Back-up Control of Truck-Trailer Vehicles with Practical Constraints: Computing Time Delay and Quantization

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Received December 8, 2015; Accepted December 22, 2015; Published December 31, 2015

** Regular Paper*

***Abstract*:** In this paper, we present implementation of backward movement control of truck-trailer vehicles using a fuzzy mode-based control scheme considering practical constraints and computational overhead. We propose a fuzzy feedback controller where output is predicted with the delay of a unit sampling period. Analysis and design of the proposed controller is very easy, because it is synchronized with sampling time. Stability analysis is also possible when quantization exists in the implementation of fuzzy control architectures, and we show that if the trivial solution of the fuzzy control system without quantization is asymptotically stable, then the solutions of the fuzzy control system with quantization are uniformly ultimately bounded. Experimental results using a toy truck show that the proposed control system outperforms a conventional system.

***Keywords*:** Fuzzy control, Vehicle control, Time-delay computation

1. Introduction

Controlling the backward movement of articulated vehicles, such as a tractor-trailer, has been adopted as a testbed for a variety of control-design methods [1]. Backward movement control of computer-simulated articulated vehicles has been realized via intelligent controls, such as fuzzy control or neural control or both [2, 3]. However, stability of the control systems has not been elaborately analyzed in the literature. Since the problem of asymptotic stabilization for backward motion was addressed by Tanaka and Sano [11], simulation and experimental results for single- and multiple-trailer cases using a fuzzy controller have been presented [1-4, 17-19, 20, 21].

Recently, experimental results for backward movement control of a truck-trailer have been reported [1, 4, 21, 22], where intelligent modeling and control of a truck-trailer was accomplished using visual sensing for state feedback of the system. A charge-coupled device (CCD) camera has been used to sense the state [1, 4, 22], and the concept of parallel distributed compensation (PDC) has been adopted to design a fuzzy controller from the corresponding Takagi-Sugeno(TS) fuzzy model. Practical, multiobjective design of a truck-trailer control system was presented [4]. Practical constraints, such as avoidance of steering angle saturation, removal of measurement noise, the jackknife phenomenon, and so on, were considered. Linear matrix inequality (LMI)-based fuzzy control was presented to achieve a multi-objective control design satisfying these practical constraints. Also, a sensor reduction technique for implementation of a control system was presented [22].

These constraints may naturally be considered in control design problems; however, more considerable and unavoidable constraints, such as computing time delay and quantization, which seriously affect performance and stability of a closed loop control system, have not been considered up to now.

Computing time delay could happen in implementation of the control system, especially in complex sensing systems like truck-trailer control. Computing time delay, unlike the delayed system from the internal delayed property of the vehicle, raises an unpredictable effect on the stability and performance of the overall control system.

While computing time delay, the control system works depending on the control input at the previous sampling, and until the next control is generated, the vehicle moves during a long period of computing time. This results in unavoidable oscillation in the system, because the control system does not work in real time due to the unpredictable computing time delay. More seriously, if the control system comes into the uncontrollable region during the delay, the generated control law does not guarantee stability. In order to avoid the computing time delay in the control process, sometimes, the vehicle cannot continuously move, and occasionally stops during the sensing state at every sampling [1, 4, 22].

Quantization is also unavoidable in implementation of a digital control system. It increases quantization error in the feedback states and the enforced control input, which also results in an unpredictable result on control performance. In all the above-mentioned research, it was supposed that no quantization existed, and therefore, in real control systems, one cannot guarantee their stability. Therefore, the influence of quantization error on the stability and performance of the control system should be analyzed for acceptable control performance.

In this paper, we additionally consider two practical constraints: computing time delay due to the control process, and the quantization effect from digital implementation of the control architecture. The proposed control design guarantees stability under the existence of a certain delay and quantization effect.

Digital fuzzy control systems can be defined as hybrid dynamic systems, which usually consist of interconnection of a continuous-time and a discrete-time fuzzy controller [15]. The analysis and design of such fuzzy systems have been of continuing interest for several decades. Since the TS fuzzy model was presented [5, 7, 17], a number of variants of various kinds of TS fuzzy model-based controllers have been proposed [6, 8], where systematic design of the fuzzy controller can be possible. The stability of most fuzzy systems could be determined by Lyapunov stability analysis, and the LMI-based approach has recently been used to determine the existence of a common positive definite matrix [9, 10]. However, these results do not take into account computing time delay and quantization effect in digital implementation of fuzzy control systems. In this paper, for backward-movement control of a truck-trailer vehicle, we propose a design method for a fuzzy feedback controller that guarantees the stability of the system in the presence of computing time delay, and we investigate qualitative stability analysis of the digital fuzzy control systems with quantization in both the controller and the interconnection elements.

To this end, we first study the design method of a digital fuzzy controller (DFC) with consideration of computing time delay as the practical constraint. If the system has a considerable amount of computing time delay, the analysis and design of the controller are very difficult, because it makes the output of the controller unsynchronized with the sampling time. We propose a fuzzy feedback controller where output is delayed with the unit sampling period and is predicted using current states and the control input to the fuzzy control system at the previous sampling instant. Analysis and design of the controller become very easy, because the output of the proposed controller is synchronized with the sampling time. Therefore, the proposed control system can be designed using conventional methods, such as PDC- [11] and LMI-based analysis.

We then study the qualitative effects of quantization of the proposed digital fuzzy control system. We show that if the trivial solution of the fuzzy control system without quantization is asymptotically stable, then the solutions of the digital fuzzy control system with quantization are uniformly ultimately bounded.

To verify the validity and effectiveness of the scheme, the proposed fuzzy feedback controller is applied to back-up control of a truck-trailer vehicle, considering the constraints of computing time delay and quantization.

2. Related Work

In a discrete-time fuzzy system with control input to the fuzzy model, the dynamic properties of each subspace can be expressed with the following fuzzy IF-THEN rules.

$$\begin{aligned}&\text{Rule } i: \ \textit{If } x_1(k) \textit{ is } M_{i1} \ \cdots \ \textit{and } x_n(k) \textit{ is } M_{in}\\ &\qquad \textit{THEN } \mathbf{x}(k+1) = \mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\end{aligned} \tag{1}$$

where $i = 1, 2, \cdots, r$, M_{ij} is the i,j-th fuzzy set, $\mathbf{x}(k) = \begin{bmatrix} x_1(k) & x_2(k) & \cdots & x_n(k)\end{bmatrix}^T \in \Re^n$ represents the state vector of the fuzzy system, and $\mathbf{u}(k) = \begin{bmatrix} u_1(k) & u_2(k) & \cdots & u_m(k)\end{bmatrix}^T \in \Re^m$ represents the input of the fuzzy system. If the set of $(\mathbf{x}(k), \mathbf{u}(k))$ is given the output of the fuzzy system described in (1) can be obtained as follows:

$$\begin{aligned}\mathbf{x}(k+1) &= \frac{\sum_{i=1}^{r} w_i(k)\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\}}{\sum_{i=1}^{r} w_i(k)}\\ &= \sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\}\end{aligned} \tag{2}$$

where $w_i(k) = \prod_{j=1}^{n} \bar{M}_{ij}(x_j(k))$, $\bar{M}_{ij}(x_j(k))$ represents the grade of membership of $x_j(t)$ in M_{ij}, and $h_i(k) = w_i(k) / \sum_{i=1}^{r} w_i(k)$.

Based on the PDC concept, the fuzzy controller is distributively designed according to the corresponding rule of the fuzzy model [12]. Therefore, the PDC for the fuzzy model in (1) can be expressed as follows:

$$\mathbf{u}(k) = -\frac{\sum_{i=1}^{r} w_i(k)\,\mathbf{F}_i\mathbf{x}(k)}{\sum_{i=1}^{r} w_i(k)} = -\sum_{j=1}^{r} h_j(k)\,\mathbf{F}_j\mathbf{x}(k) \tag{3}$$

Substituting (3) into (2) gives the following closed-loop discrete-time fuzzy system.

$$\begin{aligned}\mathbf{x}(k+1) &= \sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x}(k) - \mathbf{B}_i\sum_{j=1}^{r} h_j(k)\mathbf{F}_j\mathbf{x}(k)\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(k)\, h_j(k)\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(k)\end{aligned} \tag{4}$$

Defining $\mathbf{G}_{ij} = \mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j$, we have

$$\begin{aligned}\mathbf{x}(k+1) &= \sum_{i}^{r} h_i(k)h_i(k)\mathbf{G}_{ii}\mathbf{x}(k) \\ &\quad + 2\sum_{i<j}^{r} h_i(k)h_j(k)\left\{\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right\}\mathbf{x}(k)\end{aligned} \tag{5}$$

The stability condition can be obtained per the following theorem.

Theorem 1: The equilibrium point of the closed loop discrete-time fuzzy system in (5) is asymptotically stable if there exists a common positive definite matrix $\mathbf{P}$ that satisfies the following inequalities for all *i* and *j* except the set (i, j) satisfying $h_i(k)\cdot h_j(k) = 0$.

$$\mathbf{G}_{ii}^T\mathbf{P}\mathbf{G}_{ii} - \mathbf{P} < \mathbf{0} \tag{6a}$$

$$\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T \mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right) - \mathbf{P} \le \mathbf{0}\,,\ for\ i<j\,. \tag{6b}$$

The proof of this theorem was given by Tanaka and Wang [12].

If $\mathbf{B} = \mathbf{B}_1 = \mathbf{B}_2 = \cdots = \mathbf{B}_r$ in fuzzy system (2) is satisfied, the closed loop system (4) can be obtained as follows:

$$\begin{aligned}\mathbf{x}(k+1) &= \sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x}(k) - \mathbf{B}\sum_{j=1}^{r} h_j(k)\mathbf{F}_j\mathbf{x}(k)\} \\ &= \sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i - \mathbf{B}\mathbf{F}_i\}\mathbf{x}(k) \\ &= \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{x}(k)\end{aligned} \tag{7}$$

where $\mathbf{G}_i = \mathbf{A}_i - \mathbf{B}\mathbf{F}_i$. Theorem 2 can be applied to stability analysis of the closed loop system in (7).

Theorem 2: The equilibrium point for the discrete-time fuzzy system, expressed in (2), is asymptotically stable if there exists a common positive definite matrix $\mathbf{P}$ satisfying the following inequalities.

$$\mathbf{G}_i^T\mathbf{P}\mathbf{G}_i - \mathbf{P} < \mathbf{0}\,,\quad for\quad i = 1,2,\cdots,r \tag{8}$$

The proof of this theorem was also given by Tanaka and Wang [12].

To prove the stability of the discrete-time fuzzy control system in Theorem 1 and Theorem 2, the common positive definite matrix $\mathbf{P}$ must be solved. LMI theory can be applied to solved $\mathbf{P}$, as in Boyd et al. [13]. The stability condition of Theorem 1 can be transformed into the LMI feasibility problem as follows.

LMI feasibility problem for the stability condition of Theorem 1: The problem of finding $\mathbf{P}$ that satisfies the LMIs, $\mathbf{P} > \mathbf{0}$ and $\mathbf{G}_i^T\mathbf{P}\mathbf{G}_i - \mathbf{P} < \mathbf{0}$, $i = 1,2,\cdots,r$ or proving the unfeasibility in case $\mathbf{A}_i \in \Re^{n\times n}$, $i = 1,2,\cdots,r$ was put forward by Tanaka and Wang [12].

In order to guarantee the stability of the closed loop system in (2), the design of a PDC fuzzy controller (3) can be used to solve the following LMI feasibility problem using Schur complements [13].

LMI feasibility problem equivalent to the PDC design problem (Case I): The problem is finding $\mathbf{X} > \mathbf{0}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_r$ that satisfy the following inequalities.

$$\begin{bmatrix}\mathbf{X} & \{\mathbf{A}_i\mathbf{X} - \mathbf{B}_i\mathbf{M}_i\}^T \\ \mathbf{A}_i\mathbf{X} - \mathbf{B}_i\mathbf{M}_i & \mathbf{X}\end{bmatrix} > \mathbf{0},\quad i = 1,2,\cdots,r \tag{9}$$

$$\begin{bmatrix}\mathbf{X} & \frac{1}{2}\{\mathbf{L}_i + \mathbf{L}_j\}^T \\ \frac{1}{2}\{\mathbf{L}_i + \mathbf{L}_j\} & \mathbf{X}\end{bmatrix} > \mathbf{0},\ for\ i<j \tag{10}$$

where $\mathbf{L}_i = \mathbf{A}_i\mathbf{X} \text{-} \mathbf{B}_i\mathbf{M}_i$, $\mathbf{X} = \mathbf{P}^{-1}$, $\mathbf{M}_1 = \mathbf{F}_1\mathbf{X}$, $\mathbf{M}_2 = \mathbf{F}_2\mathbf{X}$, $\cdots$, and $\mathbf{M}_r = \mathbf{F}_r\mathbf{X}$ [12].

If $\mathbf{B} = \mathbf{B}_1 = \mathbf{B}_2 = \cdots = \mathbf{B}_r$ is satisfied, the design of the PDC fuzzy controller in (3) is equivalent to solving the following LMI feasibility problem.

LMI feasibility problem equivalent to the PDC design problem (Case II): The problem is finding $\mathbf{X} > \mathbf{0}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_r$ that satisfy the following inequality:

$$\begin{bmatrix}\mathbf{X} & \{\mathbf{A}_i\mathbf{X} - \mathbf{B}_i\mathbf{M}_i\}^T \\ \mathbf{A}_i\mathbf{X} - \mathbf{B}_i\mathbf{M}_i & \mathbf{X}\end{bmatrix} > \mathbf{0},\ i = 1,2,\cdots,r \tag{11}$$

where $\mathbf{X} = \mathbf{P}^{-1}$, $\mathbf{M}_1 = \mathbf{F}_1\mathbf{X}$, $\mathbf{M}_2 = \mathbf{F}_2\mathbf{X}$, $\cdots$, and $\mathbf{M}_r = \mathbf{F}_r\mathbf{X}$ [12].

The feedback gain matrices $\mathbf{F}_1, \mathbf{F}_2, \cdots, and\ \mathbf{F}_r$ and the common positive definite matrix $\mathbf{P}$ can be given by the LMI solutions, $\mathbf{X}$ and $\mathbf{M}_1, \mathbf{M}_2, \cdots, \mathbf{M}_r$, as follows:

$$\mathbf{P} = \mathbf{X}^{-1},\ \ \mathbf{F}_1 = \mathbf{M}_1\mathbf{X}^{-1},\ \ \mathbf{F}_2 = \mathbf{M}_2\mathbf{X}^{-1},\ \cdots,\ \text{and}\ \mathbf{F}_r = \mathbf{M}_r\mathbf{X}^{-1} \tag{12}$$

3. Practical Constraint: Computing Time Delay

In practical control systems, a considerable computing time delay can occur in processing both sensor and controller parts. Let τ be the sum of all time delays. The ideal fuzzy controller given in (3) can be described with time delay τ as

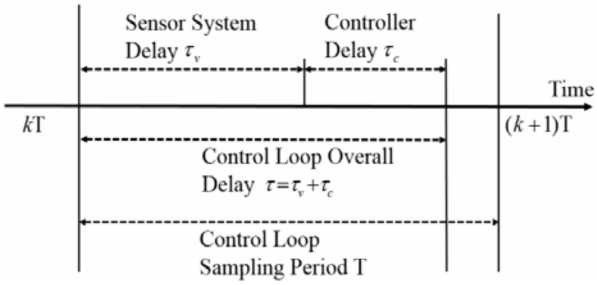

Fig. 1. Timing diagram of the fuzzy control loop.

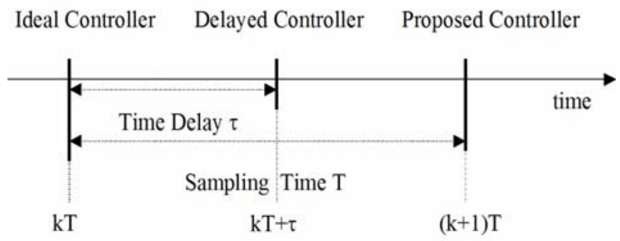

Fig. 2. Output timing of the controllers: the ideal, the delayed, and the proposed controller.

$$\begin{aligned}&\text{Rule } j: \ \textit{If } x_1(kT) \textit{ is } M_{j1} \ \cdots \ \textit{and } x_n(kT) \textit{ is } M_{jn} \\ &\qquad \textit{THEN } \mathbf{u}(kT+\tau) = -\mathbf{F}_j \mathbf{x}(kT)\end{aligned} \tag{13}$$

where $j = 1, 2, \cdots, r$.

Because the time delay makes the output of the controller unsynchronized with the sampling time, Theorem 1 cannot be applied to this system. In this paper, we propose a DFC that has the following fuzzy rules with consideration for the time delay of the fuzzy system given in (4).

$$\begin{aligned}&\text{Rule } j: \ \textit{If } x_1(k) \textit{ is } M_{j1} \ \cdots \ \textit{and } x_n(k) \textit{ is } M_{jn} \\ &\qquad \textit{THEN } \mathbf{u}(k+1) = \mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\end{aligned} \tag{14}$$

where $j = 1, 2, \cdots, r$.

In this scheme, the output of the fuzzy controller is delayed by a unit sampling period and is predicted. Hence, analysis and design of the controller becomes straightforward because the output of the proposed controller is synchronized with the sampling time.

The output of the DFC in (14) is inferred as

$$\begin{aligned}\mathbf{u}(k+1) &= \frac{\sum_{j=1}^{r} w_j(k) \ \{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\}}{\sum_{j=1}^{r} w_j(k)} \\ &= \sum_{j=1}^{r} h_j(k) \ \{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\}\end{aligned} \tag{15}$$

The general timing diagram of the fuzzy control loop is shown in Fig. 1, where T represents the sampling period of the control loop. τ_v and τ_c are the delays of the sensor and the fuzzy controller, respectively. Therefore, the output of the controller is applied to the fuzzy model after overall delay $\tau = \tau_v + \tau_c$.

The output timings of an ideal controller, a delayed controller, and the proposed controller are shown in Fig. 2. In the ideal controller, it is assumed that there is no computing time delay. If this controller is implemented in practical systems, time delay τ is added, according to (13). Analysis and design of this system with the delayed controller are complicated, because the output of the controller is not synchronized with the sampling time.

On the other hand, the analysis and design of the proposed controller are strainghtforward because the controller output is synchronized with the sampling time delayed by a unit sampling period. In the proposed controller, we can realize a control algorithm during the time interval $T - \tau_v$ in Fig. 1. During this time interval, a sophisticated algorithm, such as a fuzzy or a nonlinear control algorithm, can be efficiently realized in real time.

Combining the fuzzy model in (5) with the DFC in (15), the closed loop system is given as

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{u}(k+1) \end{bmatrix} = \sum_{i=1}^{r} h_i(k) \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{E}_i & \mathbf{D}_i \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{u}(k) \end{bmatrix} \tag{16}$$

Defining the new state vector as $\mathbf{w}(k) = \begin{bmatrix} \mathbf{x}(k) & \mathbf{u}(k) \end{bmatrix}^T$, the closed loop system in (16) can be modified to

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k) \mathbf{G}_i \mathbf{w}(k) \tag{17}$$

where $\mathbf{G}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{E}_i & \mathbf{D}_i \end{bmatrix}$.

Hence, the stability condition of the closed loop system in (17) becomes equivalent to the sufficient condition of Theorem 1, and the stability can be determined by solving the ***LMI feasibility problem about the stability condition of Theorem 1***. Also, the design problem of the DFC guaranteeing the stability of the closed loop system can be transformed into the ***LMI feasibility problem***. To do this, the design problem of the DFC is transformed into the design problem of the PDC fuzzy controller.

PDC design problem equivalent to DFC design problem: The problem is designing the PDC fuzzy controller $\mathbf{v}(k) = -\sum_{j=1}^{r} h_j(k) \ \overline{\mathbf{F}}_j \mathbf{w}(k)$ where the fuzzy system $\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k) \{\overline{\mathbf{A}}_i \mathbf{w}(k) + \overline{\mathbf{B}} \mathbf{v}(k)\}$ is given, where $\overline{\mathbf{A}}_i = \begin{bmatrix} \mathbf{A}_i & \mathbf{B}_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, $\overline{\mathbf{B}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$, and $\overline{\mathbf{F}}_j = -\begin{bmatrix} \mathbf{E}_j & \mathbf{D}_j \end{bmatrix}$.

Therefore, using the same notation in Section 2, the design problem of the DFC becomes equivalent to the ***LMI feasibility problem***.

LMI feasibility problem equivalent to DFC design problem: The problem is finding $\mathbf{X} > \mathbf{0}$ and $\mathbf{M}_1$,

$\mathbf{M}_2,\cdots,\mathbf{M}_r$ that satisfy following inequality:

$$\begin{bmatrix} \mathbf{X} & \{\bar{\mathbf{A}}_i\mathbf{X}-\bar{\mathbf{B}}\,\mathbf{M}_i\}^T \\ \bar{\mathbf{A}}_i\mathbf{X}-\bar{\mathbf{B}}\,\mathbf{M}_i & \mathbf{X} \end{bmatrix} > \mathbf{0},\ i=1,2,\cdots,r \quad (18)$$

where $\mathbf{X}=\mathbf{P}^{-1}$, $\mathbf{M}_1=\bar{\mathbf{F}}_1\mathbf{X}$, $\mathbf{M}_2=\bar{\mathbf{F}}_2\mathbf{X}$, $\cdots$, and $\mathbf{M}_r=\bar{\mathbf{F}}_r\mathbf{X}$.

The feedback gain matrices $\bar{\mathbf{F}}_1,\bar{\mathbf{F}}_2,\cdots,\bar{\mathbf{F}}_r$ and the common positive definite matrix $\mathbf{P}$ can be given by the LMI solutions, $\mathbf{X}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$, as follows:

$$\mathbf{P}=\mathbf{X}^{-1},\ \bar{\mathbf{F}}_1=\mathbf{M}_1\mathbf{X}^{-1},\ \bar{\mathbf{F}}_2=\mathbf{M}_2\mathbf{X}^{-1},\ \cdots and,\ \bar{\mathbf{F}}_r=\mathbf{M}_r\mathbf{X}^{-1} \quad (19)$$

Therefore, the control gain matrices $\mathbf{D}_1,\cdots,\mathbf{D}_r$ *and*, $\mathbf{E}_1,\cdots,\mathbf{E}_r$ of the proposed DFC can be obtained from the feedback gain matrices $\bar{\mathbf{F}}_1,\bar{\mathbf{F}}_2,\cdots,\bar{\mathbf{F}}_r$.

4. Practical Constraint: Quantization

In the implementation of digital fuzzy controllers, the quantization process is unavoidable. In this section, we investigate the nonlinear effects caused by the quantization process.

If $x\in\Re$ is the input and $Q(x)$ the output of a quantizer, the quantization process can be formulated as follows:

$$Q(x)=x+p(x) \quad (20)$$

where $p(x)$ represents the nonlinear error of quantization. Although there are many types of quantization, we will concentrate on the most commonly used fixed-point quantization, which can be characterized as

$$|p(x)|<\varepsilon \quad (21)$$

where positive constant ε represents the upper bound of quantization error.

Therefore, the quantized state $Q_{\mathbf{x}}(\mathbf{x}(k))$ with respect to system state $\mathbf{x}\in\Re^n$ can be defined as

$$\mathbf{x}_q(k)=Q_{\mathbf{x}}(\mathbf{x}(k))=\begin{bmatrix} Q_x(x_1(k)) \\ Q_x(x_2(k)) \\ \vdots \\ Q_x(x_n(k)) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} + \begin{bmatrix} p_x(x_1(k)) \\ p_x(x_2(k)) \\ \vdots \\ p_x(x_n(k)) \end{bmatrix} = \mathbf{x}(k)+\mathbf{p}_{\mathbf{x}}(\mathbf{x}(k)) \quad (22)$$

where $\|\mathbf{p}_{\mathbf{x}}(\mathbf{x}(k))\|\le\varepsilon_{\mathbf{x}}$.

A similar definition for the quantized controller $Q_{\mathbf{u}}(\mathbf{u}(k))$ with respect to control input $\mathbf{u}\in\Re^m$ is given as

$$\mathbf{u}_q(k)=Q_{\mathbf{u}}(\mathbf{u}(k))=\begin{bmatrix} Q_u(u_1(k)) \\ Q_u(u_2(k)) \\ \vdots \\ Q_u(u_m(k)) \end{bmatrix} = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_n(k) \end{bmatrix} + \begin{bmatrix} p_u(u_1(k)) \\ p_u(u_2(k)) \\ \vdots \\ p_u(u_m(k)) \end{bmatrix} = \mathbf{u}(k)+\mathbf{p}_{\mathbf{u}}(\mathbf{u}(k)) \quad (23)$$

where $\|\mathbf{p}_{\mathbf{u}}(\mathbf{u}(k))\|\le\varepsilon_{\mathbf{u}}$.

In practical digital control systems, the TS fuzzy model (24a) and controller (24b) include the quantized terms as

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}_{\mathbf{q}}(k)\} \quad (24a)$$

$$\mathbf{u}_q(k+1)=Q_u\left(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u}_{\mathbf{q}}(k)+\mathbf{E}_i\mathbf{x}_{\mathbf{q}}(k)\}\right) \quad (24b)$$

In fuzzy model (24a), the control input necessarily should be quantized, because the control law is applied by digital/analog (D/A) converter; however, $\mathbf{x}(k)$ is the internal states in the fuzzy model, and thus, should not be quantized.

In fuzzy control dynamics, $\mathbf{u}_{\mathbf{q}}(k)$ is the quantized input at the previously applied sampling time, and $\mathbf{x}_{\mathbf{q}}(k)$ is the quantized feedback state by sensors and A/D converter. The control input at the next sampling time, $\mathbf{u}(k+1)$, should be quantized, because the generated control input would be applied to the fuzzy model by D/A converter. Hence, the calculated control law should be quantized, and then, the proposed digital fuzzy controller in (15) can be transformed into

$$\begin{aligned}\mathbf{u}_{\mathbf{q}}(k+1)&=Q_{\mathbf{u}}\left(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u}_{\mathbf{q}}(k)+\mathbf{E}_i\mathbf{x}_{\mathbf{q}}(k)\}\right)\\ &=\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u}_{\mathbf{q}}(k)+\mathbf{E}_i\mathbf{x}_{\mathbf{q}}(k)\}\\ &\quad+Q_{\mathbf{u}}\left(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u}_{\mathbf{q}}(k)+\mathbf{E}_i\mathbf{x}_{\mathbf{q}}(k)\}\right)\\ &\quad-\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u}_{\mathbf{q}}(k)+\mathbf{E}_i\mathbf{x}_{\mathbf{q}}(k)\}\end{aligned} \quad (25)$$

The state $\mathbf{x}(k)$ in the fuzzy model in (24) and the state $\mathbf{x}_q(k)$ in the fuzzy controller in (25) need to be unified to derive a closed-loop equation. Therefore, we apply the quantization operator to the equation of the fuzzy model in (24) as

$$
\begin{aligned}
\mathbf{x_q}(k+1) &= Q_{\mathbf{x}}(\mathbf{x}(k+1)) \\
&= Q_{\mathbf{x}}\left(\sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\}\right) \\
&= \sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x_q}(k)+\mathbf{B}_i\mathbf{u_q}(k)\} \\
&\quad +\mathbf{p_x}\left(\sum_{i=1}^{r} h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\}\right) \\
&\quad -\sum_{i=1}^{r} h_i(k)\mathbf{A}_i\mathbf{p_x}(\mathbf{x}(k))
\end{aligned} \tag{26}
$$

From (26) and (25), the state space model of the quantized closed-loop system can be obtained as

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) + \Delta(k) \tag{27}$$

where $\mathbf{w}(k) = \begin{bmatrix}\mathbf{x_q}(k) & \mathbf{u_q}(k)\end{bmatrix}^T$ represents the augmented state, $\mathbf{G}_i = \begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i \\ \mathbf{E}_i & \mathbf{D}_i\end{bmatrix}$, and $\Delta(k) = \begin{bmatrix}\Delta_{\mathbf{x}}(k) \\ \Delta_{\mathbf{u}}(k)\end{bmatrix}$.

If any reference signal or noise exists, the state space model in (27) can be rewritten as

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) + \Delta(k) + \mathbf{r}(k) \tag{28}$$

where $\mathbf{r}(k)$ represents a by-product of the reference signal or noise.

As a follow-up step, we analyzed the stability of the digital fuzzy systems with consideration of quantization effects. Let us define the norm $\|\bullet\|_{\mathbf{P}}$ in $\Re^n$. In order to obtain the stability condition for the closed-loop system in (28), we define the norm $\|\bullet\|_{\mathbf{P}}$ in $\Re^n$ as

$$\|\mathbf{w}(k)\|_{\mathbf{P}} = (\mathbf{w}^T(k)\mathbf{P}\mathbf{w}(k))^{\frac{1}{2}} \tag{29}$$

where $\mathbf{P} \in \Re^{n\times n}$ is a symmetric positive-definite matrix.

Definition 1: The digital fuzzy system in (27) is said to be uniformly ultimately bounded with bound α if and only if for any $\beta > 0$ there exists $T(\beta) > 0$, independent of $K \ge 0$, such that whenever $|\mathbf{w}_K| \le \beta$ and $k \ge T(\beta)$, one has $|\mathbf{w}_{k+K}| \le \alpha$ [15].

Remark 1: Uniform ultimate boundness is similar to uniform asymptotic stability, except that the attracting point $x = 0$ is now replaced by an attracting set given by $\{x \in \Re^n : |x| \le \alpha\}$.

Theorem 3: If the following two conditions are satisfied, there exists a very small positive constant δ, such that $\|\Delta(k)\|_{\mathbf{P}} < \delta$ for all integers k and positive constant J, such that the closed loop system (28) is uniformly ultimately bounded by $J\delta$.

(i) There exists a common positive-definite matrix P for the system

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) \tag{30}$$

that satisfies sufficient condition (3) in Theorem 1.

(ii) There exists $\bar{\delta}\rangle 0$ $r(k) \in B_{\bar{\delta}}\{x \mid \|x\|\langle\bar{\delta}\}$ for $k > 0$.

Proof: If there exists a common positive definite matrix $\mathbf{P}$ satisfying sufficient condition (3) in Theorem 1, $V(\mathbf{w}(k)) = \|\mathbf{w}(k)\|_{\mathbf{P}} = (\mathbf{w}^T(k)\mathbf{P}\mathbf{w}(k))^{\frac{1}{2}}$ can be a norm Lyapunov function for the system in (30).

Since the system in (30) satisfies the asymptotical stability by assumption, there exists a constant c, such that

$$
\begin{aligned}
\Delta V_{(30)}(\mathbf{w}(k)) &= V(\mathbf{w}(k+1)) - V(\mathbf{w}(k)) \\
&= \left\|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k)\right\|_{\mathbf{P}} - \|\mathbf{w}(k)\|_{\mathbf{P}} \\
&\le (c-1)\|\mathbf{w}(k)\|_{\mathbf{P}} = (c-1)\ V(\mathbf{w}(k)) \quad \textit{for}\ 0 < c < 1
\end{aligned}
$$

where $\Delta V_{(30)}(\mathbf{w}(k))$ denotes the first forward difference along the solution of the system in (30).

The first forward difference for the closed loop system in (28) can be given as

$$
\begin{aligned}
\Delta V_{(28)}(\mathbf{w}(k),k) &= \left\|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) + \Delta(k) + \mathbf{r}(k)\right\|_{\mathbf{P}} \\
&\quad - \|\mathbf{w}(k)\|_{\mathbf{P}} \\
&\le \left\|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k)\right\|_{\mathbf{P}} \\
&\quad - \|\mathbf{w}(k)\|_{\mathbf{P}} + \|\Delta(k)\|_{\mathbf{P}} + \|\mathbf{r}(k)\|_{\mathbf{P}} \\
&\le (c-1)\ V(\mathbf{w}(k)) + \|\Delta(k)\|_{\mathbf{P}} + \|\mathbf{r}(k)\|_{\mathbf{P}} \\
&\le (c-1)\ V(\mathbf{w}(k)) + \delta + \|\mathbf{r}(k)\|_{\mathbf{P}}
\end{aligned}
$$

Therefore, whenever $\mathbf{w}(K)$ is selected so that $V(\mathbf{w}(K)) = \|\mathbf{w}(K)\|_{\mathbf{P}} \le \beta$, then $V(\mathbf{w}(K))$ must be less than the solution of the following comparison equation [17]:

$$X_{k+1} - X_k = (c-1)\,X_k + \delta + \|\mathbf{r}(k)\|_{\mathbf{P}} \quad \textit{for}\ k \ge K \tag{31}$$

where $X_K = \beta$.

The solution of the comparison Eq. (31) can be obtained as

$$X_{k+K} = c^k\beta + \frac{1-c^k}{1-c}\delta + \sum_{j=0}^{k} c^{k-j}\|\mathbf{r}(j+K)\|_{\mathbf{P}} \tag{32}$$

Because $0 < c < 1$ in (32), $c^k \to 0$ as $k \to \infty$ and $\gamma_k(K) \equiv \sum_{j=0}^{k} c^{k-j}\|\mathbf{r}(j+K)\|_{\mathbf{P}}\ \langle\ \bar{\delta}\sum_{j=0}^{k} c^{k-j} = \bar{\delta}\frac{1-c^{k+1}}{1-c}$.

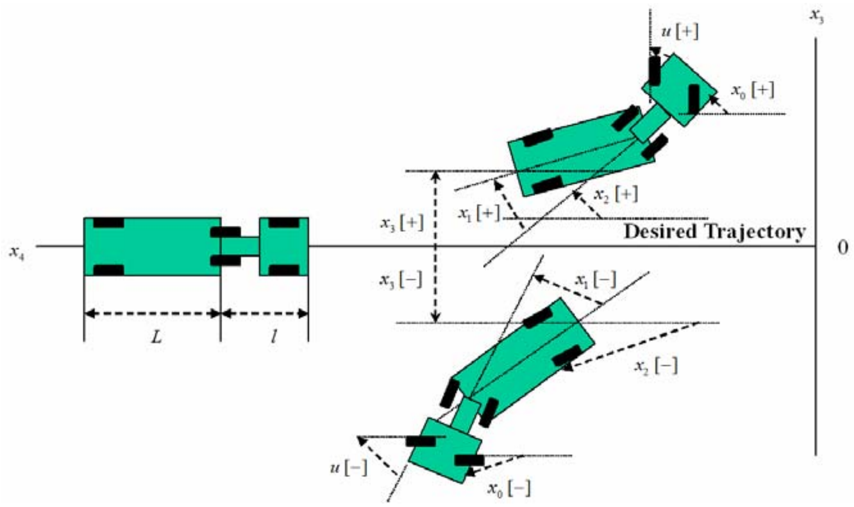

Fig. 3. A truck-trailer model and the coordinate system.

Now, we can say that V_{k+K} uniformly converges to $\frac{1}{1-c}(\delta+\bar{\delta})$ for $K \geq 0$ as $k \to \infty$ from the comparison Eq. in (31), and $J = \frac{1}{1-c}(1+\frac{\bar{\delta}}{\delta})$ will also converge because $V(\mathbf{w}_{k+K}) \leq X_{k+K}$.

If the closed loop system (27) is uniformly ultimately bounded by $J\delta$, the system converges to the attracting set $\{\mathbf{w} \in \Re^{m+n} : \|\mathbf{w}\|_{\mathbf{P}} \leq J\delta\}$, not to the equlibrium point $\mathbf{w} = \mathbf{0}$. If $\Delta(k) = 0$ in closed-loop system (27) (that is, quatization error does not exist), the constant δ is zero, and the attracting set converges to $\mathbf{w} = \mathbf{0}$. In this case, the closed-loop system is asymptotically stable.

Theorem 3 shows the analysis of qualitative characteristics of the fuzzy control systems considering quantization. The positive constant J is related to the equation $\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k)$, not to $\Delta(k)$. Therefore, once the digital fuzzy control system is stably designed, it does not diverge, although quantization error exists, and the smaller the quatization errors become, the more the asymptotic stability is guaranteed.

5. Backward Movement Control of Truck-Trailer Vehicles

In this section, we apply the proposed controller for backward movement control of a truck-trailer system with the proposed constraints, including computing time delay, and we investigate the quantization effect on control performance.

5.1 Model a Truck-trailer System

Tokunaga and Ichihashi derived a model of a truck-trailer system [16].

$$
\begin{aligned}
x_0(k+1) &= x_0(k) + vT/l \tan[u(k)] \\
x_1(k) &= x_0(k) - x_2(k) \\
x_2(k+1) &= x_2(k) + vT/L \sin[x_1(k)] \\
x_3(k+1) &= x_3(k) \\
&\quad + vT\cos[x_1(k)]\sin[\{x_2(k+1)+x_2(k)\}/2] \\
x_4(k+1) &= x_4(k) \\
&\quad + vT\cos[x_1(k)]\cos[\{x_2(k+1)+x_2(k)\}/2]
\end{aligned}
\tag{33}
$$

where $u(k)$ represents the steering angle of the truck, l the length of the truck, L the length of the trailer, T the sampling time, and v the constant backward speed.

x_0 : angle of truck
$x_1(k)$: angle difference between truck and trailer
$x_2(k)$: angle of trailer
$x_3(k)$: vertical position of rear end of trailer
$x_4(k)$: horizontal position of rear end of trailer

Fig. 3 shows the schematic diagram of this system.

Tanaka and Sano defined the state vector as $\mathbf{x}(k) = [x_1(k) \quad x_2(k) \quad x_3(k)]^T$ in truck-trailer model (33) and expressed the fuzzy model using fuzzy rules [11].

$$
\begin{aligned}
&\text{Rule 1: } \textit{If } x_2(k) + \{vT/2L\}\, x_1(k) \textit{ is } M_1 \\
&\qquad \textit{THEN } \mathbf{x}(k+1) = \mathbf{A}_1\mathbf{x}(k) + \mathbf{B}_1 u(k) \\
&\text{Rule 2: } \textit{If } x_2(k) + \{vT/2L\}\, x_1(k) \textit{ is } M_2 \\
&\qquad \textit{THEN } \mathbf{x}(k+1) = \mathbf{A}_2\mathbf{x}(k) + \mathbf{B}_2 u(k)
\end{aligned}
\tag{34}
$$

where $\mathbf{A}_1 = \begin{bmatrix} 1-\frac{vT}{L} & 0 & 0 \\ \frac{vT}{L} & 1 & 0 \\ \frac{v^2T^2}{2L} & vT & 1 \end{bmatrix}$, $\mathbf{A}_2 = \begin{bmatrix} 1-\frac{vT}{L} & 0 & 0 \\ \frac{vT}{L} & 1 & 0 \\ \frac{dv^2T^2}{2L} & dvT & 1 \end{bmatrix}$,

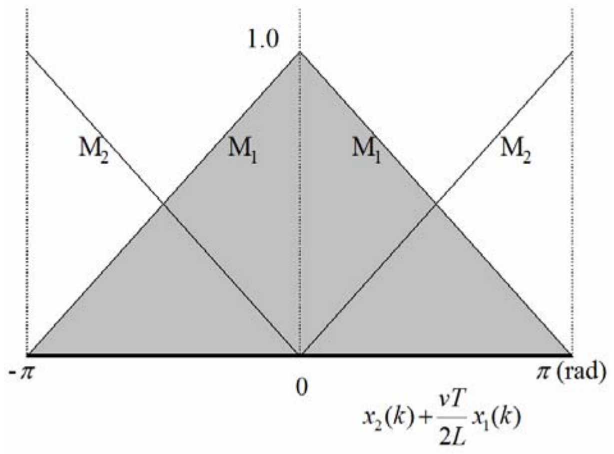

Fig. 4. Membership function.

and $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2=\begin{bmatrix} \frac{vT}{l} & 0 & 0 \end{bmatrix}^T$. $l=0.15[m]$, $L=0.38[m]$, $v=-1.0\ [m/s]$, $T=2.0\ [s]$, and $d=10^{-2}/\pi$. Fig. 4 shows the membership function of the premise in the fuzzy rules in (34). The experimental setup is shown in Fig. 5, where the states of the controlled system are observed by an angle detection sensor, a potentiometer and a CCD camera similar to other research [1, 3].

The computing time delay inevitably arises in generation of the control law because of the image processing and the fuzzy control rule computation. The quantization effect results from A/D conversion processing and image processing from state sensing. The computing time delay and quantization degrade the performance of backward movement control. Such degradation effects will be shown in the experimental results.

5.2 Backward Movement Control of an Articulated Vehicle Using Conventional Control

In this subsection, experimental results of backward movement control of a truck-trailer vehicle are given using the conventional discrete-time fuzzy controller without considering computing time delay and quantization effect.

To solve the backward parking problem in (34), the PDC fuzzy controller can be designed as follows:

$$\begin{aligned} &\text{Rule 1: } \mathit{If}\ x_2(k)+\{vT/2L\}\cdot x_1(k)\ \mathit{is}\ M_1 \\ &\qquad \mathit{THEN}\ u(k)=\mathbf{F}_1^T\mathbf{x}(k) \\ &\text{Rule 2: } \mathit{If}\ x_2(k)+\{vT/2L\}\cdot x_1(k)\ \mathit{is}\ M_2 \\ &\qquad \mathit{THEN}\ u(k)=\mathbf{F}_2^T\mathbf{x}(k) \end{aligned} \tag{35}$$

where $\mathbf{F}_1=\begin{bmatrix} 1.2837 \\ -0.4139 \\ 0.0201 \end{bmatrix}$, and $\mathbf{F}_2=\begin{bmatrix} 0.9773 \\ -0.0709 \\ 0.0005 \end{bmatrix}$.

A Ricatti equation for linear discrete systems was used to determine these feedback gains. The detailed derivation of these feedback gains come from Tanaka and Sano [11].

Substituting (35) into (34) yields the following closed loop system, because $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2$.

$$\mathbf{x}(k+1)=\sum_{i=1}^{2}h_i(k)\mathbf{G}_i\mathbf{x}(k) \tag{36}$$

where $\mathbf{G}_2=\begin{bmatrix} 0.448 & 0.296 & -0.014 \\ -0.364 & 1 & 0 \\ 0.116\times10^{-2} & -0.637\times10^{-2} & 1 \end{bmatrix}$, and

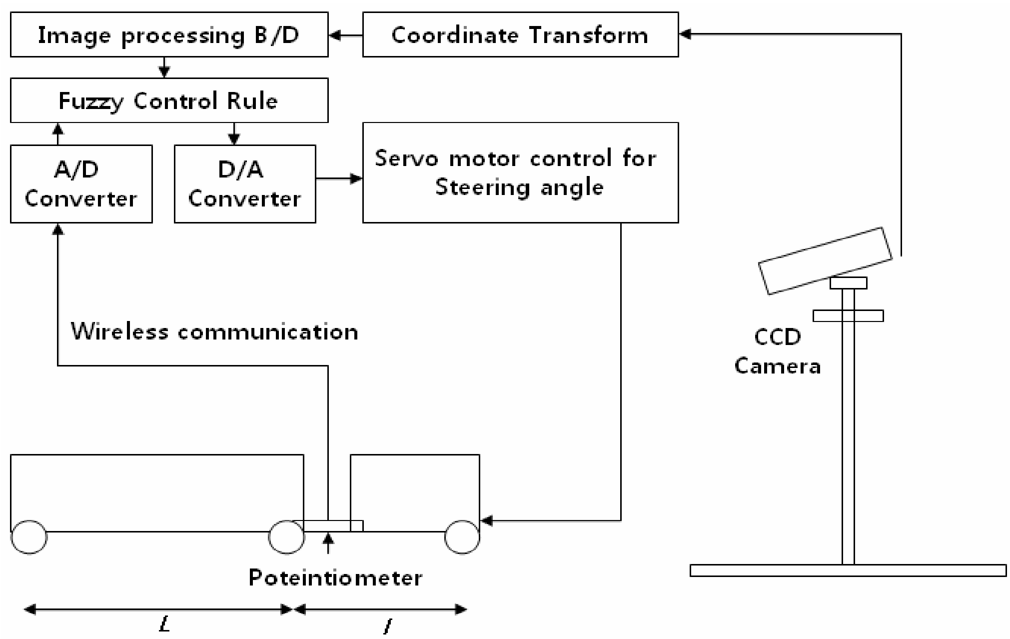

Fig. 5. Experimental setup for the backing-up control of truck-trailer type vehicle.

Table 1. The initial conditions of the truck-trailer system.

CASE	$x_1(0)$ [deg]	$x_2(0)$ [deg]	$x_3(0)$ [m]
CASE I	0	0	1
CASE II	-90	135	-0.5

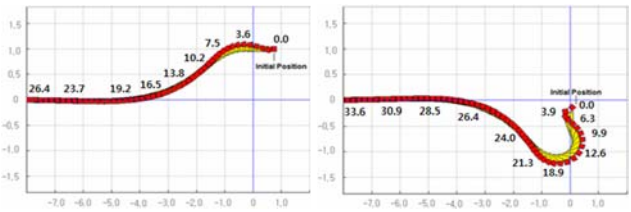

Fig. 6. Experimental results by conventional discrete-time fuzzy control.

$$\mathbf{G}_1 = \begin{bmatrix} 0.448 & 0.296 & -0.014 \\ -0.364 & 1 & 0 \\ 0.364 & -2 & 1 \end{bmatrix}.$$

Since there exists the common positive matrix $\mathbf{P}$, which satisfies the sufficient stability condition in (3), the closed loop system is asymptotically stable on the whole. That is, backward parking can be accomplished for all initial conditions if the computing time delay does not exist. The common positive matrix is given as

$$\mathbf{P} = \begin{bmatrix} 113.9 & -92.61 & 2.540 \\ -92.61 & 110.7 & -3.038 \\ 2.540 & -3.038 & 0.5503 \end{bmatrix}$$

However, with a real system, the ideal fuzzy controller in (35) can be described with the following rules due to the computing time delay τ.

$$\begin{aligned} &\text{Rule 1}: \ \mathit{If}\, x_2(kT) + vT/\{2L\} \cdot x_1(kT) \ \mathit{is}\ M_1 \\ &\qquad \mathit{THEN}\ u(kT+\tau) = \mathbf{F}_1^T \mathbf{x}(kT) \\ &\text{Rule 2}: \ \mathit{If}\, x_2(kT) + vT/\{2L\} \cdot x_1(kT) \ \mathit{is}\ M_2 \\ &\qquad \mathit{THEN}\ u(kT+\tau) = \mathbf{F}_2^T \mathbf{x}(kT) \end{aligned} \tag{37}$$

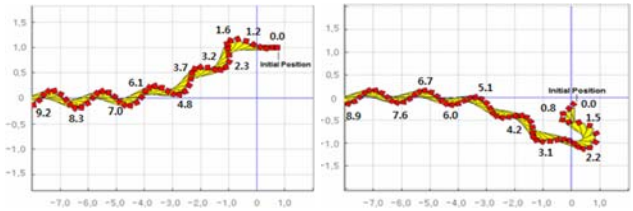

Fig. 7. Experimental results by conventional discrete time fuzzy control.

Two initial conditions used for the experiments of the truck-trailer system are given in Table 1.

The experiments were executed where maximum time delay τ is half the sampling time (τ=1 [sec]). In the revised version, for comparison with the existing control approach, we additionally conducted experiments using the method proposed by Tanaka et al. [4], in which the experimental results for back-up control of a truck trailer was first shown. In the experiments, the TS fuzzy model-based controller was adopted, and because the processing time for detecting the position of a truck trailer was long (that is, due to the computing time delay), the vehicle was stopped during position detection at every sampling. After position detection, the steering was controlled by a motor, and the vehicle moved shortly thereafter.

Fig. 6 shows the back-up control results based on the methods of Tanaka et al. [4], in which the overall control processing time is as long as the intermittent movement, because of the computing time delay, while the backing up of the truck trailer was successfully accomplished.

In order to show the influence of computing time delay on control performance, and the effectiveness of the proposed control scheme by a comparative experimental results, the control results based on the same control law as Tanaka et al. [4] without intermittent back-up movement, unlike the scheme of Tanaka et al. [4], has been presented. That is, the truck-trailer made successive movements at a constant backward speed, and during the computing time delay, the system disengaged from real-time control. The backing up control results are presented in Fig. 7 and Fig. 8. While the overall control processing time was considerably reduced, unlike the control results from inter-

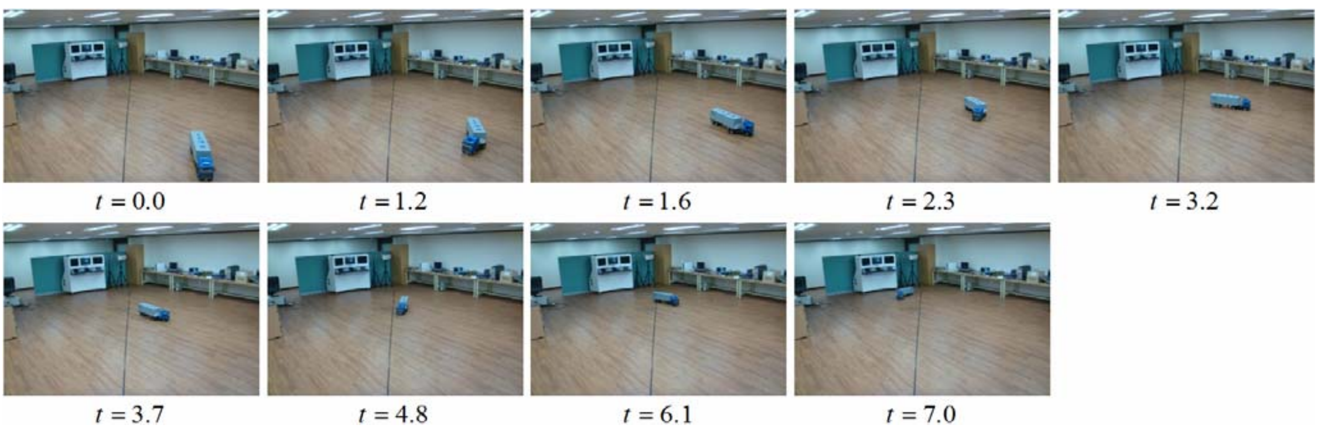

Fig. 8. The photograph of experimental results by conventional fuzzy control.

mittent movement, in the experiments, serious oscillation occurred because of the computing time delay due to visual sensing and image processing for detection of the state.

5.3 The Proposed Fuzzy Control System with Consideration of Computing Time Delay and Quantization Effect

In this subsection, we design a DFC considering computing time delay, and we analyze quantization effects on the control system. Following the DFC design technique in Section 3, we can construct a DFC for the backing up control problem as follows:

$$\begin{aligned}&\text{Rule 1}: \ \ \textit{If}\ x_2(k)+\{vT/2L\}\cdot x_1(k)\ \textit{is}\ M_1\\&\qquad \textit{THEN}\ u(k+1)=\mathbf{D}_1u(k)+\mathbf{E}_1\mathbf{x}(k)\\&\text{Rule 2}: \ \ \textit{If}\ x_2(k)+\{vT/2L\}\cdot x_1(k)\ \textit{is}\ M_2\\&\qquad \textit{THEN}\ u(k+1)=\mathbf{D}_2u(k)+\mathbf{E}_2\mathbf{x}(k)\end{aligned} \tag{38}$$

Combining (34) with (38), the augmented closed loop system is given as

$$\mathbf{w}(k+1)=\sum_{i=1}^{2}h_i(k)\mathbf{G}_i\mathbf{w}(k) \tag{39}$$

where $\mathbf{G}_1=\begin{bmatrix}\mathbf{A}_1 & \mathbf{B}_1\\ \mathbf{E}_1 & \mathbf{D}_1\end{bmatrix}$ and $\mathbf{G}_2=\begin{bmatrix}\mathbf{A}_2 & \mathbf{B}_2\\ \mathbf{E}_2 & \mathbf{D}_2\end{bmatrix}$.

To obtain the control gain matrices $\mathbf{D}_1, \mathbf{D}_2, \mathbf{E}_1, \textit{and}\ \mathbf{E}_2$ guaranteeing the stability of the closed loop system in (39), we solve the ***LMI feasibility problem that is equivalent to the DFC design problem*** as follows.

The problem is finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1$, $\mathbf{M}_2$ that satisfy the following inequalities:

$$\begin{bmatrix}\mathbf{X} & \{\bar{\mathbf{A}}_i\mathbf{X}-\bar{\mathbf{B}}\mathbf{M}_i\}^T\\ \bar{\mathbf{A}}_i\mathbf{X}-\bar{\mathbf{B}}\mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0},\ \ \textit{for}\ i=1,2. \tag{40}$$

where $\bar{\mathbf{A}}_i=\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$ and $\bar{\mathbf{B}}=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}$.

The matrices $\mathbf{X}$, $\mathbf{M}_1, \textit{and}\ \mathbf{M}_2$ in LMIs are determined using a convex optimization technique offered by Nesterov and Nemirovsky [14].

$$\mathbf{X}=\begin{bmatrix}157.0056 & 61.9680 & -1.6565 & 220.727\\ 61.9680 & 50.4822 & 69.8423 & 53.4329\\ -1.6565 & 69.8423 & 489.4416 & -2.3866\\ 220.7727 & 53.4329 & -2.3866 & 442.6866\end{bmatrix},$$

$$\mathbf{M}_1=\begin{bmatrix}-96.3672 & -43.1521 & 41.8056 & -5.8356\end{bmatrix},$$

$$\mathbf{M}_2=\begin{bmatrix}-116.3143 & -66.0021 & 1.3065 & -22.9842\end{bmatrix}.$$

The feedback gains and a common positive definite matrix $\mathbf{P}$ are determined by the relationship in (19) as follows.

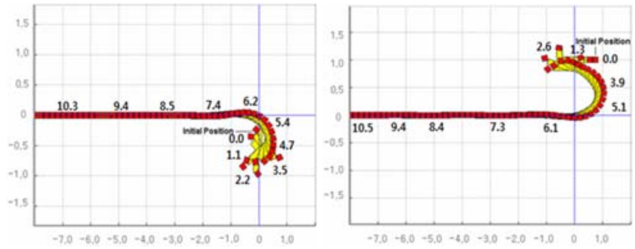

Fig. 9. Experimental results from the proposed control.

$$\mathbf{P}=\mathbf{X}^{-1}=\begin{bmatrix}0.0995 & -0.1036 & 0.0149 & -0.0370\\ -0.1036 & 0.1373 & -0.0198 & 0.0350\\ 0.0149 & -0.0198 & 0.0049 & -0.0050\\ -0.0370 & 0.0350 & -0.0050 & 0.0165\end{bmatrix},$$

$$\begin{aligned}\bar{\mathbf{F}}_1&=\mathbf{M}_1\mathbf{X}^{-1}=-\begin{bmatrix}\mathbf{E}_1 & \mathbf{D}_1\end{bmatrix}\\&=\begin{bmatrix}-3.9047 & 2.6765 & -0.3020 & 1.5869\end{bmatrix}\end{aligned} \tag{41}$$

and

$$\begin{aligned}\bar{\mathbf{F}}_2&=\mathbf{M}_2\mathbf{X}^{-1}=-\begin{bmatrix}\mathbf{E}_2 & \mathbf{D}_2\end{bmatrix}\\&=\begin{bmatrix}-3.8624 & 2.1564 & -0.3102 & 1.6123\end{bmatrix}\end{aligned}$$

Therefore, the closed loop system is asymptotically stable on the whole, and the control gain matrices are given by the ***PDC design problem equivalent to DFC design problem*** as $\mathbf{E}_1=\begin{bmatrix}3.9047 & -2.6765 & 0.3020\end{bmatrix}$, $\mathbf{D}_1=-1.5869$, $\mathbf{D}_2=-1.6123$, and $\mathbf{E}_2=\begin{bmatrix}3.8624 & -2.1564 & 0.3102\end{bmatrix}$.

Next, we analyzed the stability of the fuzzy control system with consideration for quantization. The quantization problem is unavoidable, because of the A/D conversion processes using potentiometers in order to control the truck-trailer system.

There exists a common positive definite matrix $\mathbf{P}$ in (41) for the closed loop system (39) and $\mathbf{r}(k)=\mathbf{0}$, since it is a regulation problem. Hence, all the sufficient conditions of Theorem 3 are satisfied. Therefore, we can say that the closed loop fuzzy system is uniformly ultimately bounded and does not diverge.

Fig. 9 shows with precision the experimental results of the designed DFC with the computing time delay ($\tau=1$ [sec]) and quantization effect, $\varepsilon=10^{-2}$. In the implementation of the control system, the quantization arises in state feedback by the A/D converter and camera sensor, for which quantization error depends on their resolution. It can also arise in the D/A converter for applying the generated control input to the truck-trailer system, and the error depends on the resolution of the D/A converter. In the experiments, we used A/D and D/A converters with 16-bit resolution, and because the manipulated bound is 2π, we can determine that the bound of the quantization error is within 0.006 degrees. Also, because we adopted a camera sensor with 1024×768 pixel resolution, the movement range is within $8m\times 2m$, and the bound of the quantization error is within $0.008m\times 0.003m$. Therefore, we can say that the overall bound of the quantization error can be within 0.01. As mentioned above,

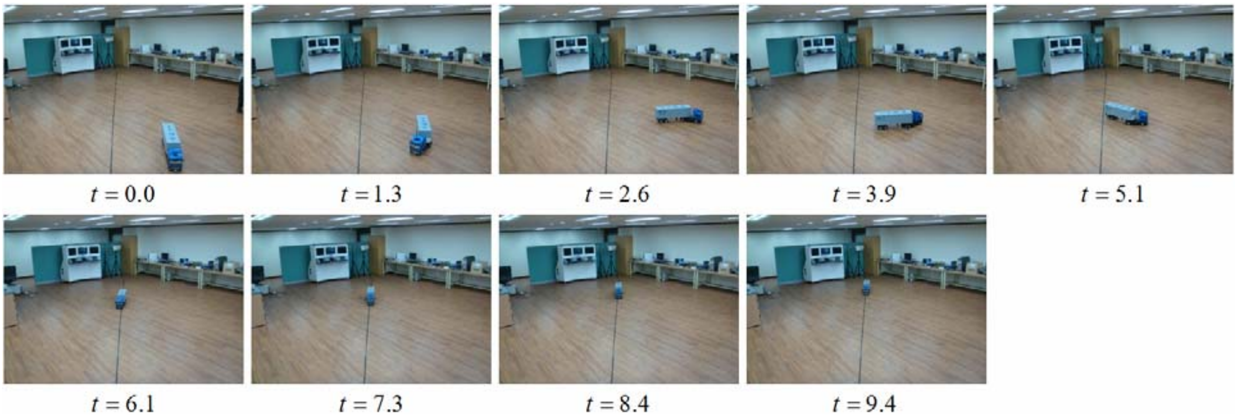

Fig. 10. The photograph of experimental results by conventional fuzzy control.

note that the bounds of the quantization error do not affect control stability, but only the uniform stable bound, and as well, the control designer need not estimate it.

Fig. 10 shows selected experimental results for Case I. As can be seen in the figures, backward parking is successfully accomplished, compared with Fig. 7, although a considerable computing time delay exists. However, due to the quantization effects, the solution of the present feedback control system seems to have some oscillation with a small amplitude. Thus, we can say that the closed-loop system converges to some small neighborhood of the origin.

6. Conclusion

In this paper, we presented backward movement control of a truck-trailer vehicle using a fuzzy model-based control. The practical constraints, including computing time delay due to control processing time and quantization from digital implementation of the control architecture, were considered, and a control design that guarantees stability under their existence was proposed. The stability of the system is guaranteed despite existence of the computing time delay, so real-time control processing could be possible. Furthermore, we proved that quantization has the effect of replacing a convergence of solutions to the origin by convergence to some small neighborhood of the origin. Experimental results show that the proposed fuzzy controller effectively achieves backward movement control of a truck-trailer vehicle, although considerable computing time delay exists. We also show that in the presence of quantization, the control system is uniformly ultimately bounded.

Acknowledgement

This work was supported in part by the Technology Innovation Program (Development of Smart Video/Audio Surveillance SoC & Core Component for Onsite Decision Security System) under Grant 10047788, and by Institute for Information & Communications Technology Promotion (IITP) grant funded by the Korean Government (MSIP) (B0101-15-0525, Development of global multi-target tracking and event prediction techniques based on real-time large-scale video analysis).

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