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LIGHTLIKE SUBMANIFOLDS OF A SEMI-RIEMANNIAN MANIFOLD WITH A SEMI-SYMMETRIC NON-METRIC CONNECTION

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Abstract. We study the geometry of r-lightlike submanifolds M of a semi-Riemannian manifold $\bar{M}$ with a semi-symmetric non-metric connection subject to the conditions; (a) the screen distribution of M is totally geodesic in M, and (b) at least one among the r-th lightlike second fundamental forms is parallel with respect to the induced connection of M. The main result is a classification theorem for irrotational r-lightlike submanifold of a semi-Riemannian manifold of index r admitting a semi-symmetric non-metric connection.

1. Introduction

The geometry of lightlike submanifolds is used in mathematical physics, in particular, in general relativity since lightlike submanifolds produce models of different types of horizons (event horizons, Cauchy's horizons, Kruskal's horizons). The universe can be represented as a four dimensional Lorentz submanifold (spacetime) embedded in an $(n+4)$-dimensional semi-Riemannian manifold. Lightlike hypersurfaces are also studied in the theory of electromagnetism [1]. Thus, large number of applications but limited information available, motivated us to do research on this subject matter. Duggal-Bejancu [1] and Kupeli [2] developed the general theory of degenerate (lightlike) submanifolds. They constructed a transversal vector bundle of lightlike submanifold and investigated various properties of these manifolds. Duggal-Jin [3] studied totally umbilical lightlike submanifold of a semi-Riemannian manifold. Ageshe and Chafle [4] introduced the notion of a semi-symmetric non-metric connection on a Riemannian manifold. Yaşar, Çöken and Yücesan [5] and Jin [6] studied lightlike hypersurfaces in semi-Riemannian manifolds admitting a semi-symmetric non-metric connections. The geometry of half lightlike submanifolds of a semi-Riemannian manifold with semi-symmetric non-metric connection was studied

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by Jin [7], and Jin and Lee [8]. However, a general notion of lightlike submanifolds of an semi-Riemannian manifold with a semi-symmetric non-metric connection is relatively new one as yet.

The objective of this paper is to study the geometry of irrotational r-lightlike submanifolds M of a semi-Riemannian manifold $\bar{M}$ admitting a semi-symmetric non-metric connection subject to the conditions; (a) the screen distribution $S(TM)$ is totally geodesic in M, and (b) at least one among the r-th lightlike second fundamental forms h_i^ℓ is parallel with respect to the induced connection ∇ of M. We have the following result:

Theorem 1.1. *Let M be an m-dimensional irrotational r-lightlike submanifold of a semi-Riemannian manifold $\bar{M}$ of index r admitting a semi-symmetric non-metric connection. If the screen distribution $S(TM)$ is totally geodesic in M and at least one among the r-th lightlike second fundamental forms h_i^ℓ is parallel with respect to the induced connection ∇ of M, then M is locally a product manifold $M_r \times M_p \times M_s$, where M_r, M_p and M_s are leaves of some integrable distributions of M, where $r + p + s = m$.*

2. Semi-symmetric non-metric connections

Let $(\bar{M}, \bar{g})$ be a semi-Riemannian manifold. A connection $\bar{\nabla}$ on $\bar{M}$ is called a *semi-symmetric non-metric connection* [4] if $\bar{\nabla}$ and its torsion tensor $\bar{T}$ satisfy

$$(\bar{\nabla}_X \bar{g})(Y, Z) = -\pi(Y)\bar{g}(X, Z) - \pi(Z)\bar{g}(X, Y), \tag{2.1}$$

$$\bar{T}(X, Y) = \pi(Y)X - \pi(X)Y, \tag{2.2}$$

for any vector fields X, Y and Z on $\bar{M}$, where π is a 1-form associated with a non-zero vector field ζ by $\pi(X) = \bar{g}(X, \zeta)$ for any vector field X on $\bar{M}$.

Let (M, g) be an m-dimensional lightlike submanifold of an $(m+n)$-dimensional semi-Riemannian manifold $(\bar{M}, \bar{g})$. We follow Duggal-Bejancu [1] for notations and results used in this paper. The radical distribution $Rad(TM) = TM \cap TM^\perp$ is a vector subbundle of the tangent bundle TM and the normal bundle $TM^\perp$, of rank $r\,(1 \le r \le \min\{m, n\})$. Then, in general, there exist two complementary non-degenerate distributions $S(TM)$ and $S(TM^\perp)$ of $Rad(TM)$ in TM and $TM^\perp$ respectively, called the *screen* and *co-screen distribution* on M, and we have the following two decompositions

$$TM = Rad(TM) \oplus_{orth} S(TM)\,;\; TM^\perp = Rad(TM) \oplus_{orth} S(TM^\perp), \tag{2.3}$$

where the symbol $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a lightlike submanifold by $(M, g, S(TM), S(TM^\perp))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. Let $tr(TM)$ and $ltr(TM)$ be complementary (but not orthogonal) vector bundles to TM in $T\bar{M}_{|M}$ and $TM^\perp$ in $S(TM)^\perp$ respectively and let $\{N_1, \ldots, N_r\}$ be a lightlike basis of $ltr(TM)$ consisting of smooth sections of $S(TM)^\perp$ [1] such that

$$\bar{g}(N_i, \xi_j) = \delta_{ij}, \quad \bar{g}(N_i, N_j) = \bar{g}(X, N_i) = \bar{g}(W, N_i) = 0,$$

for all $X \in \Gamma(S(TM))$ and $W \in \Gamma(S(TM^{\perp}))$, where the set $\{\xi_1, \cdots, \xi_r\}$ is a lightlike basis of $Rad(TM)$. Then the tangent bundle $T\bar{M}$ is decomposed as follow:

$$\begin{aligned} T\bar{M} &= TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \\ &= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}). \end{aligned} \quad (2.4)$$

We say that a lightlike submanifold $(M, g, S(TM), S(TM^{\perp}))$ of $\bar{M}$ is

(1) *r-lightlike* if $1 \leq r < \min\{m, n\}$;
(2) *co-isotropic* if $1 \leq r = n < m$;
(3) *isotropic* if $1 \leq r = m < n$;
(4) *totally lightlike* if $1 \leq r = m = n$.

The above three classes (2)∼(4) are particular cases of the class (1) as follows $S(TM^{\perp}) = \{0\}$, $S(TM) = \{0\}$ and $S(TM) = S(TM^{\perp}) = \{0\}$ respectively. The geometry of r-lightlike submanifolds is more general form than that of the other three type submanifolds. For this reason, we consider only r-lightlike submanifolds $M \equiv (M, g, S(TM), S(TM^{\perp}))$, with the following local quasi-orthonormal field of frames of $\bar{M}$:

$$\{\xi_1, \cdots, \xi_r, N_1, \cdots, N_r, F_{r+1}, \cdots, F_m, W_{r+1}, \cdots, W_n\},$$

where $\{\xi_1, \cdots, \xi_r\}$ and $\{N_1, \cdots, N_r\}$ are lightlike bases of $Rad(TM)$ and $ltr(TM)$ respectively, and $\{F_{r+1}, \cdots, F_m\}$ and $\{W_{r+1}, \cdots, W_n\}$ are orthonormal bases of $S(TM)$ and $S(TM^{\perp})$ respectively. We use the following range of indices:

$$\begin{aligned} &i, j, k, ... \in \{1, ..., r\}; &&a, b, c, ... \in \{r+1, ..., m\}; \\ &A, B, C, ... \in \{1, ..., m\}; &&\alpha, \beta, \gamma, ... \in \{r+1, ..., n\}. \end{aligned}$$

Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition (2.3). For an r-lightlike submanifold, the local Gauss-Weingartan formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + \sum_{i=1}^{r} h_i^{\ell}(X, Y) N_i + \sum_{\alpha=r+1}^{n} h_{\alpha}^{s}(X, Y) W_{\alpha}, \quad (2.5)$$

$$\bar{\nabla}_X N_i = -A_{N_i} X + \sum_{j=1}^{r} \tau_{ij}(X) N_j + \sum_{\alpha=r+1}^{n} \rho_{i\alpha}(X) W_{\alpha}, \quad (2.6)$$

$$\bar{\nabla}_X W_{\alpha} = -A_{W_{\alpha}} X + \sum_{i=1}^{r} \phi_{\alpha i}(X) N_i + \sum_{\beta=r+1}^{n} \theta_{\alpha\beta}(X) W_{\beta}, \quad (2.7)$$

$$\nabla_X PY = \nabla_X^{*} PY + \sum_{i=1}^{r} h_i^{*}(X, PY) \xi_i, \quad (2.8)$$

$$\nabla_X \xi_i = -A_{\xi_i}^{*} X - \sum_{j=1}^{r} \sigma_{ij}(X) \xi_j, \quad (2.9)$$

for any $X,\, Y \in \Gamma(TM)$, where ∇ and ∇^* are induced linear connections on TM and $S(TM)$ respectively, the bilinear forms h_i^ℓ and h_α^s on M are called the *local lightlike second fundamental form* and *local screen second fundamental form* on TM respectively, h_i^* is called the *local second fundamental form* on $S(TM)$. A_{N_i}, $A^*_{\xi_i}$ and A_{W_α} are linear operators on TM and τ_{ij}, $\rho_{i\alpha}$, $\phi_{\alpha i}$, $\theta_{\alpha\beta}$ and σ_{ij} are 1-forms on TM.

Using (2.1), (2.2) and (2.5), we show that

$$\begin{aligned}(\nabla_X g)(Y,Z) &= \sum_{i=1}^r \{h_i^\ell(X,Y)\,\eta_i(Z) + h_i^\ell(X,Z)\,\eta_i(Y)\} \qquad (2.10)\\ &\qquad - \pi(Y)g(X,Z) \;-\; \pi(Z)g(X,Y),\\ T(X,Y) &= \pi(Y)X - \pi(X)Y, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM). \qquad (2.11)\end{aligned}$$

and each h_i^ℓ and h_α^s are symmetric on TM, where T is the torsion tensor with respect to the induced connection ∇ and η_i is a 1-form on TM such that

$$\eta_i(X) = \bar g(X, N_i), \quad \forall\, X \in \Gamma(TM), \;\; i \in \{1, \cdots, r\}.$$

From the facts $h_i^\ell(X,Y) = \bar g(\bar\nabla_X Y, \xi_i)$ and $h_\alpha^s(X,Y) = \epsilon_\alpha \bar g(\bar\nabla_X Y, W_\alpha)$, we know that h_i^ℓ and h_α^s are independent of the choice of $S(TM)$. Taking $Y = \xi_i$ to this equations, we get

$$h_i^\ell(X,\xi_j) + h_j^\ell(X,\xi_i) = 0, \quad h_\alpha^s(X,\xi_i) \;=\; -\epsilon_\alpha \phi_{\alpha i}(X), \quad \forall\, X \in \Gamma(TM).$$

From the first equation of this, we have $h_i^\ell(X,\,\xi_i) = 0$ and $h_i^\ell(\xi_j, \xi_k) = 0$. The above local second fundamental forms of M and $S(TM)$ are related to their shape operators by

$$\begin{aligned}h_i^\ell(X,Y) &= g(A^*_{\xi_i}X, Y) + \lambda_i g(X,Y) - \sum_{k=1}^r h_k^\ell(X,\xi_i)\eta_k(Y), \qquad (2.12)\\ &\bar g(A^*_{\xi_i}X, N_j) = 0, \quad \tau_{ji}(X) - \sigma_{ij}(X) = \lambda_i \eta_j(X),\\ \epsilon_\alpha h_\alpha^s(X,Y) &= g(A_{W_\alpha}X, Y) + \epsilon_\alpha \nu_\alpha g(X,Y) - \sum_{i=1}^r \phi_{\alpha i}(X)\eta_i(Y), \qquad (2.13)\\ &\bar g(A_{W_\alpha}X, N_i) = \epsilon_\alpha\{\rho_{i\alpha}(X) - \nu_\alpha \eta_i(X)\},\\ h_i^*(X, PY) &= g(A_{N_i}X, PY) + \mu_i g(X, PY) + \pi(PY)\eta_i(X), \qquad (2.14)\\ &\eta_j(A_{N_i}X) + \eta_i(A_{N_j}X) + \mu_i \eta_j(X) + \mu_j \eta_i(X) = 0,\\ &\epsilon_\beta \theta_{\alpha\beta} = -\epsilon_\alpha \theta_{\beta\alpha}, \quad \forall\, X,\, Y \in \Gamma(TM),\end{aligned}$$

where $\epsilon_\alpha = \bar g(W_\alpha, W_\alpha)$ is the sign (± 1) of W_α, and $\lambda_i = \pi(\xi_i)$, $\mu_i = \pi(N_i)$ and $\nu_\alpha = \epsilon_\alpha \pi(W_\alpha)$ are smooth functions. From (2.9), we know that $A^*_{\xi_i}$ are $S(TM)$-valued for any i.

Definition 1. We say that $S(TM)$ is *totally geodesic* [1] in M if $h_i^* = 0$ for all i. M is said to be *irrotational* [2] if $\bar\nabla_X \xi_i \in \Gamma(TM)$ for any $X \in \Gamma(TM)$ and $\xi_i \in \Gamma(Rad(TM))$.

Note that M is irrotational if and only if

$$h^\ell_j(X,\xi_i)=0,\quad h^s_\alpha(X,\xi_i)=\phi_{\alpha i}=0,\quad \forall\, X\in\Gamma(TM). \tag{2.15}$$

In this case, replacing X by ξ_j to (2.12), we have $h^\ell_i(X,\,\xi_j)=g(A^*_{\xi_i}\xi_j,\,X)$. Thus we have $A^*_{\xi_i}\xi_j=0$. This implies that each ξ_j is an eigenvector field of $A^*_{\xi_i}$ corresponding to the eigenvalue 0. If M is irrotational and $S(TM)$ is totally geodesic, then we have

Theorem 2.1. *Let M be an irrotational r-lightlike submanifold of a semi-Riemannian manifold $\bar M$ admitting a semi-symmetric non-metric connection. If $S(TM)$ is totally geodesic in M, then M is locally a product manifold $M_r\times M_{m-r}$ where M_r and M_{m-r} are leaves of the integrable distributions $Rad(TM)$ and $S(TM)$ of M respectively and $m=dim\,M$.*

Proof. As M is irrotational, we have $A^*_{\xi_i}\xi_j=0$. From this and (2.9), we show that $Rad(TM)$ is an auto-parallel distribution on M. Also, as $S(TM)$ is totally geodesic in M, we have $\nabla_XY=\nabla^*_XY$ for all $X,\,Y\in\Gamma(S(TM))$. This implies that $S(TM)$ is also an auto-parallel distribution on M. Thus, by the decomposition theorem of de Rham [9], we have $M=M_r\times M_{m-r}$ where M_r and M_{m-r} are leaves of the integrable distributions $Rad(TM)$ and $S(TM)$ of M respectively. □

3. Proof of Theorem 1.1

By Theorem 2.1, we know that M is locally a product manifold $M=M_r\times M_{m-r}$ where M_r and M_{m-r} are leaves of the integrable distributions $Rad(TM)$ and $S(TM)$ respectively. As the index of $\bar M$ is r, $S(TM)$ is a Riemannian vector bundle. Now we assume that the lightlike second fundamental form h^ℓ_1 is parallel, i.e., $\nabla_Xh^\ell_1=0$, without loss generality. Then we set $\xi_1=\xi$, $\lambda_1=\lambda$ and $h^\ell_1=h^\ell$. From (2.12) we deduce the following equation

$$h^\ell(X,Y)=g(A^*_\xi X,Y)+\lambda g(X,Y),\quad \forall\, X,\,Y\in\Gamma(TM). \tag{3.1}$$

Applying ∇_X to $h^\ell(Y,\xi)=0$ and using (2.9), $(2.15)_1$ and (3.1), we have

$$g(A^*_\xi X,A^*_\xi Y)=\lambda g(A^*_\xi X,Y),\quad \forall\, X,\,Y\in\Gamma(TM). \tag{3.2}$$

By $(2.15)_1$, we show that ξ is an eigenvector field of A^*_ξ corresponding to the eigenvalue 0. As A^*_ξ is $S(TM)$-valued real self-adjoint operator, A^*_ξ have $(m-r)$ real orthonormal eigenvector fields in $S(TM)$ and is diagonalizable. Consider a frame field of eigenvectors $\{\xi_1,\,\ldots,\,\xi_r,\,E_{r+1},\,\ldots,\,E_m\}$ of A^*_ξ such that $\{E_{r+1},\,\ldots,\,E_m\}$ is an orthonormal frame field of $S(TM)$ and $A^*_\xi E_a=\kappa_aE_a$ for each $a\in\{r+1,\,\ldots,\,m\}$. Put $X=Y=E_a$ in (3.2), each κ_a is a solution of the quadratic equation

$$x^2-\lambda\,x=0. \tag{3.3}$$

This equation has at most two distinct solutions 0 and λ. Thus there exists a number $p\in\{0,\,\ldots,\,m-r\}$ such that $\kappa_{r+1}=\cdots=\kappa_{r+p}=0$ and $\kappa_{r+p+1}=\cdots=\kappa_{m-r}=\lambda$, by renumbering if necessary.

In case $p = 0$ or $p = m - r$: As $M = M_r \times M_{m-r} \cong M_r \times M_{m-r} \times \{x\}$ for any $x \in M$, we show that $M_p = \{x\}$ and $M_s = M_{m-r}$ if $p = 0$, or $M_p = M_{m-r}$ and $M_s = \{x\}$ if $p = m - r$. Thus Theorem 1.1 is true in this case.

In case $0 < p < m - r$: We show that $\lambda \neq 0$. Consider the following distributions D_p and D_s on M, and their projections D_p^{sm} and D_s^{sm} on $S(TM)$ respectively such that

$$\begin{aligned} D_p &= \{X \in \Gamma(TM) \mid A^*_\xi X = 0 \text{ and } PX \neq 0\}, & D_p^{sm} &= PD_p, \\ D_s &= \{U \in \Gamma(TM) \mid A^*_\xi U = \lambda PU \text{ and } PU \neq 0\}, & D_s^{sm} &= PD_s. \end{aligned}$$

Clearly we show that $D_p \cap D_s = \{0\}$ and $D_p^{sm} \cap D_s^{sm} = \{0\}$ as $\lambda \neq 0$.

For any $X \in \Gamma(D_p)$ and $U \in \Gamma(D_s)$, we get $A^*_\xi PX = A^*_\xi X = 0$ and $A^*_\xi PU = A^*_\xi U = \lambda PU$. This imply $PX \in \Gamma(D_p^{sm})$ and $PU \in \Gamma(D_s^{sm})$. Thus P maps $\Gamma(D_p)$ onto $\Gamma(D_p^{sm})$ and $\Gamma(D_s)$ onto $\Gamma(D_s^{sm})$. Since PX and PU are eigenvector fields of the real self-adjoint operator A^*_ξ corresponding to the different eigenvalues 0 and λ respectively, we have $g(PX,\, PU) = 0$. From the facts $g(X,U) = g(PX,PU) = 0$ and $h^\ell(X,U) = g(A^*_\xi X,U) + \lambda g(X,U) = \lambda g(X,U) = 0$, we show that $D_p \perp_g D_s$ and $D_p \perp_{h^\ell} D_s$ respectively.

Since $\{E_a\}_{r+1 \leq a \leq r+p}$ and $\{E_b\}_{r+p+1 \leq b \leq m}$ are vector fields of D_p^{sm} and D_s^{sm} respectively, D_p^{sm} and D_s^{sm} are mutually orthogonal vector subbundle of $S(TM)$ and $rank\, S(TM) = m - r$, we show that D_p^{sm} and D_s^{sm} are non-degenerate distributions of rank p and rank $(m-r-p)$ respectively. This result implies $S(TM) = D_p^{sm} \oplus_{orth} D_s^{sm}$.

From (3.2), we show that $A^*_\xi(A^*_\xi - \lambda P) = (A^*_\xi - \lambda P)A^*_\xi = 0$. Let $Y \in Im\, A^*_\xi$, then there exists $X \in \Gamma(TM)$ such that $Y = A^*_\xi X$. Then we have $(A^*_\xi - \lambda P)Y = 0$ and $Y \in \Gamma(D_s)$. Thus $Im\, A^*_\xi \subset \Gamma(D_s)$. Since the morphism A^*_ξ maps $\Gamma(TM)$ onto $\Gamma(S(TM))$, we have $Im\, A^*_\xi \subset \Gamma(D_s^{sm})$. By duality, we also have $Im(A^*_\xi - \lambda P) \subset \Gamma(D_p^{sm})$.

For any $X,\, Y \in \Gamma(D_p^{sm})$ and $U,\, V \in \Gamma(D_s^{sm})$, applying ∇_X to $h^\ell(U,V) = 2\lambda g(U,V)$ and ∇_U to $h^\ell(X,Y) = \lambda g(X,Y)$ and then, using (2.10), (3.1) and the facts $\nabla h^\ell = 0$ and $D_p^{sm} \perp_g D_s^{sm}$, we have $(X\lambda)g(U,V) = 0$ and $(U\lambda)g(X,Y) = 0$, i.e., $X\lambda = 0$ and $U\lambda = 0$. This imply $Z\lambda = 0$ for all $Z \in \Gamma(S(TM))$. Thus λ is a constant on $S(TM)$.

For any $X,\, Y,\, Z \in \Gamma(D_p^{sm})$, applying ∇_Z to $h^\ell(X,Y) = \lambda g(X,Y)$ and using (3.1) and the facts $\nabla h^\ell = 0$ and λ is a non-zero constant on $S(TM)$, we have $(\nabla_Z g)(X,Y) = 0$, i.e.,

$$\pi(X)g(Y,Z) + \pi(Y)g(X,Z) = 0. \tag{3.4}$$

due to (2.10). Using this equation and the fact D_p^{sm} is non-degenerate, we have

$$\pi(X)Y = -\pi(Y)X. \tag{3.5}$$

Taking the skew-symmetric part of (3.4) for X and Z, we get $\pi(X)g(Y,Z) = \pi(Z)g(X,Y)$, from which we have $\pi(X)Z = \pi(Z)X$. Replacing Z by Y to this result, we obtain

$$\pi(X)Y = \pi(Y)X. \tag{3.6}$$

From (3.5) and (3.6), we obtain $\pi(X) = 0$ for all $X \in \Gamma(D_p^{sm})$. By duality, we have $\pi(U) = 0$ for all $U \in \Gamma(D_s^{sm})$. Thus $\pi = 0$ on $S(TM)$ and $\nabla g = 0$ on $S(TM)$.

For any $X,\, Y \in \Gamma(D_p^{sm})$ and $U,\, V \in \Gamma(D_s^{sm})$, applying ∇_X to $h^\ell(Y,U) = 0$ and ∇_V to $h^\ell(Y,U) = 0$ and using (2.10), (3.1) and the facts $\nabla h^\ell = 0$ and $\nabla g = 0$ on $S(TM)$, we have

$$g(A_\xi^* \nabla_X Y,\, U) = 0, \quad g((A_\xi^* - \lambda P)\nabla_V U,\, Y) = 0.$$

Since $Im\, A_\xi^* \subset \Gamma(D_s^{sm})$ and D_s^{sm} is non-degenerate, we have $A_\xi^* \nabla_X Y = 0$. Thus $\nabla_X Y \in \Gamma(D_p)$. By duality, we have $\nabla_V U \in \Gamma(D_s)$. As $S(TM)$ is totally geodesic in M, this results imply that $\nabla_X Y \in \Gamma(D_p^{sm})$ for all $X,\, Y \in \Gamma(D_p^{sm})$ and $\nabla_V U \in \Gamma(D_s^{sm})$ for all $U,\, V \in \Gamma(D_s^{sm})$. Thus D_p^{sm} and D_s^{sm} are integrable and auto-parallel distributions with respect to the connections ∇ on M and ∇^* on $S(TM)$.

Since the leaf M^* of $S(TM)$ is a Riemannian manifold and $S(TM) = D_p^{sm} \oplus_{orth} D_s^{sm}$, where D_p^{sm} and D_s^{sm} are auto-parallel distributions with respect to the induced connection ∇^* on $S(TM)$, by the decomposition theorem of de Rham [9], we have $M_{m-r} = M_p \times M_s$, where M_p and M_s are leaves of D_p^{sm} and D_s^{sm} respectively. Thus we have Theorem 1.1.

Corollary 3.1. *Let M be a lightlike hypersurface or 1-lightlike submanifold of a Lorentz manifold $\bar{M}$ admitting a semi-symmetric non-metric connection. If the screen distribution $S(TM)$ is totally geodesic in M and the* (*lightlike*) *second fundamental form of M is parallel, then M is locally a product manifold $L \times M_p \times M_s$, where L is a null curve tangent to the radical distribution $Rad(TM)$, and M_p and M_s are leaves of some integrable distributions of M and $p + s = dim\, S(TM)$.*

Remark 1. Instead of the condition *M is an r-lightlike submanifold* of Theorem 1.1, even though we use the condition *M is a co-isotropic submanifold*, it is easy to find that we can establish the same result Theorem 1.1. But, for isotropic or totally lightlike submanifolds M, Theorem 1.1 can not establish because $S(TM) = \{0\}$ and h_i^ℓ does not exist for all i.

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