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Spectral Analysis of Rectangular, Hanning, Hamming and Kaiser Window for Digital Fir Filter

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Abstract

Digital filters are extensively used in the world of communication. In order to design a digital finite impulse response (FIR) filter that satisfies all the required conditions is challenging. In this paper, design techniques of digital low pass FIR filters using Rectangular window method, Hamming window, Hanning window, and Optimal Parks McClellan method are presented. The stability, number of components required and filter coefficients are demonstrated for different design techniques. It is demonstrated that filter design using hamming window is comparatively better than rectangular and hanning window though the components required for all of the windowing technique are same, hamming shows higher stability. The stability is shown with the help of magnitude and phase spectrum of each window. Simulation is carried out using MATLAB and comparisons are made entirely based on the output of the simulation.

Keywords: *DSP, finite impulse response, infinite impulse response*.

1. Introduction

Filtering is one of the most powerful tools of DSP. Digital filters are represented using difference equations and are capable of performing that specifications which are tremendously difficult, to achieve with an analog implementation. In addition, the characteristics of a digital filter can be easily changed under software control. Therefore, they are widely used in adaptive filtering applications in communications such as echo cancellation in modems, noise cancellation, and speech recognition [6]. Since the characteristics of a digital filter can be easily changed under software control. Almost all digital systems use signal filtering to remove unwanted noise, to provide spectral shaping, or to perform signal detection or analysis. Two types of filters provide these functions are finite impulse response (FIR) filters and infinite impulse response (IIR) filters [8],[9]. Typical filter applications include signal preconditioning, band selection, and low pass filtering. A.

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FIR has more advantage over IIR Filter [9] in terms of finite and infinite IR filter, excursiveness, impulse response, stability, likewise.

FIR filters have a transfer function of a polynomial in z- and is an all-zero filter in the sense that zeroes in the z-plane determine the frequency response magnitude characteristic. The Z transform of an N-point FIR filter is given by $\boldsymbol{H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}}$ (1)

FIR filters are particularly used for applications where exact linear phase response is required. The FIR filter is generally implemented in a non-recursive way which guarantees a stable filter. FIR filter design essentially consists of approximation problem and realization problem.

The approximation stage takes the specification and gives a transfer function through four steps. A desired or ideal response is chosen, usually in the frequency domain. An allowed class of filters is chosen (e.g. the length for a FIR filters). These are a measure of quality of approximation, and a method or algorithm to find the best transfer function. The realization part deals with choosing the structure to implement the transfer function which may be in the form of circuit diagram or in the form of a program [2]. There are essentially three well-known methods for FIR filter design namely as: The window method, The frequency sampling technique, and Optimal filter design method [1].

Those were the days when, most of the digital filter's applications have been limited to audio and high-end image processing. With advances in process technologies and digital signal processing methodologies, digital filters are now cost-effective in the IF range and in almost all video markets. Digital filters are commonly used for audio frequencies for two reasons. First, digital filters for audio are superior in price and performance to the analog alternative. Second, audio Analog-to-Digital Converters (A/Ds) and Digital-to-Analog Converters (DACs) can be manufactured with high accuracy and are available at low cost. Thus, the combined cost of filtering and conversion (if necessary) is low. The cost trades are much more difficult in the 1 MHz to 100MHz signal range, such as the IF ranges of many radio receivers [6].

2. Methodology

The window method the desired frequency response specification $\boldsymbol{H_d(w)}$, corresponding unit sample response $\boldsymbol{h_d(n)}$ is determined using these relations

$$\boldsymbol{h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn} dw}; \tag{2}$$

$$\boldsymbol{H_d(w) = \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-jwn}} \tag{3}$$

In general, unit sample response $\boldsymbol{h_d(n)}$ obtained from the equation 2 is infinite in duration, so it must be truncated at some point say $\boldsymbol{n = M-1}$ to yield an FIR filter of length (i.e. 0 to $\boldsymbol{M-1}$).This truncation of $\boldsymbol{h_d(n)}$ to length $\boldsymbol{M-1}$ is same as multiplying $\boldsymbol{h_d(n)}$ by the rectangular window defined as

$$\boldsymbol{w(n) = \begin{cases} a - b\cos\left(\frac{2\pi(n+1)}{N+1}\right) + c\cos\left(\frac{4\pi(n+1)}{N+1}\right) & n = 0, 1, .., N-1 \\ 0, & elsewhere \end{cases}} \tag{4}$$

$$\boldsymbol{h(n) = h_d(n) w(n)} \text{ and } \boldsymbol{h(n) = \begin{cases} h_d(n) & , 0 \le n \le M-1 \\ 0 & , elsewhere \end{cases}} \tag{5}$$

Now, the multiplication of the window function $\boldsymbol{w(n)}$ with $\boldsymbol{h_d(n)}$ is equivalent to convolution of $\boldsymbol{H_d(w)}$ with $\boldsymbol{W(w)}$, where $\boldsymbol{W(w)}$ is the frequency domain representation of the window function

$$\boldsymbol{W(w) = \sum_{n=0}^{M-1} w(n)e^{-jwn}} \tag{6}$$

Thus the convolution of $\boldsymbol{H_d(w)}$with $\boldsymbol{W(w)}$ yields the frequency response of the truncated FIR filter [6]

$$\boldsymbol{H(w) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(T)W(w-T)dw} \tag{7}$$

The frequency response can also be obtained using

$$\boldsymbol{H(w) = \sum_{n=0}^{M-1} h(n)e^{-jwn}} \tag{8}$$

Direct truncation of $\boldsymbol{h_d(n)}$ to M terms to obtain $\boldsymbol{h(n)}$leads to the Gibbs phenomenon effect which manifests itself as a fixed percentage overshoot and ripple before and after an approximated discontinuity in the frequency response due to the non-uniform convergence of the Fourier series at a discontinuity. Thus the frequency response obtained using equation (8) contains ripples in the frequency domain. In order to reduce the ripples, instead of multiplying $\boldsymbol{h_d(n)}$with a rectangular window $\boldsymbol{w(n)}$,$\boldsymbol{h_d(n)}$is multiplied with a window function that contains a taper and decays toward zero gradually, instead of abruptly as it occurs in a rectangular window. As multiplication of sequences $\boldsymbol{h_d(n)}$and $\boldsymbol{w(n)}$ in time domain is equivalent to convolution of $\boldsymbol{H_d(w)}$and $\boldsymbol{W(w)}$ (in the frequency domain, it has the effect of smoothing $\boldsymbol{H_d(w)}$. The several effects of windowing the Fourier coefficients of the filter on the result of the frequency response of the filter are: a) major effect is that discontinuities in $\boldsymbol{H(w)}$becometransition bands between values on either side of the discontinuity, b) The width of the transition bands depends on the width of the main lobe of the frequency response of the window function, $\boldsymbol{w(n) i.e. W(w)}$, c) Since the filter frequency response is obtained via convolution relation, it is clear that the resulting filters are never optimal in any sense, d) As $\boldsymbol{M}$ (the length of the window function) increases, themain lobe width of $\boldsymbol{W(w)}$ is reduced which reduces the widthof the transition band, but this also introduces more ripple in the frequency response, and e) The window function eliminates the ringing effects at band edge and does result in lower side lobes at the expense of an increase in the width of the transition band of the filter [5],[6].

The FIR filter design process via window functions can be split into several steps. These are: a) Defining filter specifications; b) Specifying a window function according to the filter specifications; c) Computing the order of the filter required for a given set of specifications; d) Computing the window function coefficients; e) Computing the ideal filter coefficients according to the order of the filter; f) Computing FIR filter coefficients according to the obtained window function and ideal filter coefficients; g) If the resulting filter has too wide or too narrow transition region, it is necessary to change the filter order by increasing or decreasing it according to needs, and after that steps 4, 5 and 6 are iterated as many times as needed [2].

Defining filter specifications is to find the desired normalized frequencies ($\boldsymbol{w_c, w_{c1}\ and\ w_{c2}}$), transition width and stopband attenuation. The window function and filter order are both specified according to these parameters some of the windows commonly used are as: Rectangular window:

$$w_R(n) = \begin{cases} \boldsymbol{1} & \boldsymbol{for n = 0, 1, 2, \ldots, M-1} \\ \boldsymbol{0} & \boldsymbol{elsewhere} \end{cases} \tag{9}$$

Bartlett window: $$W_T(n) = \begin{cases} 1 - \frac{2\left|n - \frac{M-1}{2}\right|}{M-1} for n = 0, 1, \ldots, M-1 \end{cases} \quad (10)$$

Blackmann window: $$W_B(n) = \begin{cases} 0.42 - 0.5 cos \frac{2\pi n}{M-1} + 0.8\, cos \frac{4\pi n}{M-1} for n = 0, 1, \ldots, M-1 \\ 0 \quad elsewhere \end{cases} \quad (11)$$

Hamming window: $$W_H(n) = \begin{cases} 0.54 - 0.46\, cos \frac{2\pi n}{M-1} for n = 0, 1, \ldots, M-1 \\ 0 \quad elsewhere \end{cases} \quad (12)$$

Hanning window: $$W_{HN}(n) = \begin{cases} \frac{1}{2}\left(1 - cos \frac{2\pi n}{M-1}\right) for n = 0, 1, \ldots. M-1 \\ 0 \quad elsewhere \end{cases} \quad (13)$$

Kaiser window with parameter β:

$$w_{k^{(n)}} = \begin{cases} \frac{I_0\left\{\beta\left[1 - \left(\frac{n-\alpha}{\alpha}\right)^2\right]^{\frac{1}{2}}\right\}}{I_0(\beta)} for\ 0 \le n \le M \\ 0 \quad elsewhere \end{cases} \quad (14)$$

Here, $\alpha = \frac{M}{2}$, and I_0 is the zeroth order modified Bessel function of the first kind. In the above expression, it may be observed that Kaiser window $w_k(n)$ has two parameters: length$(M + 1)$ and shape parameter β can be selected independently in Kaiser Window [3]. The modified Bessel function of the first kind is given as,

$$I_0(x) = 1 + \frac{0.25x^2}{(1!)^2} + \frac{(0.25x^2)^2}{(2!)^2} + \frac{(0.25x^2)^3}{(3!)^3} + \cdots \quad (15)$$

The most popular and widely used window functions are; Rectangular window, Hanning window, hamming window and Kaiser window. The Rectangular window response provides sidelobes which gives rise to ripples in passband and stopband. The amplitude of the ripples is determined by the amplitude of the sidelobes. For the rectangular window, the amplitude of the sidelobes is unaffected by the length of the window. But the main lode width of rectangular window is narrower and higher. For the fixed length the Hanning window has significantly lower side-lobe amplitude but the main lobe width is wider compared to Rectangular window. The Hamming window also has the same main lobe width of Hanning window but it generates lesser oscillations in the side lobes than Hanning window. Hence Hamming window is generally preferred rather than Hanning window. The Kaiser window is a kind of adjustable window function which provides independent control of the main lobe width and ripple ratio but the Kaiser window has the disadvantage of higher computational complexity due to the use of Bessel functions [4] [6].

The Bartlett window reduces the overshoot in the designed filter but spreads the transition region considerably. The Harming, Hamming and Blackman windows use progressively more complicated cosine functions to provide a smooth truncation of the ideal impulse response and a frequency response that looks better. The best window results probably come from using the Kaiser window, which has a shape parameter β that allows adjustment of the compromise between the overshoot reduction and transition region width spreading.

Special attention should be paid to the fact that minimum attenuation of window function and that of the filter designed using that function are different in most cases. The difference, i.e. additional attenuation occurs under the process of designing a filter using window functions. This affects the stopband attenuation to become additionally higher, which is very desirable. However, a drawback of this method is that the minimum stopband attenuation is fixed for each function [2][6].

3. Simulayion and Result

In simulation specification has been used: Passband ripple $(\delta_p) = 0.05$; Stopband ripple $(\delta_s) = 0.04$; Passband edge frequency $(w_p) = 1500$ rad/sec; Stopband edge frequency $(w_s) = 2000$ rad/sec; Sampling frequency $(f_s) = 9000$Hz; Order of the filter $(N) = 256$; The spectrum of the window used in digital FIR filter design were as shown in figure 2,3,4,5,6,7.

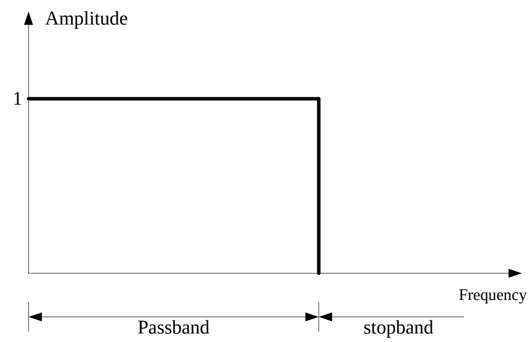

Figure 1. Ideal magnitude spectrum of digital LPF

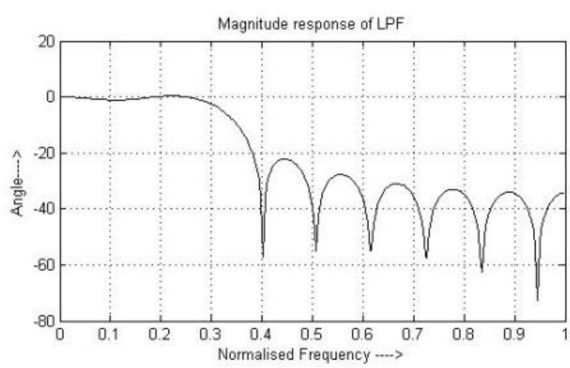

Figure 2. Magnitude spectrum of rectangular window

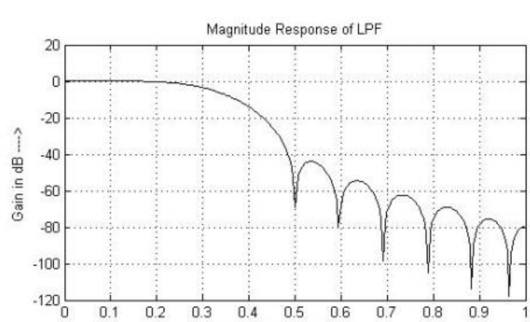

Figure 3. : Magnitude spectrum of hanning window

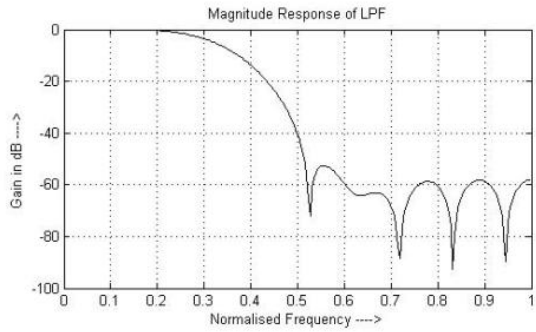

Figure 4. Magnitude spectrum of hamming window

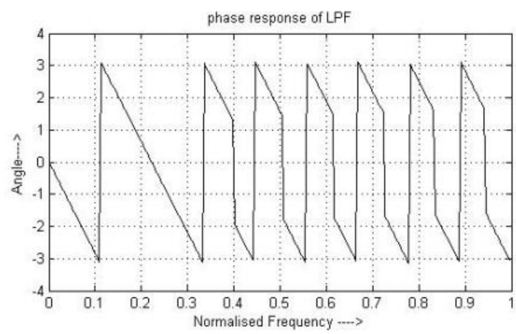

Figure 5. Phase spectrum of rectangular window

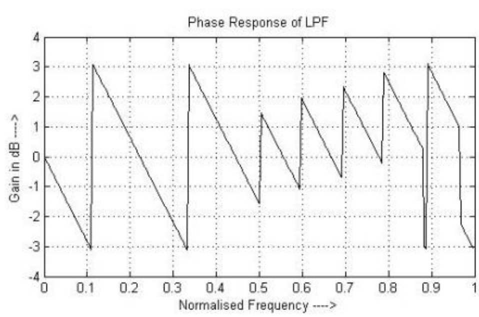

Figure 6. Phase spectrum of hanning window

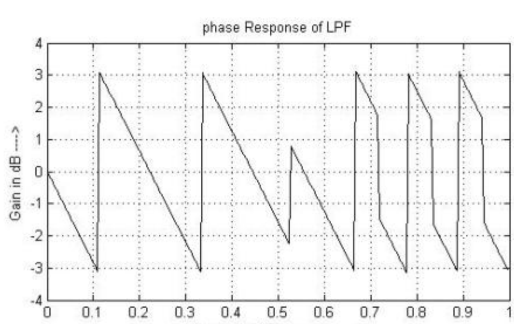

Figure 7. Phase spectrum of hamming window

4. Conclusion

FIR filter design is abundantly pronounced where linear phase characteristic is required. Comparison is made based on the spectrum of the window. Though the magnitude of the ripple in the passband and stop band were defined in the inception of the simulation, ripples in the passband are less in hamming windows as compared to other two techniques (as shown in figure 2, figure 3 and figure 4). FIR filter design by using hamming window is stable as compared to rectangular and hanning windows techniques. This is because of the linear phase characteristics of the filter incorporating hamming window.

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