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A Note on Relay Feedback Identification Under Static Load Disturbances

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Abstract – Obtaining the parameters for PID controllers based on limit cycle information for the process in a relay controlled feedback loop has become an accepted practical procedure. If the form of the plant transfer function is known, exact expressions for the limit cycle frequency and amplitude can be derived so that their measurements, assumed error free, can be used to calculate the true parameter value. In the literature, parameter estimation for an assumed form of the plant transfer function has generally been considered for disturbance free cases, except a recently published work of the author. In this paper additional simulation results are reported on exact parameter estimation from relay auto-tuning under static load disturbances.

Keywords: Process control, Mathematical modelling, Parameter identification, Simulation, Limit cycle, Disturbance

1. Introduction

Using relay feedback control for parameter estimation of an assumed form of plant transfer function has become an accepted practical procedure. For literature on relay feedback identification, interested readers can refer to [1] and literature in there.

In the past studies, it is generally assumed that there are no load disturbances into the relay feedback control system during parameter estimation procedure. However, load disturbances are quite frequently encountered in practical situations. Therefore, use of the expressions obtained for load disturbance free situations may lead to significant errors in the estimates under static load disturbances. Kaya [1] gave the conditions for a limit cycle to occur in a relay feedback control under static load disturbances and showed that the expressions obtained from the A-Function Method [2] for disturbance free cases can simply be improved by the inclusion of the term $dG(0)$ so that they can be used for parameter estimation under static load disturbances. There, the expressions for a specific case of the stable first order plus dead time (FOPDT) transfer function were given. Later, Kaya and Atherton [3] provided expressions for unstable FOPDT transfer function too. Expressions for stable and unstable second order plus dead time (SOPDT) transfer functions were also given. However, the effect of disturbances on parameter estimations was not investigated.

The aim of this paper is twofold: First, a simpler approach for estimating the plant transfer function gain and disturbance magnitude is given. Second, simulation examples are provided to illustrate that if the disturbance exists during parameter estimation procedure and the effect of static load disturbance is not considered in the expressions then large errors in estimates must be expected.

2. Parameter Estimation under Static Load Disturbances

Consider the relay feedback control system under static load disturbances given in Fig. 1. The plant is assumed to have one of the following transfer function forms

$$G_1(s) = \frac{Ke^{-Ls}}{Ts+1} \tag{1}$$

$$G_2(s) = \frac{Ke^{-Ls}}{(T_1 s+1)(T_2 s+1)} \tag{2}$$

$$G_3(s) = \frac{Ke^{-Ls}}{Ts-1} \tag{3}$$

$$G_4(s) = \frac{Ke^{-Ls}}{(T_1 s-1)(T_2 s+1)} \tag{4}$$

When the A-Function method is used for parameter estimation, four equations can be obtained [1]. These four equations are sufficient to identify the unknown parameters K, T and L for the stable and unstable FOPDT transfer functions and the disturbance magnitude, d. However, initial guesses must be provided for these unknowns to

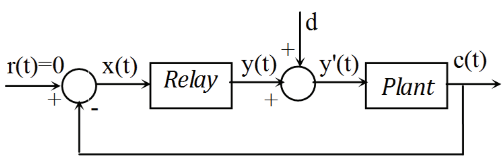

Fig. 1. Relay control under static load disturbances

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solve the obtained nonlinear equations, simultaneously. Furthermore, for the stable and unstable SOPDT transfer functions, one more equation is needed for five unknowns, namely K, T_1, T_2, L and d. Therefore, to reduce the number of unknowns and make the solution easier, Fourier analysis can be used to identify K and d. Kaya [1] suggested two approaches for this purpose. In the first approach, it was assumed that K can be obtained from measurements on the relay and plant outputs in conjunction with Fourier analysis, before the disturbance enters the system. Then, d was found from Fourier analysis; as the steady-state occurs with the disturbance exists. This approach may not be practical. In the second approach, the result from [4] was adopted to find d, where an extra relay test with equal heights was performed. Then, Fourier analysis under steady-state operation was used to estimate K. Clearly, the disadvantage of this approach is to require a second relay test, which is time consuming.

Here, it is assumed that the steady-state gain can be calculated from

$$K = G(0) = \frac{\int_t^{t+P} c(t)dt}{\int_t^{t+P} y'(t)dt} \tag{5}$$

where $c(t)$ and $y'(t)$ are the plant output and input, respectively, and P is the period of the limit cycle.

Once steady-state operation occurs, the disturbance magnitude can be calculated from

$$d = \frac{1}{G(0)P}\int_t^{t+P} c(t)dt - \frac{(h_1\Delta t_1 + h_2\Delta t_2)}{P} \tag{6}$$

Therefore, with K and d found, respectively, from eqns. (5) and (6), the expressions and procedure provided in [3] can be used to identify the remaining parameters of any plant transfer functions given in eqns. (1)-(4). For convenience, expressions to be used in parameter estimations are given in the appendix. Interested readers can refer to [1] to see the procedure for obtaining these equations.

3. Simulation Examples

In the first example, for different static-load disturbance magnitudes, the parameters of the FOPDT plant transfer function are computed using the expressions which is, respectively, not taking the effect of load disturbance into account [1] and taking the effect of load disturbance into account [3]. Hence, it is shown that large errors in the estimates are resulted when the effect of load disturbance is not considered in the expressions. In the second example, high order stable and unstable plant transfer functions under static load disturbances are modeled by the stable FODPT or SOPDT and unstable FOPDT or SOPDT to show that the models obtained are satisfactory in the sense of controller design in most cases.

Example 1: Consider an FOPDT plant transfer function of $e^{-10s}/(30s+1)$ with the normalized dead time ratio $L/T=0.33$. For different load disturbance magnitudes, relay heights and hysteresis values, relay feedback tests were carried out. Table 1 lists measured limit cycle parameters and estimated parameters when the expressions given in [1] which do not take the effect of load disturbance into consideration, are used.

Several conclusions can be derived from Table 1. First, as the disturbance magnitudes gets larger the error in the estimates gets larger too, which is expected. Second, increasing the bias in the relay increases the error in the estimates. Third, using a relay with hysteresis always results in slightly less accurate estimations for the time delay. This holds also for the time delay estimates when the bias in the relay is relatively small. However, when the bias in the relay is increased, a relay with hysteresis gives slightly better estimates for the time delay. In the table, exact solutions are obtained for the gain K for all cases, as

Table 1. Identification for example 1 ($L/T = 0.333$)

Δ	d	h_1=0.7, h_2=-0.5				h_1=0.7, h_2=-0.3			
		Limit cycle parameters	K	T	L	Limit cycle parameters	K	T	L
0.0	0.05	$\omega=0.173$ $\Delta t_1=14.712$ $a_{max}=0.213$ $a_{min}=-0.128$	1.000	29.284	9.435	$\omega=0.153$ $\Delta t_1=12.707$ $a_{max}=0.213$ $a_{min}=-0.071$	1.000	28.080	8.820
	0.10	$\omega=0.169$ $\Delta t_1=13.977$ $a_{max}=0.227$ $a_{min}=-0.113$	1.000	28.030	8.570	$\omega=0.140$ $\Delta t_1=12.054$ $a_{max}=0.227$ $a_{min}=-0.057$	1.000	25.068	7.503
	0.15	$\omega=0.161$ $\Delta t_1=13.312$ $a_{max}=0.241$ $a_{min}=-0.099$	1.000	26.153	7.511	$\omega=0.125$ $\Delta t_1=11.464$ $a_{max}=0.241$ $a_{min}=-0.043$	1.000	20.821	6.398
0.1	0.05	$\omega=0.117$ $\Delta t_1=21.360$ $a_{max}=0.284$ $a_{min}=-0.199$	1.000	29.154	9.378	$\omega=0.093$ $\Delta t_1=19.512$ $a_{max}=0.284$ $a_{min}=-0.143$	1.000	27.626	8.880
	0.10	$\omega=0.113$ $\Delta t_1=20.248$ $a_{max}=0.298$ $a_{min}=-0.188$	1.000	27.659	8.481	$\omega=0.082$ $\Delta t_1=18.470$ $a_{max}=0.298$ $a_{min}=-0.128$	1.000	24.016	8.006
	0.15	$\omega=0.107$ $\Delta t_1=19.250$ $a_{max}=0.313$ $a_{min}=-0.171$	1.000	25.402	7.504	$\omega=0.067$ $\Delta t_1=17.534$ $a_{max}=0.313$ $a_{min}=-0.114$	1.000	19.829	7.834

Table 2. Identification for example 1 ($L/T = 0.167$)

Δ	d	h_1=0.7, h_2=-0.5				h_1=0.7, h_2=-0.3			
		Limit cycle parameters	K	T	L	Limit cycle parameters	K	T	L
0.0	0.05	$\omega=0.323$ $\Delta t_1=7.643$ $a_{\max}=0.115$ $a_{\min}=-0.069$	1.000	29.229	4.422	$\omega=0.275$ $\Delta t_1=6.497$ $a_{\max}=0.115$ $a_{\min}=-0.038$	1.000	27.843	3.819
	0.10	$\omega=0.310$ $\Delta t_1=7.219$ $a_{\max}=0.123$ $a_{\min}=-0.061$	1.000	27.864	3.552	$\omega=0.247$ $\Delta t_1=6.130$ $a_{\max}=0.123$ $a_{\min}=-0.031$	1.000	24.287	2.572
	0.15	$\omega=0.294$ $\Delta t_1=6.839$ $a_{\max}=0.131$ $a_{\min}=-0.054$	1.000	25.782	2.519	$\omega=0.212$ $\Delta t_1=5.802$ $a_{\max}=0.131$ $a_{\min}=-0.023$	1.000	18.851	1.694
0.1	0.05	$\omega=0.163$ $\Delta t_1=14.887$ $a_{\max}=0.200$ $a_{\min}=-0.154$	1.000	29.131	4.359	$\omega=0.121$ $\Delta t_1=13.849$ $a_{\max}=0.200$ $a_{\min}=-0.123$	1.000	27.475	3.889
	0.10	$\omega=0.156$ $\Delta t_1=14.037$ $a_{\max}=0.208$ $a_{\min}=-0.146$	1.000	27.580	3.451	$\omega=0.104$ $\Delta t_1=13.047$ $a_{\max}=0.208$ $a_{\min}=-0.115$	1.000	23.469	3.135
	0.15	$\omega=0.147$ $\Delta t_1=13.280$ $a_{\max}=0.215$ $a_{\min}=-0.138$	1.000	25.203	2.500	$\omega=0.082$ $\Delta t_1=12.333$ $a_{\max}=0.215$ $a_{\min}=-0.108$	1.000	18.728	3.232

Tabe 3. Estimated models for example 2

Case	Process	Measured limit cycle parameters	FOPDT Model	SOPDT Model
a	$\frac{e^{-2s}}{(s+1)^2}$	$\omega=0.776$ $a_{\max}=0.892$ $a_{\min}=-0.594$ $\Delta t_1=3.630$	$\frac{e^{-2.55s}}{(1.64s+1)}$	$\frac{e^{-2.00s}}{(1.00s+1)^2}$
b	$\frac{1}{(s+1)^{20}}$	$\omega=0.149$ $a_{\max}=1.054$ $a_{\min}=-0.680$ $\Delta t_1=19.667$	$\frac{e^{-16.07s}}{(5.02s+1)}$	$\frac{e^{-13.44s}}{(3.53s+1)^2}$
c	$\frac{e^{-s}}{(s+1)(s^2+6s+10)}$	$\omega=1.414$ $a_{\max}=0.0724$ $a_{\min}=-0.0596$ $\Delta t_1=2.119$	$\frac{0.10e^{-1.50s}}{(1.38s+1)}$	$\frac{0.10e^{-1.18s}}{(0.42s+1)(1.04s+1)}$
d	$\frac{4e^{-3s}}{(10s-1)(s+1)^2}$	$\omega=0.197$ $a_{\max}=2.526$ $a_{\min}=-1.888$ $\Delta t_1=12.206$	$\frac{4.00e^{-5.06s}}{(12.01s-1)}$	$\frac{4.00e^{-4.27s}}{(10.96s-1)(0.75s+1)}$

eqn. (5) given in this paper is used for the identification. Of course, using the expressions given in [3], which are provided in the appendix, in conjunction with eqns. (5) and (6) resulted in exact solutions to three significant for all cases. It should also be noted that the disturbance magnitude d was estimated from eqn. (6) exactly to three significant to use in estimations.

Now consider an FOPDT plant transfer function of $e^{-5s}/(30s+1)$ with the normalized dead time ratio $L/T=0.167$. Similar results as in the previous case are listed in Table 2. Conclusions drawn from Table 1 are confirmed. However, comparing the estimates in the parameters for two cases, namely $L/T=0.333$ and $L/T=0.167$, it is observed that results are now slightly worse than the previous case. When the expressions considering the effect of load disturbances given in [3], also provided in the appendix, are used, again exact solutions to three significant are obtained.

Example 2: In this example several typical process transfer functions are considered to show the effectiveness of the method in modelling higher order process transfer functions for controller design. Table 3 lists some typical process transfer functions, the measured limit cycle parameters and the corresponding models found using a FOPDT or SOPDT model. Relay parameters used for

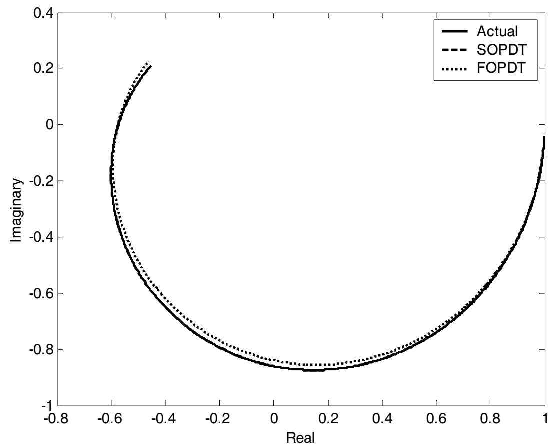

Fig. 2. Nyquist plots for case a

stable processes were h_1=1, h_2= -0.8 and $\Delta=0.2$. For processes with complex and unstable poles, case c and d, the relay parameters were selected as h_1=1, h_2= -1 and $\Delta=0$. The disturbance magnitude in the simulation was put at $d=0.1$ and it was estimated exactly to three significant, using eqn. (6) for all cases. The Nyquist curves of the actual plant transfer functions, the FOPDT and SOPDT model transfer functions are shown in Figs. 2-5, which illustrates good matching, where the phase near to 180^o, hence showing that the method can give reasonable

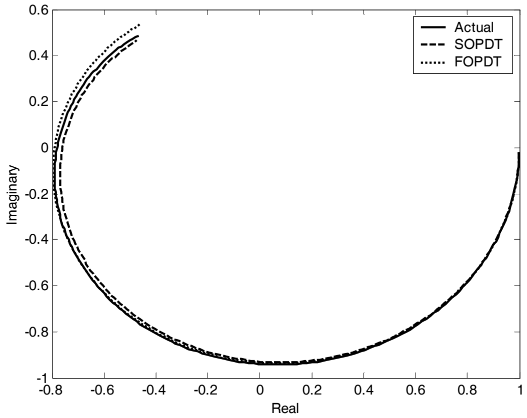

Fig. 3. Nyquist plots for case b

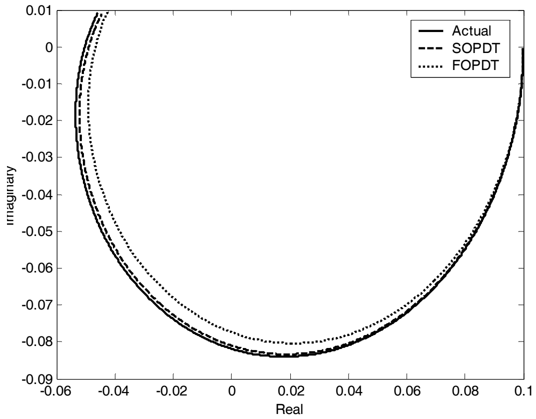

Fig. 4. Nyquist plots for case c

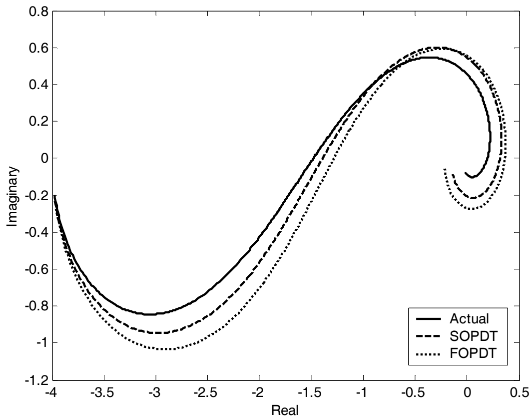

Fig. 5. Nyquist plots for case d

models for higher-order plant transfer functions with FOPDT or SOPDT plant transfer functions for controller design.

5. Conclusion

The paper has given a simple procedure for finding the plant transfer function gain and disturbance magnitude to be used in conjunction with expressions obtained by the author's previous works to help estimating unknown parameters of the stable and unstable FOPDT and SOPDT plant transfer functions. Simulation examples are provided to show the effect of static load disturbances on estimations. It is illustrated by examples that large errors can be expected if the effect of static load disturbances is not considered in the expressions.

Appendix:

The expressions used to estimate the unknown parameters of stable/unstable FOPDT/SOPDT plant transfer functions using relay feedback control when static load disturbances exist are provided here for convenience. In the expressions, $\lambda = \omega T$, $\lambda_1 = \omega T_1$ and $\lambda_2 = \omega T_2$.

A.1 Identification for stable FOPDT:

$$K\left(\frac{-\omega\Delta t_1}{2}+\frac{\pi e^{L/T}(e^{\Delta t_1/T}-1)}{(e^{2\pi/\lambda}-1)}\right) = \frac{-\pi}{(h_1-h_2)}\left(dG_1(0)+\Delta+\frac{G_1(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right) \tag{A1}$$

$$K\left(\frac{(\omega\Delta t_1-2\pi)}{2}+\frac{\pi e^{L/T}(e^{(-\omega\Delta t_1+2\pi)/\lambda}-1)}{(e^{2\pi/\lambda}-1)}\right) = \frac{\pi}{(h_1-h_2)}\left(dG_1(0)-\Delta+\frac{G_1(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right) \tag{A2}$$

$$a_{\min} = dG_1(0)+\frac{G_1(0)(h_1\Delta t_1+h_2\Delta t_2)}{P} + \frac{(h_1-h_2)K}{\pi}\left(\frac{-\omega\Delta t_1}{2}+\frac{\pi(e^{\Delta t_1/T}-1)}{(e^{2\pi/\lambda}-1)}\right) \tag{A3}$$

$$a_{\max} = dG_1(0)+\frac{G_1(0)(h_1\Delta t_1+h_2\Delta t_2)}{P} + \frac{(h_1-h_2)K}{\pi}\left(\frac{-\omega\Delta t_1}{2}+\frac{\pi e^{2\pi/\lambda}(1-e^{-\Delta t_1/T})}{(e^{2\pi/\lambda}-1)}\right) \tag{A4}$$

A.2 Identification for stable SOPDT:

$$K\left(\begin{array}{l}\dfrac{-\omega\Delta t_1}{2}+\dfrac{\pi T_2}{T_1-T_2}\dfrac{e^{L/T_2}(1-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)} \\ -\dfrac{\pi T_1}{T_1-T_2}\dfrac{e^{L/T_1}(1-e^{\Delta t_1/T_1})}{(e^{2\pi/\lambda_1}-1)}\end{array}\right) = \frac{-\pi}{(h_1-h_2)}\left(dG_2(0)+\Delta+\frac{G_2(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right) \tag{A5}$$

$$K\begin{pmatrix}\dfrac{(\omega\Delta t_1-2\pi)}{2}+\dfrac{\pi T_2}{T_1-T_2}\dfrac{e^{L/T_2}(1-e^{(-\omega\Delta t_1+2\pi)/\lambda_2})}{(e^{2\pi/\lambda_2}-1)}\\-\dfrac{\pi T_1}{T_1-T_2}\dfrac{e^{L/T_1}(1-e^{(-\omega\Delta t_1+2\pi)/\lambda_1})}{(e^{2\pi/\lambda_1}-1)}\end{pmatrix}=\frac{\pi}{(h_1-h_2)}\left(dG_2(0)-\Delta+\frac{G_2(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right)\quad (A6)$$

$$a_{\min}=dG_2(0)+\frac{G_2(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\begin{pmatrix}\dfrac{-\omega\Delta t_1+2\pi}{2}+\dfrac{\pi T_2}{T_1-T_2}\dfrac{e^{\theta_2/\lambda_2}(e^{2\pi/\lambda_2}-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)}\\-\dfrac{\pi T_1}{T_1-T_2}\dfrac{e^{\theta_2/\lambda_1}(e^{2\pi/\lambda_1}-e^{\Delta t_1/T_1})}{(e^{2\pi/\lambda_1}-1)}\end{pmatrix}\quad (A7)$$

$$a_{\max}=dG_2(0)+\frac{G_2(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\begin{pmatrix}\dfrac{-\omega\Delta t_1}{2}+\dfrac{\pi T_2}{T_1-T_2}\dfrac{e^{\theta_1/\lambda_2}(1-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)}\\-\dfrac{\pi T_1}{T_1-T_2}\dfrac{e^{\theta_1/\lambda_1}(1-e^{\Delta t_1/T_1})}{(e^{2\pi/\lambda_1}-1)}\end{pmatrix}\quad (A8)$$

where,

$$\theta_1=\frac{T_1T_2\omega}{T_2-T_1}\log\left(\frac{(1-e^{\Delta t_1/T_2})(e^{2\pi/\lambda_1}-1)}{(1-e^{\Delta t_1/T_1})(e^{2\pi/\lambda_2}-1)}\right)$$

$$\theta_2=\frac{T_1T_2\omega}{T_2-T_1}\log\left(\frac{(e^{2\pi/\lambda_2}-e^{\Delta t_1/T_2})(e^{2\pi/\lambda_1}-1)}{(e^{2\pi/\lambda_1}-e^{\Delta t_1/T_1})(e^{2\pi/\lambda_2}-1)}\right)$$

A.3 Identification for unstable FOPDT:

$$K\left(\frac{\omega\Delta t_1}{2}-\frac{\pi e^{-L/T}(e^{-\Delta t_1/T}-1)}{(e^{-2\pi/\lambda}-1)}\right)=\frac{-\pi}{(h_1-h_2)}\left(dG_3(0)+\Delta+\frac{G_3(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right).\quad (A9)$$

$$K\left(\frac{(-\omega\Delta t_1+2\pi)}{2}-\frac{\pi e^{-L/T}(e^{(\omega\Delta t_1-2\pi)/\lambda}-1)}{(e^{-2\pi/\lambda}-1)}\right)=\frac{\pi}{(h_1-h_2)}\left(dG_3(0)-\Delta+\frac{G_3(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right)\quad (A10)$$

$$a_{\min}=dG_3(0)+\frac{G_3(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\left(\frac{\omega\Delta t_1}{2}+\frac{\pi(1-e^{-\Delta t_1/T})}{(e^{-2\pi/\lambda}-1)}\right)\quad (A11)$$

$$a_{\max}=dG_3(0)+\frac{G_3(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\left(\frac{\omega\Delta t_1}{2}+\frac{\pi e^{(\omega\Delta t_1-2\pi)/\lambda}(1-e^{-\Delta t_1/T})}{(e^{-2\pi/\lambda}-1)}\right)\quad (A12)$$

A.4 Identification for unstable SOPDT:

$$K\begin{pmatrix}\dfrac{\omega\Delta t_1}{2}+\dfrac{\pi T_1}{T_1+T_2}\dfrac{e^{-L/T_1}(1-e^{-\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1}-1)}\\+\dfrac{\pi T_2}{T_1+T_2}\dfrac{e^{L/T_2}(1-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)}\end{pmatrix}=\frac{-\pi}{(h_1-h_2)}\left(dG_4(0)+\Delta+\frac{G_4(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right)\quad (A13)$$

$$K\begin{pmatrix}\dfrac{(2\pi-\omega\Delta t_1)}{2}+\dfrac{\pi T_1}{T_1+T_2}\dfrac{e^{-L/T_1}(1-e^{(\omega\Delta t_1-2\pi)/\lambda_1})}{(e^{-2\pi/\lambda_1}-1)}\\+\dfrac{\pi T_2}{T_1+T_2}\dfrac{e^{L/T_2}(1-e^{(-\omega\Delta t_1+2\pi)/\lambda_2})}{(e^{2\pi/\lambda_2}-1)}\end{pmatrix}=\frac{\pi}{(h_1-h_2)}\left(dG_4(0)-\Delta+\frac{G_4(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}\right)\quad (A14)$$

$$a_{\min}=dG_4(0)+\frac{G_4(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\begin{pmatrix}\dfrac{\omega\Delta t_1-2\pi}{2}+\dfrac{\pi T_1}{T_1+T_2}\dfrac{e^{-\theta_2/\lambda_1}(e^{-2\pi/\lambda_1}-e^{-\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1}-1)}\\+\dfrac{\pi T_2}{T_1+T_2}\dfrac{e^{\theta_2/\lambda_2}(e^{-2\pi/\lambda_2}-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)}\end{pmatrix}\quad (A15)$$

$$a_{\max}=dG_4(0)+\frac{G_4(0)(h_1\Delta t_1+h_2\Delta t_2)}{P}+\frac{(h_1-h_2)K}{\pi}\begin{pmatrix}\dfrac{\omega\Delta t_1}{2}+\dfrac{\pi T_1}{T_1+T_2}\dfrac{e^{-\theta_1/\lambda_1}(1-e^{-\Delta t_1/T_1})}{(e^{-2\pi/\lambda_1}-1)}\\+\dfrac{\pi T_2}{T_1+T_2}\dfrac{e^{\theta_1/\lambda_2}(1-e^{\Delta t_1/T_2})}{(e^{2\pi/\lambda_2}-1)}\end{pmatrix}\quad (A16)$$

where,

$$\theta_1=\frac{T_1T_2\omega}{T_1+T_2}\log\left(\frac{(1-e^{-\Delta t_1/T_1})(e^{2\pi/\lambda_2}-1)}{(1-e^{\Delta t_1/T_2})(e^{-2\pi/\lambda_1}-1)}\right)$$

$$\theta_2=\frac{T_1T_2\omega}{T_1+T_2}\log\left(\frac{(e^{-2\pi/\lambda_1}-e^{-\Delta t_1/T_1})(e^{2\pi/\lambda_2}-1)}{(e^{2\pi/\lambda_2}-e^{\Delta t_1/T_2})(e^{-2\pi/\lambda_1}-1)}\right)$$

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