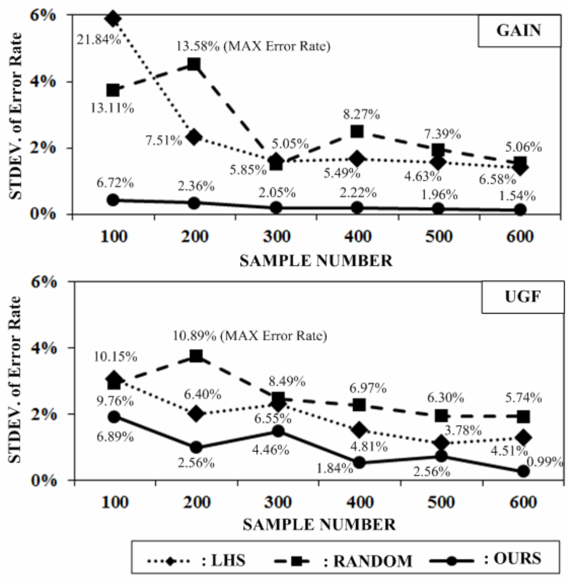


Fig. 22. Fluctuation of error rate as sample growing.

analysis, and shows that all has under about 5% error rate except for PSRR (power supply rejection ratio). In PSRR case, since standard deviation value is very smaller than mean value, error rate may increase in comparison with other performances. However, we can identify that it has lower error rate than other methods. In error rate case in Table 2, it is possible that each method has similar error rates for some performance, because we use average error rate occurred during the simulations. In other words, both high and low error rate can occur as an outcome of random pairing result in case of random and LHS method is used.

For this phenomenon, we can identify each method's reliability by confirming the fluctuation in the variance error rate. Fig. 22 shows the average fluctuation width of variation error rate of GAIN and UFG for LHS, RANDOM and OURS.

The numerical values of each point show the maximum error rates that occur corresponding to the sample size. Generally, we can identify that range of fluctuation of variance decreases with an increase in the number of sples. If range of fluctuation of variance is small, this means that Fast MC's reliability is high. In proposed Fast MC, regardless of the number of samples, we can see that the range of fluctuation of variance is within about 2%, and it has a low value than that of other method, in term of the max error rate.

Fig. 23 shows error rate of PM and UFG's standard

Table 2. AC and DC performance analysis of OP-AMP

AC (Alternating Current) : (Variance error rate of standard MC compare with reference MC)											
N : 100	GAIN	UGF	PM	CMRR	PSRR	N : 200	GAIN	UGF	PM	CMRR	PSRR
LHS	8.18%	6.59%	4.09%	5.34%	8.68%	LHS	4.15%	3.17%	3.99%	4.47%	7.63%
RANDOM	6.39%	4.86%	4.63%	6.93%	15.57%	RANDOM	4.56%	3.75%	2.47%	3.64%	4.66%
QMC	0.84%	11.34%	9.52%	5.97%	9.85%	QMC	6.82%	2.04%	4.27%	0.81%	24.48%
OURS	4.91%	2.40%	1.77%	4.83%	8.03%	OURS	1.78%	1.06%	0.84%	3.00%	0.96%
N : 300	GAIN	UGF	PM	CMRR	PSRR	N : 400	GAIN	UGF	PM	CMRR	PSRR
LHS	3.15%	2.64%	3.54%	4.41%	7.49%	LHS	2.83%	2.16%	1.92%	4.32%	4.95%
RANDOM	3.19%	4.38%	3.58%	3.85%	5.46%	RANDOM	3.00%	3.53%	2.54%	2.17%	7.92%
QMC	3.19%	0.81%	6.63%	2.69%	11.97%	QMC	1.21%	0.76%	3.47%	0.91%	5.57%
OURS	1.77%	2.75%	1.70%	2.84%	6.59%	OURS	1.99%	0.82%	0.52%	1.75%	5.21%

DC (Direct Current)							
N : 100	OFFSET	OUTPUT_RANGE	POWER	N : 200	OFFSET	OUTPUT_RANGE	POWER
LHS	4.51%	8.34%	5.60%	LHS	4.16%	4.89%	3.82%
RANDOM	5.55%	4.33%	5.46%	RANDOM	3.16%	3.94%	3.69%
QMC	2.39%	23.51%	6.48%	QMC	1.23%	3.69%	2.75%
OURS	4.34%	2.12%	5.83%	OURS	3.81%	2.22%	3.18%
N : 300	OFFSET	OUTPUT_RANGE	POWER	N : 400	OFFSET	OUTPUT_RANGE	POWER
LHS	3.02%	4.97%	4.01%	LHS	3.14%	3.58%	3.15%
RANDOM	3.37%	4.45%	4.12%	RANDOM	2.66%	4.03%	3.69%
QMC	9.61%	3.84%	8.92%	QMC	2.55%	5.47%	3.91%
OURS	2.36%	0.61%	3.83%	OURS	1.85%	0.76%	1.86%

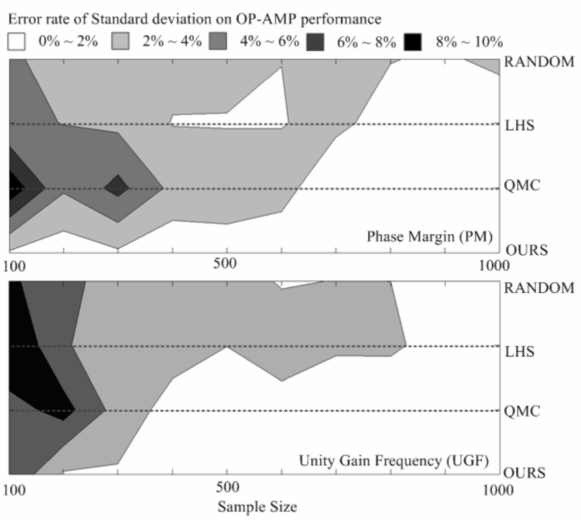


Fig. 23. Error rate of Stdev. on OP-AMP performance.

deviation according to the sample size in OP-AMP.

We define the confidence level as the 2% error rate of a MC that used as reference method, and we can see that the proposed Fast MC, with a smaller sample size than the conventional Fast MC, could approach the confidence level.

V. CONCLUSIONS

This paper proposed an effective Fast MC system for performance analysis of a circuit. The system implements a sampling method and pairing method, as we should consider sample extraction in 1-D and sample projection in 2-D. To improve the 2-D projection of critical parameters that have a significant effect on circuit performance, the effective dimension was applied using the sample size that was used on simulation. The pairing results of critical parameters could reduce the integration error rate of MC by maintaining a low discrepancy. Compared to the LHS and QMC method that were used in conventional circuit analysis, it reduces the dependence on sample size and increases the dimensions. It can be effectively applied to aging analysis and statistical performance analysis of a circuit. Calculating the effective dimension using average discrepancy can reduce the optimization cost of LDS. However, this may not guarantee the best performance. The performance of the proposed Fast MC can be improved by applying to the optimal LDS's discrepancy for effective dimension.

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