Surface Color Measurement Uncertainties

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We present a surface color measurement including quantities of surface color, methods, and uncertainty evaluation. Based on a relation between spectral reflectance and surface color, we study how an uncertainty of spectral reflectance propagates to surface color. In analyzing the uncertainty propagation, we divide the uncertainty into uncorrelated components, fully correlated components, and correlated components with spectrally varying correlations. As an experimental example, we perform spectro-reflectometric measurements for ceramic color plates. With measured spectral reflectance and its uncertainty evaluation, we determine surface color and analyze uncertainties of the ceramic color plates.

I. INTRODUCTION

Measurements of surface color have been developed to quantify color stimulus based on understanding of human color perception. For fundamentals of colorimetry, the Commission International de l'Eclairage (CIE) defined the standard physical data of illuminants and the standard observer data on spectral sensitivities of human eyes. [1] With these bases and surface reflectance of an object, the surface color of the object is quantified and described. The CIE tristimulus values were defined as basic quantities to describe color stimulus, and various chromaticity coordinates and color spaces were developed for its better description. In particular, much attention has been paid to mathematical models describing colors and color differences to be analogous to those that a human perceives, which results in the uniform color spaces of CIE (L*a*b*) and CIE (L*u*v*) and the color-difference formula of CIEDE2000. [1, 2] With these mathematical models, various methods and instruments for measuring surface color were developed and disseminated to the color industry. [2-4] Also, as novel material technology has grown and its application has extended to commercial products to cause complex color perception such as gloss, pearlescence, texture, and goniochromatism, new measurement techniques have been developed to characterize material properties with relevance to the complex color perception. [5-8] On account of such development of colorimetry and its industrial applications, surface color measurements have been investigated in terms of an uncertainty. An uncertainty in surface color measurements is interpreted as a detection threshold of color difference, .i.e., it is analogous to the color discrimination of human eyes. However, a few publications report correlations among spectral components in detail, although an intrinsic spectral overlap of color stimulus causes a correlation and makes a large contribution to an uncertainty of surface color. [9, 10]

In this paper, we present an uncertainty analysis on spectro-reflectometric surface color measurement. We focus on where an uncertainty occurs in a spectro-reflectometric measurement and how the uncertainty propagates to surface color. First, we introduce quantities and formulas of surface color in Section 2.1. and explain measurement methods of a color-filter-employed method and a spectro-reflectometric method in comparison with each other in Section 2.2. In Section III, we explain an uncertainty analysis of surface color. We derive relations between the quantities of surface color and relations between uncertainties of surface color and those of spectral reflectance corresponding to the measurand of a spectro-reflectometric measurement in Sections 3.1. and 2. We refer to ref. [9] in dealing with correlations

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among spectral components. An uncertainty evaluation method for spectro-reflectometric measurements is described in Section 3.3. In Section IV, we perform spectro-reflectometric surface color measurements for various color plates as an experimental example. Quantities and uncertainties of surface color are determined and analyzed with the measurement results of spectral reflectance.

II. SURFACE COLOR

2.1. Quantities of Surface Color

Human eyes respond to visible electromagnetic radiation in a wavelength range from 360 nm to 830 nm. When light enters an eye, the light is converted into electrical signals by photoreceptors on the retina and the signals are transmitted to the brain via optic nerves. Among two types of photoreceptors, i.e. cones and rods, cones are mainly responsible for color perception due to three cone types with different spectral sensitivities. The three cone types are called long-, middle-, and short-wavelength sensitive cones, respectively, according to their positions of spectral sensitivities in the visible wavelength range. Humans perceive two stimuli as identical color when the two stimuli activate cones to make the same signals. [11] The CIE defines tristimulus values on color stimulus as sums of products of the CIE color matching functions and color stimulus functions at each wavelength in the entire visible wavelength range from 360 nm to 830 nm as follows:

$$X = k\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{x}(\lambda)\Delta\lambda, \qquad X_{10} = k_{10}\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{x}_{10}(\lambda)\Delta\lambda,$$
$$Y = k\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{y}(\lambda)\Delta\lambda, \qquad Y_{10} = k_{10}\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{y}_{10}(\lambda)\Delta\lambda,$$
$$Z = k\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{z}(\lambda)\Delta\lambda, \qquad Z_{10} = k_{10}\sum_{\lambda}\phi_{\lambda}(\lambda)\overline{z}_{10}(\lambda)\Delta\lambda.$$
$$(1)$$

Here, X, Y, and Z are the CIE tristimulus values for the CIE 1931 standard colorimetric observer and X_{10}, Y_{10}, and Z_{10} are the CIE tristimulus values for the CIE 1964 standard colorimetric observer. The CIE defined specified virtual observers of color stimuli as colorimetric fundamentals, which are the CIE standard colorimetric observers obtained through color matching experiments using primaries of red, green, and blue. [1, 2] $\overline{x}(\lambda)$, $\overline{y}(\lambda)$, and $\overline{z}(\lambda)$ are the color matching functions of the CIE 1931 standard colorimetric observer for 2° visual field and $\overline{x}_{10}(\lambda)$, $\overline{y}_{10}(\lambda)$, and $\overline{z}_{10}(\lambda)$ are the color matching functions of the CIE 1964 standard colorimetric observer for 10° visual field. $\phi_{\lambda}(\lambda)$ denotes the spectral distribution of color stimuli which corresponds to a product of relative spectral distribution of the CIE standard illuminant and spectral reflectance of the object in the case of a reflecting object. The CIE standard illuminant is a reference illuminant that includes the CIE standard

illuminant A and the CIE standard illuminant D65. $\Delta(\lambda)$ is a wavelength interval in summing products of the CIE color matching functions and the color stimulus functions, which is generally chosen to be 1 nm or 5 nm. Normalization factors of k and k_{10} are determined to satisfy $Y = 100$ for an object with spectral reflectance equal to 1 over all the visible wavelengths.

Chromaticity coordinates were developed to transform the tristimulus values into two-dimensional coordinates. The CIE 1931 chromaticity coordinates (x, y, z) are given by normalizing the CIE tristimulus values of (X, Y, Z) with their sum, i.e.,

$$x = \frac{X}{X+Y+Z}, \qquad y = \frac{Y}{X+Y+Z}, \qquad z = \frac{Z}{X+Y+Z}.$$
$$(2)$$

With this transformation, two-dimensional description of chromaticity is possible using (x, y) along with a tristimulus value Y to indicate luminance. Similarly, the CIE 1964 chromaticity coordinates (x_{10}, y_{10}, z_{10}) are derived from X_{10}, Y_{10}, and Z_{10}.

The CIE recommends uniform coordinates in cases where a perceptually uniform color space is required. [1, 2] Among uniform color spaces, the CIE 1976 ($L*a*b*$) is an opponent-type system; $a*$ is a coordinate that represents greenness-redness, $b*$ represents yellowness-blueness, and $L*$ corresponds to luminance characteristics. $L*a*b*$ coordinates are defined with terms of the tristimulus values as follows:

$$L* = 116f(Y/Y_n) - 16$$
$$a* = 500[f(X/X_n) - f(Y/Y_n)]$$
$$b* = 200[f(Y/Y_n) - f(Z/Z_n)].$$
$$(3)$$

Here, $f(X/X_n)$, $f(Y/Y_n)$, and $f(Z/Z_n)$ are given with the following equations of

$$f(X/X_n) = (X/X_n)^{1/3} \qquad \text{if } (X/X_n) > (24/116)^3$$
$$f(X/X_n) = (841/108)(X/X_n) + 16/116 \quad \text{if } (X/X_n) \le (24/116)^3$$
$$f(Y/Y_n) = (Y/Y_n)^{1/3} \qquad \text{if } (Y/Y_n) > (24/116)^3$$
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$$f(Z/Z_n) = (Z/Z_n)^{1/3} \qquad \text{if } (Z/Z_n) > (24/116)^3$$
$$f(Z/Z_n) = (841/108)(Z/Z_n) + 16/116 \quad \text{if } (Z/Z_n) \le (24/116)^3,$$
$$(4)$$

where X_n, Y_n, and Z_n are the tristimulus values of a perfectly reflecting diffuser, namely, the tristimulus values of a light source with Y_n equal to 100.

2.2. Measurement Method

There are two methods in measuring surface color. One uses color filters to mimic a colorimetric observer so that it directly determines tristimulus values. The other is a spectro-

reflectometric measurement, which measures spectral reflectance and determines tristimulus values as well as chromaticity coordinates through the calculations of Eqs. (1)~(4). The color-filter-employed method is practical in field experiments, because the equipment, which consists of a color-filter set and a light source, is small, easy to use, and rapidly operating. However, the color-filter-employed method is inferior to the spectro-reflectometric method in precision and accuracy. Furthermore, the spectro-reflectometric method is advanced in determining chromaticity coordinates under any pair of an observer and an illuminant as being based on a spectral reflectance measurement, whereas the color-filter employed method is restricted to a specified pair of an observer and an illuminant. In the following chapter, we evaluate a measurement uncertainty of surface color through an analysis of a spectro-reflectometric measurement.

III. UNCERTAINTY ANALYSIS

3.1. Tristimulus Values

To evaluate uncertainties of the tristimulus values, we analyze uncertainties of the spectro-reflectometric measurement and their propagation to the tristimulus values. The tristimulus values of (X, Y, Z) of Eq. (1) are written as Eq. (5) to show spectral reflectance corresponding to measurand of a spectro-reflectometric measurement:

$$X = \sum_{i=1}^{m} W_{X,i}\rho_i \qquad Y = \sum_{i=1}^{m} W_{Y,i}\rho_i \qquad Z = \sum_{i=1}^{m} W_{Z,i}\rho_i .$$

$$(5)$$

Here, i is an index of wavelength with a uniform spacing, ρ_i is the reflectance of a test sample at the ith wavelength, and $W_{X,i}$, $W_{Y,i}$, and $W_{Z,i}$ are weighting factors of normalized tristimulus values at the ith wavelength which are products of the color matching functions of the CIE 1931 standard colorimetric observer, the relative spectral distributions of

the CIE illuminant, and the normalization factors. With the weighting factors of $W_{X10,i}$, $W_{Y10,i}$, and $W_{Z10,i}$ which correspond to products of the color matching functions of the CIE 1964 standard colorimetric observer, the relative spectral distributions of the CIE illuminant, and the normalization factors, the tristimulus values of (X_{10}, Y_{10}, Z_{10}) are described in a similar way;

$$X_{10} = \sum_{i=1}^{m} W_{X10,i}\rho_i, \; Y_{10} = \sum_{i=1}^{m} W_{Y10,i}\rho_i, \; Z_{10} = \sum_{i=1}^{m} W_{Z10,i}\rho_i.$$

In Fig. 1(a) and (b) are plotted the weighting factors as a function of wavelength for the CIE illuminants A and D65, respectively. A combined standard uncertainty of a tristimulus value X, denoted by $u_c(X)$, is expressed with uncertainties of reflectance by differentiating Eq. (5). Uncertainties with correlations among wavelength components and uncorrelated uncertainties are combined differently according to the error propagation formula as shown in Eq. (6) [12]:

$$u_c^2(X) = \sum_{i=1}^{m} W_{X,i}^2 \cdot u^2(\rho_i)_{\text{uncorr}}$$

$$(6)$$

$$+ \sum_{i=1}^{m}\sum_{j=i}^{m} W_{X,i} \cdot W_{X,j} \cdot r(\rho_i, \rho_j) \cdot u(\rho_i)_{\text{corr}} \cdot u(\rho_j)_{\text{corr}}.$$

Here, $u(\rho_i)_{\text{uncorr}}$ is an uncorrelated uncertainty of ρ_i, $u(\rho_i)_{\text{corr}}$ and $u(\rho_j)_{\text{corr}}$ are correlated uncertainties of reflectance at wavelength index i and j, respectively, and $r(\rho_i, \rho_j)$ is a correlation coefficient between $u(\rho_i)_{\text{corr}}$ and $u(\rho_j)_{\text{corr}}$.

Let us look at a spectro-reflectometric measurement to analyze uncertainties of the tristimulus values. Spectral reflectance of a sample is determined by comparison with that of a reference reflectance standard such as polytetrafluoroethylene (PTFE) or pressed barium sulphate plates. Therefore, a mathematical model of a spectro-reflectometric measurement is given by

$$\rho_i = \rho_{R,i} \frac{S_i - Z_i}{R_i - Z_i},$$

$$(7)$$

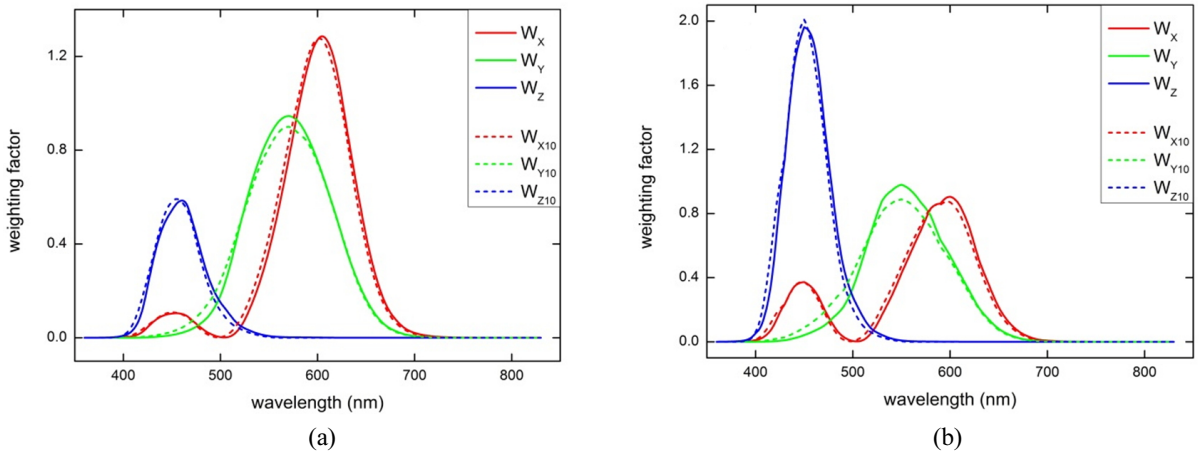(a)   (b)

FIG. 1. Graphs of weighting factors as a function of wavelength for the CIE A illuminant (a) and the CIE D65 illuminant (b).

where $\rho_{R,i}$ is the reflectance of a reference reflectance standard, S_i and R_i are the measured signals of a test sample and a reference, respectively, and Z_i is the signal in the absence of a sample at the ith wavelength. A combined standard uncertainty of spectral reflectance at the ith wavelength, $u_c(\rho_i)$, is obtained by differentiating Eq. (7), which is expressed with the uncertainties of the reference and the signals as follows:

$$u_c^2(\rho_i) \simeq \left[\left(\frac{\partial\rho_i}{\partial\rho_{R,i}}\right)^2 u^2(\rho_{R,i}) + \left(\frac{\partial\rho_i}{\partial S_i}\right)^2 u^2(S_i) + \left(\frac{\partial\rho_i}{\partial R_i}\right)^2 u^2(R_i)\cdot\right.$$
$$\left.+\left(\frac{\partial\rho_i}{\partial Z_i}\right)^2 u^2(Z_i) + 2\left(\frac{\partial\rho_i}{\partial S_i}\right)\cdot\left(\frac{\partial\rho_i}{\partial R_i}\right)\cdot r(S_i,R_i)\cdot u(S_i)\cdot u(R_i)\right].$$

(8)

Here, the correlation of Z_i with other signals is approximated as zero, and the sensitivity coefficients are given as

$$\frac{\partial\rho_i}{\partial\rho_{R,i}} = \frac{S_i - Z_i}{R_i - Z_i}, \qquad \frac{\partial\rho_i}{\partial S_i} = \frac{\rho_{R,i}}{R_i - Z_i}$$
$$\frac{\partial\rho_i}{\partial R_i} = -\rho_{R,i}\frac{S_i - Z_i}{(R_i - Z_i)^2}, \qquad \frac{\partial\rho_i}{\partial Z_i} = \rho_{R,i}\frac{S_i - Z_i}{(R_i - Z_i)^2}.$$

(9)

The uncorrelated uncertainty $u(\rho_i)_{\text{uncorr}}$ in Eq. (6) includes uncertainty components of sample non-uniformity, repeatability of measurement, signal stability, signal nonlinearity, and uncorrelated uncertainties of a reference standard. The correlated uncertainty components of $u(\rho_i)_{\text{corr}}$ correspond to reproducibility due to misalignment between a sample and a reference, stray light, wavelength inaccuracy, and uncertainties of a reference standard with correlations among wavelength components. Among the correlated components, the misalignment of a sample against a reference and the correlated uncertainties of a reference standard are considered to be fully correlated so that their correlation coefficients correspond to 1. The correlation coefficient of stray light is derived from Eqs. (8) and (9). To further look into the uncertainty of spectral reflectance, Eq. (8) is combined with Eq. (9) and expressed as a relative uncertainty, $u_c(\rho_i)\,/\,\rho_i$, as follows:

$$\frac{u_c^2(\rho_i)}{\rho_i^2} \simeq \frac{u^2(\rho_{R,i})}{\rho_{R,i}^2} + \frac{u^2(S_i)}{(S_i - Z_i)^2} + \frac{u^2(R_i)}{(R_i - Z_i)^2} + \left\{\frac{S_i - R_i}{(R_i - Z_i)(S_i - Z_i)}\right\}^2 u^2(Z_i)$$
$$-2\frac{1}{(S_i - Z_i)(R_i - Z_i)}\cdot r(S_i,R_i)\cdot u(S_i)\cdot u(R_i).$$

(10)

Stray light contributes to both the signals of a sample and that of a reference, and the uncertainties are considered to be fully correlated. Therefore, an uncertainty of stray light $u(\rho_i)_{\text{stray light}}$ is given by

$$\frac{u^2(\rho_i)_{\text{stray light}}}{\rho_i^2} \simeq \left\{\frac{u(S_i)_{\text{stray light}}}{(S_i - Z_i)} - \frac{u(R_i)_{\text{stray light}}}{(R_i - Z_i)}\right\}^2$$

(11)

with stray light uncertainties of signals, $u(S_i)_{\text{stray light}}$ and $u(R_i)_{\text{stray light}}$. With an approximation of $u(S_i)_{\text{stray light}} \simeq u(R_i)_{\text{stray light}}$, the correlation coefficient of $r(\rho_i,\ \rho_j)_{\text{stray light}}$ is estimated from the expanded formula of Eq. (11) as follows [9]:

$$r(\rho_i,\rho_j)_{\text{stray light}} = \text{sgn}\left(\frac{1}{(S_i - Z_i)} - \frac{1}{(R_i - Z_i)}\right)\cdot\ \text{sgn}\left(\frac{1}{(S_j - Z_j)} - \frac{1}{(R_j - Z_j)}\right)$$
$$= \text{sgn}\left(\frac{\rho_{R,i}}{\rho_i} - 1\right)\cdot\ \text{sgn}\left(\frac{\rho_{R,j}}{\rho_j} - 1\right).$$

(12)

Here, $\text{sgn}(x) = +1$ for $x>0$, $\text{sgn}(x) = -1$ for $x<0$, and $\text{sgn}(x) = 0$ for $x=0$.

Wavelength inaccuracy of equipment effects all spectral measurements over the entire wavelength range. An uncertainty of spectral reflectance originated from wavelength inaccuracy, $u(\rho_i)_\lambda$, is described with an uncertainty of wavelength inaccuracy, $u(\lambda)$, i.e.,

$$u(\rho_i)_\lambda \simeq \left(\frac{\partial\rho_i}{\partial\lambda}\right)u(\lambda)$$

(13)

A correlation among wavelength components depends on variation of spectral reflectance with wavelength change; the correlation coefficient between two wavelength components is approximated as +1, -1, or 0, for increase, decrease, or zero contribution to reflectance variation, respectively, since wavelength inaccuracy has systematic components such as a wavelength offset. [9] So, the correlation coefficient $r(\rho_i,\ \rho_j)_\lambda$ is expressed with

$$r(\rho_i,\rho_j)_\lambda = \text{sgn}\left(\frac{\partial\rho_i}{\partial\lambda}\right)\cdot\ \text{sgn}\left(\frac{\partial\rho_j}{\partial\lambda}\right).$$

(14)

Using the Eqs. of (11)~(14), Eq. (6) of $u_c(X)$ is written as the following equation:

$$u_c^2(X) = \sum_{i=1}^m W_{X,i}^2\cdot u^2(\rho_i)_{\text{uncorr}} + \left\{\sum_{i=1}^m W_{X,i}\cdot u(\rho_i)_{\text{fully corr}}\right\}^2$$
$$+ \sum_{i=1}^m\sum_{j=1}^m\left\{W_{X,i}\cdot W_{X,j}\cdot\text{sgn}\left(\frac{\rho_{R,i}}{\rho_i} - 1\right)\cdot\text{sgn}\left(\frac{\rho_{R,j}}{\rho_j} - 1\right)\cdot u(\rho_i)_{\text{stray light}}\cdot u(\rho_j)_{\text{stray light}}\right\}$$
$$+ \sum_{i=1}^m\sum_{j=1}^m\left\{W_{X,i}\cdot W_{X,j}\cdot\text{sgn}\left(\frac{\partial\rho_i}{\partial\lambda}\right)\cdot\text{sgn}\left(\frac{\partial\rho_j}{\partial\lambda}\right)\cdot\frac{\partial\rho_i}{\partial\lambda}\cdot\frac{\partial\rho_j}{\partial\lambda}\cdot u^2(\lambda)\right\}.$$

(15)

Here, $u(\rho_i)_{\text{fully corr}}$ includes the fully-correlated uncertainty

terms of the misalignment of a sample against a reference and the correlated uncertainties of a reference standard. We note that the uncertainties of Y and Z are described in the same way. We analyze each uncertainty component in Section 3.3.

3.2. Chromaticity Coordinates

Uncertainties of the chromaticity coordinates are obtained from those of tristimulus values according to the error propagation formula. [12] By differentiating Eq. (2), the uncertainties of the CIE 1931 (x, y) are given as

$$u_c^2(x) = \left(\frac{\partial x}{\partial X}\right)^2 u^2(X) + \left(\frac{\partial x}{\partial Y}\right)^2 u^2(Y) + \left(\frac{\partial x}{\partial Z}\right)^2 u^2(Z)$$
$$+2\left(\frac{\partial x}{\partial X}\right)\left(\frac{\partial x}{\partial Y}\right)u(X,Y) + 2\left(\frac{\partial x}{\partial X}\right)\left(\frac{\partial x}{\partial Z}\right)u(X,Z) + 2\left(\frac{\partial x}{\partial Y}\right)\left(\frac{\partial x}{\partial Z}\right)u(Y,Z),$$
$$u_c^2(y) = \left(\frac{\partial y}{\partial X}\right)^2 u^2(X) + \left(\frac{\partial y}{\partial Y}\right)^2 u^2(Y) + \left(\frac{\partial z}{\partial Z}\right)^2 u^2(Z)$$
$$+2\left(\frac{\partial y}{\partial X}\right)\left(\frac{\partial y}{\partial Y}\right)u(X,Y) + 2\left(\frac{\partial y}{\partial X}\right)\left(\frac{\partial y}{\partial Z}\right)u(X,Z) + 2\left(\frac{\partial y}{\partial Y}\right)\left(\frac{\partial y}{\partial Z}\right)u(Y,Z),$$

$$(16)$$

where the sensitivity coefficients correspond to

$$\frac{\partial x}{\partial X} = \frac{Y+Z}{(X+Y+Z)^2}, \quad \frac{\partial x}{\partial Y} = -\frac{X}{(X+Y+Z)^2}, \quad \frac{\partial x}{\partial Z} = -\frac{X}{(X+Y+Z)^2},$$
$$\frac{\partial y}{\partial X} = -\frac{Y}{(X+Y+Z)^2}, \quad \frac{\partial y}{\partial Y} = \frac{X+Z}{(X+Y+Z)^2}, \quad \frac{\partial y}{\partial Z} = -\frac{Y}{(X+Y+Z)^2}.$$

$$(17)$$

And the covariance of $u(X, Y)$ with the following form of

$$u(X,Y) = \sum_{i=1}^{m}\left(\frac{\partial X}{\partial \rho_i}\right)\left(\frac{\partial Y}{\partial \rho_i}\right)u^2(\rho_i)$$
$$+\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left[\frac{\partial X}{\partial \rho_i}\frac{\partial Y}{\partial \rho_j} + \frac{\partial X}{\partial \rho_j}\frac{\partial Y}{\partial \rho_i}\right] \cdot r(\rho_i, \rho_j) \cdot u(\rho_i) \cdot u(\rho_j)$$
$$= \sum_{i=1}^{m} W_{X,i} \cdot W_{Y,i} \cdot u^2(\rho_i)$$
$$+\sum_{i=1}^{m-1}\sum_{j=i+1}^{m}\left[W_{X,i} \cdot W_{Y,j} + W_{X,j} \cdot W_{Y,i}\right] \cdot r(\rho_i, \rho_j) \cdot u(\rho_i) \cdot u(\rho_j)$$

$$(18)$$

is expressed with Eq. (19), depending on the type of the correlation coefficient:

$$u(X,Y) = \sum_{i=1}^{m} W_{X,i} \cdot W_{Y,i} \cdot u^2(\rho_i)_{\text{uncorr}} + \sum_{i,j=1}^{m} W_{X,i} \cdot W_{Y,j} \cdot u(\rho_i)_{\text{full corr}} \cdot u(\rho_j)_{\text{full corr}}$$
$$+\sum_{i,j=1}^{m} W_{X,i} \cdot W_{Y,j} \cdot \text{sgn}\left(\frac{\rho_{R,i}}{\rho_i}-1\right) \cdot \text{sgn}\left(\frac{\rho_{R,j}}{\rho_j}-1\right) \cdot u(\rho_i)_{\text{stray light}} \cdot u(\rho_j)_{\text{stray light}}$$
$$+\sum_{i,j=1}^{m} W_{X,i} \cdot W_{Y,j} \cdot \text{sgn}\left(\frac{\partial \rho_i}{\partial \lambda}\right) \cdot \text{sgn}\left(\frac{\partial \rho_j}{\partial \lambda}\right) \cdot u^2(\lambda).$$

$$(19)$$

The covariance of $u(X, Z)$ and $u(Y, Z)$ are given with a similar form.

The uncertainties of CIE 1976 $(L^*a^*b^*)$ is derived by differentiating Eq. (3) in a similar way, which is given by

$$u_c^2(L^*) = \left(\frac{\partial L^*}{\partial Y}\right)^2 u^2(Y),$$
$$u_c^2(a^*) = \left(\frac{\partial a^*}{\partial X}\right)^2 u^2(X) + \left(\frac{\partial a^*}{\partial Y}\right)^2 u^2(Y) + 2\left(\frac{\partial a^*}{\partial X}\right)\left(\frac{\partial a^*}{\partial Y}\right)u(X,Y),$$
$$u_c^2(b^*) = \left(\frac{\partial b^*}{\partial Y}\right)^2 u^2(Y) + \left(\frac{\partial b^*}{\partial Z}\right)^2 u^2(Z) + 2\left(\frac{\partial b^*}{\partial Y}\right)\left(\frac{\partial b^*}{\partial Z}\right)u(Y,Z),$$

$$(20)$$

with the sensitivity coefficients of

$$\frac{\partial L^*}{\partial Y} = \frac{116}{3}\left(\frac{1}{Y^2 Y_n}\right)^{1/3}, \text{ if } (Y/Y_n) > (24/116)^3,$$
$$\frac{\partial L^*}{\partial Y} = \frac{116 \times 841}{108}\left(\frac{1}{Y_n}\right), \text{ if } (Y/Y_n) \le (24/116)^3,$$
$$\frac{\partial a^*}{\partial X} = \frac{500}{3}\left(\frac{1}{X^2 X_n}\right)^{1/3}, \text{ if } (X/X_n) > (24/116)^3,$$
$$\frac{\partial a^*}{\partial X} = \frac{500 \times 841}{108}\left(\frac{1}{X_n}\right), \text{ if } (X/X_n) \le (24/116)^3,$$
$$\frac{\partial a^*}{\partial Y} = \frac{-500}{3}\left(\frac{1}{Y^2 Y_n}\right)^{1/3}, \text{ if } (Y/Y_n) > (24/116)^3,$$
$$\frac{\partial a^*}{\partial Y} = -\frac{500 \times 841}{108}\left(\frac{1}{Y_n}\right), \text{ if } (Y/Y_n) \le (24/116)^3,$$
$$\frac{\partial b^*}{\partial Y} = \frac{200}{3}\left(\frac{1}{Y^2 Y_n}\right)^{1/3}, \text{ if } (Y/Y_n) > (24/116)^3,$$
$$\frac{\partial b^*}{\partial Y} = \frac{200 \times 841}{108}\left(\frac{1}{Y_n}\right), \text{ if } (Y/Y_n) \le (24/116)^3,$$
$$\frac{\partial b^*}{\partial Z} = -\frac{200}{3}\left(\frac{1}{Z^2 Z_n}\right)^{1/3}, \text{ if } (Z/Z_n) > (24/116)^3,$$
$$\frac{\partial b^*}{\partial Z} = -\frac{200 \times 841}{108}\left(\frac{1}{Z_n}\right), \text{ if } (Z/Z_n) \le (24/116)^3.$$

$$(21)$$

3.3. Uncertainty Evaluation of Spectral Reflectance

In the previous sections we looked into how uncertainties of spectral reflectance propagate into tristimulus values and chromaticity coordinates. In this section, we introduce evaluation methods of uncertainties in spectral reflectance measurements. First, we evaluate an uncorrelated uncertainty $u(\rho)_{\text{uncorr}}$ shown in Eqs. (15) and (19). The uncorrelated uncertainty terms are non-uniformity of sample, repeatability of measurement, signal stability, signal nonlinearity, and uncorrelated uncertainties of a reference standard. The sample non-uniformity is evaluated by measuring variation of spectral reflectance with change of light-illuminating position in a sample. Measurement repeatability, signal stability, and signal nonlinearity depend on the performance of equipment in use. We evaluate the uncertainties of repeatability and signal stability through repetitive measurements for a fixed sample and measurements

of signal variance with time, respectively. To evaluate the nonlinearity of signals, we measure light signals which pass through several apertures with adjusting of an illumination level using neutral density filters: Difference between a signal for 100% open aperture and a sum of two signals for 50% open aperture determines an uncertainty of signal nonlinearity. The uncorrelated uncertainties of a reference standard are referred to its calibration certificate. Second, let's look into the fully-correlated uncertainty terms of $u(\rho)_{\text{fully corr}}$. The misalignment of a sample against a reference causes an uncertainty, of which correlations among wavelength components are considered to be equal. We evaluate variance of spectral reflectance after realignment of a sample as fixing light-illuminating position in the sample. The correlated uncertainties of the reference are assumed to be fully correlated, which are referred to the calibration certificate.

Next, we evaluate an uncertainty of stray light $u(\rho)_{\text{stray light}}$. Stray light occurs in a spectrally resolving device such as a spectrograph or a monochromator, and contributes to both of the signals of a sample and a reference as shown in Eq. (11). Uncertainties of signals due to stray light at wavelength of λ, $u(S(\lambda))_{\text{stray light}}$ and $u(R(\lambda))_{\text{stray light}}$, are given with integrations of stray light components, [13] i.e.,

$$u\left(S(\lambda)\right)_{\text{stray light}} = \int_{\lambda_{DLL}}^{\lambda-\Delta\lambda} S\left(\lambda'\right)_{\text{stray light}} d\lambda' + \int_{\lambda+\Delta\lambda}^{\lambda_{DHL}} S\left(\lambda'\right)_{\text{stray light}} d\lambda'$$

$$u\left(R(\lambda)\right)_{\text{stray light}} = \int_{\lambda_{DLL}}^{\lambda-\Delta\lambda} R\left(\lambda'\right)_{\text{stray light}} d\lambda' + \int_{\lambda+\Delta\lambda}^{\lambda_{DHL}} R\left(\lambda'\right)_{\text{stray light}} d\lambda'.$$

(22)

Here, $S(\lambda')_{\text{stray light}}$ and $R(\lambda')_{\text{stray light}}$ are stray light components at wavelength of λ'. The uncertainties are expressed with an integral form to show its integration range. λ_{DLL} and λ_{DHL} are the lower and the upper limit of detection, and $\Delta\lambda$ is a spectral bandwidth of equipment. By combining Eqs. (22) and (11), the relative uncertainty is written as

$$\frac{u\left(\rho(\lambda)\right)_{\text{stray light}}}{\rho(\lambda)} \simeq \left\{ \int_{\lambda_{DLL}}^{\lambda-\Delta\lambda} \frac{\rho(\lambda')}{\rho(\lambda)} \cdot \frac{R(\lambda')_{\text{stray light}}}{R(\lambda)} d\lambda' \right. $$
$$ + \int_{\lambda+\Delta\lambda}^{\lambda_{DHL}} \frac{\rho(\lambda')}{\rho(\lambda)} \cdot \frac{R(\lambda')_{\text{stray light}}}{R(\lambda)} d\lambda' \right\}$$
$$ - \left\{ \int_{\lambda_{DLL}}^{\lambda-\Delta\lambda} \frac{R(\lambda')_{\text{stray light}}}{R(\lambda)} d\lambda' \right.$$
$$ \left. + \int_{\lambda+\Delta\lambda}^{\lambda_{DHL}} \frac{R(\lambda')_{\text{stray light}}}{R(\lambda)} d\lambda' \right\},$$

(23)

and simplified as Eq. (24) with an assumption that the uncertainty of stay light $R(\lambda') / R(\lambda)$ is constant:

$$\frac{u\left(\rho(\lambda)\right)_{\text{stray light}}}{\rho(\lambda)} \simeq u(\lambda,\lambda') \left\{ \frac{1}{\rho(\lambda)} \int_{\lambda_{DLL}}^{\lambda-\Delta\lambda} \rho(\lambda') d\lambda' \right.$$
$$ \left. + \frac{1}{\rho(\lambda)} \int_{\lambda+\Delta\lambda}^{\lambda_{DHL}} \rho(\lambda') d\lambda' - \left(\lambda_{DHL} - \lambda_{DLL} - 2\Delta\lambda\right) \right\}.$$

(24)

Here, $u(\lambda, \lambda')$ is an uncertainty of stray light with a constant value, i.e. $u(\lambda, \lambda') \equiv R(\lambda') / R(\lambda)$. To evaluate $u(\lambda, \lambda')$, we measure the spectral distribution of light which passes through equipment using a laser source with a sharp spectral peak; a ratio of a signal level at another wavelength to a signal at the laser wavelength corresponds to an uncertainty of heterochromatic stray light. Finally, wavelength inaccuracy of equipment is determined using standards of wavelength calibration. The uncertainty includes uncertainties of the calibration standard and readouts.

IV. EXAMPLE OF SURFACE COLOR MEASUREMENT

In this chapter, we show an example of surface color measurements. We used a spectrophotometer (Lambda1050, Perkin Elmer) for the spectro-reflectometric measurement. Spectral reflectance of a sample was determined to be relative to a reference standard with 99% reflectance (Spectralon, Labsphere), which was calibrated traceable to the KRISS spectral reflectance scale. We summarize the uncertainties of the spectro-reflectometric measurements in Table 1. We note that we exclude the uncertainty terms of sample non-uniformity, misalignment between sample and reference, and measurement repeatability in the Table 1, since they depend on a sample and a signal magnitude. Figure 2 shows samples of matte color plates (Ceramic colour standard, Ceram). We measured spectral reflectance under a reflection measurement geometry of 8°:de, i.e., light was incident at 8° and reflected flux was collected hemispherically using an integrating sphere with specular reflection excluded. [1]

In Fig. 3 are plotted the measured data of white, black, red, green, cyan, and blue color plates, and in Table 2 are

TABLE 1. Uncertainties of spectro-reflectometric measurement

System/Parameter	Standard uncertainty
Spectrophotometer	
Components	
Signal stability	0.02%
Signal nonlinearity	0.12% for (360~799) nm, 0.21% for (800~830) nm
Correlated Components	
Stray light	7×10^{-7}
Wavelength inaccuracy	0.08 nm
Reference standard	
Uncorrelated uncertainty	0.11%~0.17% (depending on wavelength)
Correlated uncertainty	0.31%~0.35% (depending on wavelength)

summarized the uncertainties described in Section 3.3. Signal noise is the dominant source in the uncorrelated uncertainty so that the black plate with a small signal-to-noise ratio shows the larger uncorrelated uncertainty. Similarly, the red and blue plates with partially low reflectance have large uncorrelated uncertainty in a wavelength range under 550 nm and from 500 nm to 670 nm, respectively. For the fully correlated uncertainty, systematic effects of a reference standard measurement contribute dominantly so that the samples

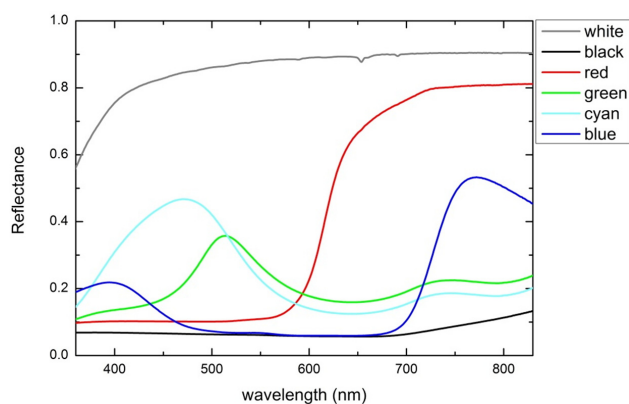FIG. 2. Color plates used in the surface color measurements.

FIG. 3. Spectral reflectance of color plates.

show equal uncertainty of 0.3%~0.4%, which depends on wavelength. The uncertainties of stray light and wavelength inaccuracy vary depending on the spectral shape of reflectance. Figure 4 shows the uncertainty of stray light (solid curve) and the uncertainty of wavelength inaccuracy (dashed curve) as a function of wavelength for each sample. As shown in Figs. 3 and 4, the contribution of stray light becomes large when a sample has large variance in spectral reflectance, which is verified in the cases of red and blue plates. The uncertainty of wavelength inaccuracy is large when a sample has rapidly-changing slope in spectral reflectance, which is revealed at the red and the blue plates.

With these measured data, we calculated the tristimulus values and the chromaticity coordinates for the CIE D65 illuminant and the CIE 1931 standard colorimetric observer according to the equations in Section 2.1. and the uncertainties using the equations derived in Section 3.1. and 2, which are summarized in Table 3. In order to look into the uncertainty of each color plate, uncertainty components of CIE 1976 ($L^*a^*b^*$) are plotted in Fig. 5. The uncorrelated uncertainty terms contribute the uncertainties of all the color plates regardless of spectral shapes. The contributions for uncertainties of a^* and b^* are larger than that of L^*, since the uncertainties of two tristimulus values propagate to those of a^* and b^* and the covariance between those two tristimulus values are comparatively small whereas only $u(Y)$ propagates to $u(L^*)$ as shown in Eq. (20). The fully correlated uncertainty terms also contribute to the uncertainties of all the color plates. However, the contributions for uncertainties of a^* and b^* are smaller than that of L^* with the large covariance due to the full correlation among the wavelength components and the sign of sensitivity coefficients (Eqs. (19)~(21)). As described in the uncertainty of spectral reflectance, the uncertainties of stray light and wavelength inaccuracy depend on the spectral shape. The achromatic samples of white and black plates have small uncertainties of stray light and wavelength inaccuracy when compared with the chromatic samples of red, green, cyan, and blue plates. Most of the chromatic samples have larger uncertainties in stray light and wavelength inaccuracy than the achromatic samples over the wavelength range. In addition,

TABLE 2. Uncertainties of spectral reflectance

Color plate	Relative standard uncertainty [a] (%)				
	Uncorrelated uncertainty	Fully correlated uncertainty	Uncertainty of stray light	Uncertainty of wavelength inaccuracy	Combined uncertainty
White	0.2%~0.3%	0.3%~0.4%	<0.02%	<0.1%	0.4%~0.5%
Black	0.2%~0.6%		<0.02%	<0.1%	0.4%~0.7%
Red	0.2%~0.4%		<0.10%	<0.3%	0.4%~0.6%
Green	0.2%~0.5%		<0.03%	<0.1%	0.4%~0.7%
Cyan	0.2%~0.3%		<0.03%	<0.2%	0.4%~0.5%
Blue	0.2%~0.3%		<0.07%	<0.3%	0.4%~0.6%

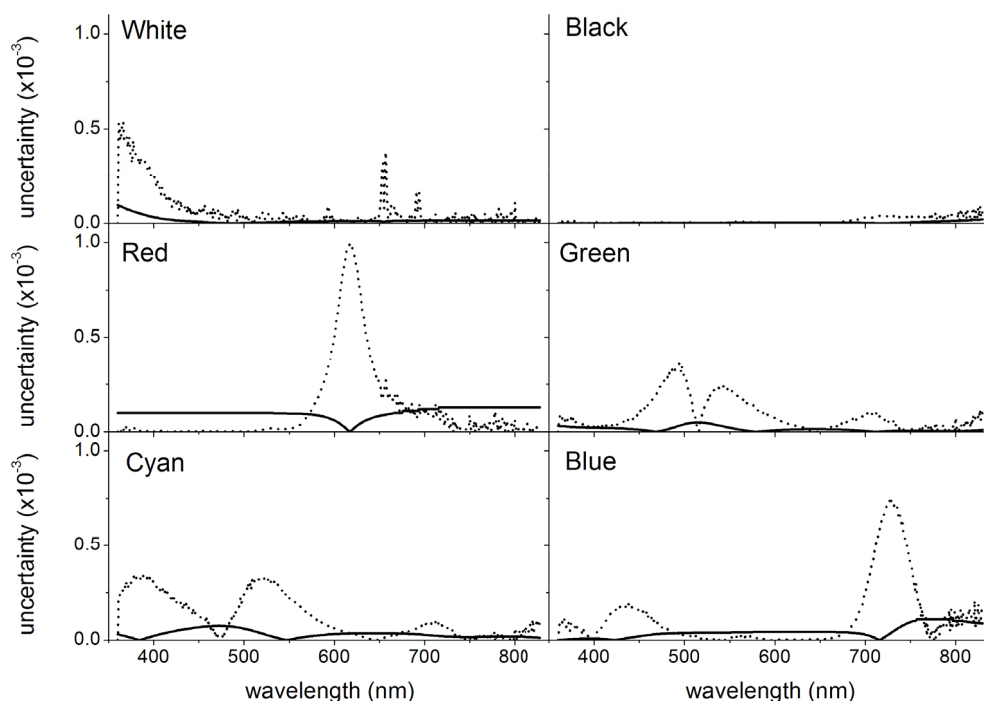[a] Relative standard uncertainties depend on wavelength.

FIG. 4. Uncertainties of stray light and wavelength inaccuracy in spectral reflectance (Solid and dashed curves are uncertainties caused by stray light and wavelength inaccuracy, respectively.).

TABLE 3. Values and uncertainties of tristimulus values and chromaticity coordinates for color plates

| Color plate	Tristimulus values and Chromaticity coordinates[a]							
	X	Y	Z	x	y	L^*	a^*	b^*
White	83.19	87.79	90.46	0.3182	0.3358	95.07	-0.48	3.49
Black	5.70	6.04	7.14	0.3018	0.3201	29.52	-0.53	-2.16
Red	23.31	16.79	11.09	0.4554	0.3280	47.99	37.15	16.93
Green	18.53	25.18	20.92	0.2867	0.3896	57.25	-25.81	10.88
Cyan	20.34	23.88	47.45	0.2219	0.2605	55.96	-11.13	-27.56
Blue	7.08	6.61	13.27	0.2626	0.2453	30.91	8.20	-18.28

[a]The values are calculated for the CIE D65 illuminant and the CIE 1931 standard colorimetric observer.

Color plate	Standard uncertainty							
	u(X)	u(Y)	u(Z)	u(x)	u(y)	u(L^*)	u(a^*)	u(b^*)
White	0.26	0.28	0.29	0.00003	0.00004	0.12	0.02	0.02
Black	0.02	0.02	0.02	0.00004	0.00006	0.05	0.01	0.01
Red	0.08	0.06	0.04	0.00019	0.00004	0.07	0.06	0.04
Green	0.06	0.08	0.07	0.00004	0.00005	0.08	0.03	0.02
Cyan	0.07	0.08	0.15	0.00002	0.00009	0.08	0.03	0.04
Blue	0.02	0.02	0.04	0.00008	0.00013	0.05	0.02	0.04

the spectral positions of the weighting factors on the CIE D65 illuminant shown in Fig. 1(b) are responsible for the large uncertainties of the chromatic samples, which is verified through comparison between the white and the green plates.

V. CONCLUSION

In summary, we investigated how a spectral reflectance measurement determines surface color with focus on an
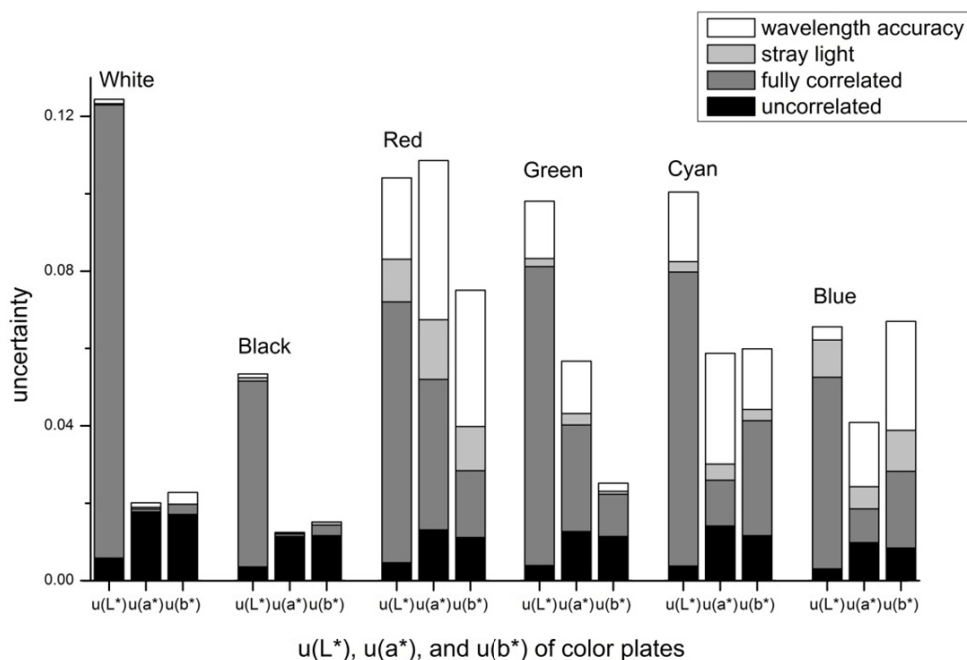
FIG. 5. Uncertainties of CIE 1976 ($L^*a^*b^*$) for each color plate.

analysis of an uncertainty. Since surface color possesses correlations among wavelength components which arise from spectral shapes of color matching functions, we divided the uncertainty terms by their correlations, i.e., an uncorrelated uncertainty, a fully correlated uncertainty, and a correlated uncertainty with spectrally varying correlation. The latter one includes uncertainty components of stray light and wavelength inaccuracy, of which correlations depend on difference in reflectance between a sample and a reference standard, and the slope of spectral reflectance, respectively. In order to show an example in practice, we performed spectro-reflectometric measurements for ceramic colour standards using a spectrophotometer and a reference standard calibrated traceable to the KRISS spectral reflectance scale. With determination of the uncertainty of spectral reflectance, we showed how the uncertainty of spectral reflectance propagates to that of surface color. It is expected that this thorough study for surface color measurements should provide reliability to color application techniques and allow effective approach to complex color measurements.

ACKNOWLEDGMENT

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