Journal of the Korean Institute of Illuminating and Electrical Installation Engineers (2015) 29(2) : 40~46
http://dx.doi.org/10.5207/JIEIE.2015.29.2.040 ISSN 1229-4691(Print) ISSN 2287-5034(Online)

논문 29-2-6

Control of a Three-Phase Voltage Source Inverter using Model Predictive Control of Laguerre Functions

Uk-Rae Cho* · Wang-Cheol Cha · Joung-Ho Park · Ho-Jeon Shin · Jae-Cheol Kim**

Abstract

This paper presents a method of controlling a three-phase VSI (Voltage Source Inverter) using MPC (Model Predictive Control) designed using Laguerre functions. It also provides a model of the three-phase VSI and its resistive-inductive load and then an overview of MPC design using Laguerre functions. The biggest challenge in using MPC is the high number of computations involved, which makes online implementation difficult. On the other hand, the LMPC (Laguerre Model Predictive Control) reduces the number of computations made and so online implementation becomes possible where traditional MPC would be unteneble. The simulation results from MATLAB are also provided.

Key Words : Inverter, LMPC, MPC, Laguerre Function, PWM, MIMO, MATLAB

1. Introduction

The control of three-phase inverters has been mostly performed by using PWM (Pulse Width Modulation) methods. Recently, however, more advanced control techniques including MPC have been used. The fact that a three-phase VSI is a system that is nonlinear, has a finite set of outputs and is MIMO (Multi-Input Multi-Output) makes MPC a control technique of natural choice. However, MPC involves many computations during its optimization and prediction stages, which may make online implementation difficult. This is a big challenge in using LMPC to control an inverter and many such "fast" systems [1-3].

Many articles have been written on the subject of controlling the load current of a three-phase VSI using MPC. Amongst them two prominent ones spring to mind: the studies by Hameed et al and Khalif et al use a special type of MPC called FS-MPC (Finite Set MPC) to control the load current. In both papers, the output voltage vector is shown to contain set seven output voltages, each voltage corresponding to a switching state [*Sa*, *Sb*, *Sc*] = [0, 0, 0] to [1, 1, 1] and noting that states [0, 0, 0] and [1, 1, 1] give the same output. Thus, at each sampling instant, seven output voltages are calculated and compared to the

* Main author : Ph.D. from the Dept. of Electrical Engineering from Soong Sil University
** Corresponding author : Professor of Electrical Engineering from Soong Sil University
Tel : 061_931_6391, Fax : 061_931_6399
E-mail : ukrae@lycos.co.kr
Received : 2014. 10. 13
Accepted : 2014. 12. 2

reference voltage. The output voltage closest to the reference voltage is chosen to be the predicted voltage and is used in the following sampling instant. This method provides rapid computations due to the known fixed number of output voltages at each instant [1-3]. Calculations have to be made in all elements at each instant, which increase the computation burden. It turns out that a trade-off between rapid computation and high accuracy in reference tracking is necessary. This balance may not be easy to achieve. The method presented in this paper does not consider the fact that there is a finite set of output voltages. Rather, it considers the three-phase VSI just like any other ordinary dynamic system with unfixed output set [1,7].

This study uses a special type of MPC designed using Laguerre functions to control the load current. This type of LMPC was shown in [PAPER] to reduce the computation burden, but this study will show that it actually tracks the reference signal very well [1-4].

2. Three-Phase Inverter Model

In the figure below

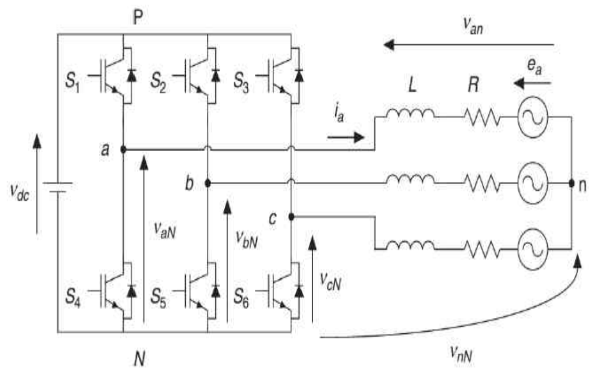

Fig 1. Inverter connected to grid

the switching signals at points a, b, and c are:

$$S_a = \begin{cases} 1 & S_1 = ON \quad S_4 = OFF \\ 0 & S_1 = OFF \quad S_4 = ON \end{cases}$$
$$S_b = \begin{cases} 1 & S_2 = ON \quad S_5 = OFF \\ 0 & S_2 = OFF \quad S_5 = ON \end{cases}$$
$$S_c = \begin{cases} 1 & S_3 = ON \quad S_6 = OFF \\ 0 & S_3 = OFF \quad S_6 = ON \end{cases}$$

Each voltages are points a, b and c are given $V_a = S_a V_{dc}$, $V_b = S_b V_{dc}$ and $V_c = S_c V_{dc}$, respectively.

2.1 Load Model

For each phase, the differential equation of the load current is given by (1)

$$L\frac{di(t)}{dt} + Ri(t) = V_{dc}(t)$$

Or

$$\frac{di(t)}{dt} = -\frac{R}{L}i(t) + \frac{1}{L}V_{dc}(t) \tag{1}$$

The equation (1) can be re-written in a stationary $\alpha-\beta$ frame and where I_α, I_β, V_α and V_β are obtained using the Clarke transformation [5-7]. (X stands for current or voltage)

$$\begin{bmatrix} X_\alpha \\ X_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} X_a \\ X_b \\ X_c \end{bmatrix} \tag{2}$$

2.2 Discretization

One of the many tenets deployed in the control of complex systems like the three-phase VSI using MPC is to use models that reduce the computation burden that is necessary when it comes to online implementation. It is also agreed upon that a linear model performs better in this respect than the nonlinear models they represent. For this reason and the fact that digital devices are typically used in implementation, this study uses linear discrete models and the Euler equation [5-7].

3. LMPC

A model predictive control designed using Laguerre functions is developed and summarized in [7]. The author presented this design technique for both single-variable and multi-variable systems.

Fortunately, the derivations for the single-variable case and the multi-variable case are very similar and knowing one would give enough insight into the derivation of the other. For brevity only the single-variable case is presented and it is hoped that this is enough to set forth the LMPC design technique [5-7].

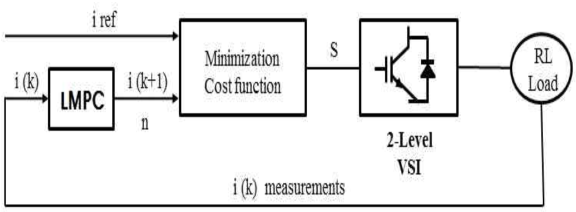

Fig 2. LMPC block diagram

Consider a plant with m inputs, u outputs and k states as described by equation (3) below [5-7]

$$x_m(k+1) = A_m x_m(k) + B_m u(k)$$

$$y(k) = C_m x(k) \tag{3}$$

Where $u(k)$ is the input variable, $y(k)$ is the process output and $x_m(k)$ is the state vector. The plant described by equation (3) can be expressed in augmented state space form as [5-7]

$$x(k+1) = A_x(k) + B\Delta u(k)$$

$$y(k) = C_x(k) \tag{4}$$

where $A = \begin{bmatrix} A_m & 0_m \\ C_m A_m & I_{q \times q} \end{bmatrix}$, $B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}$, $C = \begin{bmatrix} 0_m & I_{q \times q} \end{bmatrix}$

and the state $x(k) = \begin{bmatrix} \Delta x_m(k) & y(k) \end{bmatrix}^T$

3.1 LMPC Design Framework

The model predictive control designed using Laguerre functions is designed by the overall flow below and this paper was also written based on this flow chart.

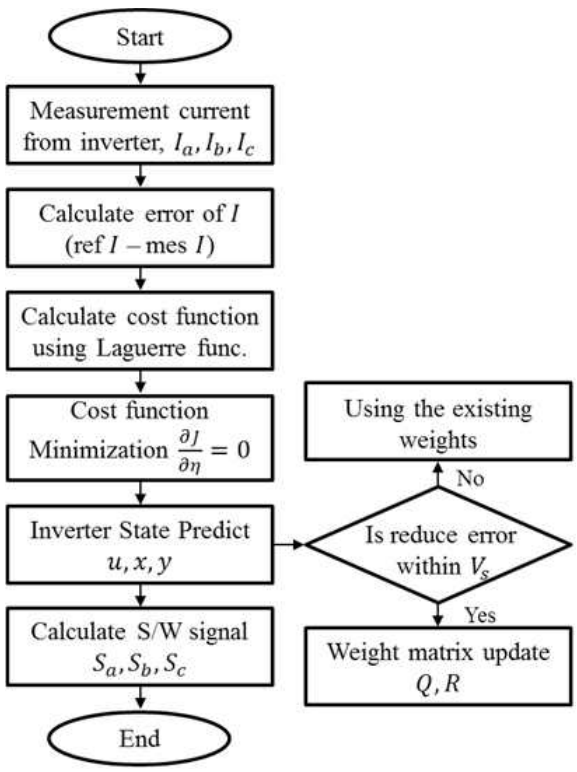

Fig 3. LMPC flow chart

At sampling instant k_i, the state variable $x(k_i)$ is available through the measurement. The future control trajectory is given by [5-7]

$$\Delta U = \left[\Delta u(k_i), \Delta u(k_i+1), \ldots, \Delta u(k_i+N_c-1) \right] \tag{5}$$

where N_c is the control horizon.

The sequence representing future state variables is

$$x(k_i+1|k_i), x(k_i+2|k_i), \ldots, x(k_i+m|k_i), \ldots \; x(k_i+N_p|k_i)$$

and $x(k_i+m|k_i)$ represents the predicted state variable at instant k_i+m given the current information $x(k_i)$. N_p is called the prediction horizon. Typically this is $N_c \le N_p$ because it is difficult to control what has not been predicted. All the output variables can be grouped together in one output vector as follows [5-7]

$$Y = \left[y(k_i+1|k_i), y(k_i+2|k_i), \ldots y(k_i+N_p|k_i) \right]^T \tag{6}$$

In designing an MPC using Laguerre functions the control trajectory ΔU is expressed using a set of orthonormal functions called Laguerre functions. Since the state and output vectors can also be described in terms of ΔU, it follows that they too can be expressed using Laguerre functions. At sampling instant k_i, the control trajectory is described using Laguerre functions as in equation (7) [7].

$$\Delta u(k_i+k) = \sum_{j=1}^{N} c_j(k_i) l_j(k_i) \tag{7}$$

where k = 0, 1, 2, $\cdots N_c$ and $l_j(k_i)$ is the inverse z-transform of the jth term in the discrete Laguerre network. Equation (7) can be rewritten as [6-7]

$$\Delta u(k_i+k) = L(k)^T \eta \tag{8}$$

where $L(k) = \left[l_1(k), l_2(k), \ldots, l_N(k) \right]^T$, $\eta = \left[c_1, c_2, \ldots, c_N \right]^T$

and the coefficients $c_1, c_2, \ldots, c_N$ are obtained from system data. At an arbitrary future instant m, the state is described using Laguerre functions as [5-7]

$$x(k_i+m|k_i) = A^m x(k_i) + \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T \eta \tag{9}$$

Similarly, the output is described as

$$y(k_i+m|k_i) = CA^m x(k_i) + \sum_{i=0}^{m-1} CA^{m-i-1} BL(i)^T \eta \tag{10}$$

3.2 Cost Function

The cost function is used to choose the optimal control (voltage) trajectory ΔU to bring the predicted output (load current) as close as possible to the set-point [5-7]. The cost function

$$J = \sum_{m=1}^{N_p} x(k_i+m|k_i)^T Q x(k_i+m|k_i) + \eta^T R_L \eta \tag{11}$$

minimizes the error between the set-point signal and the output in the shortest possible time by carefully tuning the weighting matrices $Q \ge 0$ and $R_L > 0$.

Cost function minimization is the state variable equation (9) and can be rewritten as [5-7];

$$\phi(m)^T = \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T \tag{12}$$

By substituting equation (12) into (11) and performing the partial derivative $\partial J / \partial \eta = 0$, the Laguerre coefficients vector is found to be [5-7]

$$\Omega = \sum_{m=1}^{NP} \phi(m) Q \phi(m)^T + R_L \quad \text{and} \quad \Psi = \phi(m) Q A^m \qquad (13)$$

In RHC (Receding Horizon Control) only the first term in ΔU, that is $\Delta U(k_i)$, is implemented at instant k_i. The rest of the sequence is ignored. The state vector of the most recent measured value of the control signal is calculated. This procedure is repeated in real-time to give the receding horizon control law [5-7].

4. Simulation Results

The simulation parameters are the most commonly used by solar power plant and it was this design that was produced. A 500kW inverter is currently the most commonly used, and a 185㎟ cable was based on the 300m. The parameters used for simulation are shown in the following table.

Table 1. Parameter

Parameter	Value
Load resistance, R	0.06Ω
Load inductance, L	0.76mH
DC link voltage, V_{dc}	750V
Reference current	780A
Sampling time, T_δ	5 msec

The simulation results show the following features. Figs 4, 6 and 8 depict a situation in which no constraints and disturbances have been applied whereas Figs 5, 7 and 9 depict a situation where both constraints and disturbances have been applied.

From Fig. 4 it is observed that the controller is able to track the reference signal well. During the first 0.002seconds an overshoot in the predicted current I_δ was observed due to the fact that the starting point for the controller is 0Amps while that of the reference signal is 780Amps. In trying to quickly move to 780Amps the controller produces large changes in the control signal (voltage) as seen in Fig. 6. However, after this initial period (0.002sec) the controller tracks the reference signals with an error of less than 10 percent as can be seen in Fig. 8.

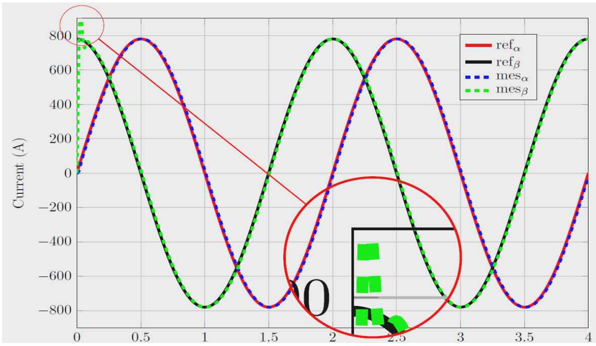

Fig 4. Load current without disturbance

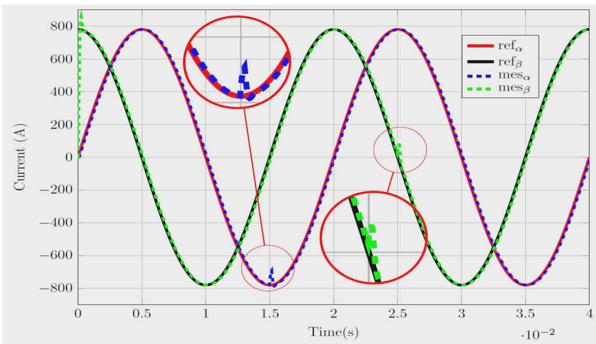

Fig 5. Load current with disturbance

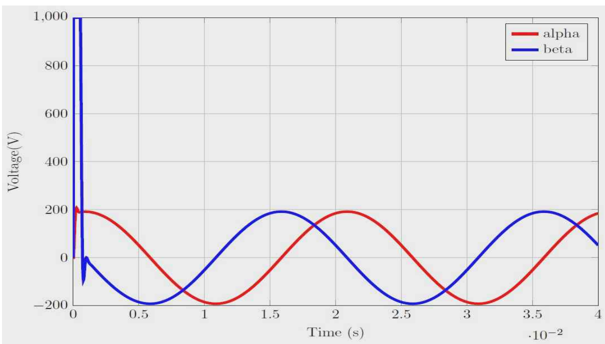

Fig 6. Control voltage without constraints

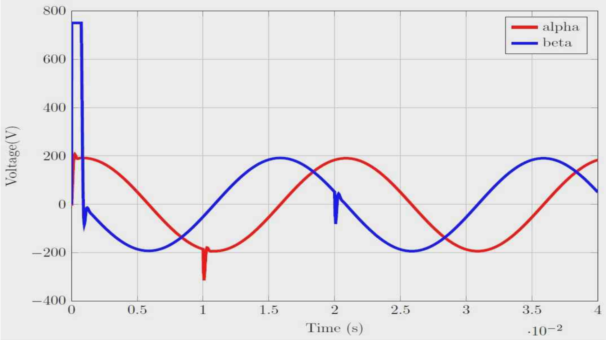

Fig 7. Constrained control voltage

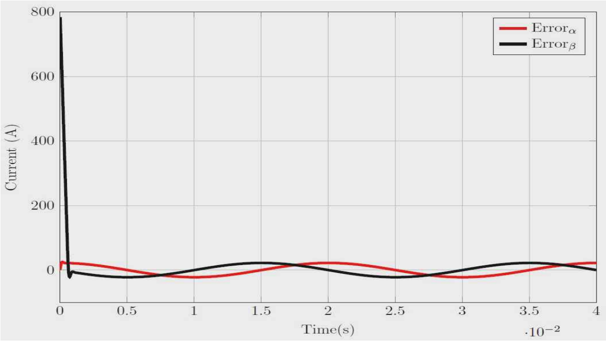

Fig 8. Error in load current without disturbance

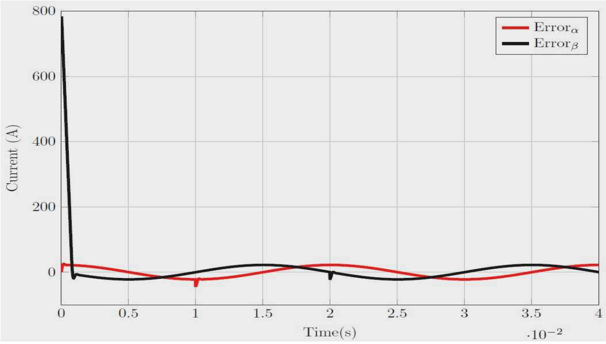

Fig 9. Error in load current with disturbance

This error can be reduced further by using a smaller sampling period or a higher order discrete model. In the second simulation, the control signal is constrained to within 780volts as presented in Fig. 7. In the same simulation disturbances have been applied at 0.001sec and 0.002sec as shown in Fig. 5. It is observed that the controller rejects those disturbances and tracks the reference signals as required. In addition, Figs 5, 7 and 9 depict a situation in which both constraints and disturbances have been applied, but immediately controlled.

5. Conclusion

This paper has presented a solution to control a three-phase inverter using a model predictive controller designed with Laguerre functions (LMPC). A quadratic cost function similar to that used in LQR (Linear Quadratic Regulator) has been used. The results from the simulations indicate that the controller performs very well and is feasible as well. With this perceived feasibility LMPC offers the added advantage that it can handle systems where rapid sampling and more complicated process dynamics are required. LMPC also reduces the number of parameters required for accurate prediction when using the traditional MPC approach. LMPC approximates the desired trajectory using a set of Laguerre functions over a long control horizon instead of making exact calculations for each time step. This is a large advantage in that with the reduced number of parameters, on-line implementation might be possible where the traditional MPC would have failed. Further research on the power grid is needed.

References

[1] William J. Palm III, System Dynamics, Second Edition, McGraw Hill, 2008.
[2] J.M. Maciejowski, Predictive Control with Constraints, First Edition, Prentice Hall, 2002.
[3] T. Kailath, Linear Systems, Prentice Hall, 1980.
[4] B.D.O Anderson, J.M Moore, Linear Optimal Control, Prentice Hall, 1971.
[5] Y. Gao, C.G.Lee and K.T. Chong Receding horizon tracking control for wheeled mobile robots with time-delay, Journal of Mechanical Science and Technology 22 (2008) 2403 2416.9.
[6] J.A Rossiter and Liuping Wang Exploiting Laguerre functions to improve the feasibility/performance comp-

romise in MPC, Journal 47th IEEE Conference on Decision and Control Cancun, Mexio, Dec. 9-11, 2008.
〔7〕 Tim Barry and Liuping Wang, A Model-free Predictive Controller with Laguerre Polyomials, 2004 6th Asian Control Conference.

◇ 저자소개 ◇

Uk-Rae Cho
He received an M.S. degree in the Dept. of Electronic Engineering from Dong-a University, Korea, in 2000 and a Ph.D. from the Dept. of Electrical Engineering from Soong Sil University, Korea, in 2010.

Wang-Cheol Cha
He received an M.S. degree in the Dept. of Electronic Engineering from Han-Yang University, Korea, in 2009 and a Ph.D. from the Dept. of Electrical Engineering from Soong Sil University, Korea, in 2010.

Joung-Ho Park
He received an M.S. degree in the Dept. of Electrical Engineering from Han-Yang University, Korea, in 1998 and a Ph.D. from the Dept. of Electrical Engineering from Soong Sil University, Korea, in 2010.

Ho-Jeon Shin
She received an M.S. degree in the Dept. of Electronic Engineering from Soong Sil University, Korea, in 2011 and a Ph.D. from the Dept. of Electrical Engineering from Soong Sil University, Korea, in 2011.

Jae-Cheol Kim
He received an M.S. and Ph.D. degree in Electrical Engineering from Seoul National University in 1983 and 1987, respectively. He joined the Department of Electrical Engineering, Soong Sil University, Korea, in 1988, where he is now a Professor. He is Chairman of the Korea Committee of KIIEE.