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What Kinds of Mathematics Learning are related to Prospective Elementary School Teachers' Mathematics Pedagogical Content Knowledge?

KANG, Eun Kyung[1)]

The statement, 'Taking more mathematics would result a better mathematics teacher.' sounds plausible. However, it is questionable that how much of taking university level of mathematics such as abstract algebra and real analysis would affect to teach elementary mathematics well. Would a mathematician be a better teacher for elementary students to teach mathematics than who has been prepared to teach elementary mathematics? This paper reports the effects of opportunities to learn tertiary level mathematics and school level mathematics on pre-service primary school teachers' mathematics pedagogical content knowledge. The study analyzed Teacher Education and Development Study in Mathematics 2008 (TEDS-M 2008) database using multiple regression. Prospective primary teachers who have been prepared as generalist were the focus of the study. The results support future elementary teachers might need to have opportunities to revisit school mathematics they are going to teach.

Key words: Prospective Elementary School Teacher, Teacher Education, Opportunities to Learn

Ⅰ. Introduction

It is usually assumed that primary mathematics is easy to teach and prospective teachers who enter teacher education program have enough knowledge for teaching elementary school mathematics. Many teacher education programs provide future elementary teachers to learn advanced mathematics topics such as modern algebra. Of course elementary school teachers need to view elementary mathematics with mathematically rigorous eyes, however, recently mathematics teacher educators claim

1) Seoul Yangjae Elementary School

future teachers need to learn mathematics for teaching. This might stems from the question, ‘to what extend such opportunities to learn tertiary level mathematics are helpful for elementary school teacher candidates?’.

One of the goals of teacher education programs might be improving future teachers’ pedagogical content knowledge and content knowledge. Prospective teachers need to have opportunities to develop pedagogical content knowledge before they teach as in-service teachers (Song & Pang, 2008). Then what kinds of features of teacher education programs might be effective to improve prospective primary school teachers’ mathematics pedagogical content knowledge? It is a still ongoing discussion about what prospective teachers need to learn (Seo, 2010). However, especially for educating prospective primary school teachers, studies about ‘What kinds of learning mathematics would be helpful for prospective primary school teachers to be well prepared to teach elementary mathematics?’ might provide valuable information for teacher education field. Would taking more advanced mathematics be helpful for prospective primary teachers’ mathematics pedagogical content knowledge? Would providing primary school teacher candidates to think more about mathematics that they are going to teach be helpful for their mathematics pedagogical content knowledge?

Ⅱ. Theoretical Backgrounds

1. Mathematics content knowledge as a source of Mathematics pedagogical content knowledge

The study of teachers’ mathematical knowledge; mathematical content knowledge and pedagogical content knowledge has been in a great interest in mathematics education research for decades as they affect students’ achievement. Many studies supports teachers’ mathematics content knowledge has an effect on their teaching practice and in return, teaching practice has an impact on students’ achievement. Many studies confirm teachers’ mathematical knowledge for teaching is positively related to the quality of mathematics teaching practice and students’ performance (Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Rowland, Huckstep, and Thwaites, 2005; Tchoshanow, 2011; Tchoshanov, Lesser, and Salazar, 2008; Warfield, 2001).

Shulman (1987) proposed three categories of teachers’ content knowledge that is needed for teaching; subject matter content knowledge, pedagogical content knowledge, and curricular knowledge. Shulman (1986, 1987) first used the term, Pedagogical Content Knowledge (PCK), claiming that teaching requires beyond content knowledge, for example, how to represent the subject to students to help them understand.

Many researchers are studying about developing Shulman’s (1987) Pedagogical Content Knowledge (PCK) for teaching mathematics, for example, Ball, Thames, & Phelps (2008) and Hill, Ball, & Schilling (2008) tried to frame knowledge that is needed for teaching by proposing a domain map for Mathematical Knowledge for Teaching (MKT). After analyzing what teachers do to teach mathematics, Ball et al (2008) defined MKT as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball, Thames, & Phelps, 2008, p.395). Hill et al (2008) defined Knowledge of Content and Students (KCS) is a primary element in Shulman's PCK and content knowledge is intertwined with knowledge of how students think about, know, or learn the subject.

Regarding the relationship between content knowledge and pedagogical content knowledge, many scholars agreed content knowledge is a basic source for pedagogical content knowledge as content knowledge is an important source for pedagogical decision making (Morine-Dershimer and Kent, 1999; Shulman, 1987). And content knowledge and pedagogical content knowledge are closely related as pedagogical content knowledge cannot be considered without content knowledge (Baumert et al., 2010) yet they are distinguishable (Blömeke, 2012). Similarly, Rollnick, Bennett, Rhemtula, Dharsey, and Ndlovu (2008) claimed improving pedagogical content knowledge requires development of content knowledge in the first place. Thus in this study, mathematics content knowledge was considered as one of the predictors for mathematics pedagogical content knowledge.

2. Teachers' learning mathematics for teaching

The relationship between 9th grade teachers' knowledge of algebra and students achievements was investigated by Begle (1972). Begle and his colleagues developed two sets of tests for teachers; a test for real number system which is related to 9th grade algebra and a test for modern algebra such as groups, rings, and fields, and two sets of tests for students; a test for algebraic computation and a test for understanding of algebraic concepts of ninth grade algebra.

Using correlational analysis, what Begle (1972) found was that teachers' achievement of modern algebra was not statistically significantly related to 9th graders' performance in algebraic computation and understanding of 9th grade algebra. This implies just taking advanced mathematics courses might not have direct impact on developing teachers ' mathematical knowledge for teaching.

Hill and Ball (2004) developed and conducted assessments of measuring teachers' mathematics knowledge for teaching before and after California's Mathematics Professional Development Institutes (MPDIs). One of the features of MPDIs was participant teachers were given opportunities to discuss elementary level mathematics problems that have many challenges. The authors concluded the opportunity might affect the assessment result after taking MPDIs which was higher than the result before taking MPDIs. Similarly, after reviewing studies about teachers' professional development, Oh (2012) concluded teachers need beyond content knowledge, for example, experiences to connect their content knowledge in school settings. This implies having teachers to understand school mathematics which is they are teaching might be a pre-requisite for effective teaching practice.

Ⅲ. Methods

1. Data Sources

The study employs Teacher Education and Development Study in Mathematics 2008 (TEDS-M 2008) database which was conducted by International Association for the Evaluation of Educational Achievement (IEA), by a group of researchers at Michigan State University, and the Australian Council for Educational Research. TEDS-M is the first cross-national study to provide data on the knowledge; mathematics content knowledge and mathematics pedagogical content knowledge that future primary, lower-secondary, and secondary school teachers have acquired in their mathematics teacher education.

TEDS-M studied the variation in the nature and impact of teacher education programs within and across countries. The study collected data from representative samples of pre-service teacher education programs, future primary and lower-secondary teachers in the programs, and their teacher educators from 17 participating countries to inform policy and practice in teacher education (for more information, see Tatto, Schwille, Senk, Ingvarson, Peck, & Rowley, 2008).

TEDS-M data provides the results of mathematics content knowledge assessment, mathematics pedagogical content knowledge assessment, and survey results from the participating prospective teachers and teacher educators.

The whole TEDS-M data set included 17 countries and was collected from prospective primary and secondary teachers who were at the end of their teacher education programs so the database is cross sectional in nature. The author has attended TEDS-M data workshop which was provided by Michigan State University to learn the structure of the database and how to analyze it.

For the purpose of the present study, a subset of the data was analyzed. Data from prospective primary teachers who have been prepared to teach grade 1-6 as generalists (in TEDS-M database, TARGETP=2) is the focus of the present study. In this study, all the available data of primary prospective teachers from 6 countries; Chinese Taipei (Taiwan), Philippine, Singapore, Spain, Switzerland, and USA were analyzed.

2. Brief Description of Participant Population

Over all, female was dominant in the population in the present study which reflects many of prospective primary teachers were female. The percentage of female prospective primary generalist teachers was over 70% in all the six countries. And the mean age of generalist prospective primary teachers of the six countries was in between 20 to 30 which was quite young population and this is understandable when we consider that they were prospective teachers at the end of their teacher education program. <Table 1> shows the results of descriptive statistics of the participant population.

<Table 1> Description of Participant Population

Country (Number of Institutions)	Age (years)				Gender		
	N	*M* (*SD*)	Min.	Max.	*N*	% Female	% Male
Chinese Taipei (Taiwan) (11)	921	23.26 (2.12)	21	54	923	72.3	27.7
Philippines (33)	588	20.90 (1.76)	18	33	589	78.6	21.2
Singapore (1)	260	26.35 (4.75)	20	45	260	75.8	24.2
Spain (45)	1092	23.15 (4.36)	20	54	1092	79.9	20.1
Switzerland (13)	809	23.84 (3.55)	22	40	811	83.5	16.5
USA (48)	949	25.48 (6.44)	21	55	950	92.1	7.8

Note. Source: TEDS-M 2008 IEA

3. Design and Analysis

The study was designed to study the following research questions,

1. What is the relationship between mathematics content knowledge and mathematics pedagogical content knowledge?
2. What kinds of opportunities to learn mathematics are related to prospective primary teachers’ mathematics pedagogical content knowledge?

using multiple regression analysis. The data was aggregated at institution level because it is reasonable to assume that opportunities to learn might vary by institution rather than individual level (Meinck and Rodriguez, 2011).

A. Description of variables

For the study, mathematics pedagogical content knowledge (variable name in TEDS-M database, MPCK) was selected as dependent variable in the TEDS-M database and MPCKI was used to represent aggregated assessment result MPCK at institution level. Seven variables were identified as predictors; mathematics content knowledge (MCK), opportunities to learn tertiary level math - geometry (MFB1GEOM), opportunities to learn tertiary level math - discrete structures & logic (MFB1DISC), opportunities to learn tertiary level math - continuity & functions (MFB1CONT) , Opportunities to learn tertiary level math - probability & statistics (MFB1PRST), opportunities to learn school level math - numbers measurement geometry (MFB2SLMN), and opportunities to learn school level math - functions probability calculus (MFB2SLMF).

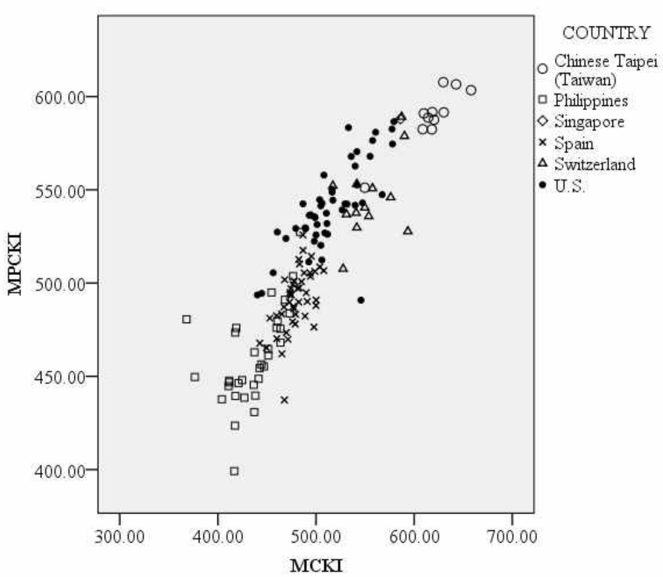
Among the predictors, MCKI was used to represent aggregated MCK at institution level. Four variables; MFB1GEOM , MFB1DISC , MFB1CONT , MFB1PRST, were selected to represent opportunities to learn tertiary level mathematics and the variable names were used in the present study are TG, TD, TC, and TP respectively. Two variables: MFB2SLMN, MFB2SLMF, were selected to represent opportunities to learn school level mathematics from TEDS-M database. Among the variables for school level mathematics, MFB2SLMN is related to the topics that are taught in primary schools; numbers, measurement, and geometry, so MFB2SLMN represents mathematics that the prospective primary teachers will teach in their future teaching. Thus its variable name was changed as PM to express 'opportunities to learn Primary level Mathematics'. The variable in TEDS-M database MFB2SLMF is related to the topics that are usually taught in secondary schools. The topics includes Functions, Relations, and Equations; Data representation, Probability, and Statistics; Validation, Structuring, and Abstracting. And it is not the topics the prospective primary teachers will teach. Thus its variable name was changed as SM to express 'opportunities to learn Secondary level Mathematics'. <Table 2> summarizes the explanation of variables in this study.

For the assessment results of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK), Item Response Theory (IRT) was applied so that they were centered at 500 and standard deviation of 100. For opportunities to learn of tertiary level mathematics and school level mathematics, Racsh scale score were applied 10 for neutral position so above 10 indicates agree for the survey question and below 10 indicates disagree for the survey question.

<Table 2> Description of the Variables in the Study

Description	Variables in	
	TEDS-M database	this study
Mathematics Content Knowledge	MCK	MCKI
Mathematics Pedagogical Content Knowledge	MPCK	MPCKI
Opportunities to Learn (OTL) - Tertiary Level Math - Geometry	MFB1GEOM	TG
Opportunities to Learn (OTL) - Tertiary Level Math - Discrete Structures & Logic	MFB1DISC	TD
Opportunities to Learn (OTL) - Tertiary Level Math - Continuity & Functions	MFB1CONT	TC
Opportunities to Learn (OTL) - Tertiary Level Math - Probability & Statistics	MFB1PRST	TP
Opportunities to Learn (OTL) - School Level Math - Numbers Measurement Geometry	MFB2SLMN	PM
Opportunities to Learn (OTL) - School Level Math - Functions Probability Calculus	MFB2SLMF	SM

First, to see how would MCKI predict the level of MPCKI, scatter plot of the aggregated at the institution level data was produced by using SPSS statistics software version 21 (SPSS IBM, New York, U.S.A) as shown in [Figure 1].



[Figure 1] Scatter plot of MCKI and MPCKI

All the variables in the study were scaled as continuous and ordinal in nature and the relation between MCKI and MPCKI was linear so multiple regression was appropriate to study the research questions. Thus multiple regression was conducted with MPCKI as response variable and MCKI as predictor and added other predictors of opportunities to learn mathematics at teacher education institution.

B. Missing data

Missing data was excluded list wise automatically in the analysis using statistics software SPSS version 21 (SPSS IBM, New York, U.S.A) because the database was cross sectional in nature and there was no pattern found in missing data so it can be assumed the missing were completely at random.

C. Multiple regression analysis

Multiple regression analysis was conducted to see whether the level of mathematics content knowledge, opportunities to learn tertiary level mathematics and school level mathematics predict the level of mathematics pedagogical content knowledge.

The following is the statistical full model tested.

$$MPCKI = \beta_0 + \beta_1 MCKI + \beta_2 TG + \beta_3 TD + \beta_4 TC + \beta_5 TP + \beta_6 PM + \beta_7 SM + \varepsilon$$

where,

β_0 is intercept of MPCKI which means the value of MPCKI when the value of all predictors are zero, β_1, β_2, β_3, β_4, β_5, β_6, β_7 are the partial coefficient representing the slope of the line relating to the corresponding predictor when all the other predictors are held fixed, and ε is random error component.

In conducting multiple regression analysis, one of the issues that needs to be considered is multicollinearity when predictors are collinear. Multicollinearity might cause problems such as misleading individual *p*-values and wider confidence intervals on the regression coefficients. To detect multicollinearity, correlation matrix was conducted and investigated whether there is high correlation between predictors (rule of thumb is $|r| > .7$) Also Tolerance and Variance Influence Factor (VIF) was computed and investigated whether there is a tolerance of less than 0.2 or 0.1 and VIF of 5 or10 and above in analysis (O'Brien 2007).

To select the final model, forward selection, backward elimination, and stepwise regression model selection methods were used. Forward selection starts from empty model with no predictor and add variables one by one until the model cannot be improved significantly by adding another variable. Backward elimination starts with the full model then eliminates the least significant variable one by one until all the variables in the model are significant at a given α for testing. Stepwise regression is the mix of forward selection and backward elimination so each time when a new variable is added, the significance of each variables is re-tested. The process continues until no more variables can be added or can be removed.

4. Limitations

The interpretation of the present study is limited to the end of teacher education program because TEDS-M data does not have information about the prospective teachers' entering status. Consequently, there should be no attempts to consider the change of the prospective teachers' change in knowledge and belief during their teacher education program.

Also the interpretation of the study is limited to the participating six countries and it might be hard to be generalized because the participating countries does not necessarily represent whole population of prospective primary teachers in non-participating countries.

Teachers' beliefs do not mean their teaching practice necessarily. One of the reasons for this is the belief measurements were from self professed survey result. Also there could be many other reasons for inconsistent teacher belief and teaching practices as many studies found, for example, standardized test, teaching context, school environment etc.

Ⅳ. Results

It seems the more knowledgeable in mathematics, the better to be prepared to be a teacher. Many mathematics teacher educators agree that future teachers need to have opportunities to learn mathematics at the level that they are going to teach rather than just taking advanced level mathematics. This does not necessarily mean learning advanced mathematics topic is not needed for future primary school teachers. Because they need to be able to view elementary mathematics with mathematically rigorous eyes.

To begin with, intercorrelations and descriptive statistics were conducted to have general information about the data in the study and to see whether there is multicollinearity between the predictors. <Table 3> summarizes the intercorrelation analysis results.

<Table 3> Summary of Intercorrelations, Means, and Standard Deviations for Scores on the MCK, MPCK, TG, TD, TC, TP, PM, and SM

Measure	1	2	3	4	5	6	7	8
1. MCK								
2. MPCK	.89**							
3. TG	-.38**	-.51**						
4. TD	-.23**	-.37**	.68**					
5. TC	-.18*	-.23**	.39**	.45**				
6. TP	-.16	-.28**	.50**	.56**	.19*			
7. PM	-.33**	-.22**	.22**	-.09	-.18*	.01		
8. SM	-.41**	-.33**	.32**	.23**	.04	.30**	.61**	
M	498.32	512.36	2.21	3.41	1.82	1.51	2.72	1.99
SD	55.65	44.80	0.57	0.79	0.88	0.28	0.27	0.49

Note. N=151 (institutions). Source IEA TEDS-M 2008.
$*p < .05$. $**p < .01$.

The 151 teacher education institutions that have programs for prospective primary teachers to be prepared as generalist in the six countries showed just about the mean MCKI achievement of the whole participant institutions for prospective primary teachers (M = 498.32). On the contrary, the 151 teacher education institutions showed slightly higher MPCKI performance than the whole participant institutions for prospective primary teachers (M = 512.36).

Regarding to the opportunities to learn mathematics tertiary/school level mathematics at the teacher education institution, the means were much below from 10 and this means that the prospective primary teachers of the participant teacher education institutions relatively strongly disagree that they had opportunities to learn tertiary/school level mathematics.

Opportunities to learn tertiary/school level mathematics are negatively correlated or not statistically significantly associated with MCKI and MPCKI in general. The results of the intercorrelations indicate that there might be multicollinearity between the predictors, TG and TD because correlation between TG and TD was .68 ($p < .01$) which is close to .07. Thus collinearity statistics and Variance Inflation Factor (VIF) values were investigated to see whether there are considerable level of multicolliniarity.

1. Model Selection

Forward, backward, and stepwise model selection methods were used and all of the methods ended up with the same final model. I report and compare three models that were in forward selection procedure in <Table 4>.

<Table 4> Predictors of Mathematics Pedagogical Content Knowledge (MPCKI)

	MPCKI			
			Model 3	
Variable	Model 1 *b*	Model 2 *b*	*b*	VIF
Constant	153.80	220.24	159.33	
MCKI	0.72	0.66	0.68	1.26
TG		-16.03	-17.03	1.18
PM			18.48	1.14
R^2	.80	.84	.85	
F	592.26**	374.16**	269.10**	
ΔR^2		.04	.01	
ΔF		32.17**	10.57**	

Note. N=151 (institutions) *p<.01.

TD, TC, TP, and SM were excluded in the model selection process because they were not significantly predict MPCKI in the model.

2. The Final Model

According to the model selection procedure above, model 3 was selected as the final model. The final model equation is as follows.

$$MPCKI = 159.33 + 0.68MCKI - 17.03TG + 18.48PM + \varepsilon$$

<Table 5> Summary of the Final Model

	MPCKI		
Variable	*b*	SE *b*	*b**
Constant	159.33**	25.51	
MCKI	0.68**	0.03	.85**
TG	-17.03**	2.76	-.22**
PM	18.48*	5.68	.11**
R^2		.85	
F		269.10**	

Note. *p < .01. **p < .001.

The results of the multiple regression indicate the three predictors; mathematics content knowledge (MCKI), opportunities to learn tertiary level math-geometry (TG), and opportunities to learn primary school level mathematics (PM) explain 85.7% of the variance of MPCKI (R^2=.85, F(3,147)=269.10, $p < .001$). It was found that MCKI significantly positively predicts MPCKI (b =.68, $p < 0.001$). When the level of MCKI increases by one unit, MPCKI increases by 0.68 and when the level of TG and PM are at the same level and it was significant at α = .01.

Among the opportunities to learn tertiary level mathematics; TG, TD, TC, and TP, opportunities to learn tertiary level geometry (TG) was found to negatively predict MPCKI and it was significant. When TG increases by one unit MPCKI decreases by 17.03 (b = -.17.03, $p < 0.001$) when the level of MCKI and PM are held constant and it was significant at α = .01. Other opportunities to learn tertiary level mathematics; TD, TC, and TP, were found not to significantly predict the level of MPCKI. This means that taking tertiary level mathematics did not significantly predict their level of mathematics pedagogical content knowledge. Furthermore, when prospective primary teachers who have been prepared as generalist take more tertiary level geometry, it will negatively predict prospective teachers level of pedagogical content knowledge.

Among the opportunities to learn school level mathematics; PM and SM, PM was found to positively predict the level of MPCKI (b =14.48, $p < 0.001$) when MCKI and TG were held constant and it was statistically significant at α=.05. SM was found not to statistically significantly predict the level of mathematics pedagogical content knowledge. Considering that PM contains topics that are usually taught at elementary school such as numbers and geometry, and SM contains topics that are taught at high school such as calculus, the result indicates when the prospective primary teachers less disagree that they have taken more school level mathematics that they are going to teach, it will positively affect the prospective teachers' level of mathematics pedagogical content knowledge.

V. Conclusions and Discussions

One cannot teach what the person does not know. As expected, mathematics content knowledge is an important source for mathematics pedagogical content knowledge and the results of this study confirms it. Knowing what teachers will teach is a pre-requisite even though it does not automatically connected to teaching practice. The study investigated what kinds of opportunities to learn mathematics might be related to primary teacher candidates' mathematics pedagogical content knowledge. The results of this study supports that to have prospective primary teachers to be prepared to teach mathematics well, improving teachers' own mathematics knowledge that is related to elementary mathematics could be the starting point.

The results of the study share lines with previous research and confirm studies about knowing mathematics for teaching. Even though prospective primary teachers have already learnt elementary school mathematics as students, elementary teacher candidates do need experiences with primary mathematics for teaching rather than master advanced topics in mathematics. This does not necessarily imply learning university level of mathematics is not helpful for improving knowledge of mathematics that is needed for teaching primary school mathematics because teachers do need to see school level mathematics with mathematical rigor. Learning advanced mathematics might be helpful in other ways, for example, if it is connected to teaching school level mathematics.

Considering the data analyzed in this study consists of 6 countries which include low performing and high performing countries, the situation in each country might be different from the findings of this study. It needs further study for each country considering their own environments and condition of education. Also the results of this study is suggestive. If different analysis method was applied, the result might not be the same that was analyzed in this study.

More studies are needed in the area of opportunities to learn for prospective elementary school teachers. For example, in what ways school level mathematics need to be taught to future teachers? What and how do prospective teachers learn during the courses for learning school level mathematics? Just revisiting formulas or procedures or focused on conceptual understandings of elementary school mathematics might not be helpful. Studies about these kinds of questions might provide information for teacher educators to design teacher education programs.

References

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, *90*, 449-466.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of Teacher Education*, *59*(5), 398–407.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4–14.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 133–180. doi: 10.3102/0002831209345157

Begle, E. G. (1972). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Washington, DC: Mathematical Association of America and National Council of Teachers of Mathematics.

Blömeke, S. (2012). Does greater teacher knowledge lead to student orientation? The relationship between teacher knowledge and teacher beliefs. In J. König (Ed.), Teachers' Pedagogical Beliefs (pp. 15–35). Retrieved from https://play.google.com/books/reader?id=MYyRBYGIYTQC&printsec=frontcover&output=reader&authuser=0&hl=en&pg=GBS.PA3

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal of Research in Mathematics Education*, *35*, 330-351.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*(2), 371–406.

IBM Corp. (Released 2012). *IBM SPSS Statistics for Windows, Version 21.0*. Armonk, NY:IBM Corp.

Meinck, S., & Rodriguez, M. (2011, May). Particular considerations for correlation analysis using data from the Teacher Education and Development Study in Mathematics (TEDS-M). Paper presented at the annual meeting of the 55th Annual Conference of the Comparative and International Education Society, Fairmont Le Reine Elizabeth, Montreal, Quebec, Canada. Retrieved from http://www.allacademic.com/meta/p492570_index.html

Morine-Dershimer, G., & Kent, T. (1999). The complex nature and sources of teachers' pedagogical content knowledge. In J. Gess-Newsome & N. G. Lederman (Eds.), *Examining pedagogical content knowledge* (pp. 21–50). Dordrecht, The Netherlands: Kluwer.

O' brien, R. M. (2007). A caution regarding rules of thumb for variance inflation factors. *Quality & Quantity*, *41*(5), 673-690.

Oh, Y. (2012). A research on teachers' professional development of mathematics, *Journal of Elementary Mathematics Education in Korea*, *16*(3), 389-401.

Rollnick, M., Bennett, J., Rhemtula, M., Dharse, N., & Ndlovu, T. (2008). The place of subject matter knowledge in pedagogical content knowledge: A case study of South Africa teachers teaching the amount of substance and chemical equilibrium. *International Journal of Science Education*, *30*(10), 1365-1387.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*, 255–281.

Seo, D. Y.(2010) A comparative study between the lectures on the practices of mathematics education in the courses for pre-service elementary teachers of two university in United States and Korea - Focusses on two professors' cases - . *Journal of Elementary Mathematics Education in Korea*, *14*(3), 547-565.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1–22.

Song, K. & Pang, J. (2008). An analysis of pre-service teachers' pedagogical content knowledge about decimal calculation. *Journal of Elementary Mathematics Education in Korea*, *12*(1), 1-25.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework.* East 100 Lansing: Teacher Education and Development International Study Center, College of Education, Michigan State University.

Tatto, M. T., Schwille J., Senk, S., Ingvarson, L., Rowley, G., Peck, R., Bankov, K., Rodriguez, M., & Reckase, M. (2012). *Policy, practice, and readiness to teach primary and secondary mathematics in 17 countries: Findings from the IEA Teacher Education and Development Study in Mathematics (TEDS-M).* Amsterdam, the Netherlands: International Association for the Evaluation of Educational Achievement.

Tatto, M. T., & Senk, S. (2011). The mathematics education of future primary and secondary teachers: methods and findings from the Teacher Education and

Development Study in Mathematics. *Journal of Teacher Education*, *62*(2), 121–137.

Tchoshanov, M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics, 76*, 141–164. doi:10.1007/s10649-010-9269-y

Tchoshanov, M. A., Lesser, L., & Salazar, J. (2008). Teacher knowledge and student achievement: Revealing patterns. *Journal of Mathematics Education Leadership, 10*(2), 38–48.

Warfield, J. (2001). Where mathematics content knowledge matters. In T. Wood, B. S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 135–155). Mahwah, NJ: Erlbaum.

<국문 초록>

예비 초등 교사의 수학 교수를 위한 내용 지식과 관련 있는 수학 학습은 무엇인가?

강은경[2)]

‘수학 수업을 더 들을수록 더 나은 수학 교사가 될 것이다.’ 라는 주장은 정당하게 들린다. 하지만 대학 수준의 수학, 예를 들어, 추상 대수나 해석학 같은 수학을 듣는 것이 어느 정도 초등 수학을 잘 가르치는데 영향을 미칠까 하는 데에는 의문이 생긴다는 주장이 일고 있다. 수학자가 초등 수학을 가르치도록 교육 받은 사람보다 나은 초등교사일 수 있는가? 이 논문은 대학 수준의 수학을 배우는 것과 학교 수준의 수학을 배우는 것이 예비 초등 교사들의 수학 교수를 위한 내용지식에 미치는 영향에 대하여 연구하였다. 이 연구에는 Teacher Education and Development Study in Mathematics에서 제공하는 데이터베이스를 다중회귀 분석방법을 사용하여 분석하였다. 초등 전 과목을 다 가르치도록 교육받은 예비 초등 교사들이 연구의 대상이며 교사교육을 이미 다 받은 시점에서 데이터가 수집되었다. 데이터 분석 결과는 예비 초등 교사들이 그들이 앞으로 가르치게 될 초등 수학을 다시 한 번 접해 볼 기회를 갖는 것이 수학 교수를 위한 내용 지식에 도움이 될 것이라는 것을 보여준다.

주제어: 예비 초등학교 교사, 교사 교육, 학습 기회

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2) eunkyung.kang@gmail.com