

Kernel Fisher Discriminant Analysis for Indoor Localization

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Abstract

In this paper we introduce Kernel Fisher Discriminant Analysis (KFDA) to transform our database of received signal strength (RSS) measurements into a smaller dimension space to maximize the difference between reference points (RP) as possible. By KFDA, we can efficiently utilize RSS data than other method so that we can achieve a better performance.

Keywords: localization, fingerprinting, Kernel Fisher Discriminant Analysis, particle filter, radio map.

1. INTRODUCTION

Indoor localization has attracted much attention in recent years. There have been several proposals such as Received Signal Strength (RSS)-based localization, time-of-arrival (TOA), direction-of-arrival (DOA), etc. The RSS-based localization (or also called fingerprinting localization) was introduced by RADAR[1]. Time-of-arrival (TOA) and direction-of-arrival (DOA) methods are proposed by [2] and [3], respectively. In general, TOA and DOA have a better performance than the RSS-based localization. Specifically, those mean error is reportedly less than 1.5m. However, those are too complex requiring some extra devices for deployment. The RSS-based localization algorithms have been extensively studied as an inexpensive solution for indoor localization recently [4]. Despite its merits in technical and economical aspects, the most serious challenge in the RSS-based localization is fluctuation of received signal strength (e.g., AP's signal in WLAN based localization).

In [1], [4] the received signal strength is used directly without any processing and the noise is not removed effectively, thus causing larger localization error. In [5], they used principal component analysis (PCA) to transform RSS to a new set of variables, namely principal components (PCs). By transforming into the new space, noise is reduced, consequently leading to improvement in performance. But in actual PCA is not a very good method for classification problem, its objective is to reduce the dimensionality (number of variables) of the dataset but retain most of the original variability in the data. Due to the multi-path propagation of indoor environment, the received signal strength is always fluctuating. It makes fingerprinting of adjacent locations very similar. By applying kernel Fisher discriminant analysis (KFDA) [6], we can transform our fingerprinting data into a new space, in which the fingerprinting of each location can be effectively separated. In this paper we propose KFDA method for indoor localization that transforms fingerprinting data into a new space which facilitates indoor localization. .

2. INDOOR LOCALIZATION VIA KERNEL FISHER DISCRIMINANT ANALYSIS

The KFDA is an extension of linear discriminant analysis (LDA) and is known to be effective in classification [6]. In general, localization based on fingerprinting can be viewed as a classification problem in a sense that we need to estimate current position to be the one whose data set is most similar to current measurements.

2.1 Localization Algorithm

Our proposed localization algorithm consists of two phases: preliminary localization and KFDA-based localization. Such a two-step localization is similar in spirit to [7]. In the preliminary localization, a set of possible reference points (RPs) near target position is selected using a light-weight coarse localization algorithm. Then, the second-step localization based on KFDA takes the vectors of RSS measurements associated with the selected RPs as inputs. The KFDA gives a projection matrix that transforms the vectors of RSS measurements for the selected RPs into the new space [6]. In the new space the RSS measurements vectors of each RP are easily separated, thereby facilitating locating mobile node (MN)'s position.

We consider the fifth floor of our department building as a testbed for evaluating performance of our localization scheme. The layout of our testbed is divided into many RPs and each of those is associated with a unique position as shown in Figure 1. At each RP, several RSS measurements from access points (APs) are taken offline and are stored in a database named radio map.

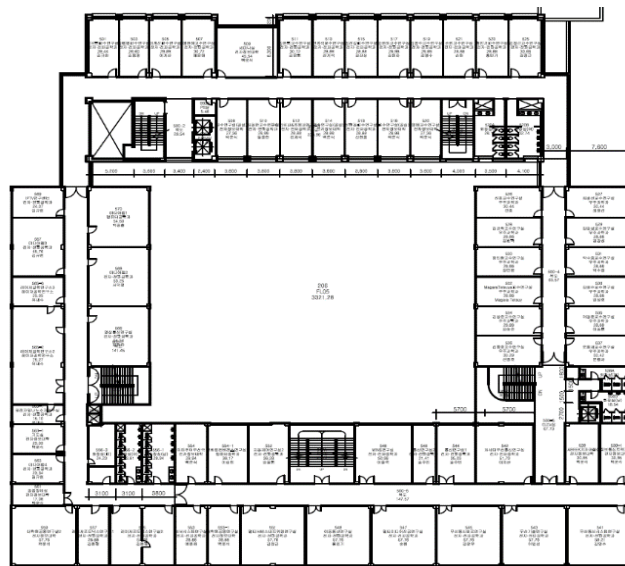


Figure 1. Floor plan of our testbed with illustration of some positions with coordinates.

A. Preliminary Localization Phase

Our positioning system is intended to operate in a moderately large indoor environment, e.g., campus building and shopping mall, so the size of radio map is expected to be big. If our KFDA-based localization is directly applied to such environments, it may require large computing resources and take a problematic processing time due to large database. In order to address such issues, we organized the measurement vectors in the radio map in clusters via K-means clustering [7] in the offline phase.

Given a set of measurements vectors $\psi_1, \psi_2, \dots, \psi_n$ where n denotes the total number of measurement in the radio map, the K-means clustering algorithm randomly choose the k centroids first $n \gg k$, one for each cluster. Then, each measurement vector associates itself with the nearest centroid, thereby forming k clusters. Once the initial clusters are established, the K-means clustering algorithm uses an iterative algorithm that minimizes the objective function below known as within-cluster sum of squares:

$$J = \sum_{j=1}^k \sum_{i=1}^{n_j} \|\psi_i^{(j)} - c_j\|^2 \quad (1)$$

where c_j is the centroid of j th cluster, ψ_i^j is i th measurement vector of j th cluster, and n_j is the number of measurement vectors associated with j th cluster. After evaluation of (1), a new set of k centroids are obtained. This process repeats until the centroids get stabilized.

B. Online Localization Phase

When the MN needs to be located, it first takes RSS measurements m_0 times from APs around and calculates the average value for each AP. The vector of average RSS values is denoted by ψ_0 :

$$\psi_0 = [\psi_{1,0}, \psi_{2,0}, \dots, \psi_{L,0}]^T \quad (2)$$

where $\psi_{i,0}$ is the average RSS value for i th AP. We next calculate the Euclidean distance between online measurement ψ_0 and the corresponding centroid vectors of the clusters. In order to avoid the edge problem - selecting the wrong cluster when MN is located in the edge between two clusters, we may select multiple clusters that satisfy best the minimum distance criterion below [7]:

$$\operatorname{argmin}_c \|\psi_0 - c_j\|^2 \quad (3)$$

where c_j denotes the centroid of j th cluster. Once (3) gives multiple candidate clusters, those are now used as the input of KFDA based localization that will be discussed in detail in the next section.

2.2 Kernel Fisher Discriminant Analysis for Localization

Since APs are deployed in wide area, the set of AP seen by an MN varies depending on its location. The candidate measurement vectors in the clusters selected by (3) may see a different profile of hearable APs. To use an input of KFDA, we modify the measurement vectors by dropping the measurements from the APs that are not hearable in other measurement. That is, we consider only measurements from the APs that are in common to all the measurement vectors. Let K denote the total number of RP's in the clusters selected by (3), and S denote the number of APs hearable by all candidate RPs. Then, the input that goes to the KFDA is:

$$\Psi = \begin{bmatrix} \psi_{1,1}^{(1)}, \psi_{1,2}^{(1)}, \dots, \psi_{1,S}^{(1)} \\ \psi_{2,1}^{(1)}, \psi_{2,2}^{(1)}, \dots, \psi_{2,S}^{(1)} \\ \vdots \\ \psi_{m,1}^{(K)}, \psi_{m,2}^{(K)}, \dots, \psi_{m,S}^{(K)} \end{bmatrix}^T \quad (4)$$

where $\psi_{i,j}^{(k)}$ is the RSS of j th AP on i th measurement at k th RP. The dimension of the matrix Ψ is $S \times (Km)$ where S is the number of APs hearable to all RPs selected by (3), K is the number of RP, and m is the number of measurements for each RP.

Let Φ be a Gaussian kernel mapping from Ψ into feature space F [6]. A projection matrix W that projects Ψ^Φ into the lower subspace Y :

$$Y = W^T \Psi^\Phi \quad (5)$$

The matrix W is obtained such that it maximizes $J(W)$ given by

$$J(W) = \frac{W^T S_B^\Phi W^T}{W^T S_W^\Phi W^T} \quad (6)$$

where S_B^Φ and S_W^Φ are the between and within class scatter matrices [6] in the feature space F , respectively, and are defined by

$$S_B^\Phi = \sum_{j=1}^K m(\mu_j^\Phi - \mu^\Phi)^T \quad (7)$$

$$S_W^\Phi = \sum_{j=1}^K \sum_{i=1}^m m(\Phi(\psi^i) - \mu_j^\Phi)(\Phi(\psi^i) - \mu_j^\Phi)^T \quad (8)$$

In the equations above μ^Φ , is global mean of measurement vectors in Ψ ,

$$\mu^\Phi = \frac{1}{K.m} \sum_{i=1}^{K.m} \Phi(\psi^i) \quad (9)$$

and μ_j^Φ is mean of measurement vectors of a RP,

$$\mu_j^\Phi = \frac{1}{m} \sum_{i=1}^m \Phi(\psi^i) \quad (10)$$

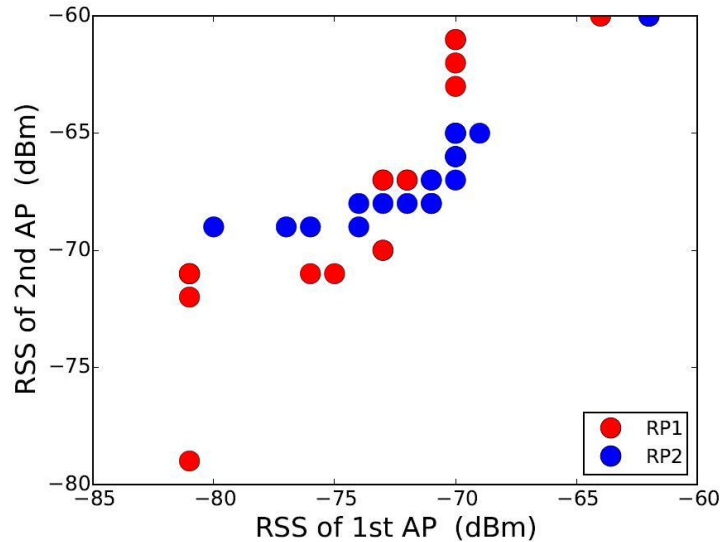


Figure 2. Raw values of RSS of Wifi AP.

In online phase, MN measures RSS values of APs m times and calculates the vector of average values $\tilde{\psi}_0$ given by

$$\tilde{\psi}_0 = [\psi_{1,0}, \psi_{2,0}, \dots, \psi_{S,0}]^T \quad (11)$$

Then, $\tilde{\psi}_0$ is projected into the subspace via KFDA, i.e.,

$$x = W^T \tilde{\psi}_0 \quad (12)$$

Finally, we estimate the location of MN by a i th RP with the minimum distance to x , i.e.,

$$\operatorname{argmin}_Y \|x - Y_i\|, \forall i \in K \quad (13)$$

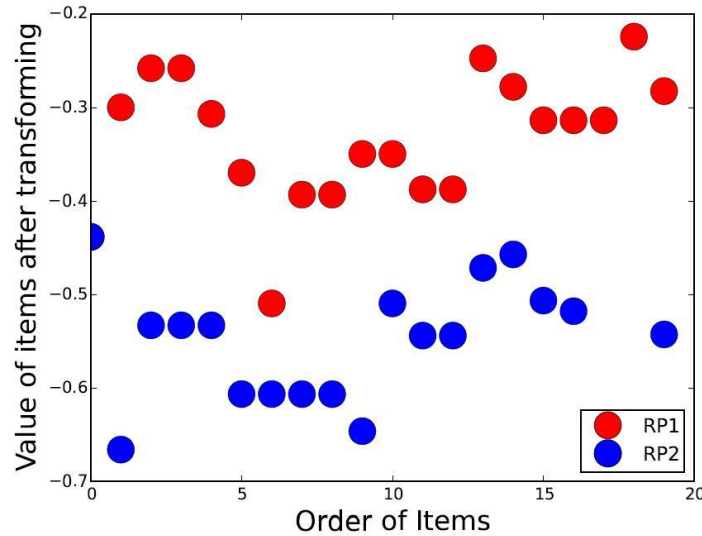


Figure 3. RSS of Wifi AP after transforming by KFDA.

Figure 2 showed raw fingerprint data, because of strong noise property of indoor environment, the fingerprint data is not unique for each location, it will make big error if we directly use the raw fingerprint data. By applying KFDA, we can easily separate fingerprint data of each location as in Figure 3, so we can effectively improve localization accuracy.

3. PARTICLE FILTER BASED FINGERPRINTING LOCALIZATION

Particle filter (PF) has widely applied in many fields. It also can work well for the general tracking problem, it has good performance competitive with other method especially in noise case. In this paper we introduce how to apply PF into fingerprinting based localization. Our problem is given all measurements for AP's RSS from beginning up to time t as in Figure 4, we try to find MN's location, alternatively the posterior PDF of MN's location. In this case, we measured RSS over time, so for easy to discriminate with fingerprint measurement in section 2, we used the different notation.

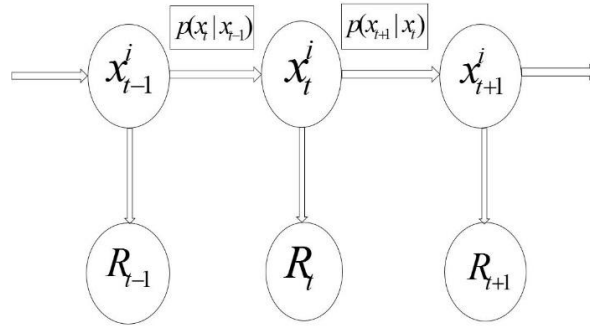


Figure 4. Particle filter diagram.

Let's consider the starting time t_0 is when MN determined its cluster in the previous section. For initial, we set $p(x_0|R_0)$ be the prior PDF of MN's location at state $t = t_0$, MN can be at any RP in this cluster with the same probability. Then at each state, MN measures AP's RSS values R_t for m_t times, then PF used them to estimate the posterior PDF $p(x_t^i|R_{0:t-1})$ of MN's location x_t by the predict and update process used respectively as in [8]:

$$p(x_t|R_{0:t-1}) = \int p(x_{t-1}|R_{0:t-1})p(x_t|x_{t-1})dx_{t-1} \quad (14)$$

$$p(x_t|R_{0:t}) = \frac{p(R_t|x_t)p(x_t|R_{0:t-1})}{\int p(R_t|x_t)p(x_t|R_{0:t-1})dx_t} \quad (15)$$

where we consider $p(x_{t-1}|R_{0:t-1})$ as the posterior PDF of x_{t-1} , $p(x_t|x_{t-1})$ is the transition probability function, $p(x_t^i|R_{0:t-1})$ is the prior PDF of x_t and $p(R_t|x_t)$ is the observation likelihood function. In actual, it is very hard to exactly calculate these PDFs. So we can use PF algorithm to approximate the prior and posterior PDF with particles as:

$$p(x_t|R_{0:t-1}) \approx \sum_{i=1}^N w_{t-}^i \delta(x_t - x_t^i) \quad (16)$$

$$p(x_t|R_{0:t}) \approx \sum_{i=1}^N w_t^i \delta(x_t - x_t^i) \quad (17)$$

where $\delta()$ is the Dirac function, N is the number of particles, $\{x_{t-1}^i, w_{t-}^i | i = 1, \dots, N\}$ denotes a random measure-particles to approximate the prior PDF $p(x_t^i|R_{0:t-1})$, $\{x_t^i, w_t^i | i = 1, \dots, N\}$ denotes a random measure-particles to approximate the posterior PDF $p(x_t^i|R_{0:t-1})$, x_t^i is a sample of x_t location with its normalized predicted weight w_{t-}^i and updated weight w_t^i . From [8], the particle weight w_t^i of sample x_t^i state can be updated by:

$$w_t^i = w_{t-}^i \times p(R_t|x_t^i) \quad (18)$$

Then we need to find observation likelihood function ($R_t|x_t$). At state t , MN do N samples $x_t^i, i = 1, \dots, N$ of x_t with given measurement R_t , $p(R_t|x_t)$ can be calculated as:

$$p(R_t|x_t^i) = 1/d(R_t, R(RP_j \equiv x_t^i)) \quad (19)$$

where $R(RP_j \equiv x_t^i)$ is fingerprinting data at j th RP which is x_t^i sample location, $d(R_t, R(RP_j \equiv x_t^i))$ is Euclidean distance between measurement R_t and fingerprinting data at x_t^i . After that, we normalized these

observation likelihood probability for sum of them is equal to 1. Now, we can calculate the particle weight w_t^i of sample x_t^i , to make sure that sum of them equal to 1, we also need to normalize them as follows:

$$\bar{w}_t^i = \frac{w_t^i}{\sum_{j=1}^N w_t^j} \quad (20)$$

The particle set of MN's location is achieved, so we can estimate MN's location x_t as follows:

$$\hat{x}_t = \sum_{i=1}^N \bar{w}_t^i \times x_t^i \quad (21)$$

In theory of PF, people usually set N be very large to best approximate the posterior PDF of objective as possible as they can. In our localization problem, the distribution of location of MN at state t is a discrete functions, MN can be anywhere in the selected cluster and the number of RPs K is not large, so we can set $N = 2K$ to save the computing time and still remain good performance.

The degeneration phenomenon is a common problem in PF, when we only run a small number of iterations, there is a particle have a negligible weight versus the others. To avoid this problem, we define the effective sample size as in [8]:

$$N_{eff} = \frac{1}{\sum_{i=1}^N w_t^i} \quad (22)$$

If $N_{eff} < N_{Th}$, we do the resampling. We remove particles with small weight, only focus on particles have large weight. We do a new sample step, reset all weights of particles as $w_t^i = 1/N$.

4. PERFORMANCE EVALUATION

In order to evaluate performance of our localization scheme, we consider the testbed introduced in Figure 1, we assign a RP for every one meter following the corridor on the floor and the total 200 RPs cover the testbed. For each RP, twenty RSS measurements are conducted and stored in the database. Our MN is a linux-based laptop, running Ubuntu 12.04. We randomly selected about 50 RPs in radio map for doing localization experiments. As a primary performance metric, we use localization error that refers to the Euclidean distance between MN's actual location and estimated location.

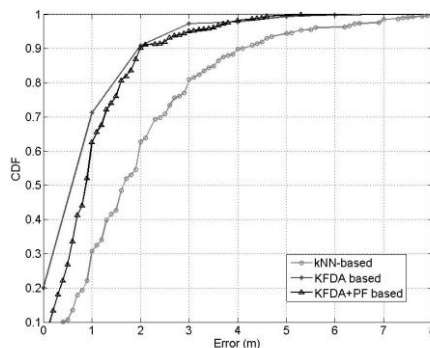


Figure 5. CDF of localization error for our proposal and kNN based method as a function of number of offline measurement.

Our proposal is compared against the well-known fingerprinting approach named kNN method (also known as RADAR) in [1]. The number of neighbor in the kNN method is set to 3. For our proposal methods, we tested for two cases: KFDA-based only and KFDA+PF-based localization. Figure 5 shows cumulative distribution function (CDF) of average localization error for three different methods. In KFDA-based and KFDA+PF-based method, 90% of error is less than 2 meters, whereas around 65% error is less than 2 meter in kNN method. Table 1 shows mean and variance of localization error for two localization schemes under comparison. As shown in the table, a noticeable improvement is observed at KFDA-based and KFDA+PF-based method in terms of mean and variance. By using PF, we can utilize these estimation locations from the previous steps to improve the localization accuracy.

Table 1. Error Statistics for KFDA-based and kNN method.

Method	Mean(m)	Var
kNN ($k = 3$)	2.16	2.84
KFDA-based	1.24	1.32
KFDA+PF-based	1.12	0.89

As pointed out in [9], constructing the radio map to cover wide indoor area is not trivial. They introduced a method to reduce the labor cost for constructing the radio map by reducing time for measurement APs' RSS. This work is really useful, especially when we wanted to deploy the positioning system in a wide area. However, the localization accuracy will be sacrificed. Obviously, more measurements are expected to be beneficial because of its statistical stability. However, it cannot be achieved without the expense of significant label cost.

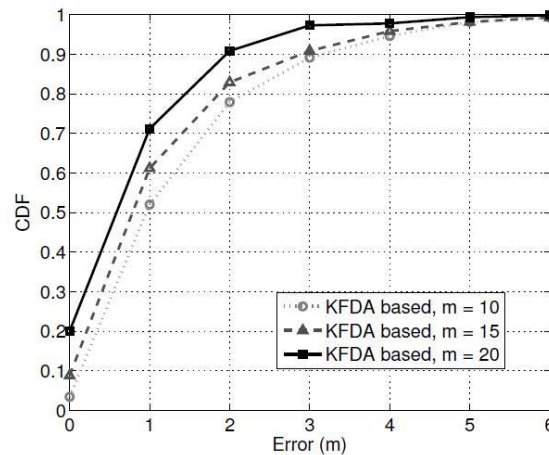


Figure 6. CDF of localization error with KFDA-based method for different number of offline measurement.

Thus, the size of the radio map is one of system parameters of interest. For this purpose, we wanted to test our proposed method in different number of offline measurements, to find the optimal number of offline measurement to maintain both aspects of performance and reducing labor cost, in this case, we only tested for KFDA-based method for saving time. Figure 6 shows mean error of KFDA-based method for different numbers of offline measurement $m = 10, 15, 20$. Consider the mean error, we are probably interested in

error of 1 and 2. For these errors, a significant improvement is over served by increasing the number of offline measurement. Specifically, probability gain of about 0.1 is achieved by five more measurements. However, it only occurred when the number of offline measurement is small and when it is larger than a certain number, the mean error doesn't decrease much. This number is depend on properties of environment testbed, in our testbed it is around 20.

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