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TOTAL MEAN CORDIAL LABELING OF SOME CYCLE RELATED GRAPHS

R. PONRAJ* AND S. SATHISH NARAYANAN

ABSTRACT. A Total Mean Cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \to \{0, 1, 2\}$ such that $f(xy) = \left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, $xy \in E(G)$, and the total number of 0, 1 and 2 are balanced. That is $\left|ev_f(i) - ev_f(j)\right| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. If there is a total mean cordial labeling on a graph G, then we will call G is Total Mean Cordial. Here, We investigate the Total Mean Cordial labeling behaviour of prism, gear, helms.

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Key words and phrases : prism, gear, helms, join of graphs, product of graphs.

1. Introduction

Terminology and notations in graph theory we refer Harary [2]. New terms and notations shall, however, be specifically defined whenever necessary. By a graph $G = (V, E)$ we mean a finite, undirected graph with neither loops nor multiple edges. The product graph $G_1 \times G_2$ is defined as follows: Consider any two points $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V = V_1 \times V_2$. Then u and v are adjacent in $G_1 \times G_2$ whenever $[u_1 = v_1 \text{ and } u_2 \text{ adj } v_2]$ or $[u_2 = v_2 \text{ and } u_1 \text{ adj } v_1]$. The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$. The order and size of G are denoted by p and q respectively. Ponraj, Ramasamy and Sathish Narayanan [3] introduced the concept of Total Mean Cordial labeling of graphs and studied about their behavior on Path, Cycle, Wheel and some more standard graphs. In [4], Ponraj and Sathish Narayanan proved that $K_n^c + 2K_2$ is Total Mean Cordial if and only if $n = 1$ or 2 or 4 or 6 or 8. Also in [5], Ponraj, Ramasamy and Sathish

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Narayanan studied about the Total Mean Cordiality of Lotus inside a circle, bistar, flower graph, $K_{2,n}$, Olive tree, P_n^2, $S(P_n \odot K_1)$, $S(K_{1,n})$. In this paper, we investigate the Total Mean Cordiality of some cycle related graphs. Let x be any real number. Then the symbol $\lceil x \rceil$ stands for the smallest integer greater than or equal to x.

2. Main results

Definition 2.1. A Total Mean Cordial labeling of a graph $G = (V, E)$ is a function $f : V(G) \rightarrow \{0, 1, 2\}$ such that $f(xy) = \left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, $xy \in E(G)$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x ($x = 0, 1, 2$). If there exists a total mean cordial labeling on a graph G, we will call G is Total Mean Cordial.

Prisms are graphs of the form $C_m \times P_n$. We now look into the graph prism $C_n \times P_2$.

Theorem 2.2. *Prisms are Total Mean Cordial.*

Proof. It is clear that $p + q = 5n$. Let $V(C_n \times P_2) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(C_n \times P_2) = \{u_1 u_n, v_1 v_n\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\}$.

Case 1. $n \equiv 0 \pmod 6$.

Let $n = 6t$ and $t > 0$. Define a map $f : V(C_n \times P_2) \rightarrow \{0, 1, 2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 2t \\ f(u_{2t+1+i}) & = f(v_{2t+1+i}) & = 2 & 1 \leq i \leq 2t-1 \\ f(u_{4t+i}) & = f(v_{4t+i}) & = 1 & 1 \leq i \leq 2t-1 \end{array}$$

$f(u_{2t+1}) = 0$, $f(v_{2t+1}) = f(v_{6t}) = 2$, $f(u_{6t}) = 1$. In this case $ev_f(0) = ev_f(1) = ev_f(2) = 10t$.

Case 2. $n \equiv 1 \pmod 6$.

Let $n = 6t + 1$ and $t > 0$. Define a map $f : V(C_n \times P_2) \rightarrow \{0, 1, 2\}$ by

$$\begin{array}{lll} f(u_i) & = 0 & 1 \leq i \leq 2t+2 \\ f(u_{2t+2+i}) & = f(v_{2t+2+i}) = 2 & 1 \leq i \leq 2t \\ f(u_{4t+2+i}) & = f(v_{4t+2+i}) = 1 & 1 \leq i \leq 2t-1 \\ f(v_i) & = 0 & 1 \leq i \leq 2t \end{array}$$

$f(v_{2t+1}) = f(v_{2t+2}) = 1$. Here $ev_f(0) = ev_f(2) = 10t + 2$, $ev_f(2) = 10t + 1$.

Case 3. $n \equiv 2 \pmod 6$.

Let $n = 6t + 2$ and $t > 0$. Define a map $f : V(C_n \times P_2) \rightarrow \{0, 1, 2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 2t+1 \\ f(u_{2t+1+i}) & = f(v_{2t+1+i}) & = 2 & 1 \leq i \leq 2t \\ f(u_{4t+1+i}) & = f(v_{4t+1+i}) & = 1 & 1 \leq i \leq 2t \end{array}$$

$f(u_{6t+2}) = 1$, $f(v_{6t+2}) = 2$. In this case $ev_f(0) = ev_f(2) = 10t + 3$, $ev_f(1) = 10t + 4$.

Case 4. $n \equiv 3 \pmod 6$.
Let $n = 6t+3$ and $t \geq 0$. A Total Mean Cordial labeling of $C_3 \times P_2$ is given in figure 1.

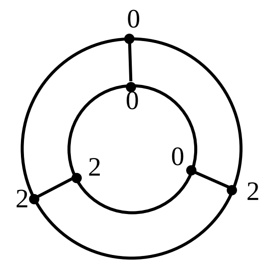

FIGURE 1.

Assume $t > 0$. Define a map $f : V(C_n \times P_2) \to \{0, 1, 2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 2t+1 \\ f(u_{2t+2+i}) & = f(v_{2t+2+i}) & = 2 & 1 \leq i \leq 2t \\ f(u_{4t+2+i}) & = f(v_{4t+2+i}) & = 1 & 1 \leq i \leq 2t \end{array}$$

$f(u_{2t+2}) = 0$, $f(u_{6t+3}) = 1$, $f(v_{2t+2}) = f(v_{6t+3}) = 2$. In this case $ev_f(0) = ev_f(1) = ev_f(2) = 10t+5$.
Case 5. $n \equiv 4 \pmod 6$.
Let $n = 6t+4$ and $t \geq 0$. A Total Mean Cordial labeling of $C_4 \times P_2$ is given in figure 2.

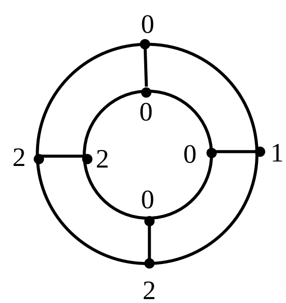

FIGURE 2.

Assume $t > 0$. Define a map $f : V(C_n \times P_2) \to \{0, 1, 2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 2t+1 \\ f(u_{2t+3+i}) & = f(v_{2t+3+i}) & = 2 & 1 \leq i \leq 2t+1 \\ f(u_{4t+4+i}) & = f(v_{4t+4+i}) & = 1 & 1 \leq i \leq 2t \\ f(u_{2t+2}) & = f(u_{2t+3}) & = 0 & \\ f(v_{2t+2}) & = f(v_{2t+3}) & = 1 & \end{array}$$

In this case $ev_f(0) = ev_f(1) = 10t + 7$, $ev_f(2) = 10t + 6$.
Case 6. $n \equiv 5 \pmod 6$.
Let $n = 6t - 1$ and $t > 0$. Define a map $f : V(C_n \times P_2) \to \{0, 1, 2\}$ by

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 2t \\ f(u_{2t+i}) & = f(v_{2t+i}) & = 2 & 1 \le i \le 2t-1 \\ f(u_{4t-1+i}) & = f(v_{4t-1+i}) & = 1 & 1 \le i \le 2t-1 \end{array}$$

$f(u_{6t-1}) = 1$, $f(v_{6t-1}) = 2$. In this case $ev_f(0) = ev_f(2) = 10t - 2$, $ev_f(1) = 10t - 1$. □

The gear graph G_n is obtained from the wheel $W_n = C_n + K_1$ where C_n is the cycle $u_1u_2 \ldots u_nu_1$ and $V(K_1) = \{u\}$ by adding a vertex between every pair of adjacent vertices of the cycle C_n.

Theorem 2.3. *The gear graph G_n is Total Mean cordial.*

Proof. Let $V(G_n) = V(W_n) \cup \{v_i : 1 \le i \le n\}$ and $E(G_n) = E(W_n) \cup \{u_iv_i, v_ju_{j+1} : 1 \le i \le n, 1 \le j \le n\} - E(C_n)$. Clearly $p + q = 5n + 1$.
Case 1. $n \equiv 0 \pmod{12}$.
Let $n = 12t$ and $t > 0$. Define a map $f : V(G_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t \\ f(u_{5t+1+i}) & = f(v_{5t+i}) & = 2 & 1 \le i \le 4t \\ f(u_{9t+1+i}) & = f(v_{9t+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

$f(u_{5t+1}) = 0$, $f(v_{12t}) = 1$.
Case 2. $n \equiv 1 \pmod{12}$.
Let $n = 12t + 1$ and $t > 0$. Assign the label to the vertices u_i $(1 \le i \le 12t)$, v_i $(1 \le i \le 12t - 1)$ as in case 1. Then put the labels 0, 1, 2 to the vertices v_{12t}, u_{12t+1}, v_{12t+1} respectively.
Case 3. $n \equiv 2 \pmod{12}$.
Let $n = 12t + 2$ and $t > 0$. Define a map $f : V(G_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t+1 \\ f(u_{5t+1+i}) & = f(v_{5t+1+i}) & = 2 & 1 \le i \le 4t \\ f(u_{9t+2+i}) & = f(v_{9t+1+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

$f(u_{9t+2}) = f(v_{12t+2}) = 2$, $f(u_{12t+2}) = 0$, $f(u_{12t+1}) = 1$.
Case 4. $n \equiv 3 \pmod{12}$.
Let $n = 12t - 9$ and $t > 0$. Define a map $f : V(G_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-4 \\ f(u_{5t-3+i}) & = f(v_{5t-4+i}) & = 2 & 1 \le i \le 4t-3 \\ f(u_{9t-6+i}) & = f(v_{9t-7+i}) & = 1 & 1 \le i \le 3t-3 \end{array}$$

$f(u_{5t-3}) = 0$, $f(v_{12t-9}) = 1$.
Case 5. $n \equiv 4 \pmod{12}$.

Let $n = 12t - 8$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-3 \\ f(u_{5t-3+i}) & = f(v_{5t-3+i}) & = 2 & 1 \leq i \leq 4t-3 \\ f(u_{9t-6+i}) & = f(v_{9t-6+i}) & = 1 & 1 \leq i \leq 3t-3 \end{array}$$

$f(u_{12t-8}) = 1$, $f(v_{12t-8}) = 2$.

Case 6. $n \equiv 5 \pmod{12}$.

Let $n = 12t - 7$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-3 \\ f(u_{5t-2+i}) & = f(v_{5t-2+i}) & = 2 & 1 \leq i \leq 4t-3 \\ f(u_{9t-4+i}) & = f(v_{9t-5+i}) & = 1 & 1 \leq i \leq 3t-3 \end{array}$$

$f(u_{5t-2}) = 0$, $f(v_{5t-2}) = 1$, $f(u_{9t-4}) = 2$, $f(v_{12t-7}) = 1$.

Case 7. $n \equiv 6 \pmod{12}$.

Let $n = 12t - 6$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-2 \\ f(u_{5t-2+i}) & = f(v_{5t-2+i}) & = 2 & 1 \leq i \leq 4t-3 \\ f(u_{9t-4+i}) & = f(v_{9t-5+i}) & = 1 & 1 \leq i \leq 3t-2 \end{array}$$

$f(u_{9t-4}) = 2$, $f(v_{12t-6}) = 2$.

Case 8. $n \equiv 7 \pmod{12}$.

Let $n = 12t - 5$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-2 \\ f(u_{5t-2+i}) & = f(v_{5t-2+i}) & = 2 & 1 \leq i \leq 4t-2 \\ f(u_{9t-4+i}) & = f(v_{9t-4+i}) & = 1 & 1 \leq i \leq 3t-3 \end{array}$$

$f(u_{12t-6}) = f(u_{12t-5}) = 1$, $f(v_{12t-6}) = 0$, $f(v_{12t-5}) = 2$.

Case 9. $n \equiv 8 \pmod{12}$.

Let $n = 12t - 4$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-2 \\ f(u_{5t-1+i}) & = f(v_{5t-1+i}) & = 2 & 1 \leq i \leq 4t-2 \\ f(u_{9t-2+i}) & = f(v_{9t-3+i}) & = 1 & 1 \leq i \leq 3t-2 \end{array}$$

$f(u_{5t-1}) = 0$, $f(v_{5t-1}) = f(v_{12t-4}) = 1$, $f(u_{9t-2}) = 2$.

Case 10. $n \equiv 9 \pmod{12}$.

Let $n = 12t - 3$ and $t > 0$. Define a map $f : V(G_n) \rightarrow \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \leq i \leq 5t-1 \\ f(u_{5t-1+i}) & = f(v_{5t-1+i}) & = 2 & 1 \leq i \leq 4t-2 \\ f(u_{9t-2+i}) & = f(v_{9t-3+i}) & = 1 & 1 \leq i \leq 3t-1 \end{array}$$

$f(u_{9t-2}) = 2$, $f(v_{12t-3}) = 2$.

Case 11. $n \equiv 10 \pmod{12}$.

Let $n = 12t - 2$ and $t > 0$. Define a map $f : V(G_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-1 \\ f(u_{5t+i}) & = f(v_{5t-1+i}) & = 2 & 1 \le i \le 4t-1 \\ f(u_{9t-1+i}) & = f(v_{9t-2+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

$f(u_{5t}) = 0$, $f(v_{12t-2}) = 2$.
Case 12. $n \equiv 11 \pmod{12}$.
Let $n = 12t - 1$ and $t > 0$. Define a map $f : V(G_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t \\ f(u_{5t+i}) & = f(v_{5t+i}) & = 2 & 1 \le i \le 4t-1 \\ f(u_{9t+i}) & = f(v_{9t-1+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

$f(u_{9t}) = 0$, $f(v_{12t-1}) = 1$.
The following table 1 shows that G_n is a Total Mean Cordial graph. □

TABLE 1.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0 \pmod{12}$	$20t+1$	$20t$	$20t$
$n \equiv 1 \pmod{12}$	$20t+2$	$20t+2$	$20t+2$
$n \equiv 2 \pmod{12}$	$20t+4$	$20t+3$	$20t+4$
$n \equiv 3 \pmod{12}$	$20t-15$	$20t-14$	$20t-15$
$n \equiv 4 \pmod{12}$	$20t-13$	$20t-13$	$20t-13$
$n \equiv 5 \pmod{12}$	$20t-11$	$20t-12$	$20t-11$
$n \equiv 6 \pmod{12}$	$20t-9$	$20t-10$	$20t-10$
$n \equiv 7 \pmod{12}$	$20t-8$	$20t-8$	$20t-8$
$n \equiv 8 \pmod{12}$	$20t-7$	$20t-6$	$20t-6$
$n \equiv 9 \pmod{12}$	$20t-5$	$20t-4$	$20t-5$
$n \equiv 10 \pmod{12}$	$20t-3$	$20t-3$	$20t-3$
$n \equiv 11 \pmod{12}$	$20t-1$	$20t-1$	$20t-2$

The helm H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle.

Theorem 2.4. *Helms H_n are Total Mean Cordial.*

Proof. Let $V(H_n) = \{u, u_i, v_i : 1 \le i \le n\}$ and $E(H_n) = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_n u_1\} \cup \{uu_i, u_i v_i : 1 \le i \le n\}$. Clearly the order and size of H_n are $2n + 1$ and $3n$ respectively.
Case 1. $n \equiv 0 \pmod{12}$.
Let $n = 12t$ and $t > 0$. Construct a vertex labeling $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t \\ f(u_{5t+1+i}) & = f(v_{5t+1+i}) & = 2 & 1 \le i \le 4t \\ f(u_{9t+1+i}) & = f(v_{9t+1+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

and $f(v_{5t+1}) = 1$.

Case 2. $n \equiv 1 \pmod{12}$.
Let $n = 12t + 1$ and $t > 0$. Define a map $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t \\ f(u_{5t+1+i}) & = f(v_{5t+1+i}) & = 2 & 1 \le i \le 4t \\ f(u_{9t+1+i}) & = f(v_{9t+1+i}) & = 1 & 1 \le i \le 3t - 2 \end{array}$$

$f(u_{5t+1}) = 0$, $f(v_{5t+1}) = 1$, $f(u_{12t}) = f(u_{12t+1}) = 1$, $f(v_{12t}) = 2$ and $f(v_{12t+1}) = 0$.
Case 3. $n \equiv 2 \pmod{12}$.
Let $n = 12t + 2$ and $t > 0$. Define a map $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t + 1 \\ f(u_{5t+1+i}) & = f(v_{5t+1+i}) & = 2 & 1 \le i \le 4t \\ f(u_{9t+1+i}) & = f(v_{9t+1+i}) & = 1 & 1 \le i \le 3t - 1 \end{array}$$

$f(u_{12t+1}) = f(u_{12t+2}) = 1$ and $f(v_{12t+1}) = f(v_{12t+2}) = 2$.
Case 4. $n \equiv 3 \pmod{12}$.
The Total Mean Cordial labeling of H_3 is given in figure 3.

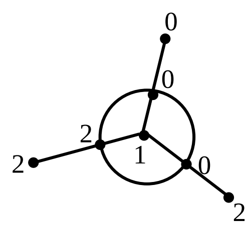

FIGURE 3.

Let $n = 12t + 3$ and $t > 0$. Define a map $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t + 1 \\ f(u_{5t+2+i}) & = f(v_{5t+2+i}) & = 2 & 1 \le i \le 4t + 1 \\ f(u_{9t+3+i}) & = f(v_{9t+3+i}) & = 1 & 1 \le i \le 3t \end{array}$$

$f(u_{5t+2}) = 0$, $f(v_{5t+2}) = 1$.
Case 5. $n \equiv 4 \pmod{12}$.
Let $n = 12t - 8$ and $t > 0$. Define a map $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t - 3 \\ f(u_{5t-3+i}) & = f(v_{5t-3+i}) & = 2 & 1 \le i \le 4t - 3 \\ f(u_{9t-6+i}) & = f(v_{9t-6+i}) & = 1 & 1 \le i \le 3t - 3 \end{array}$$

$f(u_{12t-8}) = 1$ and $f(v_{12t-8}) = 2$.
Case 6. $n \equiv 5 \pmod{12}$.
The Total Mean Cordial labeling of H_3 is given in figure 4.

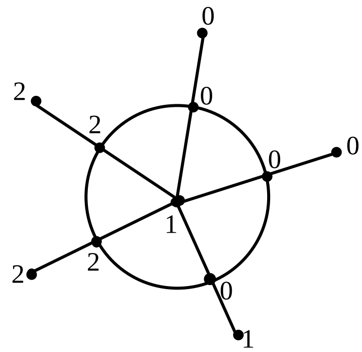

FIGURE 4.

Let $n = 12t + 5$ and $t > 0$. Define a function $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t+2 \\ f(u_{5t+3+i}) & = f(v_{5t+3+i}) & = 2 & 1 \le i \le 4t+1 \\ f(u_{9t+4+i}) & = f(v_{9t+4+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

$f(u_{5t+3}) = 0$, $f(v_{5t+3}) = 1$, $f(u_{12t+4}) = f(u_{12t+5}) = 1$ and $f(v_{12t+4}) = f(v_{12t+5}) = 2$.

Case 7. $n \equiv 6 \pmod{12}$.

Let $n = 12t - 6$ and $t > 0$. Define a function $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-2 \\ f(u_{5t-2+i}) & = f(v_{5t-2+i}) & = 2 & 1 \le i \le 4t-2 \\ f(u_{9t-4+i}) & = f(v_{9t-4+i}) & = 1 & 1 \le i \le 3t-2. \end{array}$$

Case 8. $n \equiv 7 \pmod{12}$.

Let $n = 12t - 5$ and $t > 0$. Define a function $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-2 \\ f(u_{5t-2+i}) & = f(v_{5t-2+i}) & = 2 & 1 \le i \le 4t-2 \\ f(u_{9t-4+i}) & = f(v_{9t-4+i}) & = 1 & 1 \le i \le 3t-3 \end{array}$$

$f(u_{12t-6}) = f(u_{12t-5}) = 1$, $f(v_{12t-6}) = 2$ and $f(v_{12t-5}) = 0$.

Case 9. $n \equiv 8 \pmod{12}$.

Let $n = 12t - 4$ and $t > 0$. Construct a vertex labeling $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-2 \\ f(u_{5t-1+i}) & = f(v_{5t-1+i}) & = 2 & 1 \le i \le 4t-2 \\ f(u_{9t-3+i}) & = f(v_{9t-3+i}) & = 1 & 1 \le i \le 3t-3 \end{array}$$

$f(u_{5t-1}) = 0$, $f(v_{5t-1}) = 1$, $f(u_{12t-5}) = f(u_{12t-4}) = 1$ and $f(v_{12t-5}) = f(v_{12t-4}) = 2$.

Case 10. $n \equiv 9 \pmod{12}$.

Let $n = 12t - 3$ and $t > 0$. Define $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-1 \\ f(u_{5t-1+i}) & = f(v_{5t-1+i}) & = 2 & 1 \le i \le 4t-1 \\ f(u_{9t-2+i}) & = f(v_{9t-2+i}) & = 1 & 1 \le i \le 3t-1 \end{array}$$

Case 11. $n \equiv 10 \pmod{12}$.
Let $n = 12t - 2$ and $t > 0$. Define a function $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t-1 \\ f(u_{5t+i}) & = f(v_{5t+i}) & = 2 & 1 \le i \le 4t-1 \\ f(u_{9t-1+i}) & = f(v_{9t-1+i}) & = 1 & 1 \le i \le 3t-2 \end{array}$$

$f(u_{5t}) = 0$, $f(v_{5t}) = 1$, $f(u_{12t-2}) = 1$ and $f(v_{12t-2}) = 2$.
Case 12. $n \equiv 11 \pmod{12}$.
Let $n = 12t - 1$ and $t > 0$. Define a function $f : V(H_n) \to \{0, 1, 2\}$ by $f(u) = 1$,

$$\begin{array}{llll} f(u_i) & = f(v_i) & = 0 & 1 \le i \le 5t \\ f(u_{5t+i}) & = f(v_{5t+i}) & = 2 & 1 \le i \le 4t-1 \\ f(u_{9t-1+i}) & = f(v_{9t-1+i}) & = 1 & 1 \le i \le 3t-2 \end{array}$$

$f(u_{12t-2}) = f(u_{12t-1}) = 1$ and $f(v_{12t-2}) = f(v_{12t-1}) = 2$.
The following table 2 shows that H_n is a Total Mean Cordial graph. □

Table 2.

Values of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0 \pmod{12}$	$20t + 1$	$20t$	$20t$
$n \equiv 1 \pmod{12}$	$20t + 2$	$20t + 2$	$20t + 2$
$n \equiv 2 \pmod{12}$	$20t + 3$	$20t + 4$	$20t + 4$
$n \equiv 3 \pmod{12}$	$20t + 5$	$20t + 6$	$20t + 5$
$n \equiv 4 \pmod{12}$	$20t - 13$	$20t - 13$	$20t - 13$
$n \equiv 5 \pmod{12}$	$20t + 9$	$20t + 8$	$20t + 9$
$n \equiv 6 \pmod{12}$	$20t - 9$	$20t - 10$	$20t - 10$
$n \equiv 7 \pmod{12}$	$20t - 6$	$20t - 6$	$20t - 6$
$n \equiv 8 \pmod{12}$	$20t - 7$	$20t - 6$	$20t - 6$
$n \equiv 9 \pmod{12}$	$20t - 5$	$20t - 4$	$20t - 5$
$n \equiv 10 \pmod{12}$	$20t - 3$	$20t - 3$	$20t - 3$
$n \equiv 11 \pmod{12}$	$20t - 1$	$20t - 2$	$20t - 1$

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Dr. R. Ponraj received his Ph.D in Manonmaniam Sundaranar University, Tirunelveli. He is currently an Assistant Professor at Sri Paramakalyani College, Alwarkurichi, India. His Research interest is in Discrete Mathematics.

Department of Mathematics, Sri Paramakalyani College, Alwarkurichi, Tamil Nadu, India-627412.
e-mail: ponrajmaths@gmail.com

Mr. S. Sathish Narayanan did his M.Phil in St. Johns College, Palayamkottai. He is persuing doctoral research work. His Research interest is in Graph labeling.

Department of Mathematics, Sri Paramakalyani College, Alwarkurichi, Tamil Nadu 627 412, India.
e-mail: sathishrvss@gmail.com