Paper

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The hybrid uncertain neural network method for mechanical reliability analysis

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Abstract

Concerning the issue of high-dimensions, hybrid uncertainties of randomness and intervals including implicit and highly nonlinear limit state function, reliability analysis based on the hybrid uncertainty reliability mode combining with back propagation neural network (HU-BP neural network) is proposed in this paper. Random variables and interval variables are as input layer of the neural network, after the training and approximation of the neural network, the response variables are obtained through the output layer. Reliability index is calculated by solving the optimization model of the most probable point (MPP) searching in the limit state band. Two numerical cases are used to demonstrate the method proposed in this paper, and finally the method is employed to solving an engineering problem of the aerospace friction plate. For this high nonlinear, small failure probability problem with interval variables, this method could achieve a good analysis result.

Key words: neural network, interval variables, hybrid uncertainty model, aerospace mechanism, nonlinearity reliability.

1. Introduction

Traditional reliability analysis requires the availability of probability distributions. In many engineering applications, the distributions of some variables may not be precisely known due to limited information. These variables are known with certain intervals, in other words, the hybrid uncertain model with probabilistic and interval widely exists in the practical engineering problems. Some researches make some assumptions for random distributions when using probability approach to perform the reliability analysis[1-3], and these methods bring a problem of confidence. Other techniques[4-8] also are proposed to deal with this problem. Literature[8] analyze the hybrid reliability of linear problems through a two-stage limit state function. A robust design method combining the probability and non-probability is also developed for uncertain structures[9-11]. More studies on the precise hybrid probability and interval can refer to[12-17].

All approaches for the hybrid uncertain model above are based on the explicit performance function. However, in engineering application, most problems are implicit. It means that we could not analyze reliability on the performance function directly. Response surface method (RSM) is widely used in both probability and non-probabilistic reliability problems[18,19]. A polynomial function is used to approximate the unknown implicit performance function. This technique develops well in probabilistic reliability analysis, and fairly accurate estimate of the failure probability could be obtained if the selected polynomial function fits the actual limit state well. Some researches also discuss the application of RSM in non-probabilistic reliability analysis[20]. However, for the mechanical systems, the performance function is highly nonlinear, the polynomial based RSM is not accurate enough for the real limit state function. Some researchers [21-23] also focus on the stochastic expansion method for the mixed uncertainties, but these methods are still based on the probabilistic assumption to some extent. Artificial neural network(ANN) algorithm has been rapidly developed for universal function approximations[24-26]. ANN has the stronger learning ability for fitting complex

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problems. The ANN technique was compared with the RSM and other methods for structural reliability analysis[27]. Literature performed reliability analysis of a mine pillar and geotechnical engineering structure by combining finite element model (FEM)s, ANN, first order reliability method (FORM), second order reliability method(SORM) and Monte-Carlo simulation[28-30].

In this paper, a hybrid uncertainty reliability analysis method based on the ANN is proposed. The hybrid uncertainty reliability model combines with the back propagation (HU-BP) neural network for highly nonlinear performance function reliability problems. The paper is organized as follows: the probability-interval hybrid reliability theory and ANN theory are introduced in section2 and section 3 respectively; HU-BP neural network is constructed in section 4; section 5 proposes the reliability analysis based HU-BP neural network; two numerical cases and an engineering case are carried out in section 6 to demonstrate the method, and finally is conclusion in section 7.

2. Probability-interval hybrid reliability theory

Assuming that X=(X_1, X_2, ..., X_n) is an n-dimensional random vector, and Y=(Y_1, Y_2, ..., Y_m) is an m-dimensional interval vector. The limit state function Z is defined as follows:

$$Z = g(\boldsymbol{X}, \boldsymbol{Y}) = 0 \tag{1}$$

The failure probability P_f can be defined as

$$P_f = \Pr\{g(\boldsymbol{X}, \boldsymbol{Y}) \le 0\} \tag{2}$$

Where $\Pr\{\bullet\}$ represents the probability. The interval parameters Y_i, i=1, 2, ..., m can be described as the following equations:

$$Y_i = \left[Y_i^{\mathrm{L}}, Y_i^{\mathrm{U}}\right] \tag{3}$$

$$Y_i^{c} = \frac{Y_i^{\mathrm{L}} + Y_i^{\mathrm{U}}}{2}, Y_i^{r} = \frac{Y_i^{\mathrm{U}} - Y_i^{\mathrm{L}}}{2}, \gamma_i = \frac{Y_i^{r}}{Y_i^{c}} \quad i = 1, 2, \cdots, m \tag{4}$$

Where Y_i^L, Y_i^U, Y_i^c, Y_i^r represent the lower bound, the upper bound, the middle point, the radius of the interval variables Y_i. γ_i represents the degree of uncertainty of the Y_i.

In the limit state function Z, the random variables as well as interval variables are included. The random parameter space will form a band of limit state when Z=0. In this limit state band, the boundaries can be represented as

$$\begin{aligned} &\max_Y g(\boldsymbol{X}, \boldsymbol{Y}) = 0 \\ &\min_Y g(\boldsymbol{X}, \boldsymbol{Y}) = 0 \end{aligned} \tag{5}$$

$\max_Y g(X, Y)$ and $\min_Y g(X, Y)$ are the maximum and minimum values of the performance function g over the intervals of Y. Thus the failure probability P_f can be given as

$$\begin{aligned} &P_f = \left[P_f^{\mathrm{L}}, P_f^{\mathrm{U}}\right] \\ &P_f^{\mathrm{L}} = \Pr\left\{\max_{Y} g(\boldsymbol{X}, \boldsymbol{Y}) < 0\right\} \\ &P_f^{\mathrm{U}} = \Pr\left\{\min_{Y} g(\boldsymbol{X}, \boldsymbol{Y}) < 0\right\} \end{aligned} \tag{6}$$

Where P_f^L and P_f^U represent the lower and upper bounds of the failure probability, respectively

The limit state band of the failure is as shown as in Fig. 1. When using classical FORM or SORM to solve the hybrid uncertainty model with both random variables and interval variables, the reliability index will belong to an interval and two optimization problems:

$$\begin{aligned} &\begin{cases} \beta^{\mathrm{U}} = \min_U \|\boldsymbol{U}\| \\ \text{s.t.} \quad \max_Y G(\boldsymbol{U}, \boldsymbol{Y}) = 0 \end{cases} \\ &\begin{cases} \beta^{\mathrm{L}} = \min_U \|\boldsymbol{U}\| \\ \text{s.t.} \quad \min_Y G(\boldsymbol{U}, \boldsymbol{Y}) = 0 \end{cases} \end{aligned} \tag{7}$$

Where β^U and β^L represent the upper and lower bounds of the reliability index, respectively. U is the normalization of X, and the normalization process is as :

$$\Phi(U_i) = F_{X_i}(X_i), \; i = 1, 2, \cdots, n \tag{8}$$

$$U_i = \Phi^{-1}\left[F_{X_i}(X_i)\right], \; i = 1, 2, \cdots, n \tag{9}$$

3. Artificial Neural network technique

The multi-layer perceptron, trained by the back propagation(BP), is currently the most widely used ANN. In this context, ANN is referred to as multi-layer perceptron or multi-layer feed forward ANN. An ANN consists of a set of neurons that are logically arranged into the input layer, the output layer and the hidden layer. The neurons is a processing elements whose output is calculated by multiplying its inputs by a weight vector, summing the results, and applying

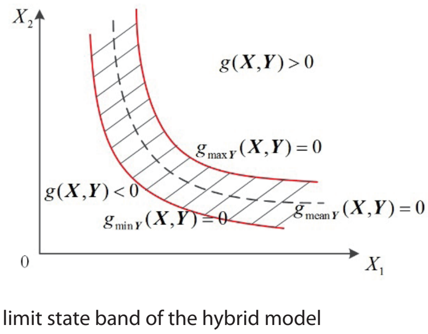

Fig. 1. The limit state band of the hybrid model

an activation function to the sum.

$$y = f\left[\sum_{k=1}^{n} x_k w_k + b_k\right] \tag{10}$$

Where w_k is the weight coefficient, b_k is threshold. The active function can be linear or non-linear. A linear activation function's output is simply equal to its input:

$$f(x) = x \tag{11}$$

The most common non-linear activation function is the logistic sigmoidal function

$$f(x) = \frac{1}{1+\exp(-\delta x)} \tag{12}$$

Where δ is called the sigmoid slope, usually δ=1

4. Hybrid uncertainty based BP neural network

In this section, a hybrid uncertainty based BP(HU-BP) neural network is defined. In order to illustrate the HU-BP neural network, this paper will use the typical three-layer neural network. Its structure is as shown in Fig. 2. There are input layer, hidden layer and output layer in the neural network. As mentioned in the section 2, in reliability analysis, the hybrid uncertainties of probability and interval are widely exist in the engineering applications, and the performance function as equation (1), there are random variables $X=(X_1, X_2, ..., X_n)$ and $Y=(Y_1, Y_2, ..., Y_m)$ in the performance function g.

Then the performance function of the HU-BP neural network could be defined as follows:

$$g(\boldsymbol{X},\boldsymbol{Y}) = f_2\left[\boldsymbol{W}_2^{\mathrm{T}} f_1\left(\boldsymbol{W}_1^{\mathrm{T}}(\boldsymbol{X},\boldsymbol{Y})\right)\right] \tag{13}$$

Where W_1 and W_2 represent the weight coefficients of the transformation.

In above equations, the "training rules" of the neural network is defined, and the next issue is the training process. In the training process, training samples for neural network is important.

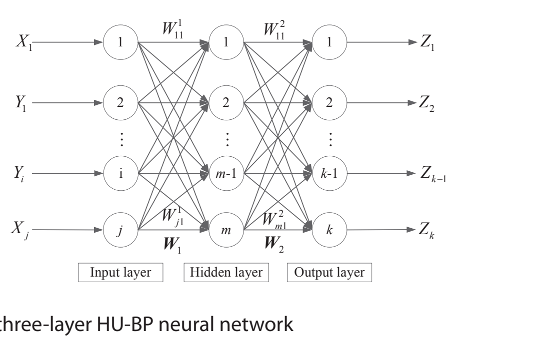

Fig. 2. A three-layer HU-BP neural network

This paper uses the improved axial experimental design method[31] to obtain samples points of X and Y, the initial sample point is selected as (μ_X, Y^C),where μ_X is the mean vector of the random variables X and Y^C is the midpoint vector of the interval variables Y, the samples of input variables $(\overline{X},\overline{Y})$ will be updated in subsequent iterations, the remaining $2n+2m$ samples points will be obtained through $\overline{X}_i \pm \kappa_x \cdot \sigma_{X_i}$, i=1, 2, ..., n and $\overline{Y}_j \pm \kappa_y \cdot Y_j^r$, j=1, 2, ..., m, where σ_{Xi} is the standard deviation of X_i and Y_j^r is the radius of Y_j, κ_x and κ_y are two sampling coefficients and generally between [1,3] can provide good computational results.

Once the input samples are identified , the output samples will obtained by FEA of the mechanical systems. And finally the train samples (($X^{(t)}$, $Y^{(t)}$), $Z^{(t)}$) are obtained.

The whole training steps of the HU-BP neural network are:

Step 1: built the structure of the BP neural network.

Step 2: select the hybrid uncertainty training sample sets (($X^{(t)}$, $Y^{(t)}$), $Z^{(t)}$), (t=1, 2, ..., S) according to section 4, and identify the initial weight coefficient $W_k^{(0)}$

Step 3: calculate $Z^{(t)}$ with the trained set ($X^{(t)}$, $Y^{(t)}$)

Step 4: reverse calculate the partial derivative $\frac{\partial e^{(t)}}{\partial \boldsymbol{W}_k^{(t)}}$, ($k$=1, 2, ..., L), where e is the final error, and $e = \frac{1}{2}\|\boldsymbol{Z}_L - \boldsymbol{T}\| = \frac{1}{2}(\boldsymbol{Z}_L - \boldsymbol{T})^{\mathrm{T}}(\boldsymbol{Z}_L - \boldsymbol{T})$, T is the ideal output of the neural network.

Step 5: the modified $W_k^{(t+1)}$ is obtained by

$\boldsymbol{W}_k^{(t+1)} = \boldsymbol{W}_k^{(t)} + \delta\boldsymbol{W}_k^{(t)}$, $\delta\boldsymbol{W}_k^{(t)} = -\eta_t \frac{\partial e^{(t)}}{\partial \boldsymbol{W}_k^{(t)}}$, k=1, 2, ..., L, η_t is the training efficient in k^{th} step.

Step 6: repeat the step 3 to 5 until all the samples are trained.

5. Reliability analysis based HU-BP neural network

The implicit limit state function containing hybrid uncertainty of probability and interval can be simultaneously

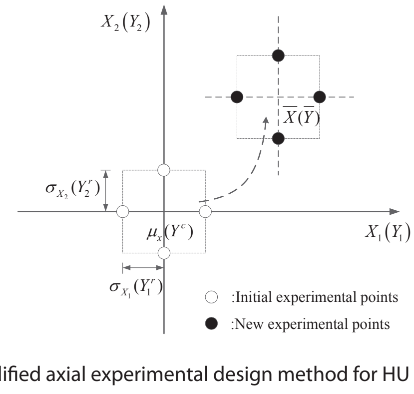

Fig. 3. Modified axial experimental design method for HU

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approximated through HU-BP neural network, then the method of HU-BP neural network based on "Most Probable Point" (MPP) is used to calculated the failure probability.

The limit state band of the failure is as shown as in Fig. 4. When using classical FORM or SORM to solve the hybrid uncertainty model with both random variables and interval variables, the reliability index will belong to an interval and two optimization problems:

$$\begin{cases} \beta^{\mathrm{U}} = \min_U \|\boldsymbol{U}\| \\ \text{s.t.} \quad \max_Y G(\boldsymbol{U},\boldsymbol{Y}) = 0 \end{cases} \qquad \begin{cases} \beta^{\mathrm{L}} = \min_U \|\boldsymbol{U}\| \\ \text{s.t.} \quad \min_Y G(\boldsymbol{U},\boldsymbol{Y}) = 0 \end{cases} \tag{14}$$

For the MPP search in the equation (14), HL-RF algorithm is widely used, and reference[32] has proposed an iHL-RF algorithm for limit state function containing random variables and interval variables. For example, we focus on the MPP for β^{U}, in iteration k+1, the MPP is given by

$$\boldsymbol{U}_{k+1} = \boldsymbol{U}_k + \alpha \boldsymbol{d}_k \tag{15}$$

Where the search direction d_k is defined by

$$\boldsymbol{d}_k = \frac{\nabla g_Y(\boldsymbol{U}_k,\boldsymbol{Y}_k)\boldsymbol{U}_k^{\mathrm{T}} - g(\boldsymbol{U}_k,\boldsymbol{Y}_k)}{\|\nabla g_Y(\boldsymbol{U}_k,\boldsymbol{Y}_k)\|^2} \nabla g_Y(\boldsymbol{U}_k,\boldsymbol{Y}_k) - \boldsymbol{U}_k \tag{16}$$

Where

$$\nabla g_Y(\boldsymbol{U}_k,\boldsymbol{y}_k) = \left(\frac{\partial g}{\partial U_1}, \frac{\partial g}{\partial U_2}, \cdots, \frac{\partial g}{\partial U_n}\right)_{U_k, Y_k}$$

The step size α is determined by minimizing the merit function defined by

$$m(\boldsymbol{U},\boldsymbol{Y}) = \frac{1}{2}\|\boldsymbol{U}\| + c\left|g(\boldsymbol{U},\boldsymbol{Y})\right| \tag{17}$$

In which the constant c should satisfy

$$c > \frac{\|\boldsymbol{U}\|}{\|\nabla g(\boldsymbol{U},\boldsymbol{Y})\|} \tag{18}$$

In above equations, intervals are assumed fixed, once the U_{k+1} is found, the interval analysis is conducted to find Y_{k+1} that maximize the performance function. The optimization model is given by

$$\max_Y g(\boldsymbol{U}_{k+1},\boldsymbol{Y}) \big| \boldsymbol{Y} \in C \tag{19}$$

where C represents the constrained interval real field. Then, the reliability analysis steps for HU-BP neural network are as follows:

Step 1: suppose that X=(X_1, X_2, ..., X_n) is the basic n-dimensional random variables in original coordinate system, then vector U=(U_1, U_2, ..., U_n) represents the random variables which are equivalent standard normal space. Y=(Y_1, Y_2, ..., Y_m) is m-dimensional interval variables.

Step 2: construct an approximate BP neural network for the structure reliability, and select train sample sets

Step 3: establish the MPP optimization model of the performance function $G(U, Y)$ and calculate the partial derivatives of design point using HU-BN neural network $\left.\frac{\partial g}{\partial X}\right|_{x_j^*}$, and then the optimization model of equation (19) is obtained;

Step 4: find Y_{k+1} by solving the optimization model of equation (19);

Step 5: check convergence, if $\left|g(\boldsymbol{U}_{k+1},\boldsymbol{Y}_{k+1})\right| \le \varepsilon_1$ and $\|\boldsymbol{U}_{k+1} - \boldsymbol{U}_k\| \le \varepsilon_2$ (ε_1 and ε_2 are small positive numbers =1.0), then $\beta^{\min} = \|\boldsymbol{U}_{k+1}\|$ and go to step 6, otherwise, k=k+1, go to step 3

Step 6: $P_f^{\min} = \Phi(-\beta^{\min})$

6. Case study

6.1 numerical case 1

Denote the reliability function as g, and g is given by

$$g = \frac{x_1 \cdot x_2^2 \cdot y_1}{(\sqrt{y_2} - 1)(y_1 - x_1)} + \exp[0.1(x_1 + x_2 - 1) + 1] - \exp[0.18(y_1 - y_2) + 2.5] + 14 \tag{20}$$

Where the above explicit limit state function is assumed to be an implicit model for simulation. And the uncertainty information of variables of x_1, x_2, y_1, y_2 are : x_1 and x_2 are random variables, they obey normal distribution, their mean value standard deviation are: μ_{x_1}=1.2, σ_{x_1}=0.12; μ_{x_1}=0.8, σ_{x_1}=0.02. y_1 and y_2 are interval variables, and $y_1 \in [4, 4.5]$, $y_2 \in [1.6, 1.64]$.

The HU-BP neural network is employed to fitting the performance, and 120 samples (x_i, y_i) are used, and equation (20) is used to compute the response value z_i=g(x_i, y_i), the train sample sets {(x_i, y_i), z_i} are obtained. A three-layer HU-BP neural network is employed and the transfer function in hidden layer is chosen as

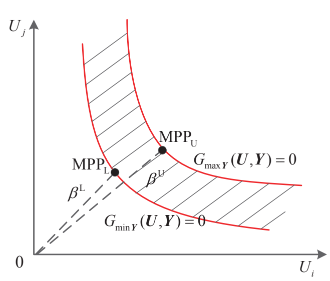

Fig. 4. Reliability index of the HU reliability model

$$f_{\tan sig}(x) = \tanh x \tag{21}$$

The output layer function is :

$$f_{purlin}(x) = x \tag{22}$$

Then the fitting results of HU-BP neural network is as shown in Fig. 5 and Fig. 6.

The reliability index of the performance with HU-BP neural network is β^L=3.30744; β^U=3.59621. In order to illustrate the method proposed by this paper furthermore, the second order RSM is also employed in the same problem. In the RSM, y_1 and y_2 are sampled as the unified in the intervals and the two MPP is also calculated. After 8 iterations, the reliability index are β^L=3.85176; β^U=4.158613. Comparing the results, it can be seen that the reliability indexes obtained by the HU-neural network are small than the results obtained by the RSM. It demonstrates that the performance function obtained by the neural network is more accurate than the RSM. In order to verify the results, the failure probability of this problem is calculated by the direct MC with 1000000 samples, and the max probability and min probability are as list in table 1. It can be seen that the failure probability of P_f^U and P_f^L obtained through the HU-BP neural network and the MC is almost the same, the relative error of reliability are: 0.132e-4 and 0.455e-4. The relative error of results of the RSM is larger, and they are 4.369e-4 and 2.524e-4 respectively. The results indicate that the HU-BP neural could get a good result in the reliability analysis

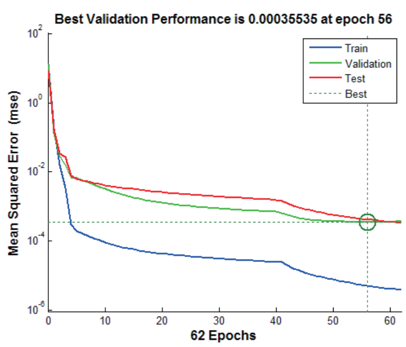

Fig. 5. Fitting result of HU-BP neural network

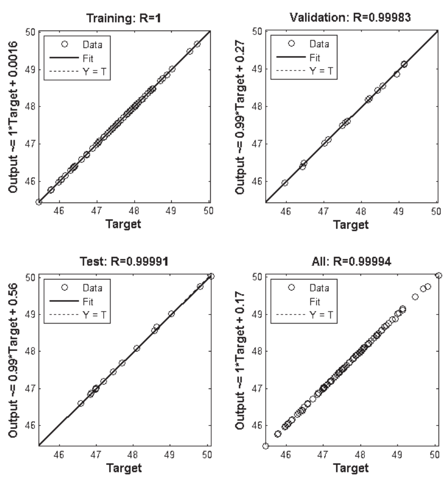

Fig. 6. Fitting results of performance function

6.2 numerical case 2

The performance function of ten-bar truss structure as shown in Fig. 7 is

$$Z = 4 - u_{2y}(\boldsymbol{A}, F_1, F_2, F_3) = 0 \tag{23}$$

Where u_{2y} is the horizontal displacement (/m) of the joint 2. The length of the horizontal and vertical bars is 1.0 m. Joint 4 is subjected to the vertical load F_1 and Joint 2 is subjected to the horizontal load F_3 and vertical load F_2. The Young's Modulus of the material E is normal variable, μ_E=2.0×106kN/m^2, variation coefficient V_E=0.01. The mean value and variation coefficient are μ_{A_i}=0.36m^2, V_{A_i}=0.1. The external load F_3 obeys normal distribution, the mean value and variation coefficient are : μ_{F_3}=10N, V_{F_3}=0.1. The external loads F_1, F_2, are interval uncertain, and $F_1 \in [76, 84]$kN, $F_2 \in [9.5, 10.5]$. Now we calculate the failure probability of the structure.

For this problem, the displacement u_{2y} could be obtained through the FEA, and the HU-BP neural network method proposed by this paper is utilized for reliability analysis. To fitting the performance function, 150 samples $(x_i, y_i)=(A, F_1, F_2, F_3)^T$ are used as input variables, and FEA model is used to compute the response value z_i=4-$u_{2y}(A, F_1, F_2, F_3)$. The

Table 1. Failure probability of different methods

Methods	P_f^U	P_f^L
Method proposed in this paper	4.834e-4	2.326e-4
RSM method	5.91e-5	2.57e-5
MC	4.966e-4	2.781e-4

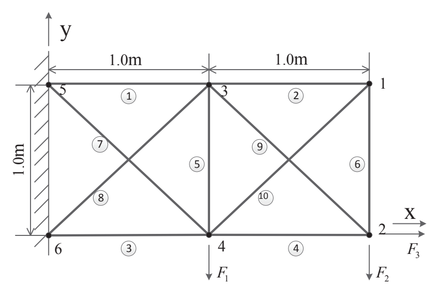

Fig. 7. Ten-bar plane structure

train sample sets $\{(x_i, y_i), z_i\}$ are obtained in a three-layer HU-BP neural network. After 12 iterations, the reliability index of this problem are obtained as: β^U=3.4721; β^L=3.3122. For the same problem, second order RSM and Monte Carlo simulation are also utilized here to compute the reliability, the RSM with 150 sample sets for fitting the performance and the results of failure probability are listed in table 2. The RSM method has 8 iterations for reliability index but it is rough than the results obtained by the method proposed by this paper. The whole computational time of the RSM is 0.79 hours, and the whole computational time of the HU-BP neural network is 1.71 hours (the computer with Intel CORE 5 CPU and 3.00GHz). The MC based on ANN method with 100000 sample sets, and the computational time is 16 hours. The results indicate that the HU-neural network could get a more precise result for high nonlinearity problem, but it cost more time than the RSM method. However, it is much more efficient than the MC method. It means that the HU-BP neural network is meaningful in the implicit high nonlinearity problems

Table 2. Reliability of different methods

Methods	R^U	R^L
Method proposed in this paper	0.9997398	0.9995335
RSM	0.9998282	0.9996631
MC based ANN	0.9997279	0.9995754

6.3 engineering case

The reliability analysis of locking mechanism in aerospace engineering.

The locking mechanism as shown in Fig. 8, it is a key part in support device which is used as the main power transmission structure between the spacecraft and transparency nose. The locking mechanism works by contacting the inner friction plates(IFP) and the outer friction plates(OFP). The friction force between IFP and OFP is achieved by the shear stress of the IFP underlying the load and the contact area S. Now we analyze the friction force reliability of the locking mechanism. Referring relative tests and discussions, the variables of the performance function are as shown in table 3

The locking mechanism is pressured with the normal load P_1 and the lateral load P_2, and their values are as shown in Fig. 9. Obviously this is an implicit high nonlinear problem, the HU-BP neural network method and FEA is combined to solve this problem. Finite element model of the locking

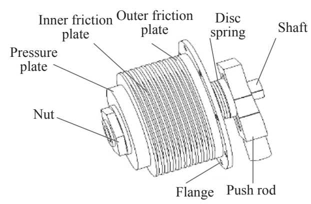

Fig. 8. Structure of the locking mechanism

Table 3. Thecharacteristic parameters of the locking mechanism

Parameters	Physical meaning	Values
E_1(Gpa)	elastic modulus of IFP	65.4
r	Passion rate	0.3
ρ(kg/m^3)	density	2.72e3
G_1(GPa)	Shear modulus of IFP	25.15
E_2(Gpa)	elastic modulus of OFP	70
G_2(GPa)	Shear modulus of OFP	24.81
c_1	Static friction coefficient	0.59
c_2	viscous damping coefficient	0.15878e9

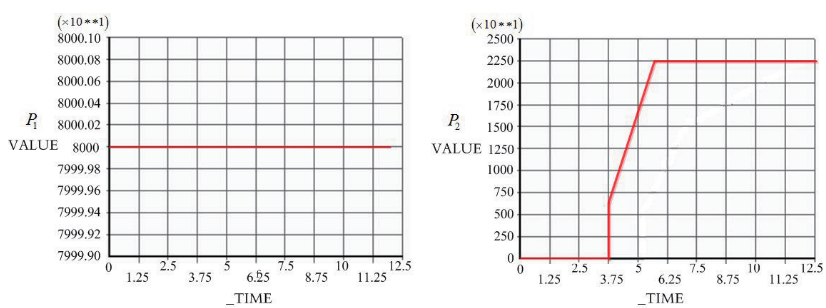

Fig. 9. Values of P_1 and P_2

mechanism can be established with ANSYS, and the work simulation process of contact and shock is carried out in the LS-DYNA. In the loading process, the P_1 is loading on the surface of the flange plate, and the P_2 is loading on side face of the OFP. The boundary condition is that the all the freedom degrees of the Pressure plate are constrained. In Fig. 10, the maximum shear stress is 12.83MPa, and the stress varying by the time is as shown in Fig. 11. Through the discretization of time interval into 84 time point, the average shear stress could be obtained as 3.1Mpa according to the table 4.

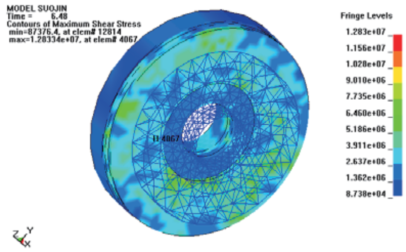

Fig. 10. Shear stress of the friction plate

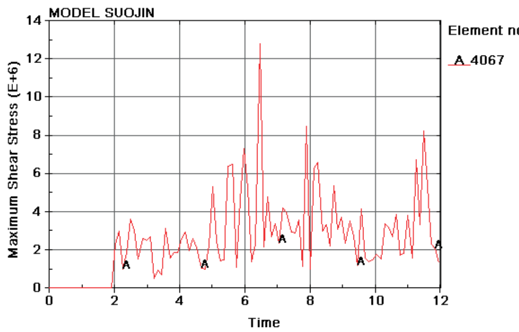

Fig. 11. Curve of shear stress with time

-Reliability simulation analysis for locking mechanism

The definition of limit state function and probabilistic characteristic and interval range of variables. The model of stress and strength interference and hybrid uncertainty performance function is applied to compute the failure probability. The limit state function is defined as

$$Z = F - F_0 = f\left(P_1, P_2, E_1, E_2, G_1, G_2, D\right) \times S - F_0 \tag{24}$$

Where $f(P_1, P_2, E_1, E_2, G_1, G_2, D)$ is the element average equivalent stress of the inner friction plate, which could be calculated by LS-DYNA. S is the area of the plate. F_0 is the allowable load.

Based on the limit historic data and the epistemic knowledge, the probabilistic and interval characters of the variables are in table 5.

-Reliability simulation analysis

The limit state function could be approximated by three-layer HU-BP neural network. The number of neurons for input layer is nine and there are eighteen hidden layers,

Table 4. Shear stress values with time under the varying loads

No.	Value(Pa)	No.	Value(Pa)	No.	Value(Pa)	No.	Value(Pa)
1	2.30337e6	22	2.11395e6	43	2.34749e6	64	4.15947e6
2	2.99647e6	23	1.05623e6	44	4.2094e6	65	1.56383e6
3	997698	24	1.01825e6	45	3.95451e6	66	1.41306e6
4	1.86108e6	25	2.2757e6	46	2.96955e6	67	1.47164e6
5	3.61296e6	26	5.35305e6	47	2.90995e6	68	1.79964e6
6	3.03982e6	27	2.44242e6	48	3.59123e6	69	1.532e6
7	1.53638e6	28	1.45343e6	49	1.13604e6	70	3.38367e6
8	2.59281e6	29	1.51117e6	50	8.51143e6	71	3.20651e6
9	2.50227e6	30	6.42199e6	51	976126	72	2.68522e6
10	2.711e6	31	6.51172e6	52	6.31719e6	73	3.87357e6
11	508333	32	1.0959e6	53	6.61351e6	74	1.71388e6
12	952073	33	4.63751e6	54	2.98379e6	75	1.80908e6
13	709136	34	7.31972e6	55	3.35098e6	76	3.83426e6
14	3.16505e6	35	5.2669e6	56	2.2195e6	77	1.58045e6
15	1.56832e6	36	1.38007e6	57	5.40534e6	78	6.76475e6
16	1.87172e6	37	2.30834e6	58	3.11468e6	79	3.34421e6
17	1.85895e6	38	1.28334e7	59	3.75113e6	80	8.28679e6
18	2.62145e6	39	2.1724e6	60	2.35026e6	81	5.33554e6
19	2.92965e6	40	4.78592e6	61	3.54081e6	82	2.3122e6
20	1.95469e6	41	2.69129e6	62	2.70612e6	83	2.09523e6
21	2.60807e6	42	3.41076e6	63	1.17879e6	84	1.32194e6

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Table 5. Probabilistic and interval character of variables

Sequence number	Variables	Unit	Mean	Coefficient of variation(γ)	Distribution
1	P_1	kN	80	0.1	normal
2	P_2	kN	22.5	0.1	normal
3	E_1	GPa	65.4	0.01	normal
4	E_2	GPa	70	0.01	normal
5	G_1	GPa	25.12	0.02	normal
6	G_2	GPa	24.81	0.02	normal
7	ρ	kg/m^3	2.72e3	0.01	normal
8	S	m^2	12.16e-3	0.02	interval
9	F_0	kN	44.6	0.06	interval

one output layer. The number of train set is 50. The relation between root mean square error(RMSE) of neural network output vector and the number of training epochs is shown as in Fig 12 and Fig 13 .

Figure 12 shows that the lowest RMSE of the output vector in the training set can be reached after 10 epochs. After approximation, the reliability index obtained by the FORM is: β_Z^L=3.901, β_Z^U=3.901.The failure probability is : P_f^L=4.321e-6, P_f^U=e-6.

To study the impact of the interval uncertainty on the failure probability, γ_s and γ_{F_0} are varied gradually from 0 to 0.1 and 0 to 0.15 respectively. Fig. 14 and Fig. 15 show the variation in the reliability index β and the failure probability P_f with the change in γ_s and γ_{F_0}. In Fig. 14, when γ_{F_0}=0.06 is selected, and the γ_s changes from 0 to 0.1, the reliability index changes from [4.025 ,4.4076] to [3.719 , 4.612], the failure probability changes from [5.42e-6, 2.67e-5] to [2.02e-6, 1.042e-4]. In Fig. 15, when γ_s=0.02 is selected, and the γ_{F_0} changes from 0 to 0.15, the reliability index changes from [4.117 ,4.376] to [3.341 ,5.012], the failure probability changes from [6.21e-6,1.98e-5] to [3.01e-7,4.189e-4]. This information suggests that the increase in uncertainty of the interval variables has led to an increased uncertainty in the reliability results of the whole system. And the results also show that in comparison with the γ_s, the impact of the uncertainty of the γ_{F_0} is more significant.

7. Conclusions

The BP neural network for hybrid uncertainty of random variables and interval variables is constructed, and the

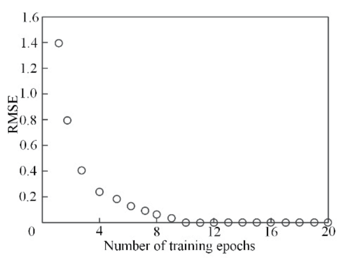

Fig. 12. RMSE of training set and the number of training epochs

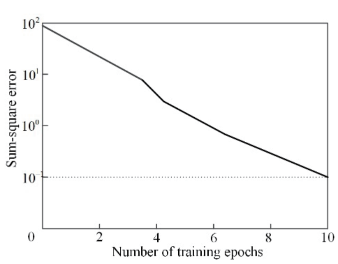

Fig. 13. Training errors vs number of epochs

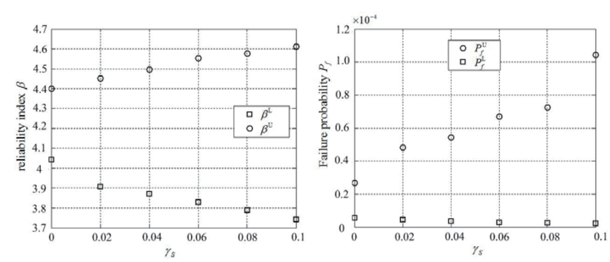

Fig. 14. effects of the uncertainty on the failure probability

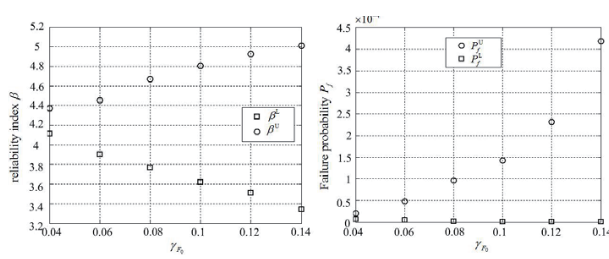

Fig. 15. Effects of the uncertainty on the failure probability

training rules is defined in the HU-BP neural network, it is suitable for approximating high nonlinear performance function with hybrid variables;

Hybrid reliability model based analysis method combining the neural network, the limit state band is introduced concerning on the interval variables in the mechanical systems, the reliability index interval based on searching for the MPP in performance function is employed to evaluate the reliability of mechanical systems. It ignored the assumption of probability distributions of the non-probabilistic variables and the reliability analysis is more confident in real engineering. The reliability interval has been proposed for the explicit function reliability problem, to the best of authors knowledge, this is first contribution in the implicit problems.

For high nonlinear, implicit problems of mechanical systems, the HU-BP neural network method provides an efficient and accurate way for reliability analysis. The impact of the interval uncertainty on the failure probability could also be obtained, and it is helpful for the mechanism design.

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