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KNOTS ADMITTING SEIFERT-FIBERED SURGERIES OVER S^2 WITH FOUR EXCEPTIONAL FIBERS

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Abstract. In this paper, we construct infinite families of knots in S^3 which admit Dehn surgery producing a Seifert-fibered space over S^2 with four exceptional fibers. Also we show that these knots are turned out to be satellite knots, which supports the conjecture that no hyperbolic knot in S^3 admits a Seifert-fibered space over S^2 with four exceptional fibers as Dehn surgery.

1. Introduction

Knots in S^3 which admit Dehn surgery of a Seifert-fibered space have been studied by many people. Infinite families of knots with lens space Dehn surgeries were constructed by Berge in [1]. He introduced a primitive curve in the boundary of a genus two handlebody and constructed 12 types of primitive/primitive or double-primitive knots which lie in a genus two Heegaard surface of S^3. These knots are called *Berge knots*. He showed in [1] that these knots admit lens space surgeries with an integral slope. There is a conjecture about the Berge knots.

Berge Conjecture (Berge [1]). *A hyperbolic knot K in S^3 has a lens space surgery if and only if K is a Berge knot, and the surgery is the corresponding integral surgery.*

Later, by introducing a Seifert curve in the boundary of a genus two handlebody, Dean described a generalization, called primitive/Seifert knots, in his thesis [3], and its published version [4]. The knots that he constructed admit Dehn surgery of a Seifert-fibered space over S^2 with three exceptional fibers. Two infinite families of hyperbolic knots which have a Seifert-fibered space over $\mathbb{RP}^2$ with two exceptional fibers as Dehn surgery are given by Eudave-Muñoz in [6]. These knots turned out to be primitive/Seifert knots. Berge and the author give complete classification of hyperbolic primitive/primitive knots and primitive/Seifert knots in [2].

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Now we focus on knots which admit Dehn surgery producing a Seifert-fibered space over S^2 with four exceptional fibers. Let K be a knot in S^3 and $K(\gamma)$ the manifold obtained by a γ-Dehn surgery of S^3. Kalliongis and Tsau in [7] showed that the connected sum of two torus knots admits a Seifert-fibered space over S^2 with four exceptional fibers. In other words, if $K = T(p,q)\#T(m,n)$, where $T(p,q)$ is a (p,q)-torus knot, then $K(pq+mn)$ is a Seifert-fibered space over S^2 with four exceptional fibers of indices p, q, m, and n. Miyazaki and Motegi in [8] gave a new family of satellite knots producing a Seifert-fibered space over S^2 with four exceptional fibers as Dehn surgery. The connected sum of two torus knots is contained in this family as a special case.

The following is a conjecture related to a Seifert-fibered surgery over S^2 with four exceptional fibers.

Conjecture. *If $K(\gamma)$ is a Seifert-fibered space for some slope γ, then $K(\gamma)$ is a Seifert-fibered space over S^2 with at most four exceptional fibers or a Seifert-fibered space over $\mathbb{RP}^2$ with no more than two exceptional fibers. Furthermore, if K is hyperbolic, then the surgery slope γ is integral, and $K(\gamma)$ cannot be a Seifert-fibered space over S^3 with four exceptional fibers.*

In this paper, we use the method of Berge and Dean to construct infinite families of knots which admit Dehn surgery of a Seifert-fibered space over S^2 with four exceptional fibers. Moreover we show that these knots are all satellite knots, which supports the above conjecture. In other words, we have the following as the main result of this paper.

Theorem 1.1. *There are infinite families of knots lying in a genus two Heegaard surface of S^3 which admit Dehn surgery producing a Seifert-fibered space over S^2 with four exceptional fibers. In particular, these families of knots are all satellite knots.*

This theorem follows immediately from Theorem 3.6 which provides six families of such knots. One of these families is the connected sum of two torus knots. Also we can guess by comparing slopes that the family of satellite knots constructed by Miyazaki and Motegi in [8] belongs to one of these families.

Throughout this paper, we denote by $S(a_1, \ldots, a_n)$ the Seifert-fibered space over a surface S with n exceptional fibers of indices $a_1, \ldots, a_n$.

2. Knots lying in a genus two Heegaard surface of S^3

Let H be a genus two handlebody, k an essential simple closed curve in ∂H, and $H[k]$ the 3-manifold obtained by adding a 2-handle to H along k. We say k is *primitive* in H if $H[k]$ is a solid torus. Equivalently k is conjugate to a free generator of $\pi_1(H)$.

Similarly, we say k is *Seifert* in H if $H[k]$ is a Seifert-fibered space and not a solid torus. Note that, since H is a genus two handlebody, that k is Seifert in H implies that $H[k]$ is an orientable Seifert-fibered space over D^2 with two exceptional fibers, or an orientable Seifert-fibered space over the Möbius band with at most one exceptional fiber.

Suppose K is a simple closed curve in a genus two Heegaard surface Σ of S^3 bounding handlebodies H and H'. K in Σ is *primitive/primitive* or *double-primitive* if it is primitive with respect to both H and H'. Similarly, K is *primitive/Seifert* if it is primitive with respect to one of H or H', and Seifert with respect to the other. Also K is *Seifert/Seifert* or *double-Seifert* if it is Seifert with respect to both H and H'.

Suppose $N(K)$ is a tubular neighborhood of K in S^3. Let γ be a component of $\partial N \cap \Sigma$ which is an essential simple closed curve in ∂N. Then the isotopy class of γ in ∂N defines the *surface slope* in ∂N. (The surface slope depends on the embedding of K in Σ, so a knot in S^3 may have more than one surface slope.)

The following lemma shows the relationship between Dehn surgery at a surface slope and adding a 2-handle to a genus two handlebody.

Lemma 2.1. *Let K be a knot lying in a genus two Heegaard surface Σ of S^3 bounding handlebodies H and H', and γ be a surface slope with respect to this embedding of K. Then $K(\gamma) \cong H[K] \cup_\partial H'[K]$.*

Proof. It follows from Lemma 2.1 in [4]. $\square$

Lemma 2.1 implies that if K is primitive/primitive, then $K(\gamma)$ has a genus one Heegaard splitting, and so $K(\gamma)$ is a lens space. If K is primitive/Seifert, then $K(\gamma)$ is either $S^2(a,b,c)$, $\mathbb{R}\mathrm{P}^2(a,b)$, or a connected sum of lens spaces. However, Eudave-Muñoz [5] proved that if a primitive/Seifert curve K is hyperbolic in S^3, then a connected sum of lens spaces cannot arise as a Dehn surgery on K. As mentioned in the introduction of this paper, hyperbolic primitive/primitive knots and primitive/Seifert knots are completely classified in [2].

If K is Seifert/Seifert, then $K(\gamma)$ is either $S^2(a,b,c,d), \mathbb{R}\mathrm{P}^2(a,b,c), K^2(a,b)$, or a graph manifold. However, for homological reasons Dehn surgery on a knot in S^3 producing $K^2(a,b)$ cannot happen. Since we are interested in constructing knots admitting Dehn surgery of $S^2(a,b,c,d)$, we focus on a Seifert curve k in a genus two handlebody H such that $H[k]$ is $D^2(a,b)$.

3. Twisted torus knots admitting Dehn surgery of $S^2(a,b,c,d)$

In this section, we give the main result of this paper. In other words, by using twisted torus knots we construct infinite families of knots which admit Dehn surgery of $S^2(a,b,c,d)$, and we show that all of these knots are satellite knots. The precise definitions of the twisted torus knots can be found in [4]. For readers, we give brief explanation on how to construct the twisted torus knots.

Let V and V' be two standardly embedded disjoint unlinked solid tori in S^3. Let $T(p,q)$ be the (p,q)-torus knot which lies in the boundary of V. Let

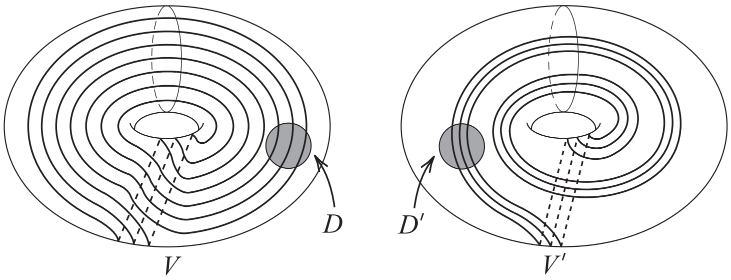

FIGURE 1. The $(7, 3)$-torus knot $T(7, 3)$ and 3 parallel copies $3T(2, 1)$ of the $(2, 1)$-torus knot.

$rT(m, n)$ be the r parallel copies of $T(m, n)$ which lies in the boundary of V'. Here we may assume that $0 < q < p$ and $m > 0$. Let D be the disk in ∂V so that $T(p, q)$ intersects D in r disjoint parallel arcs, where $0 < r \leq p+q$, and D' the disk in $\partial V'$ so that $rT(m, n)$ intersects D' in r disjoint parallel arcs, one for each component of $rT(m, n)$. Figure 1 shows the $(7, 3)$-torus knot $T(7, 3)$, 3 parallel copies $3T(2, 1)$ of the $(2, 1)$-torus knot, and the disks D and D'. We excise the disks D and D' from their respective tori and glue the punctured tori together along their boundaries so that the orientations of $T(p, q)$ and $rT(m, n)$ align correctly. The resulting one must yield a knot and is called a twisted torus knot, which is denoted by $K(p, q, r, m, n)$. Figure 2 shows the twisted torus knot $K(7, 3, 3, 2, 1)$.

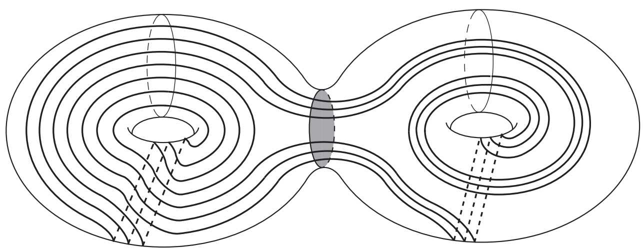

FIGURE 2. The twisted torus knot $K(7, 3, 3, 2, 1)$.

Let H be the genus two handlebody obtained from the two solid tori V and V' by identifying the two disks D and D'. Also we let $H' = \overline{S^3 - H}$, and $\Sigma = \partial H = \partial H'$. Then $(H, H'; \Sigma)$ forms a genus 2 Heegaard splitting of S^3. Thus we can consider all of the twisted torus knots as lying on this genus 2 Heegaard surface Σ bounding the two handlebodies H and H' of S^3.

The following proposition shows the surface slope of a twisted torus knot.

Proposition 3.1. *Let K be a twisted torus knot $K(p, q, r, m, n)$ described above. The surface slope of K with respect to the Heegaard surface Σ is $pq + r^2mn$.*

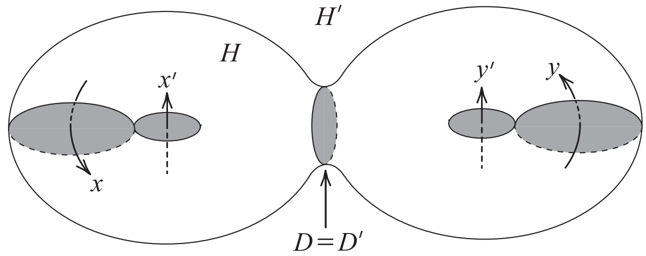

FIGURE 3. The generators of $\pi_1(H)$ and $\pi_1(H')$.

Proof. This is Proposition 3.1 in [4]. □

Suppose $K = K(p, q, r, m, n)$ is a twisted torus knot lying in a genus two Heegaard splitting $(H, H'; \Sigma)$ of S^3 as described above. Let $w_{p,q,r,m,n}$ and $w'_{p,q,r,m,n}$ be the conjugacy class of K in $\pi_1(H) = \langle x, y\rangle$ and $\pi_1(H') = \langle x', y'\rangle$ respectively, where x and y are generators in H and x' and y' are generators in H', which are dual to the cutting disks as shown in Figure 3. Then we have the following remark.

Remark 3.2. (1) $w'_{p,q,r,m,n}$ is equal to $w_{q,p,r,n,m}$ with x replaced by x' and y replaced by y'.

(2) By the construction of a twisted torus knot, $w_{p,q,r,m,n}$ ($w'_{p,q,r,m,n}$, resp.) does not depend on the parameter n (m, resp.).

The following two lemmas show which values of the parameters p, q, r, m, and n produce a primitive or a Seifert curve of $K(p, q, r, m, n)$ with respect to H and H'.

Lemma 3.3. *$K(p, q, r, m, n)$ is a primitive curve in H if and only if*

(1) $p = 1$; *or*
(2) $m = 1$ *and* $r \equiv \pm 1$ *or* $\pm q$ *mod* p.

Similarly, $K(p, q, r, m, n)$ is a primitive curve in H' if and only if

(1) $q = 1$; *or*
(2) $n = 1$ *and* $r \equiv \pm 1$ *or* $\pm p$ *mod* q.

Proof. This is Theorem 3.4 in [4]. □

Lemma 3.4. *If $K(p, q, r, 1, n)$ is a primitive curve in H, then $K = K(p, q, r, m, n)$ is a Seifert curve in H with $H[K] = D^2(p, m)$. Similarly, if $K(p, q, r, m, 1)$ is a primitive curve in H', then $K = K(p, q, r, m, n)$ is a Seifert curve in H' with $H'[K] = D^2(q, n)$.*

Proof. This is Proposition 3.6 in [4]. □

Lemmas 3.3 and 3.4 allow one to produce knots in S^3 admitting Dehn surgery of $S^2(a, b, c, d)$. If $K(p, q, r, 1, n)$ is a primitive curve in H and $K(p, q, r, m, 1)$

is a primitive curve in H', then by Lemma 3.4 $K = K(p,q,r,m,n)$ is Seifert in both H and H' such that $H[K] = D^2(p,m)$ and $H'[K] = D^2(q,n)$, and by Lemma 3.3 we can find all possible values of the parameters p, q, and r for $K(p,q,r,1,n)$ and $K(p,q,r,m,1)$ being primitive curves in H and H', respectively. It follows from Lemma 2.1 that Dehn surgery on K at a surface slope is either an $S^2(p,q,m,n)$ or a graph manifold. However, the following theorem shows that Dehn surgery on K at a surface slope is an $S^2(p,q,m,n)$.

Theorem 3.5. *Suppose $K = K(p,q,r,m,n)$ is a twisted torus knot lying in a genus two Heegaard splitting $(H, H'; \Sigma)$ of S^3 such that $K(p,q,r,1,n)$ is a primitive curve in H and $K(p,q,r,m,1)$ is a primitive curve in H'. Then $K(\gamma)$ is a Seifert-fibered space over S^2 with four exceptional fibers of indices p,q,m, and n, where γ is a surface slope which is equal to $pq + r^2mn$. Furthermore, K is a satellite knot whose companion is a torus knot $T(m,n)$.*

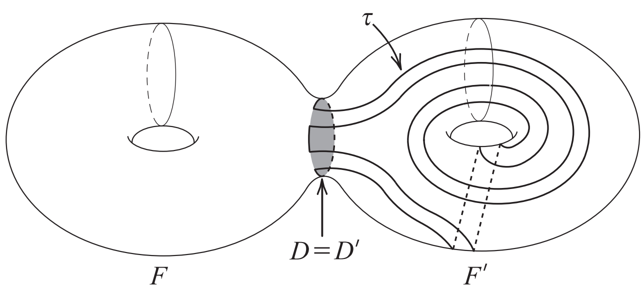

FIGURE 4. The essential annulus A in H which can be obtained by bandsumming the disk D with the band τ.

Proof. First we consider the genus two handlebody H which is constructed from two solid tori V and V' by gluing along disks D and D'. $\partial D (= \partial D')$ decomposes ∂H into two once-punctured tori F and F' which come from ∂V and $\partial V'$, respectively. Then $K \cap F'$ consists of r parallel arcs. As shown in Figure 4 considering a band τ in F' which contains the r parallel arcs, and the disk D, we can construct a properly embedded separating essential annulus A in H. In other words, the annulus A can be obtained by bandsumming the disk D with the band τ.

Cutting H apart along A yields a genus two handlebody W and a solid torus Z. Note that Z is homeomorphic to the solid torus V', and that K lies in the boundary of the genus two handlebody W as a twisted torus knot $K(p,q,r,1,n')$ for some integer n'. Since $K(p,q,r,1,n)$ is primitive in H and the primitivity does not depend on the parameter n, by Lemma 3.3, $K(p,q,r,1,n')$ is also primitive in W. Since K is a primitive curve in W and thus $W[K]$ is a solid torus, it follows that $H[K]$ is obtained by gluing the two solid tori

$W[K]$ and Z together along A. So $H[K]$ is Seifert-fibered over D^2 with ∂A as regular fibers and the cores of $W[K]$ and Z as exceptional fibers.

We need to compute the indices of the two exceptional fibers of $H[K]$. It is clear that the annulus A wraps around the solid torus Z m times longitudinally, so the core of Z is an exceptional fiber of index m. The other index can be obtained by computing $\pi_1(W[K][\beta])$, where β is one boundary component of A. We can observe that $W[K][\beta]$ is homeomorphic to $W[\beta][K]$, $W[\beta]$ is a solid torus, and K lies in the boundary of $W[\beta]$ as a torus knot $T(p,q)$. Thus $\pi_1(W[K][\beta]) = \pi_1(W[\beta][K]) = \mathbb{Z}_p$ and then the core of $W[K]$ is an exceptional fiber of index p.

We have shown that $H[K]$ is a $D^2(p,m)$ with ∂A as regular fibers. We can apply the similar argument to H'. We can construct a properly embedded essential annulus A' with $\partial A' = \partial A$, which separates the handlebody H' into a genus two handlebody W' and a solid torus Z'. Then $H'[K]$ is a $D^2(q,n)$ with $\partial A'$ as regular fibers. Since for the surface slope γ $K(\gamma) = H[K] \cup_\partial H'[K] = D^2(p,m) \cup_\partial D^2(q,n)$ and the two regular fibers coincide, $K(\gamma)$ is an $S^2(p,q,m,n)$. This completes the proof of the first part of the theorem.

Now we will show that K is a satellite knot whose companion is a torus knot $T(m,n)$. Using the separating annuli A and A', we can decompose S^3 as follows;

$$S^3 = H \cup_\partial H' \cong (W \cup_A Z) \cup_\partial (W' \cup_{A'} Z') \cong (W \cup_\partial W') \cup_T (Z \cup_{A''} Z'),$$

where $T = A \cup_\partial A'$ and $A'' = \overline{\partial Z - A} = \overline{\partial Z' - A'}$. Since $\partial A'' = \partial A$ and one boundary component of $\partial A''$ lies on the boundary of $Z(Z'$, resp.) as a torus knot $T(m,n)(T(n,m)$, resp.), $Z \cup_{A''} Z'$ is homeomorphic to $D^2(m,n)$. Therefore $(W \cup_\partial W')$ is a solid torus whose core is a torus knot $T(m,n)$. This proves that K is a satellite knot whose companion is a torus knot $T(m,n)$. $\square$

In the following theorem, which is the main theorem of this paper, we give infinite families of knots in S^3 which admit Dehn surgery of $S^2(a,b,c,d)$. Basically, using Lemma 3.3 we will find all possible values of the parameters of $K(p,q,r,m,n)$ such that $K(p,q,r,1,n)$ is a primitive curve in H and $K(p,q,r,m,1)$ is a primitive curve in H'. Recall that we may assume that $0 < q < p$ with $\gcd(p,q) = 1$, $0 < r \leq p+q$, and $m > 0$.

Theorem 3.6. *Let $K = K(p,q,r,m,n)$ be a twisted torus knot whose parameters p,q, and r, with $1 < q < p$, $\gcd(p,q) = 1$, and $0 < r \leq p+q$, satisfy one of the following values in the table, and $m > 1$, $|n| > 1$ with $\gcd(m,n) = 1$. Then K admits a Dehn surgery producing a Seifert-fibered space over S^2 with four exceptional fibers of indices p,q,m, and n at slope $pq + r^2mn$. Furthermore, K is a satellite knot whose companion is a torus knot $T(m,n)$.*

	(p, q, r)	satisfying
1	$(p, q, 1)$	$1 < q < p$
2	$(p, q, p-q)$	$1 < q < p$
3	$(p, q, p+q)$	$1 < q < p$
4	$(\alpha q + 2\epsilon, q, \alpha q + \epsilon)$	$q > 1, \alpha > 0, \epsilon = \pm 1$ with $\alpha q + 2\epsilon > 2$
5	$(3J + \epsilon, 2J + \epsilon, 4J + \epsilon)$	$J > 0, \epsilon = \pm 1$ with $2J + \epsilon > 1$
6	$((2J+\epsilon)J' + J, 2J + \epsilon, (2J+\epsilon)J' + J + \epsilon)$	$J, J' > 0, \epsilon = \pm 1$ with $2J + \epsilon > 1$

Proof. By Theorem 3.5, it suffices to show that $K(p, q, r, 1, n)$ is a primitive curve in H and $K(p, q, r, m, 1)$ is a primitive curve in H' for the values of the parameters p, q, r in the table. However here using Lemma 3.3, we find all possible values of the parameters p, q, r such that $K(p, q, r, 1, n)$ is a primitive curve in H and $K(p, q, r, m, 1)$ is a primitive curve in H'. By Lemma 3.3, $K(p, q, r, 1, n)$ is a primitive curve in H if and only if $r \equiv \pm 1$ or $\pm q$ mod p. All possible values for r satisfying the condition that $r \equiv \pm 1$ or $\pm q$ mod p are as follows:

$$r = 1,\ q,\ p-1,\ p-q,\ p+1,\ 2p-q,\ p+q,\ \text{or}\ 2p-1.$$

Similarly, $K(p, q, r, m, 1)$ is a primitive curve in H' if and only if $r \equiv \pm 1$ or $\pm p$ mod q. All possible values for r satisfying the condition that $r \equiv \pm 1$ or $\pm p$ mod q are as follows:

$$r = 1,\ p,\ \alpha q - 1,\ \alpha q + 1,\ \alpha q - p,\ p - \alpha q,\ \text{or}\ p + q,$$

where $\alpha > 0$. (if $r = \alpha q - p$, then $\alpha > 1$.)

Now we figure out which values of r satisfy both conditions. Since 1, $p - q$, $p + q$ are contained in both conditions, these cases give (1), (2), and (3) in the table.

If $r = 2p-1$, then since $r \leq p+q$, $p \leq q+1$ and thus $p = q+1$. This implies that $r = p + q$ and this case belongs to (3).

If $r = p$, then $r \not\equiv \pm 1$ or $\pm q$ mod p, which implies that $K(p, q, r, 1, n)$ is not a primitive curve in H. Similarly we can rule out the case $r = q$.

If $r = p - \alpha q$ in the second condition, then it cannot be $p - 1$, $p + 1$, or $2p - q$, otherwise it contradicts $\alpha q > 1$ or $\gcd(p, q) = 1$. Therefore in the first condition r must be equal to $p-q$ and thus $\alpha = 1$ and $r = p-q$, which belongs to (2).

The remaining values for r in the both conditions are as follows.

- the first condition: $r = p-1,\ p+1$, or $2p-q$
- the second condition: $r = \alpha q - 1,\ \alpha q + 1$, or $\alpha q - p$.

We handle each values of r in the second condition. First suppose that $r = \alpha q - 1$. Since p and q are coprime, $\alpha q - 1 \neq p - 1$. If $\alpha q - 1 = p + 1$, then $p = \alpha q - 2$ and $r = \alpha q - 1$, which belongs to the case (4) in the table. If $\alpha q - 1 = 2p - q$, then $2p = (\alpha + 1)q - 1$. Since $r = \alpha q - 1 = 2p - q \leq p + q$, $p \leq 2q$. Therefore $2p = (\alpha + 1)q - 1 \leq 4q$, which implies that $\alpha = 1, 2$, or 3. But if $\alpha = 1$, then $2p = 2q - 1$, a contradiction to $q < p$. If $\alpha = 3$, then $2p = 4q - 1$ and thus $2(p - 2q) = -1$, a contradiction. If $\alpha = 2$, then $2p = 3q - 1$

i.e., $3q-2p=1$. Therefore $p=3J+1$ and $q=2J+1$ for $J>0$. Therefore $r=4J+1$, which belongs to (5) in the table.

Second, suppose $r=\alpha q+1$. Then since p and q are coprime, $\alpha q+1\neq p+1$. If $\alpha q+1=p-1$, then $p=\alpha q+2$ and $r=\alpha q+1$, which belongs to (4) in the table. If $\alpha q+1=2p-q$, then $2p=(\alpha+1)q+1$. Since $r\leq p+q$ and thus $p\leq 2q$, $(\alpha+1)q+1\leq 4q$. This implies that $\alpha=1$ or 2. But if $\alpha=1$, then $2p=2q+1$, contradiction. If $\alpha=2$, then $3q-2p=-1$. Hence $p=3J-1$ and $q=2J-1$ for $J>1$. Therefore $r=4J-1$, which belongs to (5) in the table.

Third, suppose $r=\alpha q-p$. Note that in this case, $\alpha>1$. Since p and q are coprime, $\alpha q-p\neq 2p-q$. If $\alpha q-p=p-1$, then $2p=\alpha q+1$ and thus α must be an odd number. Put $\alpha=2J'+1$, where $J'>0$. Since $2p-\alpha q=1$, $q=2J-1$ and $p=(2J-1)J'+J$, where $J>1$. This belongs to the case (6).

If $\alpha q-p=p+1$, then $2p=\alpha q-1$ and thus α must be an odd number. Put $\alpha=2J'+1$, where $J'>0$. Since $2p-\alpha q=-1$, $q=2J+1$ and $p=(2J+1)J'+J$, where $J>0$. This belongs to the case (6). □

Remark 3.7. (1) If $K(p,q,r,m,n)$ is in the case (1) in the table, i.e., $r=1$, then $K(p,q,r,m,n)$ is the connected sum of two torus knots $T(p,q)$ and $T(m,n)$.

(2) If $K(p,q,r,m,n)$ is in the case (3) in the table, i.e., $r=p+q$, then the surface slope is $pq+(p+q)^2mn$. This enables us to conjecture by comparing slopes that the family of satellite knots constructed by Miyazaki and Motegi in [8] belongs to the case (3) in the table.

References

[1] J. Berge, *Some Knots with Surgeries Yielding Lens Spaces*, Unpublished manuscript. Univ. of Texas at Austin, 1990.

[2] J. Berge and S. Kang, *The hyperbolic* P/P, $\mathrm{P/SF_d}$, *and* $\mathrm{P/SF_m}$ *knots in* S^3, preprint.

[3] J. Dean, *Hyperbolic knots with small Seifert-fibered Dehn surgeries*, Ph.D. thesis, University of Texas at Austin, 1996.

[4] ——, *Small Seifert-fibered Dehn surgery on hyperbolic knots*, Algebr. Geom. Topol. **3** (2003), 435–472.

[5] M. Eudave-Muñoz, *Band sums of links which yield composite links. The cabling conjecture for strongly invertible knots*, Trans. Amer. Math. Soc. **330** (1992), no. 2, 463–501.

[6] ——, *On hyperbolic knots with Seifert fibered Dehn surgeries*, Topology Appl. **121** (2002), no. 1-2, 119–141.

[7] J. Kalliongis and C. M. Tsau, *Seifert fibered surgery manifolds of composite knots*, Proc. Amer. Math. Soc **108** (1990), no. 4, 1047–1053.

[8] K. Miyazaki and K. Motegi, *Seifert fibered manifolds and Dehn surgery*, Topology **36** (1997), no. 2, 579–603.

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