

**NON-EXISTENCE OF LIGHTLIKE SUBMANIFOLDS OF
INDEFINITE TRANS-SASAKIAN MANIFOLDS WITH
NON-METRIC θ -CONNECTIONS**

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ABSTRACT. We study two types of 1-lightlike submanifolds, so-called lightlike hypersurface and half lightlike submanifold, of an indefinite trans-Sasakian manifold \bar{M} admitting non-metric θ -connection. We prove that there exist no such two types of 1-lightlike submanifolds of an indefinite trans-Sasakian manifold \bar{M} admitting non-metric θ -connections.

1. Introduction

A linear connection $\bar{\nabla}$ on a semi-Riemannian manifold (\bar{M}, \bar{g}) is called a *non-metric θ -connection* if, for any vector fields X, Y and Z on \bar{M} , it satisfies

$$(1.1) \quad (\bar{\nabla}_X \bar{g})(Y, Z) = -\theta(Y)\bar{g}(X, Z) - \theta(Z)\bar{g}(X, Y),$$

where θ is a 1-form, associated with a non-vanishing smooth vector field ζ by

$$\theta(X) = \bar{g}(X, \zeta).$$

Two special cases are important for both the mathematical study and the applications to physics: (1) A non-metric θ -connection $\bar{\nabla}$ on \bar{M} is called a *semi-symmetric non-metric connection* if its torsion tensor \bar{T} satisfies

$$\bar{T}(X, Y) = \theta(Y)X - \theta(X)Y.$$

The notion of semi-symmetric non-metric connections on a Riemannian manifold was introduced by Ageshe and Chafle [1] and later studied by many authors. The lightlike version of Riemannian manifolds with semi-symmetric non-metric connections has been studied by some authors [18, 20, 21, 23, 27].

(2) A non-metric θ -connection $\bar{\nabla}$ on \bar{M} is called a *quarter-symmetric non-metric connection* if its torsion tensor \bar{T} satisfies

$$\bar{T}(X, Y) = \theta(Y)\phi X - \theta(X)\phi Y,$$

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where ϕ is a $(1, 1)$ -type tensor field. Quarter-symmetric non-metric connection was introduced by S. Golad [12], and then, has been studied by many authors [2, 4, 26].

Oubina [24] introduced the notion of an indefinite trans-Sasakian manifold equipped with an indefinite almost contact metric structure $(J, \zeta, \theta, \bar{g})$ of type (α, β) . Indefinite Sasakian manifold is an important kind of indefinite trans-Sasakian manifold with $\alpha = 1$ and $\beta = 0$. Indefinite cosymplectic manifold is another kind of indefinite trans-Sasakian manifold such that $\alpha = \beta = 0$. Indefinite Kenmotsu manifold is also an example with $\alpha = 0$ and $\beta = 1$.

The theory of lightlike submanifolds is an important research topic in differential geometry due to its application in mathematical physics, especially in the general relativity. The study of such notion was initiated by Duggal and Bejancu [6] and later studied by many authors [8, 10]. 1-lightlike submanifold is a particular case of general r -lightlike submanifold [6] such that $r = 1$. Much of its geometry will be immediately generalized in a formal way to arbitrary r -lightlike submanifolds. Moreover the theory of 1-lightlike submanifold is a simple one more than that of r -lightlike submanifold. For this reason, we study only 1-lightlike submanifolds in this paper. Although now we have lightlike version of a large variety of Riemannian submanifolds, unfortunately, the geometry of lightlike submanifolds of indefinite trans-Sasakian manifolds admitting non-metric θ -connections has not been introduced yet.

In this paper, we study two types of 1-lightlike submanifolds, so-called lightlike hypersurface and half lightlike submanifold, of indefinite trans-Sasakian manifolds \bar{M} admitting non-metric θ -connections, in which the 1-form θ and its associated vector field ζ , defined by (1.1), is identical with the structure 1-form θ and its associated vector field ζ of the indefinite almost contact structure $(J, \zeta, \theta, \bar{g})$, respectively. We prove that (1) there exist no such two types of 1-lightlike submanifolds of indefinite trans-Sasakian manifolds \bar{M} admitting non-metric θ -connections, and (2) there exist no such two types of 1-lightlike submanifolds of indefinite trans-Sasakian manifolds \bar{M} admitting either semi-symmetric non-metric connections or quarter-symmetric non-metric connections.

2. Non-existence theorem for lightlike hypersurfaces

An odd-dimensional semi-Riemannian manifold (\bar{M}, \bar{g}) is said to be an *indefinite almost contact metric manifold* ([2, 3, 4, 9, 10], [13]~[17], [19]) if there exist a structure set $\{J, \zeta, \theta, \bar{g}\}$, where J is a $(1, 1)$ -type tensor field, ζ is a vector field which is called the *structure vector field* and θ is a 1-form such that

$$(2.1) \quad J^2X = -X + \theta(X)\zeta, \quad \bar{g}(JX, JY) = \bar{g}(X, Y) - \epsilon\theta(X)\theta(Y), \quad \theta(\zeta) = 1,$$

for any vector fields X and Y on \bar{M} , where $\epsilon = 1$ or -1 according as ζ is spacelike or timelike respectively. In this case, the structure set $\{J, \zeta, \theta, \bar{g}\}$ is called an *indefinite almost contact metric structure* of \bar{M} .

In an indefinite almost contact metric manifold, we show that $J\zeta = 0$ and $\theta \circ J = 0$. Such a manifold is said to be an *indefinite contact metric manifold* if $d\theta = \Phi$, where $\Phi(X, Y) = \bar{g}(X, JY)$ is called the *fundamental 2-form* of \bar{M} . The indefinite almost contact metric structure of \bar{M} is said to be *normal* if $[J, J](X, Y) = -2d\theta(X, Y)\zeta$ for any vector fields X and Y on \bar{M} , where $[J, J]$ denotes the Nijenhuis (or torsion) tensor field of J given by

$$[J, J](X, Y) = J^2[X, Y] + [JX, JY] - J[JX, Y] - J[X, JY].$$

An indefinite normal contact metric manifold is said to be an *indefinite Sasakian manifold*. It is well known [10, 25] that an indefinite almost contact metric manifold $(\bar{M}, \bar{g}, J, \zeta, \theta)$ is indefinite Sasakian if and only if

$$(\bar{\nabla}_X J)Y = \bar{g}(X, Y)\zeta - \epsilon\theta(Y)X,$$

for any vector fields X and Y of \bar{M} , where $\bar{\nabla}$ is the Levi-Civita connection of \bar{M} with respect to the semi-Riemannian metric \bar{g} .

Definition. An indefinite almost contact metric manifold (\bar{M}, \bar{g}) is called an *indefinite trans-Sasakian manifold* [24] if there exist two smooth functions α and β such that

$$(2.2) \quad (\bar{\nabla}_X J)Y = \alpha\{\bar{g}(X, Y)\zeta - \epsilon\theta(Y)X\} + \beta\{\bar{g}(JX, Y)\zeta - \epsilon\theta(Y)JX\},$$

for any vector fields X and Y on \bar{M} . In this case, we say that $\{J, \zeta, \theta, \bar{g}\}$ is an *indefinite trans-Sasakian structure of type (α, β)* .

Replacing Y by ζ in (3.2), we get

$$(2.3) \quad \bar{\nabla}_X \zeta = -\epsilon\alpha JX + \epsilon\beta(X - \theta(X)\zeta).$$

If $\beta = 0$, then \bar{M} is called an *indefinite α -Sasakian manifold*. Indefinite Sasakian manifolds [9, 13, 14, 15] appear as examples of indefinite α -Sasakian manifolds, with $\alpha = 1$. Another important kind of indefinite trans-Sasakian manifold is that of indefinite cosymplectic manifolds [17, 22] obtained for $\alpha = \beta = 0$. If $\alpha = 0$, then \bar{M} is called an *indefinite β -Kenmotsu manifold*. Indefinite Kenmotsu manifolds [16, 19] are particular examples of indefinite β -Kenmotsu manifold, with $\beta = 1$.

Let (M, g) be a lightlike hypersurface of (\bar{M}, \bar{g}) . It is well known that the normal bundle TM^\perp of M is a vector subbundle of the tangent bundle TM , of rank 1, and coincides with the radical distribution $Rad(TM) = TM \cap TM^\perp$. A complementary vector bundle $S(TM)$ of TM^\perp in TM is non-degenerate distribution on M , which is called a *screen distribution* on M , such that

$$(2.4) \quad TM = TM^\perp \oplus_{orth} S(TM),$$

where \oplus_{orth} denotes the orthogonal direct sum. We denote such a lightlike hypersurface by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M . It is well-known [6] that, for any null section ξ of TM^\perp on

a coordinate neighborhood $\mathcal{U} \subset M$, there exists a unique null section N of a unique vector bundle $tr(TM)$ in $S(TM)^\perp$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = 0, \quad \forall X \in \Gamma(S(TM)).$$

We call $tr(TM)$ and N the *transversal vector bundle* and the *null transversal vector field* of M with respect to the screen distribution respectively. Then the tangent bundle $T\bar{M}$ of \bar{M} is decomposed as follow:

$$(2.5) \quad T\bar{M} = TM \oplus tr(TM) = \{TM^\perp \oplus tr(TM)\} \oplus_{orth} S(TM).$$

Let P be the projection morphism of TM on $S(TM)$ with respect to the decomposition (2.4). From the decompositions (2.4) and (2.5), the local Gauss and Weingartan formulas of M and $S(TM)$ are given respectively by

$$(2.6) \quad \bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N,$$

$$(2.7) \quad \bar{\nabla}_X N = -A_N X + \tau(X)N,$$

$$(2.8) \quad \nabla_X PY = \nabla_X^* PY + C(X, PY)\xi,$$

$$(2.9) \quad \nabla_X \xi = -A_\xi^* X - \sigma(X)\xi,$$

for any $X, Y \in \Gamma(TM)$, where ∇ and ∇^* are the induced linear connections on TM and $S(TM)$ respectively, B and C are the local second fundamental forms on TM and $S(TM)$ respectively, A_N and A_ξ^* are the shape operators on TM and $S(TM)$ respectively, and τ and σ are 1-forms on TM .

The induced connection ∇ on M is not metric and satisfies

$$(2.10) \quad (\nabla_X g)(Y, Z) = B(X, Y)\eta(Z) + B(X, Z)\eta(Y) \\ - \theta(Y)g(X, Z) - \theta(Z)g(X, Y),$$

for any $X, Y, Z \in \Gamma(TM)$, where η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X, N), \quad \forall X \in \Gamma(TM).$$

From the fact that $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$, we know that B is independent of the choice of the screen distribution $S(TM)$, and satisfies

$$(2.11) \quad B(X, \xi) = 0, \quad \forall X \in \Gamma(TM).$$

From this result, (2.6) and (2.9), for all $X \in \Gamma(TM)$ we obtain

$$(2.12) \quad \bar{\nabla}_X \xi = -A_\xi^* X - \sigma(X)\xi.$$

In the entire discussion of this article, we shall assume that the structure vector field ζ of \bar{M} to be unit spacelike, *i.e.*, $\epsilon = 1$, without loss of generality. In general, the structure vector field ζ of \bar{M} is decomposed by

$$\zeta = \omega + a\xi + bN,$$

where ω is a smooth vector field on $S(TM)$, and a and b are smooth functions defined by $a = \theta(N)$ and $b = \theta(\xi)$. For any $X, Y \in \Gamma(TM)$, the above two second fundamental forms B and C are related to their shape operators by

$$(2.13) \quad g(A_\xi^* X, Y) = B(X, Y) - bg(X, Y), \quad \bar{g}(A_\xi^* X, N) = 0,$$

$$(2.14) \quad \begin{aligned} g(A_N X, PY) &= C(X, PY) - ag(X, PY) - \eta(X)\theta(PY), \\ \bar{g}(A_N X, N) &= -a\eta(X), \quad \sigma(X) = \tau(X) - b\eta(X). \end{aligned}$$

Now we quote the following result by Jin [13, 16, 17]:

Lemma 2.1. *Let M be a lightlike hypersurface of an indefinite almost contact metric manifold \bar{M} . Then the distributions $J(TM^\perp)$ and $J(\text{tr}(TM))$ are vector subbundles of $S(TM)$, of rank 1.*

Theorem 2.2. *There exist no lightlike hypersurfaces of indefinite trans-Sasakian manifolds admitting non-metric θ -connections.*

Proof. Now we consider a vector field V on $S(TM)$ and a 1-form v such that

$$(2.15) \quad V = -J\xi, \quad v(X) = g(X, V), \quad \forall X \in \Gamma(TM).$$

For any $X \in \Gamma(TM)$, by (2.5) the action JX of X by J is expressed as

$$(2.16) \quad JX = FX + v(X)N,$$

where FX is the tangential component of the vector field JX . Applying $\bar{\nabla}_X$ to (2.15)₁ and using (2.2), (2.6), (2.12) and (2.16), we have

$$\begin{aligned} \nabla_X V &= F(A_\xi^* X) - \sigma(X)V + \alpha bX + \beta bFX - \beta u(X)\{\omega + a\xi\}, \\ B(X, V) &= v(A_\xi^* X), \quad \forall X \in \Gamma(TM). \end{aligned}$$

On the other hand, taking $Y = V$ in (2.13) and using (2.15)₂, we have

$$B(X, V) = v(A_\xi^* X) + bv(X), \quad \forall X \in \Gamma(TM).$$

From the last two equations, we obtain $bv(X) = 0$ for all $X \in \Gamma(TM)$. Taking $X = V$ in this result and using (2.1), we get $b = 0$. This implies that ζ is tangent to M . Replacing Y by ζ to (2.6) and using (2.3), we obtain

$$(2.17) \quad \nabla_X \zeta = \beta X - \alpha FX - \beta\theta(X)\zeta,$$

$$(2.18) \quad B(X, \zeta) = -\alpha v(X), \quad \forall X \in \Gamma(TM).$$

Applying ∇_X to $g(\zeta, \zeta) = 1$ and using (2.10), we get

$$g(\nabla_X \zeta, \zeta) = \theta(X) - aB(X, \zeta), \quad \forall X \in \Gamma(TM).$$

Substituting (2.17) into the last equation and using (2.16) and (2.18), we get $\theta(X) = 0$ for all $X \in \Gamma(TM)$. It is a contradiction as $\theta(\zeta) = 1$. Thus there exist no lightlike hypersurfaces of indefinite trans-Sasakian manifolds admitting non-metric θ -connections. \square

Corollary 2.3. *There exist no lightlike hypersurfaces of indefinite trans-Sasakian manifolds admitting either semi-symmetric non-metric connections or quarter-symmetric non-metric connections.*

3. Non-existence theorem for half lightlike submanifolds

A submanifold (M, g) of a semi-Riemannian manifold \bar{M} of codimension 2 is called a *half lightlike submanifold* if the radical distribution $Rad(TM) = TM \cap TM^\perp$ of M is a vector subbundle of the tangent bundle TM and the normal bundle TM^\perp of rank 1. Then there exists complementary non-degenerate distributions $S(TM)$ and $S(TM^\perp)$ of $Rad(TM)$ in TM and TM^\perp respectively, which are called the *screen* and *co-screen* distributions on M , such that

$$(3.1) \quad TM = Rad(TM) \oplus_{orth} S(TM), \quad TM^\perp = Rad(TM) \oplus_{orth} S(TM^\perp).$$

We denote such a half lightlike submanifold by $M = (M, g, S(TM), S(TM^\perp))$. Without loss of generality, choose a unit spacelike vector field $L \in \Gamma(S(TM^\perp))$. Consider the orthogonal complementary distribution $S(TM)^\perp$ to $S(TM)$ in $T\bar{M}$. Certainly, $Rad(TM)$ and $S(TM^\perp)$ are vector subbundles of $S(TM)^\perp$. As the co-screen distribution $S(TM^\perp)$ is non-degenerate, we have

$$S(TM)^\perp = S(TM^\perp) \oplus_{orth} S(TM^\perp)^\perp,$$

where $S(TM^\perp)^\perp$ is the orthogonal complementary to $S(TM^\perp)$ in $S(TM)^\perp$. For any null section ξ of $Rad(TM)$, there exists a uniquely defined lightlike vector bundle $ltr(TM)$ and a null vector field N of $ltr(TM)$ satisfying

$$\bar{g}(\xi, N) = 1, \quad \bar{g}(N, N) = \bar{g}(N, X) = \bar{g}(N, L) = 0, \quad \forall X \in \Gamma(S(TM)).$$

We call N , $ltr(TM)$ and $tr(TM) = S(TM^\perp) \oplus_{orth} ltr(TM)$ the *lightlike transversal vector field*, *lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to $S(TM)$ respectively [7]. Thus $T\bar{M}$ is decomposed as

$$(3.2) \quad T\bar{M} = TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \\ = \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^\perp).$$

The local Gauss and Weingarten formulas of M and $S(TM)$ are given by

$$(3.3) \quad \bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N + D(X, Y)L,$$

$$(3.4) \quad \bar{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)L,$$

$$(3.5) \quad \bar{\nabla}_X L = -A_L X + \phi(X)N,$$

$$(3.6) \quad \nabla_X PY = \nabla_X^* PY + C(X, PY)\xi,$$

$$(3.7) \quad \nabla_X \xi = -A_\xi^* X - \sigma(X)\xi,$$

for all $X, Y \in \Gamma(TM)$, where ∇ and ∇^* are induced linear connections on TM and $S(TM)$ respectively, B and D are called the *local second fundamental forms* of M , C is called the *local second fundamental form* on $S(TM)$. A_N , A_ξ^* and A_L are linear operators on TM and τ , ρ , ϕ and σ are 1-forms on TM .

Using (1.1) and (3.3), for all $X, Y, Z \in \Gamma(TM)$ we have

$$(3.8) \quad (\nabla_X g)(Y, Z) = B(X, Y)\eta(Z) + B(X, Z)\eta(Y) \\ - \theta(Y)g(X, Z) - \theta(Z)g(X, Y).$$

From the facts $B(X, Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X, Y) = \bar{g}(\bar{\nabla}_X Y, L)$, we know that B and D are independent of the choice of $S(TM)$ and satisfy

$$(3.9) \quad B(X, \xi) = 0, \quad D(X, \xi) = -\phi(X), \quad \forall X \in \Gamma(TM).$$

From this result, (3.3) and (3.7), for all $X \in \Gamma(TM)$ we obtain

$$(3.10) \quad \bar{\nabla}_X \xi = -A_\xi^* X - \sigma(X)\xi - \phi(X)L.$$

In general, the torsion vector field ζ of \bar{M} is decomposed by

$$\zeta = \omega + a\xi + bN + eL,$$

where ω is a smooth vector field on $S(TM)$, and a , b and e are smooth functions defined by $a = \theta(N)$, $b = \theta(\xi)$ and $e = \theta(L)$. For any $X, Y \in \Gamma(TM)$, the above three local second fundamental forms are related to their shape operators by

$$(3.11) \quad g(A_\xi^* X, Y) = B(X, Y) - bg(X, Y), \quad \bar{g}(A_\xi^* X, N) = 0,$$

$$(3.12) \quad g(A_L X, Y) = D(X, Y) - eg(X, Y) + \phi(X)\eta(Y),$$

$$\bar{g}(A_L X, N) = \rho(X) - e\eta(X),$$

$$(3.13) \quad g(A_N X, PY) = C(X, PY) - ag(X, PY) - \eta(X)\theta(PY),$$

$$\bar{g}(A_N X, N) = -a\eta(X), \quad \sigma(X) = \tau(X) - b\eta(X).$$

Now we quote the following result by Jin [14, 15, 19]:

Lemma 3.1. *Let M be a half lightlike submanifold of an indefinite almost contact metric manifold \bar{M} . Then the distributions $J(TM^\perp)$, $J(\text{tr}(TM))$ and $J(S(TM^\perp))$ are vector subbundles of $S(TM)$, of rank 1.*

Theorem 3.2. *There exist no half lightlike submanifolds of an indefinite trans-Sasakian manifolds admitting non-metric θ -connections.*

Proof. Now we consider three vector fields V , W and U on $S(TM)$ such that

$$(3.14) \quad V = -J\xi, \quad W = -JL, \quad U = -JN.$$

For any $X \in \Gamma(TM)$, by (3.2) the action JX of X by J is expressed as

$$(3.15) \quad JX = FX + v(X)N + w(X)L,$$

where FX is the tangential component of JX , and u and w are 1-forms given by

$$(3.16) \quad v(X) = g(X, V) \quad w(X) = g(X, W), \quad \forall X \in \Gamma(TM).$$

Applying $\bar{\nabla}_X$ to (3.14)₁ and using (2.2), (3.3), (3.10) and (3.15), we have

$$(3.17) \quad \begin{aligned} \bar{\nabla}_X V &= F(A_\xi^* X) - \sigma(X)V - \phi(X)W + \alpha bX + \beta bFX \\ &\quad - \beta v(X)\{\omega + a\xi\}, \end{aligned}$$

$$(3.18) \quad B(X, V) = v(A_\xi^* X), \quad D(X, V) = w(A_\xi^* X) - \beta\{ev(X) - bw(X)\}$$

for all $X \in \Gamma(TM)$. Taking $Y = V$ in (3.11) and using (3.16)₁, we have

$$B(X, V) = v(A_\xi^* X) + bv(X).$$

From this and (3.18)₁, we have $bv(X) = 0$ for any $X \in \Gamma(TM)$. Thus we get $b = 0$. It follows that $B(X, Y) = g(A_\xi^*X, Y)$ and $\tau = \sigma$. Applying $\bar{\nabla}_X$ to (3.14)₂ and using (2.2), (3.3), (3.5), (3.14) and (3.15), we have

$$(3.19) \quad \nabla_X W = F(A_L X) + \phi(X)U + \alpha eX + \beta eFX - \beta w(X)\{\omega + a\xi\},$$

$$(3.20) \quad B(X, W) = v(A_L X) + \beta ev(X), \quad D(X, W) = w(A_L X).$$

On the other hand, taking $Y = W$ in (3.12), we have

$$D(X, W) = w(A_L X) + ew(X).$$

From this equation and (3.20)₂, we obtain $ew(X) = 0$ for any $X \in \Gamma(TM)$. Thus we get $e = 0$. As $b = e = 0$, the structure vector field ζ of \bar{M} is tangent to M . Replacing Y by ζ to (3.3) and using (2.3), we obtain

$$(3.21) \quad \nabla_X \zeta = \beta X - \alpha FX - \beta \theta(X)\zeta,$$

$$(3.22) \quad B(X, \zeta) = -\alpha v(X), \quad D(X, \zeta) = -\alpha w(X)$$

for all $X \in \Gamma(TM)$. Applying ∇_X to $g(\zeta, \zeta) = 1$ and using (3.8), we get

$$g(\nabla_X \zeta, \zeta) = \theta(X) - \alpha B(X, \zeta), \quad \forall X \in \Gamma(TM).$$

Substituting (3.21) into the last equation and using (3.15), (3.22) and the fact that ζ is tangent to M , we have $\theta(X) = 0$ for all $X \in \Gamma(TM)$. It is a contradiction as $\theta(\zeta) = 1$. Thus there exist no half lightlike submanifolds of indefinite trans-Sasakian manifolds admitting non-metric θ -connections. \square

Corollary 3.3. *There exist no half lightlike submanifolds of an indefinite trans-Sasakian manifolds admitting either semi-symmetric or quarter-symmetric non-metric connections.*

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