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Three level constant stress accelerated life tests for Weibull distribution

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Abstract

In this paper, the maximum likelihood estimators and confidence intervals for parameters of Weibull distribution are derived under three level constant stress accelerated life tests and the assumption that a log quadratic relationship exits between stress and the scale parameter θ. The compound linear plan proposed by Kim (2006) is used to allocate the test units at each stress level, which performed nearly as good as the optimum quadratic plan and had the advantage of simplicity. Some simulation studies are given.

Keywords: Accelerated life tests, compound linear plan, confidence interval estimator, constant stress, maximum likelihood estimator, Weibull distribution

1. Introduction

Several life tests require long times because of extremely reliable units. The accelerated life tests (ALTs) such as the constant stress ALTs and the step stress ALTs are widely used to solve this problem. Some test units are put on preassigned stress levels and examined until they fail under the constant stress ALTs. Under the step stress ALTs, unfailed test units at specified time are put on the stronger stress and observed until they fail.

Meeker and Nelson (1975) proposed the optimum test conditions and sample allocation. Meeker (1984) studied the optimal design for Type I censored constant stress ALTs. Bai and Chung (1992) compared the performances of the two step stress ALTs and the constant stress partially ALTs. Khamis and Higgins (1996) evaluated compound linear plan under three-step stress test when the lifetime of test unit is exponential. Khamis (1997a, 1997b) studied the optimal plans for two level step stress and constant stress ALTs under Weibull distribution and also the M step stress ALTs with K stress variables for exponential distribution. Kim (2006) proposed the compound linear plan as an alternative to the optimal quadratic plan and compared it with the optimal plan and two other compromise plan given by Meeker (1984) and Khamis and Higgins (1996). Moon and Kim (2006) considered the confidence interval estimators (CIs) for parameters under three step stress ALTs and a two-parameter exponential distribution. Moon (2008) studied the maximum likelihood estimators (MLEs) of parameters and optimal plan for Type I censored three step stress ALTs with exponential

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distribution. Moon and Park (2009) studied MLEs of parameters and the optimal plan using data obtained from periodic inspection under type I censored three step stress ALTs. Moon (2012) obtained MLEs and their confidence regions of parameters on the Type II censored three step stress ALTs for two-parameter exponential distribution.

In this paper, we derive the MLEs and CIs of model parameters for three level constant stress ALTs under the assumption that the lifetime of test units follows an Weibull distribution and a quadratic relationship exits between stress and log(scale parameter). The model and some assumptions are given in section 2. The MLEs and CIs of parameters are obtained under three level constant stress ALTs using the compound linear plan in section 3. The optimal plan finding optimal sample proportion at each stress level is discussed in section 4. Some simulations are presented for the proposed procedures in section 5. In section 6, some conclusions are given.

2. Model and assumptions

Suppose that there are three level stresses , that is $s_0 < s_1 < s_2 < s_3$, where s_0 is the use stress. We use the notation as follows in the presentation of our results and without loss of generality.

$$x_i = \frac{s_i - s_0}{s_3 - s_0}, \qquad i = 1, 2, 3.$$

For the three level constant stress ALTs, n_i test units randomly chosen from n $(= n_1 + n_2 + n_3)$ test units are put on each stress level x_i, $i = 1, 2, 3$ and they are run until all of them fail.

Let the lifetime of test unit for any stress x_i, T_{ij}, $j = 1, 2, \cdots, n_i$, $i = 1, 2, 3$ follow an Weibull distribution with the scale parameter θ_i and the shape parameter β.

The relationship of θ_i and stress level x_i is assumed to be the log-quadratic function given by

$$\log\theta_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2, \qquad i = 1, 2, 3, \tag{2.1}$$

where α_0, α_1 and α_2 are unknown model parameters.

The probability density function(p.d.f) for Weibull distribution at each stress level x_i under the constant stress ALTs is

$$f(t, \theta_i, \beta) = \frac{\beta}{\theta_i}\left(\frac{t}{\theta_i}\right)^{\beta-1} \exp\left(-\left(\frac{t}{\theta_i}\right)^{\beta}\right), \qquad i = 1, 2, 3, \tag{2.2}$$

where $t \geq 0$, $\beta > 0$ and $\theta_i > 0$.

Let $\pi_i = n_i/n$, $i = 1, 2, 3$ be the proportion of test units and $\pi_i^* = n_i^*/n$ be the optimal proportion allocated at stress x_i.

The optimal proportion $\pi_i^* = n_i^*/n$ must be obtained minimizing the asymptotic variance of $\log\theta_0$ at the use stress x_0. But it is very complicated and troublesome to find the optimal proportions, and some authors proposed the compromise plans. Kim (2006) proposed compound linear plan under the three stress constant ALTs with a log-quadratic stress model and showed relatively high efficiency in comparison to the optimal plan and other compromise plans given by Meeker (1984), Khamis and Higgins (1996). In our case, we use the compound linear plan by Kim (2006) to allocate test units at each stress level. The compound linear plan uses the optimal simple (two level) plan twice instead of finding the optimal proportions for the three level constant stress ALTs with a log-quadratic function.

3. Maximum likelihood and confidence interval estimators

In this section, the MLEs of the model parameters α_0, α_1, α_2 and β by Newton-Rapshon method and the CIs for model parameters using their MLEs are derived.

The likelihood function and the log likelihood function based on observations t_{ij}, $j = 1, 2, \cdots, n_i$, $i = 1, 2, 3$ of test units at stress $x_i, i = 1, 2, 3$ are given by

$$L(\alpha_0, \alpha_1, \alpha_2, \beta) = \prod_{i=1}^{3} \prod_{j=1}^{n_i} \frac{\beta}{\theta_i} \left(\frac{t_{ij}}{\theta_i}\right)^{\beta-1} \exp\left(-\left(\frac{t_{ij}}{\theta_i}\right)^{\beta}\right)$$

and

$$\begin{aligned} \log L(\alpha_0, \alpha_1, \alpha_2, \beta) =& n \log \beta - n\beta \sum_{i=1}^{3} \pi_i \left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2\right) \\ &+ (\beta - 1) \sum_{i=1}^{3} \sum_{j=1}^{n_i} \log t_{ij} \\ &- \sum_{i=1}^{3} \exp\left(-\beta\left(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2\right)\right) \sum_{j=1}^{n_i} t_{ij}^{\beta}. \end{aligned} \quad (3.1)$$

The MLEs for model parameters α_0, α_1, α_2 and β can be obtained by solving the following equation in (3.2) using the Newton Raphson method .

$$\begin{aligned} \frac{\partial \log L}{\partial \alpha_k} =& - n\beta \sum_{i=1}^{3} \pi_i x_i^k + \sum_{i=1}^{3} \frac{x_i^k \sum_{j=1}^{n_i} t_{ij}^{\beta}}{{\theta_i}^{\beta}} = 0, \qquad k = 0, 1, 2 \\ \frac{\partial \log L}{\partial \beta} =& \frac{n}{\beta} - n \sum_{i=1}^{3} \pi_i \log \theta_i + \sum_{i=1}^{3} \sum_{j=1}^{n_i} \log t_{ij} + \sum_{i=1}^{3} \frac{\sum_{j=1}^{n_i} t_{ij}^{\beta} (\log \theta_i - \log t_{ij})}{{\theta_i}^{\beta}} = 0. \end{aligned} \quad (3.2)$$

The second partial derivatives of $\log L(\alpha_0, \alpha_1, \alpha_2, \beta)$ in (3.1) with respect to α_0, α_1, α_2 and β are given in (3.3) and the asymptotic Fisher information matrix is $F = (f_{kl}), k, l = 0, 1, 2, 3,$

where

$$\begin{aligned}
f_{kl} = -\frac{\partial^2 \log L}{\partial \alpha_k \partial \alpha_l} =& \beta^2 \sum_{i=1}^{3} \frac{x_i^{k+l} \sum_{j=1}^{n_i} t_{ij}^{\beta}}{{\theta_i}^{\beta}}, \qquad k,l = 0,1,2 \\
f_{k3} = -\frac{\partial^2 \log L}{\partial \alpha_k \partial \beta} =& n \sum_{i=1}^{3} \pi_i x_i^k - \sum_{i=1}^{3} \frac{x_i^k \sum_{j=1}^{n_i} t_{ij}^{\beta}}{{\theta_i}^{\beta}} \\
&+ \beta \sum_{i=1}^{3} \frac{x_i^k \sum_{j=1}^{n_i} t_{ij}^{\beta} (\log \theta_i - \log t_{ij})}{{\theta_i}^{\beta}}, \qquad k = 0,1,2 \\
f_{33} = -\frac{\partial^2 \log L}{\partial \beta^2} =& \frac{n}{\beta^2} + \sum_{i=1}^{3} \frac{\sum_{j=1}^{n_i} t_{ij}^{\beta} (\log \theta_i - \log t_{ij})^2}{{\theta_i}^{\beta}}. \qquad (3.3)
\end{aligned}$$

Let $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\beta}$ be the MLEs of α_0, α_1, α_2 and β obtained from (3.2). F can be consistently estimated by $\hat{F}$, where $\hat{F} = (\hat{f}_{kl}), k,l = 0,1,2,3$ is obtained by replacing (α_0, α_1, α_2, β) by MLEs ($\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\beta}$) in F. Then based on the asymptotic normality of ($\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\beta}$) with the estimated covariance matrix $\hat{V} = \hat{F}^{-1} = (\hat{v}_{kl}^2)$, the asymptotic $100(1-\gamma)\%$ CI can be set for α_0, α_1, α_2 and β and is given by $(\hat{\lambda} - z_{\gamma/2}\hat{v}_{\lambda}, \hat{\lambda} + z_{\gamma/2}\hat{v}_{\lambda})$, where λ means model parameter and $z_{\gamma/2}$ is the $\gamma/2$ point of the standard normal distribution.

The asymptotic variance for $\log \hat{\theta}_i = \hat{\alpha}_0 + \hat{\alpha}_1 x_i + \hat{\alpha}_2 x_i^2$ is given by

$$\begin{aligned}
\hat{\sigma}_i^2 =& (1, x_i, x_i^2) \hat{V} (1, x_i, x_i^2)' \\
=& \hat{v}_{00}^2 + x_i^2 \hat{v}_{11}^2 + x_i^4 \hat{v}_{22}^2 + 2x_i \hat{v}_{01}^2 + 2x_i^2 \hat{v}_{02}^2 + 2x_i^3 \hat{v}_{12}^2, \qquad i = 1,2,3.
\end{aligned}$$

Then the asymptotic $100(1-\gamma)\%$ CI for $\log \theta_i, i = 1,2,3$ is

$$\left\{ \log \hat{\theta}_i - z_{\gamma/2} \hat{\sigma}_i, \quad \log \hat{\theta}_i + z_{\gamma/2} \hat{\sigma}_i \right\}.$$

Finally, the asymptotic $100(1-\gamma)\%$ CIs for $\theta_i, i = 1,2,3$ and β are given by

$$\left\{ \exp\left(\log \hat{\theta}_i - z_{\gamma/2} \hat{\sigma}_i \right), \quad \exp\left(\log \hat{\theta}_i + z_{\gamma/2} \hat{\sigma}_i \right) \right\},$$

and

$$\left\{ \hat{\beta} - z_{\gamma/2} \hat{v}_{33}, \quad \hat{\beta} + z_{\gamma/2} \hat{v}_{33} \right\}.$$

4. Optimal plan

The optimal plan of the three constant stress ALTs is discussed in this section, which is very important to improve the precision of estimators of model parameters and the quality of the statistical inference at the use stress. Under the constant stress ALTs, the optimal plan means to find the optimal sample proportion allocated at each stress level $x_i, i = 1,2,3$, and the D-optimality criterion based on the minimization of the generalized asymptotic variance (GAV) is considered.

The GAV is given by

$$GAV = \frac{1}{|F|}, \tag{4.1}$$

where F is the asymptotic Fisher information matrix. Therefore, the optimal sample proportion π_1^* and π_2^* are chosen to maximize $|F|$ by solving the following equations

$$\frac{\partial |F|}{\partial \pi_1} = 0 \qquad \text{and} \qquad \frac{\partial |F|}{\partial \pi_2} = 0. \tag{4.2}$$

But, the solutions of (4.2) are not in closed forms, and above all, to find $\partial|F|/\partial\pi_i$ are very complicated and difficult. So in our case, we use the compound linear plan to decide the sample size allocated at each stress level proposed by Kim (2006) performed better than two other compromise plan by Meeker (1984), Khamis and Higgins (1996) and had the advantage of simplicity. The properties and usefulness of the compound linear plan in our case are investigated in section 5.

5. Simulation studies

In this section, simulation studies are conducted to investigate the performances of the MLEs for model parameters $\theta_i = \exp(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2), i = 1, 2, 3$ and β through the mean square error (MSE). Furthermore, the asymptotic CIs for model parameters are obtained. The size of test units allocated at each stress are decided using the compound linear plan proposed by Kim (2006).

The failure times of test units on stress $x_i, i = 1, 2, 3$ are generated for several input values of parameter θ_i, β and stress level $x_i, i = 1, 2, 3$, and the MLEs, the MSEs and the CIs for parameters are calculated to show the performances of estimators. The simulation studies were carried out for samples of sizes 200, 350, 500 generated from an Weibull distribution using different combinations of model parameters $\theta_i, i = 1, 2, 3$ and β. The effects are investigated by the sample size, the shape parameter β and the scale parameter $\theta_i, i = 1, 2, 3$. And the GAVs are obtained to observe the properties by different values of the sample size and the stress level.

Firstly, the input values of parameters are set to $\alpha_0 = 2.0$, $\alpha_1 = -2.0$, $\alpha_2 = 5.0$ and $\beta = 0.5$ for the stress level $x_0 = 0.0$, $x_1 = 0.45$, $x_2 = 0.65$, $x_3 = 1.0$ to investigate the effect of the sample size and the results are shown in Table 4.1. We can see that as the sample size n increases, the MSEs of model parameters decrease and the MLEs become close to true values, and the CIs of model parameters are to be narrower. The proportion of test units allocated at the stress $x_i, i = 1, 2, 3$ are $\pi_1^* = 0.38409$, $\pi_2^* = 0.39394$, $\pi_3^* = 0.22197$ by the compound linear plan.

Secondly, the effects of the scale parameter θ are investigated and the results are shown in Table 4.2. Then the input values of the scale parameter $\theta_i = \exp(\alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2), i = 1, 2, 3$ are given by three stress levels $x_1 = 0.3, 0.4, 0.5$, $x_2 = 0.5, 0.6, 0.7$, $x_3 = 1.0$, $\beta = 0.5$ and the sample size is $n = 200$. The proportions of test units allocated at the stress $x_i, i = 1, 2, 3$ are changed according to the stress levels. As the stress x_1 and x_2 increase, the proportion of test units at the stress x_1, x_3 decreases and increases at the stress x_2. As the stress x_1 and

x_2 increase, the scale parameters θ_1 and θ_2 decrease, the MLEs and the CIs of $\theta_i, i = 1, 2$ become more accurate. But the accuracies of the MLEs and the CIs of θ_3 and β do not depend on the values of θ_1 and θ_2. The sample sizes allocated at the stress x_1 and x_3 decrease and the sample size at the stress x_2 increases by the compound linear plan as the stress level x_1 and x_2 increase, and then the MLEs and the CIs of $\theta_i, i = 1, 2$ become more accurate.

Thirdly, the effects of the shape parameter β are investigated when $\beta = 0.5$, $\beta = 1.0$ and $\beta = 1.2$ and the results are shown in Table 4.3. The input values of parameters are $\alpha_0 = 2.0$, $\alpha_1 = -2.0$ and $\alpha_2 = 5.0$ for the stress level $x_0 = 0.0$, $x_1 = 0.45$, $x_2 = 0.65$, $x_3 = 1.0$ and the sample size $n = 200$. As the value of β increases, the MLEs and the CIs of $\theta_i, i = 1, 2, 3$ become more accurate, but the MLE and the CI of β become less accurate.

Table 5.1 The effect of the sample size

	$\theta_1 = 1.09144$				$\theta_2 = 0.24353$			
n	MEAN	MSE	LCI	RCI	MEAN	MSE	LCI	RCI
200	1.07133	0.05437	0.67765	1.69691	0.23980	0.00391	0.15300	0.37628
350	1.07546	0.03453	0.76326	1.51641	0.24427	0.00248	0.17440	0.34227
500	1.09349	0.02513	0.82054	1.45769	0.24349	0.00155	0.18352	0.32313
	$\theta_3 = 0.00674$				$\beta = 0.5$			
n	MEAN	MSE	LCI	RCI	MEAN	MSE	LCI	RCI
200	0.00808	0.00002	0.00443	0.01472	0.50195	0.00191	0.44701	0.55689
350	0.00760	0.00001	0.00484	0.01194	0.50618	0.00136	0.46454	0.54782
500	0.00720	0.00000	0.00494	0.01050	0.50456	0.00082	0.46995	0.53917

Table 5.2 The effect of the scale parameter θ

θ_1					θ_2				
true	MEAN	MSE	LCI	RCI	true	MEAN	MSE	LCI	RCI
2.58571	2.58917	0.32287	1.65660	4.05500	0.77880	0.76060	0.04095	0.46678	1.24175
1.49182	1.50039	0.10512	0.95965	2.35033	0.36788	0.37134	0.00874	0.23600	0.58507
0.77880	0.76417	0.02727	0.48384	1.20905	0.15724	0.15938	0.00184	0.10291	0.24706
θ_3					$\beta = 0.5$				
true	MEAN	MSE	LCI	RCI	true	MEAN	MSE	LCI	RCI
0.00674	0.00794	0.00002	0.00455	0.01384	0.50000	0.50486	0.00193	0.44950	0.56023
0.00674	0.00818	0.00002	0.00457	0.01468	0.50000	0.51092	0.00215	0.45516	0.56668
0.00674	0.00808	0.00002	0.00436	0.01499	0.50000	0.50438	0.00198	0.44925	0.55951

Table 5.3 The effect of the shape parameter β

	$\theta_1 = 1.09144$				$\theta_2 = 0.24353$			
β	MEAN	MSE	LCI	RCI	MEAN	MSE	LCI	RCI
0.5	1.07133	0.05437	0.67765	1.69691	0.23980	0.00391	0.15300	0.37628
1.0	1.07422	0.01407	0.85076	1.35702	0.23636	0.00152	0.18801	0.29720
1.2	1.07716	0.01004	0.88536	1.31094	0.23476	0.00132	0.19358	0.28476
	$\theta_3 = 0.00674$				β			
β	MEAN	MSE	LCI	RCI	MEAN	MSE	LCI	RCI
0.5	0.00808	0.00002	0.00443	0.01472	0.50195	0.00191	0.44701	0.55689
1.0	0.00719	0.00001	0.00518	0.01001	0.98905	0.00736	0.87658	1.10151
1.2	0.00692	0.00001	0.00520	0.00927	1.17922	0.01070	1.04018	1.31826

Table 5.4 The optimal sample proportion $\pi_i, i = 1, 2, 3$ and GAV by the stress level

x_1	x_2	π_1^*	π_2^*	π_3^*	n	n_1	n_2	n_3	GAV
					200	97	46	57	0.00001540
0.10	0.30	0.48750	0.23077	0.28173	350	170	80	100	0.00000174
					500	243	115	142	0.00000037
					200	90	51	59	0.00002059
0.15	0.35	0.45500	0.25926	0.28574	350	159	90	101	0.00000208
					500	227	129	144	0.00000045
					200	86	57	57	0.00002442
0.20	0.40	0.43333	0.28571	0.28095	350	151	99	100	0.00000268
					500	216	142	142	0.00000058
					200	83	62	55	0.00003196
0.25	0.45	0.41786	0.31034	0.27180	350	146	108	96	0.00000340
					500	208	155	137	0.00000076
					200	81	66	53	0.00004377
0.30	0.50	0.40625	0.33333	0.26042	350	142	116	92	0.00000465
					500	203	166	131	0.00000103
					200	79	70	51	0.00006402
0.35	0.55	0.39722	0.35484	0.24794	350	139	124	87	0.00000658
					500	198	177	125	0.00000149
					200	78	75	47	0.00009076
0.40	0.60	0.39000	0.37500	0.23500	350	136	131	83	0.00000991
					500	195	187	118	0.00000220
					200	76	78	46	0.00014887
0.45	0.65	0.38409	0.39394	0.22197	350	134	137	79	0.00001552
					500	192	196	112	0.00000348
					200	75	82	43	0.00024650
0.50	0.70	0.37917	0.41176	0.20907	350	132	144	74	0.00002577
					500	189	205	106	0.00000586

The optimal sample proportion $\pi_i^*, i = 1, 2, 3$ and the GAV for different values of the sample size and the stress level are presented in Table 5.4. We can see that the sample proportion π_1^* at the stress x_1 decreases from about 49% to 38% and the sample proportion π_2^* at the stress x_2 increases from about 23% to 41% for the compound linear plan as the stress x_1 and x_2 increase, and the GAV increases. It is observed that as the optimal sample proportion π_1^* at the stress level x_1 becomes nearly 0.5, the GAV decreases and as the sample size n increases, the GAV decreases.

6. Conclusion

For the three level constant stress ALTs, the MLEs and the CIs of model parameters $\theta_i, i = 1, 2, 3$ and β are derived, when the lifetime, T_{ij}, $j = 1, 2, \cdots, n_i$ of failures for stress x_i, $i = 1, 2, 3$ follows an Weibull distribution with the scale parameter θ_i and the shape parameter β, where $\log\theta_i = \alpha_0 + \alpha_1 x_i + \alpha_2 x_i^2$. In simulation studies, the accuracies and some properties of the MLEs and the CIs of model parameters are investigated and we can see some results.

As the sample size increases, the accuracies of the MLEs and the CIs of model parameters are more improved. As the values of parameter θ_1 and θ_2 increase, the MLEs and the CIs of parameters $\theta_i, i = 1, 2, 3$ approach the true values, but the accuracy of the MLE and the CI of β does not depend on the value of scale parameter. As the value of the shape parameter β

increases, not the MLE and the CI of β, but the MLEs and the CIs of $\theta_i, i = 1, 2, 3$ become more accurate. The compound linear plan proposed by Kim (2006) performed nearly well in our case.

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