

시변지연을 가지는 LPV시스템의 H_∞ 샘플데이터 제어

H_∞ Sampled-Data Control of LPV Systems with Time-varying Delay

유 아 연* · 이 상 문†
(Yajuan Liu · Sangmoon Lee)

Abstract - This paper considers the problem of sampled-data control for continuous linear parameter varying (LPV) systems. It is assumed that the sampling periods are arbitrarily varying but bounded. Based on the input delay approach, the sampled-data control LPV system is transformed into a continuous time-delay LPV system. Some less conservative stabilization results represented by LMI (Linear Matrix Inequality) are obtained by using the Lyapunov-Krasovskii functional method and the reciprocally combination approach. The proposed method for the designed gain matrix should guarantee asymptotic stability and a specified level of performance on the closed-loop hybrid system. Numerical examples are presented to demonstrate the effectiveness and the improvement of the proposed method.

Key Words : LPV system, Sampled-data control, Time-varying delay

1. Introduction

Linear parameter-varying (LPV) systems have been received consideration attention due to the fact that LPV models are useful to describe the dynamics of linear systems affected by time-varying parameters as well as to represent nonlinear systems in terms of a family of linear models. Time delays often appear in many physical, biological and engineering systems, and the existence of time delay may cause instability. Therefore, the stability analysis and controller design problem of delayed LPV systems have been received considerable attention by man researchers[1-6]. The results of stabilization and controller design of time delay systems can be classified into two types: delay-independent and delay-dependent results. It is well known that the delay dependent results are less conservative than delay-independent ones.

Because of the rapid growth of the digital hardware technologies, the sampled-data control systems, where the control signals are kept constant during the sampling period and are allowed to change only at the sampling instant, has been more important than other control approaches. Thus, many important and essential results have been reported in

the literature over the past decades[7-14]. The authors in [13] have investigated the sampled-data control for LPV system. Furthermore, the sampled-data control of the delayed LPV systems have been studied in [14]. The shortcoming of the method utilized in [13, 14] is that Jensen Inequality was employed to obtain the condition for the controller design. Recently, a lower bound lemma[15], which can get less conservative results compared with only using Jensen inequality, was proposed. Thus, it remains a space to further improve the results reported in [13, 14], which motivates this work.

The main focus of the present work is to examine the sampled-data control design for LPV systems. We propose a method for the design of sampled-data controllers for LPV systems using a parameter-independent state feedback law. The proposed design method guarantees asymptotic stability and optimized energy-to-energy gain of the closed-loop system from disturbance input to the system output. The criteria for the existence of the controllers are derived in terms of LMI (Linear Matrix Inequality). Compared with the result obtained in [13, 14], less conservative results are obtained by using new Lyapunov-Krasovskii functional and reciprocally convex approach[15]. By means of numerical simulations, it is shown that the proposed results are effective and can significantly improve the existing ones.

The rest of this paper is organized as follows. Section II provides the problem statement and preliminary. Section III presents main results for the derived sufficient conditions in terms of LMIs. Section IV shows simulation results to

† Corresponding Author : Dept. of Electronic Engineering, Daegu University, Korea.

E-mail : moony@daegu.ac.kr

* Dept. of Electronic Engineering, Daegu University, Korea.

Received : August 15, 2014; Accepted : December 17, 2014

demonstrate its effectiveness of the proposed method. Finally, Section V concludes this paper with a summarization.

The notation used in this paper is strand. \mathbf{R} denotes the set of real numbers. \mathbf{R}^n and $\mathbf{R}^{m \times k}$ are used to denote set of real vectors of dimension n and the set of m by k real matrix, respectively. For symmetric matrices X , $X > 0$ and $X < 0$, mean that X is a positive/negative definite symmetric matrix, respectively. $*$ represents the elements below the main diagonal of a symmetric matrix. $diag\{\dots\}$ denotes the diagonal matrix.

2. Problem Statement and Preliminaries

Consider the following state-space representation for a linear parameter varying (LPV) system with time-varying delay:

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + A_h(\rho(t))x(t-d(t)) \\ &\quad + B_1(\rho(t))u(t) + B_2(\rho(t))w(t), \\ z(t) &= C(\rho(t))x(t) + C_h(\rho(t))x(t-d(t)) \\ &\quad + D_1(\rho(t))u(t) + D_2(\rho(t))w(t), \\ u(t) &= Kx(t_k), \end{aligned} \quad (1)$$

$$(2)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $z(t) \in \mathbf{R}^m$ is the vector of controlled outputs, $w(k) \in \mathbf{R}^{n_w}$ is exogenous disturbance vector containing both process and measurement noise with finite energy and $u(k) \in \mathbf{R}^{n_u}$ is the control input vector. The system matrices $A(\cdot), A_h(\cdot), B_1(\cdot), B_2(\cdot),$

$C(\cdot), C_h(\cdot), D_1(\cdot), D_2(\cdot)$ are real continuous functions of a time varying parameter vector. $\rho(t) \in F_P^w$ and of appropriate trajectories defined as

$$F_P^w = \left\{ \rho : \rho(t) \in C(\mathbf{R}, \mathbf{R}_s) : \rho(t) \in F, |\dot{\rho}(t)| \leq v_i, i = 1, 2, \dots, s, t \in \mathbf{R}_+ \right\}, \quad (3)$$

where $C(\mathbf{R}, \mathbf{R}_s)$ is the set of continuous-time functions from \mathbf{R} to \mathbf{R}^s , and $\{v_i\}_{i=1}^s$ are non-negative numbers. The constraints in (3) imply that the parameter trajectories and their variations are bounded. The time varying delay satisfies $0 \leq d(t) \leq d, \dot{d}(t) \leq \mu$.

In this paper, the control signal is assumed to be generated by using a zero-order-hold (ZOH) with a sequence of hold times

$$0 \leq t_0 < t_1 < \dots < t_k < \dots < \lim_{k \rightarrow \infty} t_k = +\infty.$$

Also, the sampling is not required to be periodic, and the only assumption is that the distance between any two

consecutive sampling instants is less than a given bound[11]. Specially, it is assumed that

$$t_{k+1} - t_k \leq h,$$

for all $k \geq 0$, where h represents the upper bound of the sampling periods. Define $t_k = t - (t - h(t))$ with $h(t) = t - t_k$. Then, the system (1) can be represented as

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + A_h(\rho(t))x(t-d(t)) \\ &\quad + B_1(\rho(t))Kx(t-h(t)) + B_2(\rho(t))w(t), \\ z(t) &= C(\rho(t))x(t) + C_h(\rho(t))x(t-d(t)) \\ &\quad + D_1(\rho(t))Kx(t-h(t)) + D_2(\rho(t))w(t). \end{aligned} \quad (4)$$

The purpose of this paper is to design a proper sampled-data controller (2) such that the following conditions hold.

- 1) The system (1) with $w(t) = 0$ is asymptotically stable.
- 2) For some positive scalar γ the following condition holds

$$\|T_{wz}\|_{i,2} < \gamma,$$

3. Main Results

In this section, we present the stability and H_∞ norm performance condition for delayed LPV systems by deriving a set of linear matrix inequality conditions.

Theorem 1. For given $\gamma > 0, h > 0, d > 0, \mu$, the LPV system (1) is asymptotically stable for all $0 \leq h(t) \leq h$ and satisfies $\|z\|_{L_2} \leq \gamma \|w\|_{L_2}$, if there exist positive constant $\bar{P}, \bar{Q}, \bar{R}, \bar{S}, \bar{W}, \bar{N}$ symmetric G and any matrix $\bar{T}, \bar{T}_1, \bar{M}$ satisfying the following LMIs

$$\Xi = \begin{bmatrix} \bar{\Sigma} & \bar{\Pi} \\ * & -rI \end{bmatrix} < 0, \quad (5)$$

$$\begin{bmatrix} R & T_1 \\ * & R \end{bmatrix} \geq 0, \quad \begin{bmatrix} N & T_2 \\ * & N \end{bmatrix} \geq 0, \quad (6)$$

where

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Sigma}_1 & \bar{\Sigma}_2 \\ * & -rI \end{bmatrix}, \quad (7)$$

$$\bar{\Sigma}_1 = \begin{bmatrix} \bar{\Sigma}_{11} & \bar{\Sigma}_{12} & \bar{\Sigma}_{13} & \bar{\Sigma}_{14} & \bar{T}_2 & \bar{\Sigma}_{16} \\ * & \bar{\Sigma}_{22} & \bar{\Sigma}_{23} & \bar{\Sigma}_{24} & 0 & \bar{\Sigma}_{26} \\ * & * & \bar{\Sigma}_{33} & \bar{\Sigma}_{34} & 0 & -\lambda_2 G \\ * & * & * & \bar{\Sigma}_{44} & \bar{\Sigma}_{45} & \bar{\Sigma}_{46} \\ * & * & * & * & \bar{\Sigma}_{55} & 0 \\ * & * & * & * & * & \bar{\Sigma}_{66} \end{bmatrix}, \quad \bar{\Sigma}_2 = \begin{bmatrix} B_2 \\ \lambda_1 B_2 \\ \lambda_2 B_2 \\ 0 \\ 0 \\ \lambda_3 B_2 \end{bmatrix}$$

$$\begin{aligned}
 \bar{\Sigma}_{11} &= \bar{Q} - \bar{R} + \bar{S} + \bar{W} - \bar{N} + AG + G^T A, \\
 \bar{\Sigma}_{12} &= \bar{R} - \bar{T}_1 + B_1 M + \lambda_1 G A^T, \\
 \bar{\Sigma}_{13} &= \bar{T}_1 + \lambda_2 G A^T, \\
 \bar{\Sigma}_{14} &= \bar{N} - \bar{T}_2 + A_h G, \\
 \bar{\Sigma}_{16} &= \bar{P} - G + \lambda_3 G A^T, \\
 \bar{\Sigma}_{22} &= -2\bar{R} + \bar{T}_1 + \bar{T}_1^T + \lambda_1 (B_1 M + M^T B_1^T), \\
 \bar{\Sigma}_{23} &= \bar{R} - \bar{T}_1 + \lambda_2 M^T B_1^T, \\
 \bar{\Sigma}_{24} &= \lambda_1 A_h G, \\
 \bar{\Sigma}_{26} &= -\lambda_1 G + \lambda_3 M^T B_1^T, \\
 \bar{\Sigma}_{33} &= -\bar{Q} - \bar{R}, \\
 \bar{\Sigma}_{34} &= \lambda_2 A_h G, \\
 \bar{\Sigma}_{44} &= -(1-\mu)\bar{W} - 2\bar{N} + \bar{T}_2 + \bar{T}_2^T, \\
 \bar{\Sigma}_{45} &= \bar{N} - \bar{T}_2, \\
 \bar{\Sigma}_{46} &= \lambda_3 G A_h^T, \\
 \bar{\Sigma}_{55} &= -\bar{S} - \bar{N}, \\
 \bar{\Sigma}_{66} &= h^2 \bar{R} + d^2 \bar{N} - 2\lambda_3 G, \\
 \bar{\Pi} &= [C_1 G \ D_1 M \ 0 \ C_h G \ 0 \ 0 \ D_2]^T.
 \end{aligned}$$

Further, the sampled-data controller gain matrix in (2) is given by $K = MG^{-1}$

Proof. Consider the following Lyapunov-Krasovskii functional candidate

$$V(t) = \sum_{i=1}^6 V_i, \quad (8)$$

where

$$\begin{aligned}
 V_1 &= x^T(t) P x(t), \\
 V_2 &= \int_{t-h}^t x^T(s) Q x(s) ds, \\
 V_3 &= h \int_{-h}^0 \int_{t+\alpha}^t \dot{x}^T(s) R \dot{x}(s) ds, \\
 V_4 &= \int_{t-d}^t x^T(s) S x(s) ds, \\
 V_5 &= \int_{t-d(t)}^t x^T(s) W x(s) ds, \\
 V_6 &= d \int_{-d}^0 \int_{t+\alpha}^t \dot{x}^T(s) N \dot{x}(s) ds.
 \end{aligned}$$

Now, calculating the time derivative of $V(t)$ along the solution of

$$\dot{V}_1 = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t), \quad (9)$$

$$\dot{V}_2 = x^T(t) Q x(t) - x^T(t-h) Q x(t-h), \quad (10)$$

$$\dot{V}_3 = h^2 \dot{x}^T(t) R \dot{x}(t) - h \int_{t-h}^t \dot{x}^T(s) R \dot{x}(s) ds, \quad (11)$$

$$\dot{V}_4 = x^T(t) S x(t) - x^T(t-d) S x(t-d), \quad (12)$$

$$\dot{V}_5 \leq x^T(t) W x(t) - (1-\mu) x^T(t-d(t)) W x(t-d(t)), \quad (13)$$

$$\dot{V}_6 = d^2 \dot{x}^T(t) N \dot{x}(t) - d \int_{t-d}^t \dot{x}^T(s) N \dot{x}(s) ds. \quad (14)$$

Since $\begin{bmatrix} R & T_1 \\ * & R \end{bmatrix} > 0$ and $\begin{bmatrix} N & T_2 \\ * & N \end{bmatrix} > 0$, by employing Jensen's inequality [13, 14] and the reciprocally convex combination technique [15], one can obtain

$$-h \int_{t-h}^t \dot{x}^T(s) R \dot{x}(s) ds \leq \alpha^T(t) \Phi_1 \alpha(t), \quad (15)$$

and

$$-d \int_{t-d}^t \dot{x}^T(s) N \dot{x}(s) ds \leq \beta^T(t) \Phi_2 \beta(t), \quad (16)$$

where

$$\begin{aligned}
 \alpha^T(t) &= [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-h)], \\
 \beta^T(t) &= [x^T(t) \quad x^T(t-d(t)) \quad x^T(t-d)], \\
 \Phi_1 &= \begin{bmatrix} -R & R - T_1 & T_1 \\ * & -2R + T_1 + T_1^T & R - T_1 \\ * & * & -R \end{bmatrix}, \\
 \Phi_2 &= \begin{bmatrix} -N & N - T_2 & T_2 \\ * & -2N + T_2 + T_2^T & N - T_2 \\ * & * & -N \end{bmatrix}.
 \end{aligned}$$

On the other hand, according to (1), for any appropriately dimensioned matrix G and scalars $\lambda_1, \lambda_2, \lambda_3$ the following equation holds:

$$\begin{aligned}
 &2[x^T(t) G^{-1} + \lambda_1 x^T(t-h(t)) G^{-1} + \lambda_2 x^T(t-h) G^{-1} \\
 &+ \lambda_3 x^T(t) G^{-1}] [-\dot{x}(t) + A(\rho(t))x(t) + A_h(\rho(t))x(t-d(t)) \\
 &+ B_1(\rho(t))Kx(t-h(t)) + B_2(\rho(t))w(t)] = 0.
 \end{aligned} \quad (17)$$

Define

$$\xi^T(t) = [\alpha^T(t), \quad x^T(t-d(t)) \quad x^T(t-d), \quad \dot{x}^T(t) \quad w^T(t)],$$

From (9)-(17), one can obtain

$$\dot{V}(t) - \gamma w^T(t) w(t) + \frac{1}{\gamma} z^T(t) z(t) = \xi^T(t) \Sigma \xi(t) + \frac{1}{\gamma} z^T(t) z(t), \quad (18)$$

where

$$\Sigma = \begin{bmatrix} \Sigma_1 & \Sigma_2 \\ * & -rI \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & T_2 & \Sigma_{16} \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & 0 & \Sigma_{26} \\ * & * & \Sigma_{33} & \Sigma_{34} & 0 & -\lambda_2 G^{-1} \\ * & * & * & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} \\ * & * & * & * & \Sigma_{55} & 0 \\ * & * & * & * & * & \Sigma_{66} \end{bmatrix},$$

$$\bar{\Sigma}_2 = \begin{bmatrix} G^{-1}B_2 \\ \lambda_1 G^{-1}B_2 \\ \lambda_2 G^{-1}B_2 \\ 0 \\ 0 \\ \lambda_3 G^{-1}B_2 \end{bmatrix},$$

$$\begin{aligned} \Sigma_{11} &= Q - R + S + W - N + G^{-1}A + AG^{-1}, \\ \Sigma_{12} &= R - T_1 + G^{-1}B_1 K + \lambda_1 A^T G^{-1}, \\ \Sigma_{13} &= T_1 + \lambda_2 A^T G^{-1}, \\ \Sigma_{14} &= N - T_2 + G^{-1}A_h, \\ \Sigma_{16} &= P - G^{-1} + \lambda_3 A^T G^{-1}, \\ \Sigma_{22} &= -2R + T_1 + T_1^T + \lambda_1 (G^{-1}B_1 K + K^T B_1^T G^{-1}), \\ \Sigma_{23} &= R - T_1 + \lambda_2 K^T B_1^T G^{-1}, \\ \Sigma_{24} &= \lambda_1 G^{-1}A_h, \\ \Sigma_{26} &= -\lambda_1 G^{-1} + \lambda_3 K^T B_1^T G^{-1}, \\ \Sigma_{33} &= -Q - R, \\ \Sigma_{34} &= \lambda_2 G^{-1}A_h, \\ \Sigma_{44} &= -(1 - \mu)W - 2N + T_2 + T_2^T, \\ \Sigma_{45} &= N - T_2, \\ \Sigma_{46} &= \lambda_3 A_h^T G^{-1}, \\ \Sigma_{55} &= -S - N, \\ \Sigma_{66} &= h^2 R + d^2 N - 2\lambda_3 G^{-1}. \end{aligned}$$

By Schur complement, $\dot{v}(t) - \gamma w^T(t)w(t) + \frac{1}{\gamma} z^T(t)z(t) < 0$ is equivalent to

$$\begin{bmatrix} \Sigma & \Pi \\ * & -rI \end{bmatrix} < 0 \quad (19)$$

where $\Pi = [C_1^T \ D_1 K \ 0 \ C_h \ 0 \ 0 \ D_2]^T$.

Define $M = KG$, $\bar{P} = GPG$, $\bar{Q} = GQG$, $\bar{R} = GRG$, $\bar{S} = GSG$, $\bar{M} = GMG$, $\bar{N} = GNG$, $\bar{T}_1 = GT_1 G$, $\bar{T}_2 = GT_2 G$. Then pre and post-multiplying the matrix $\text{diag}\{G, G, G, G, G, I, I\}^T$ and $\text{diag}\{G, G, G, G, G, I, I\}$ in Eq. (19), we can get (5). This completes the proof. ■

When $A_h = C_h = 0$, then the system (1) reduces to the following system without time delay

$$\dot{x}(t) = A(\rho(t))x(t) + B_1(\rho(t))u(t) + B_2(\rho(t))w(t), \quad (20)$$

$$\begin{aligned} z(t) &= C(\rho(t))x(t) + D_1(\rho(t))u(t) + D_2(\rho(t))w(t), \\ u(t) &= Kx(t_k). \end{aligned} \quad (21)$$

Correspondingly, the sampled-data controlled system (4) reduces to

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + B_1(\rho(t))Kx(t-h(t)) + B_2(\rho(t))w(t), \\ z(t) &= C(\rho(t))x(t) + D_1(\rho(t))Kx(t-h(t)) + D_2(\rho(t))w(t). \end{aligned} \quad (22)$$

Let $V_4 = V_5 = V_6 = 0$. By using a method similar to the one employed in the proof of Theorem 1, we have the following corollary.

Corollary 1. For given $\gamma > 0, h > 0$, the LPV system (22) is asymptotically stable for all $0 \leq h(t) \leq h$ and satisfies $\|z\|_{L_2} \leq \gamma \|w\|_{L_2}$, if there exist positive constant $\bar{P}, \bar{Q}, \bar{R}$, symmetric G and any matrix \bar{T}_1, \bar{M} satisfying the following LMIs

$$\begin{bmatrix} \hat{\Sigma}_{11} & \bar{\Sigma}_{12} & \bar{\Sigma}_{13} & \bar{\Sigma}_{16} & B_2 & GC_1^T \\ * & \Sigma_{22} & \Sigma_{23} & \Sigma_{26} & \lambda_1 B_2 & 0 \\ * & * & -\bar{Q} - \bar{R} & -\lambda_2 G & \lambda_2 B_2 & -\lambda_2 G \\ * & * & * & \hat{\Sigma}_{44} & \lambda_3 B_2 & 0 \\ * & * & * & * & -\gamma I & D_2^T \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} R & T_1 \\ * & R \end{bmatrix} \geq 0, \quad (24)$$

where

$$\begin{aligned} \hat{\Sigma}_{11} &= \bar{Q} - \bar{R} + AG + G^T A, \hat{\Sigma}_{44} = -h^2 \bar{R} - 2\lambda_3 G, \text{ and} \\ \bar{\Sigma}_{12}, \bar{\Sigma}_{13}, \bar{\Sigma}_{16}, \bar{\Sigma}_{23}, \bar{\Sigma}_{26} &\text{ are the same with Theorem 1.} \end{aligned}$$

Further, the sampled-data controller gain matrix in (2) is given by $K = MG^{-1}$.

Remark 1 In Theorem 1 and Corollary 1, to find a solution satisfying LMI conditions for all parameters in the set of Eq.(3), a parameter grid method is applied for the LPV system which is a bounded function of a scheduling parameter vector. The grid LPV synthesis method has been successfully applied to synthesis controllers for systems with measurable parameters which is in a bounded set[14]. Also, we choose the constant matrix in the constructed Lyapunov functional and the controller gain matrix. If \bar{P} is a continuously differentiable matrix function in (8) or K is parameter-dependent functional, it can be easily derived in the extended results using the similar method in [13, 14].

Remark 2. Compared with the results obtained in [13, 14], the derived results in this paper can have better performance and larger sampling period, which relies on the constructed Lyapunov functional and the method of estimation of its derivative. On one hand, $V_2(t)$ and $V_4(t)$ are considered in this paper while these term are neglected in [13] and [14]. On the other hand, reciprocally convex

approach[15], which is provided more tighter than the ones based on Jensens inequality, is employed to estimate the upper bound of the derivative of Lyapunov function $V_3(t)$ and $V_6(t)$.

4. Numerical Examples

In this section, two numerical examples will be given to demonstrate the effectiveness of the proposed method.

Example 1 This example is motivated by the control of chattering during the milling process[14]. The dynamical equations of the system is

$$\begin{aligned} \ddot{x}_1 &= \frac{1}{m} [-k_1 - k \sin(\phi) \sin(\phi + \beta)] x_1 + c \frac{k_1}{m_1} x_1 \left(t - \frac{\pi}{w} \right) \\ &\quad + \frac{k_1}{m_1} x_2 + \frac{k_1}{m_1} w(t), \\ \ddot{x}_2 &= \frac{k_1}{m_2} x_1 - \frac{k_1 + k_2}{m_2} x_2 - \frac{c}{m_2} \dot{x}_2 + \frac{k_1}{m_2} u, \end{aligned} \quad (25)$$

with $m_1 = 1, k_1 = 10, k_2 = 20, c = 0.5, \beta = 70^\circ$. It is noted that $\sin(\phi + \beta) \sin(\phi) = 0.1710 - 0.5 \cos(2\phi + \beta)$. We define the scheduling parameter vector as $\rho(t) = [\rho_1(t) \ \rho_2(t)]^T$ with $\rho_1(t) = \cos(2\phi(t) + \beta(t))$ and $\rho_2(t) = w(t)$. The rotation speed w is assumed to vary between 200 rpm and 2000 rpm, and the maximum variation rate is 1000 rpm/sec. The parameter space associated with the LPV parameters is as follows

$$\begin{aligned} \rho_1(t) &\in [-1, 1], |\dot{\rho}_1| \leq 418.9(\text{rad}/\text{sec}^2), \\ \rho_2(t) &\in [20.94, 209.4](\text{rad}/\text{sec}), |\dot{\rho}_2| = 104.7(\text{rad}/\text{sec}^2) \end{aligned}$$

Consider the state vector as $x = [x_1, x_2, \dot{x}_1, \dot{x}_2]^T$, the state matrices corresponding time-delay LPV plant to be controlled are as follows

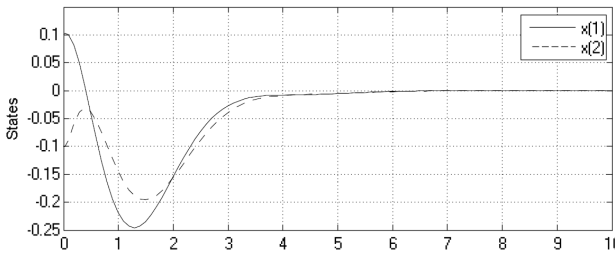


Fig. 1 State response of the delayed LPV systems in Example 1.

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10.34 + \rho_1 & 0 & 0 & 0 \\ 5 & -15 & 0 & 0.25 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{bmatrix}, \\ A_h &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.34 - \rho_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}, D_1 = 0. \end{aligned}$$

In order to reduce the computational cost, we consider constant function variables (parameter-independent). For this example, we assume that $d = 0.15, h = 0.1, \lambda_1 = 1.4, \lambda_2 = 0.5$ and $\lambda_3 = 0.2$. Solving the LMIs in Theorem 1, we obtain an H_∞ performance bound is $\gamma = 1.9$, which is less than the derived results $\gamma = 2.4$ in [14]. The corresponding controller gain matrix is

$$K = [16.9370 \quad -2.3223 \quad 7.6668 \quad -15.4072].$$

With the above controller gain, taking $w(t) = (\sin t)e^{-10t}$, Fig 1 shows the simulation results, indicating the displacement of the cutter x_1 and that of the spindle x_2 for a predefined text condition. It is apparent that the proposed controller attenuates the disturbance successfully under the variable rotational speed. The control effort required for this study is shown in Fig. 2. It is obvious that the results of the present paper yield a much smoother control effort.

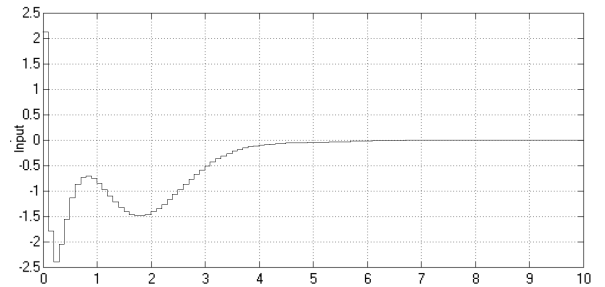


Fig. 2 Input of the delayed LPV systems Example 1.

Example 2 We consider the following linear LPV system [13]

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 2\sin(0.2t) & 1.1 + \sin(0.2t) \\ -2.2 + \sin(0.2t) & -3.3 + 0.1\sin(0.2t) \end{bmatrix} x(t) \\ &\quad + \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} w(t) + \begin{bmatrix} 2\sin(0.2t) \\ 0.1 + \sin(0.2t) \end{bmatrix} u(t), \\ z(t) &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t). \end{aligned}$$

In the model shown above, the sine term is assumed to be an LPV parameter, whose functional representation is not known a priori but can be measured in real time. Defining $\rho(t) = \sin(0.2t)$, it can be seen that an LPV state-space representation with the parameter space $\rho \in [-1, 1]$. As pointed out in [13], the optimal value γ is quite sensitive to the scalars λ_1, λ_2 and λ_3 . Taking $\lambda_1 = 1.6, \lambda_2 = 1, \lambda_3 = 2$, Table 1 show the a comparison of the H_∞ norms obtained. This indicates that the derived result in this paper is less conservative. When $h = 0.1, \gamma = 0.194$, solving the LMIs in Corollary 1, we can obtain the controller gain matrix as

$$K = [-1.8155 \quad -0.9014].$$

With the above controller gain, taking $w(t) = (\sin t)e^{-10t}$, Fig 3 and Fig 4 present the simulation result of the state response and control input of this example.

Table 1 H_∞ norm for different sampling rates.

Method	0.1	0.2	0.3
[13]	0.328	0.379	0.513
Corollary 1	0.194	0.273	0.468

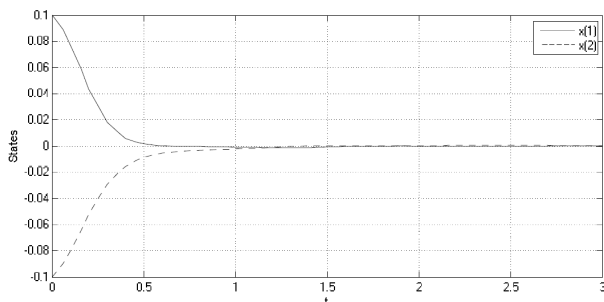


Fig. 3 State response of the delayed LPV systems in Example 2.

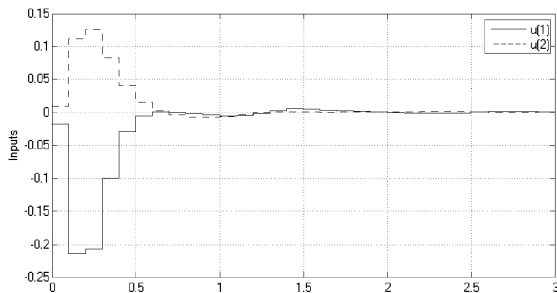


Fig. 4 Input of the delayed LPV systems Example 2.

5. Conclusions

This paper proposed a sampled-data control design for a continuous-time LPV system. By using a reciprocally convex approach, the obtained results are less conservative than the existing ones in the literatures. The proposed method guarantees asymptotic stability and optimized energy-to-energy gain of the closed-loop system from disturbance input to the system output. Finally, we demonstrate the effectiveness of the proposed method using two examples.

References

- [1] Z. Zhang, P. Tsiotras, and C. Knospe, Stability analysis of LPV time-delayed system, International journal of control, 75(7): 538-558.
- [2] F. Wu and K.M. Grigoriadis, LPV Systems with parameter-varying time delays: analysis and control, Automatica, 37(2): 221-229, 2001.
- [3] M. Sun, Y. Jia, J. Du, and S. Yuan, Delay-dependent H_∞ control for LPV systems with time delays, Int. J. Systems, Controls and Communications, 1(2): 256-265, 2008.
- [4] C. Briat, O. Sename, and J.F. Lafay. Parameter dependent statefeedback control of LPV time-delay systems with time-varying delays using a projection approach. IFAC World Congress, Seoul, 2008.
- [5] S.M. Lee, J.H. Park, D.H. Ji and S.C. Won, Robust model predictive control for LPV systems using relaxation matrices, IET Control Theory Appl., 1(6): 1567.1573-2007.
- [6] S.M. Lee, S.C. Jeong, D.H. Ji, S.C. Won, Robust model predictive control for LPV systems with delayed state using relaxation matrices, Proc. American Control Conference, 2011.
- [7] P. Shi, Filtering on sampled-data systems with parametric uncertainty, IEEE Trans. Autom. Control, 43(7): 1022-1027, 1998.
- [8] T. Chen and B.A. Francis. Optimal sampled-data control systems, volume 124. Springer Londres., 1995.
- [9] E. Fridman, A. Seuret, and J.P. Richard. Robust sampled-data stabilization of linear systems: an input delay approach. Automatica, 40(8):1441-1446, 2004.
- [10] E. Fridman, U. Shaked, and V. Suplin. Input/output delay approach to robust sampled-data H_∞ control. Systems Control Letters, 54(3):271.282, 2005.
- [11] E. Fridman, A refined input delay approach to sampled-data control, Automatica, 46(2): 421-427, 2010.

- [12] R. Zope, J. Mohammadpour, K.M. Grigoriadis, and M. Franchek. Delay-dependent H_∞ control for LPV systems with fast-varying time delays. Proc. American Control Conference, 2012.
- [13] A. Ramezanifar, J. Mohammadpour, and K. Grigoriadis. Sampled-data control of LPV systems using input delay approach. Proc. IEEE Conference on Decision and Control, to appear, 2012.
- [14] A. Ramezanifar, J. Mohammadpour, K.M. Grigoriadis, Sampled-data Control of Linear Parameter Varying Time-delay Systems Using State Feedback, Proc. American Control Conference, 2013.
- [15] P. Park, J. W. Ko, and C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, Automatica, 47(1): 235-238, 2011.

저 자 소 개



유 아 연(Yajuan Liu)

She received her B.S degree in mathematics and applied mathematics from shanxi nominal university, Linfen, China, in 2010, and M.S degree in applied mathematics from University of Science and Technology Beijing, Beijing, China, in 2012. She is currently working toward the Ph.D degree in Electronic Engineering from Daegu University, Korea.



이 상 문(Sangmoon Lee)

1973년 6월 15일생. 1999년 경북대학교 전자공학과 졸업(공학). 2006년 포항공과대학교 전기전자공학부 졸업(공학). 현재 대구대학교 전자전기공학부 조교수.

E-mail : moony@daegu.ac.kr